Physics of ultrasonography
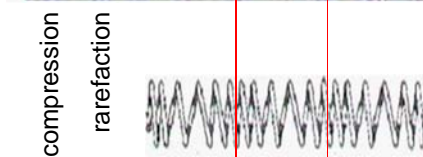

KAD 2015.02.25

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whistle

compression · rarefaction
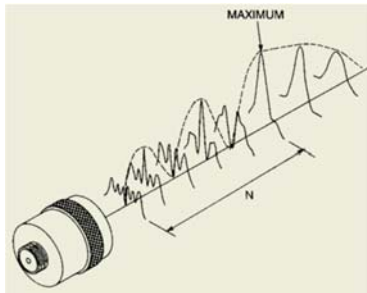
spring

function

spatial and temporal periodicity

crest

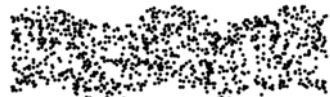λ or T

Δp_0

Δp

trough

x or t
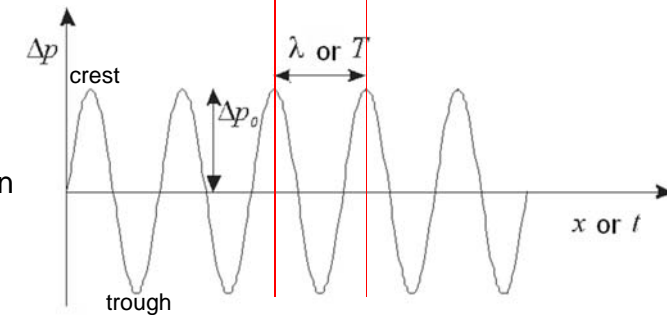
2

---

longitudinal wave
(in the interior of liquids and gases only this type)

transverse wave

hydrostatic pressure   pressure change, sound pressure
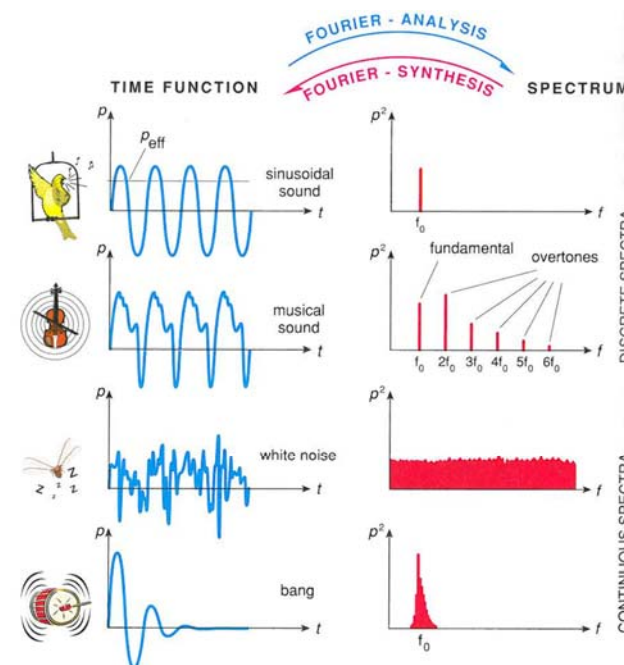
$p_{total} = p_{hydrostat} + \Delta p$

pressure   DC   + AC

amplitude   phase

$$\Delta p(t, x) = \Delta p_{max} \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda} \right) \right]$$

$c \cdot T = \lambda, \quad c = f \cdot \lambda$

Biophysics textbook, Fig. II.46.

3

FOURIER - ANALYSIS

TIME FUNCTION   SPECTRUM

FOURIER - SYNTHESIS

p_{eff}

sinusoidal sound

musical sound

white noise

bang

DISCRETE SPECTRA

CONTINUOUS SPECTRA

fundamental overtones

f_0 $2f_0$ $3f_0$ $4f_0$ $5f_0$ $6f_0$

pitch:
frequency of the fundamental

timbre (tone colour):
relative strengths of overtones/harmonics (spectrum)

Biophysics textbook, Fig. IV.23.

4

Frequency and intensity regions of sounds
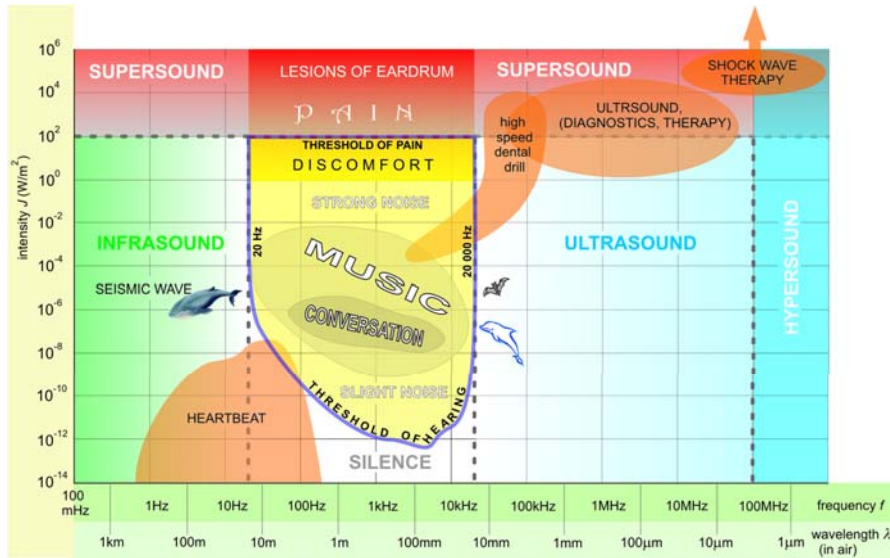
5

The role of elastic medium

$$\kappa = \frac{-\dfrac{\Delta V}{V}}{\Delta p}$$

compressibility
relative volume decrease over pressure

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

speed of sound

$$Z = \frac{p}{v} = \frac{p_{max}}{v_{max}}$$

acoustic **impedance** (definition)

$$Z_{el} = \frac{U}{I}$$

$$\boxed{Z = c\rho} = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance** (useful form)

6

Supplementary material

Acoustic impedance (way to the useful form)

$$y = y_{max} \sin\left[\omega(t - \frac{x}{c})\right]$$

$$\frac{\Delta y}{\Delta t} = v = y_{max}\,\omega \cos\left[\omega(t - \frac{x}{c})\right]$$

$$\frac{\Delta v}{\Delta t} = a = -y_{max}\,\omega^2 \sin\left[\omega(t - \frac{x}{c})\right]$$

$$ma = F$$

$$\frac{1}{V}ma = \frac{1}{A\Delta x}F$$

$$\rho a = -\frac{\Delta p}{\Delta x}$$

$$p = p_{max} \sin\left[\omega(t - \frac{x}{c})\right]$$

$$Z = \frac{p}{v} = \frac{p_{max}}{v_{max}}$$

$$\rho\, y_{max}\,\omega^2 \sin\left[\omega(t - \frac{x}{c})\right] = \frac{\Delta p}{\Delta x}$$

$$-\rho\, y_{max}\,\omega c \cos\left[\omega(t - \frac{x}{c})\right] = p$$

$$p_{max} = \rho\, y_{max}\,\omega c = \rho\, v_{max}\, c$$

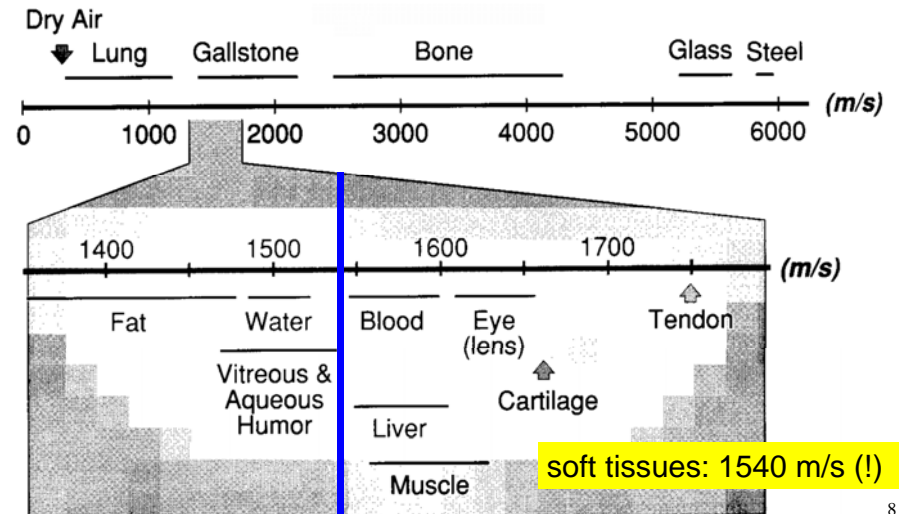$$\boxed{\frac{p_{max}}{v_{max}} = \rho c = Z}$$

7

Speed of sound/US in different media

soft tissues: 1540 m/s (!)

8

Slide 9

Assuming constant speed of US ➡ Artefacts

The image of the back-wall reflection appears in different distances, depending on the material in the finger of the rubber gloves

C_{water} = 1540 m/s, $C_{glycerol}$ = 1900 m/s, $C_{alcohol}$ = 1200 m/s

contours of the rubber glove finger on the screen

transducer

US beam

water — d_{water} water
water — $d_{glycerol}$ glycerol
water — $d_{alcohol}$ alcohol

Laboratory manual, US, Figure 15

Slide 10

Intensity of US

$$J = \frac{1}{Z}\Delta p_{eff}^2 \qquad P_{el} = \frac{1}{Z_{el}}U_{eff}^2$$

intensity = energy-current density   electric analogy

Loss of energy during propagation (absorption)

attenuation: $\alpha = 10 \cdot \lg\frac{J_0}{J}$ dB

$\alpha = 10 \cdot \mu \cdot x \cdot \lg e$ dB

μ is proportional to frequency in the diagnostic range

$$J = J_0 e^{-\mu x}$$

specific attenuation: $\boxed{\dfrac{\alpha}{f \cdot x}}$

Slide 11
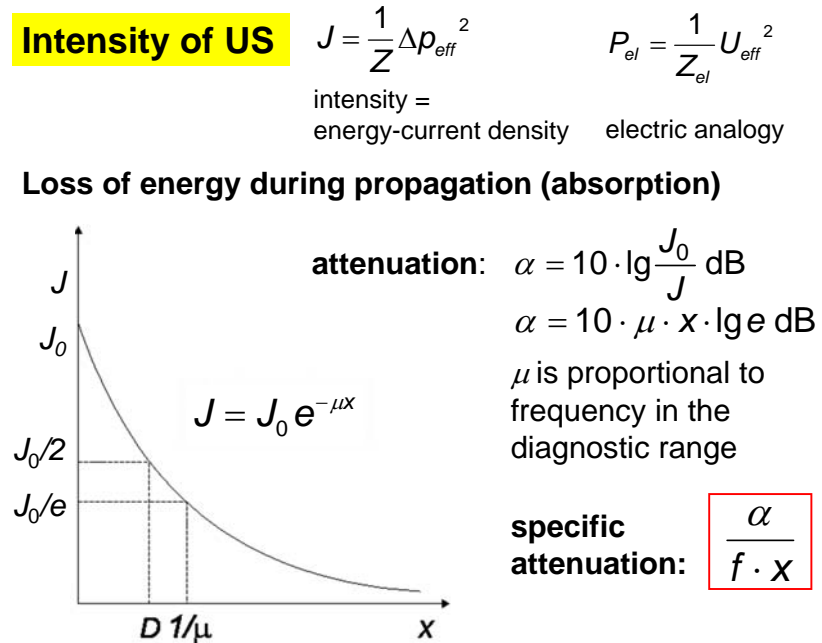
μ is proportional to frequency in the diagnostic range
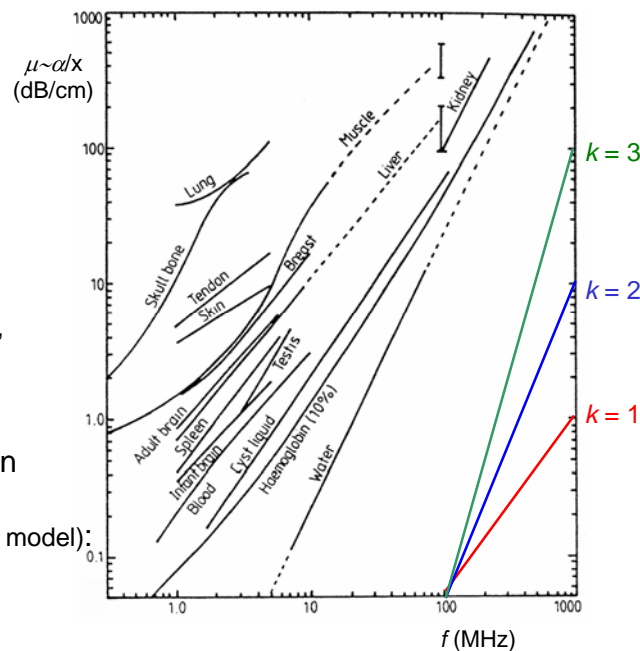
$\mu \sim f^k, \quad k \sim 1 (?)$

$\log \mu \sim k \log f$

if the graph is a linear, the power function approximation is valid

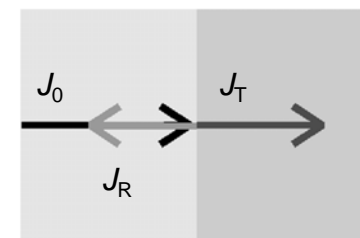specific attenuation for soft tissues (homogeneous tissue model):

$$\boxed{\frac{\alpha}{f\,x} \sim 1\frac{dB}{cm\,MHz}}$$

$\mu \sim \alpha/x$ (dB/cm)

$k = 3$
$k = 2$
$k = 1$

f (MHz)

Slide 12

Phenomena at the boundary of different media

normal/perpendicular incidence

J_0 J_T

J_R

$$J_0 = J_R + J_T$$

reflection and transmission (penetration)

skew incidence

$c_1 > c_2$ surface normal

J_0 α α β

J_R J_T
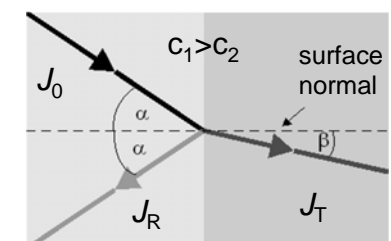
$$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

Biophysics textbook, Fig. II.47.

Reflection (normal incidence)

reflectivity:

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

boundary surface	R
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

"full" reflection:

$$Z_1 \ll Z_2, \quad R \approx 1$$

optimal coupling:

$$Z_{\text{connecting}} \approx \sqrt{Z_{\text{source}} Z_{\text{skin}}}$$

13
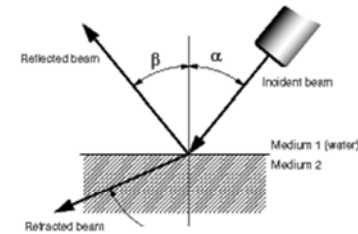
Skew incidence

$$\text{Reflexion coefficient} = R = \left[\frac{z_2 \cos\alpha - z_1 \cos\gamma}{z_2 \cos\alpha + z_1 \cos\gamma}\right]^2$$

$$\text{Refraction coefficient} = D = \frac{4 z_1 z_2 \cos^2\alpha}{(z_2 \cos\alpha + z_1 \cos\gamma)^2}$$

$$\text{Refraction angle} = \gamma = a\sin\left(\frac{c_2 \sin\alpha}{c_1}\right)$$

14

Phenomenon of skew incidence or normal incidence and skew boundaries

c_s
c_l
c_s

c_l
c_s
c_l

real position / position in the image

c_k
c_n
c_k

c_n
c_k
c_n

Biophysics textbook, Fig. on pg. 153
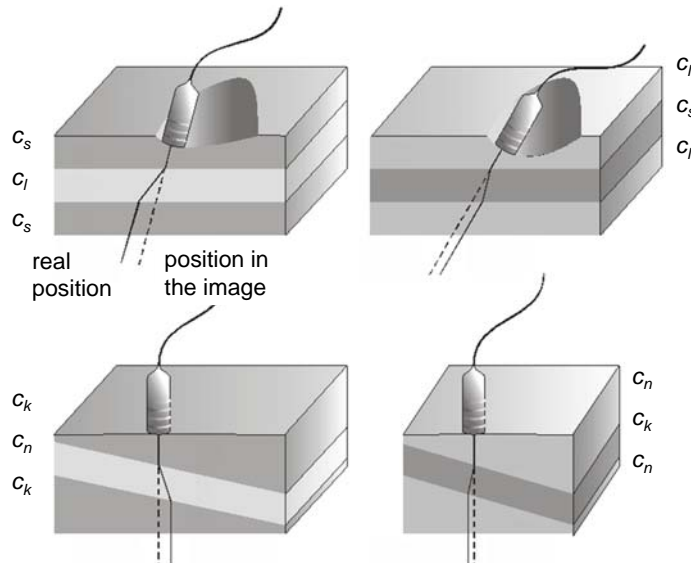
15

Absorption and reflection
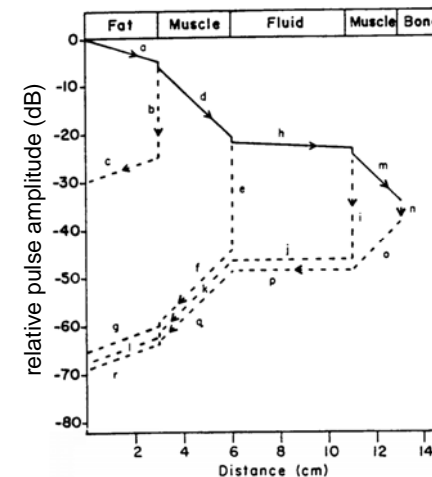
the later comes back the reflection, the deeper lays the reflecting surface and the weaker is the intensity

run time dependent amplification

TGC: time gain compensation

DGC: depth gain control

boundary surface	R	$10 \lg R$ (dB)	T	$10 \lg T$ (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

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Generation of US. Piezoelectric effect

production: inverse ~
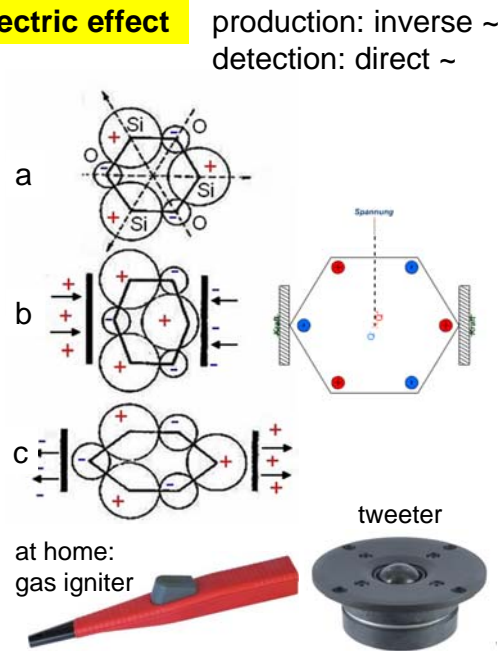detection: direct ~

source of electric signal (sine wave oscillator)+ transducer (piezo-crystal)

a

(a) Center of charge of positive and negative charges coincides.

b

(b) and (c) As a result of pressure , the charge centers are separated, i.e. a potential difference arises (direct ~).
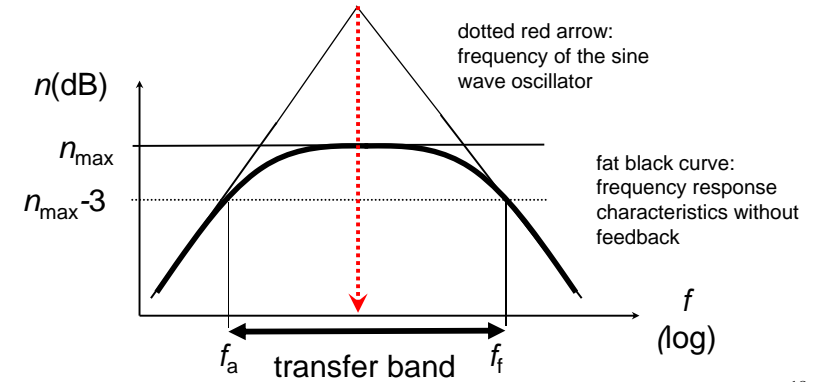The crystal is deformed when voltage is applied (inverse ~).

c

tweeter

at home:
gas igniter

Source of electric signal : sine wave oscillator

amplifier with positive feedback

$$A_{U, \text{feedback}} = \frac{A_U}{1 - \beta A_U}$$

βA_U=1, amplification = „infinity" ➡ sine wave oscillator
no input signal,   output signal: sine voltage

n(dB)

n_{max}

n_{max}-3

dotted red arrow: frequency of the sine wave oscillator

fat black curve: frequency response characteristics without feedback

f_a transfer band f_f

f
(log)

18

Ultrasound transducer

active cable

acoustic insulator

damping unit

active electrode
piezoelectric crystal, $\lambda/2$

grounded cable

grounded electrode

plastic housing

couplant layer, $\lambda/4$

receiver   transmitter
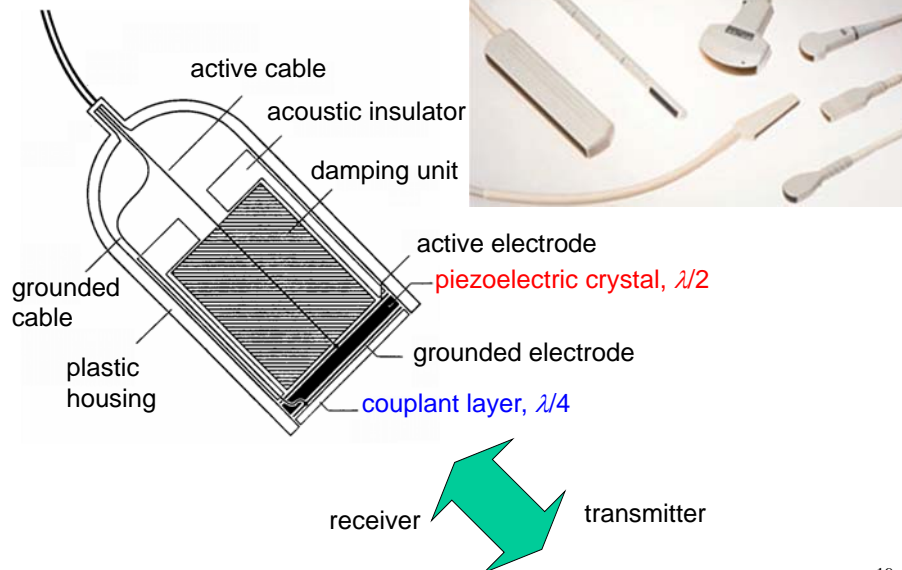
19

Characteristic of US pulses

transducer: transmitter and receiver is the same unit
time sharing mode: pulses instead of continuous wave US

skin

pulse repetition time: 1 ms

speed of propagation of US (in soft tissue): 1540 m/s

pulse repetition frequency: 1000/s = 1 kHz

transducer

pulse duration: 1 μs

frequency of US: 1-10 MHz
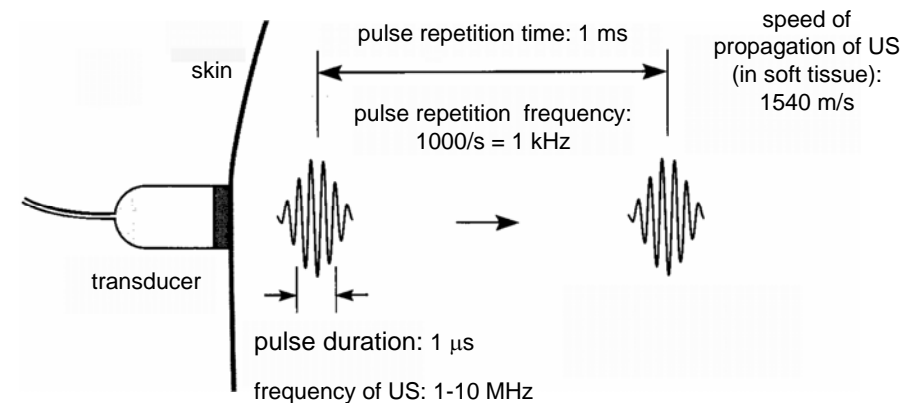
Biophysics textbook, Fig. VIII.32.
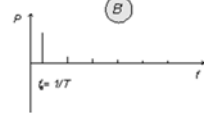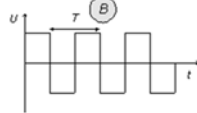
20

Slide 21

Time function **Spectrum**
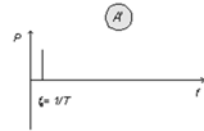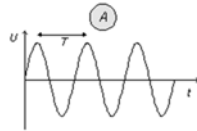
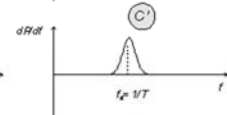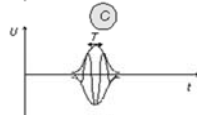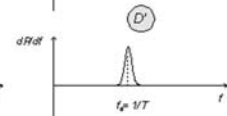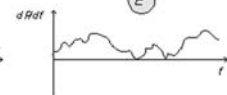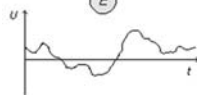|	
sine function	line spectrum (1 line)
square function	line spectrum
sine wave pocket (some „periods")	band spectrum
sine wave pocket (several „periods")	**US pulse** band spectrum
aperiodic function	continuous spectrum
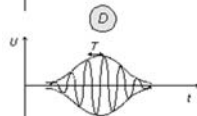
21

Slide 22

US beam shape (simplified version)

$$\frac{R^2}{\lambda}$$

2R

← near field → ← far field →
(Fresnel zone) (Fraunhofer zone)
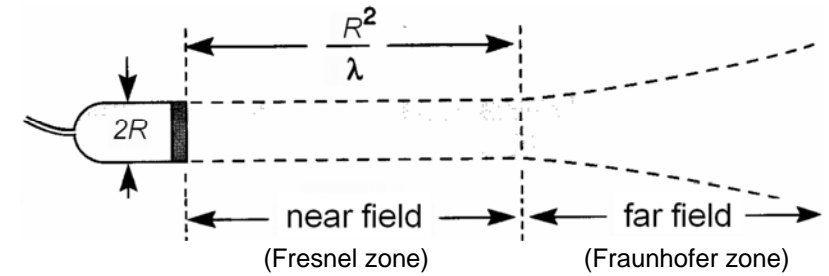
Beam shape, continuous wave US   Beam shape, pulsed wave US
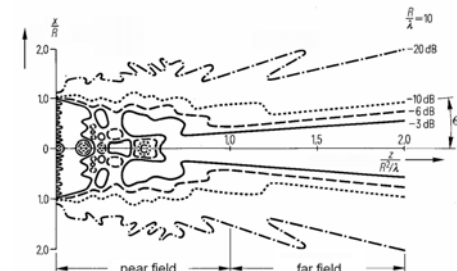
near field far field   near field far field

22

Slide 23
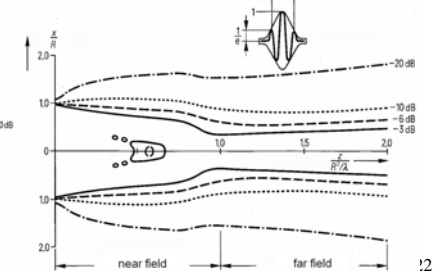
Perspective view of the US beam

MAXIMUM

J

X

change of the intensity in the axial direction

cf. Textbook. Fig. on p.505

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Slide 24
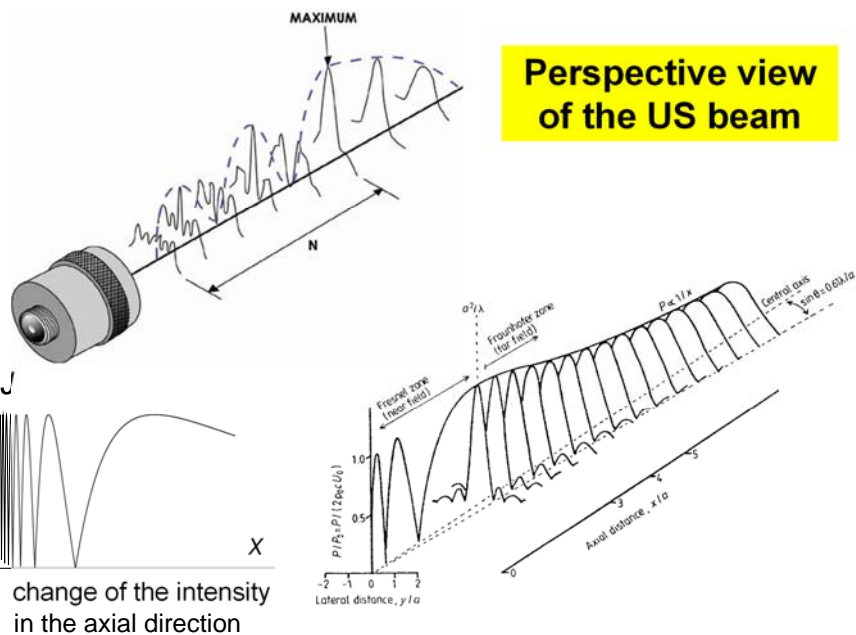
Resolving limit, resolution

Resolving limit is the distance between two object details which can be just resolved as distinct objects (the smaller the better).
Resolution (resolving power): the reciprocal of the resolving limit (the greater the better)

Axial resolving limit depends on the pulse length. Pulse length is inversly proportional to the frequency.

Lateral resolving limit is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

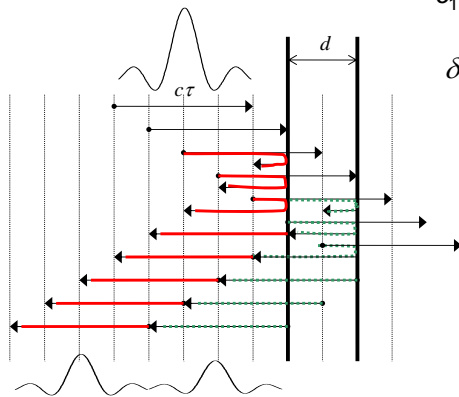|	**Typical values**		
frequency (MHz):		2	15
wavelength (in muscle) (mm):		0.78	0.1
penetration depth (cm):		12	1.6
lateral resolving limit (mm):		3.0	0.4
axial resolving limit (mm):		0.8	0.15

24

Axial resolving limit

τ :   pulse duration

$c_1 \tau \cong c_2 \tau = c\tau$ pulse length

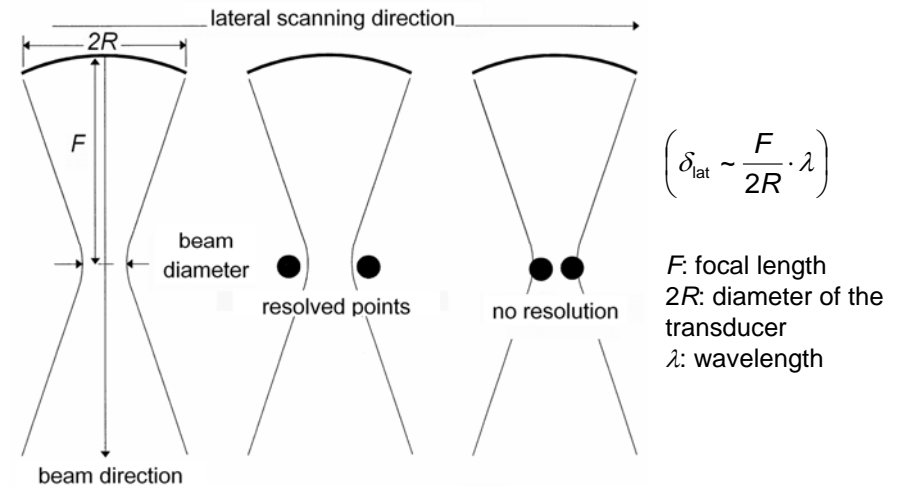$\delta_{ax} = d = \dfrac{c\tau}{2}$ resolving limit

The axial resolving limit is the half of the pulse length. The echos from the adjacent surfaces in this case just hit another.
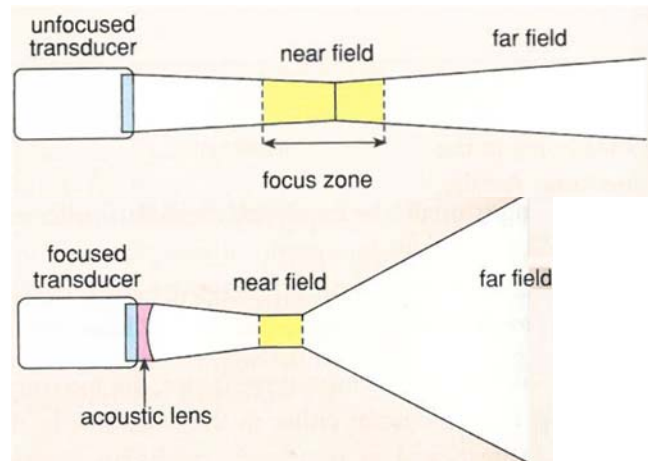
$$\tau \sim T = \dfrac{1}{f}$$
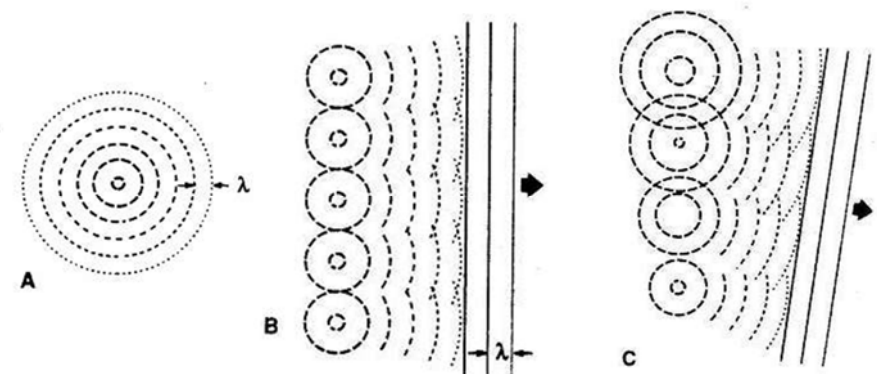
25

Lateral resolving limit

$$\left(\delta_{lat} \sim \dfrac{F}{2R} \cdot \lambda \right)$$

F: focal length
2*R*: diameter of the transducer
λ: wavelength

26

Focusing of the beam

Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

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Huygens' principle

Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.
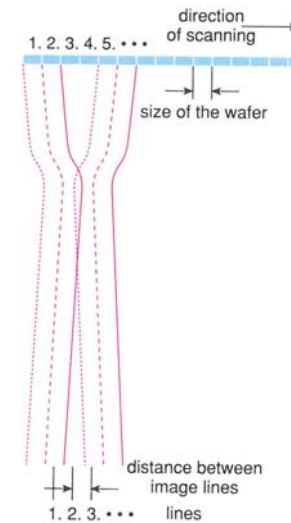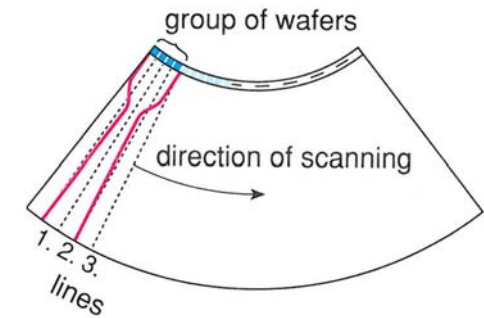
28

Electronic focusing

delay units

the time of emission

transducers

elementary waves

wave front

unfocused ultrasound beam

focused ultrasound beam

focal point #k

focal point # (k+1)

focal point #(k+2)
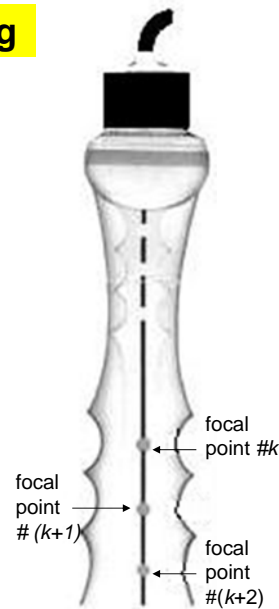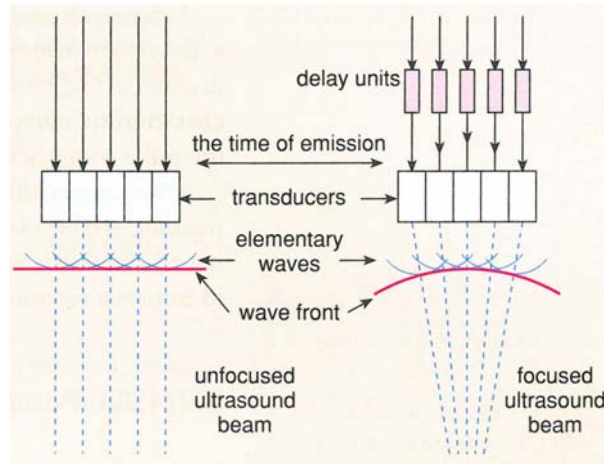
29

Scanning

multi unit linear array

multi unit curved array

direction of scanning

1. 2. 3. 4. 5. ···

size of the wafer

group of wafers

direction of scanning

1. 2. 3. lines

distance between image lines

1. 2. 3. ··· lines

30

Echo principle

1794 Spallanzani: bat's navigation

1822 Colladen measured the speed of sound in water

10 miles

bottlenose dolphin

Blowhole
Phonic Lips
Dorsal Bursae
Cranium
Melon
Bony Nares
Upper Mandible
Auditory Bullae
Lower Mandible

Outgoing Sounds

Incoming Sounds
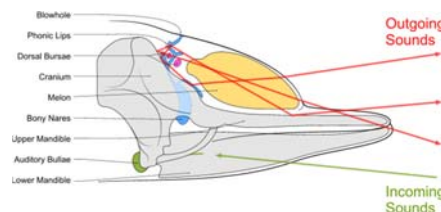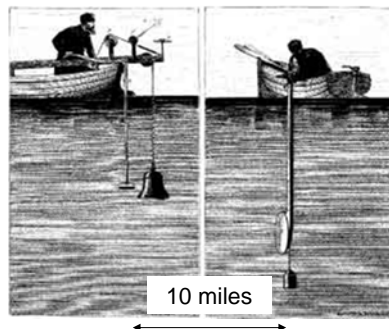
31

Echo principle

using a special US-head, short pulses are emitted in the air towards a reflecting surface, and the same US-head detects the echo signal
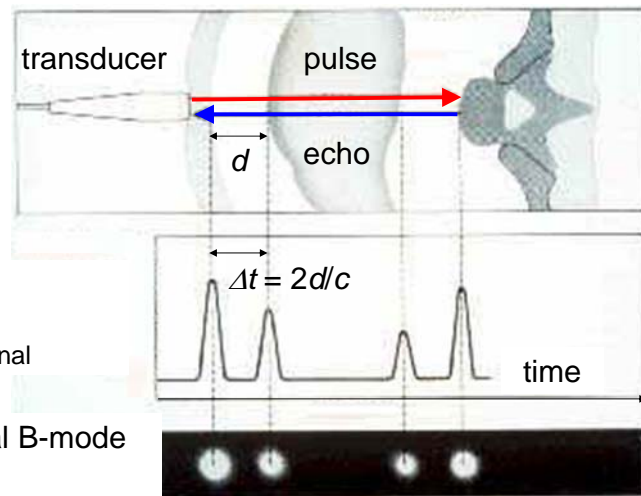

reflecting surface

US transducer
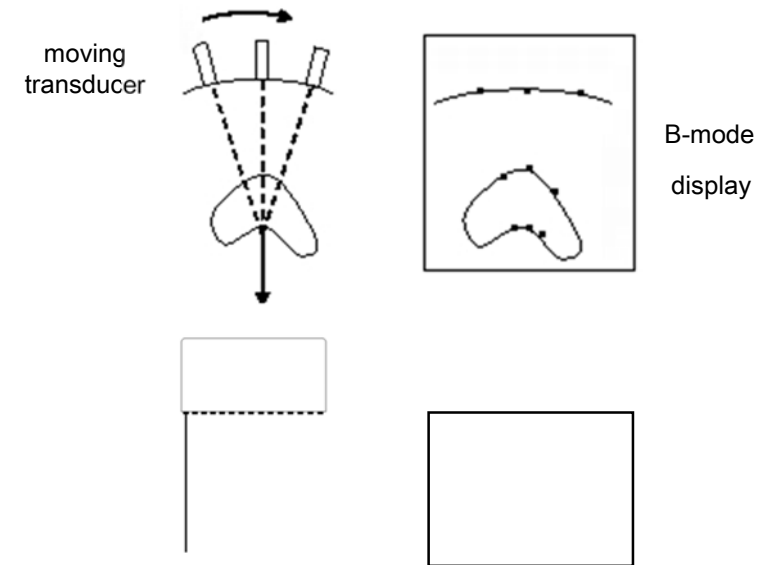
Δt

A-image (emission)

A-image (echo)

d
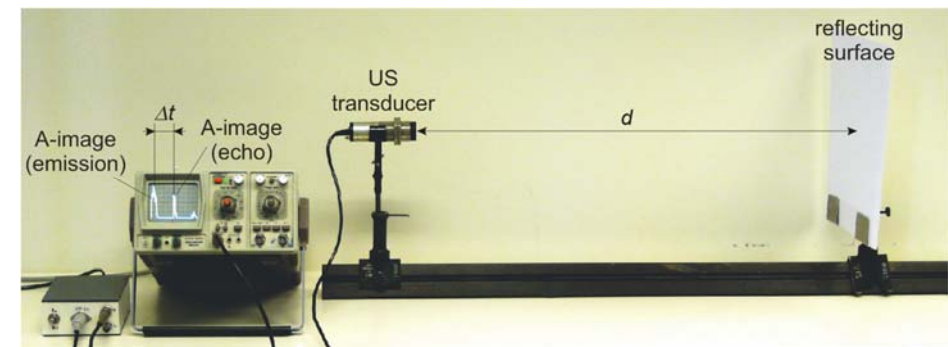
$$c\Delta t = d + d = 2d$$

32

Receiving the echos

transducer   pulse

d echo

$\Delta t = 2d/c$

time

A-mode
(**A**mplitude)
only 1-dimensional

1-dimensional B-mode
(**B**rightness)

2-dimensional B-mode

moving
transducer

B-mode

display

A- mode

1D B- mode

TM- mode

TM-mode

transducer
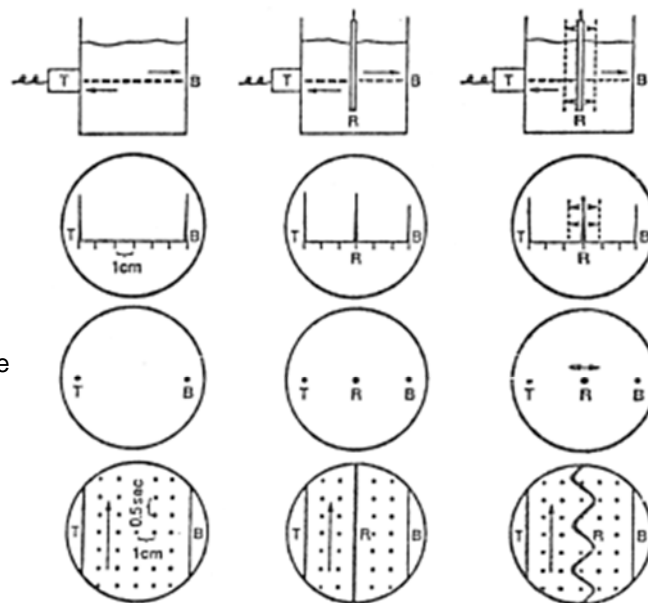
ECG signal
for reference

ECG

1   2   3   4

time

(vertical)
time-dependent
1-dimensional
B-mode

(T)M-mode

Time–
Motion

Slide 37
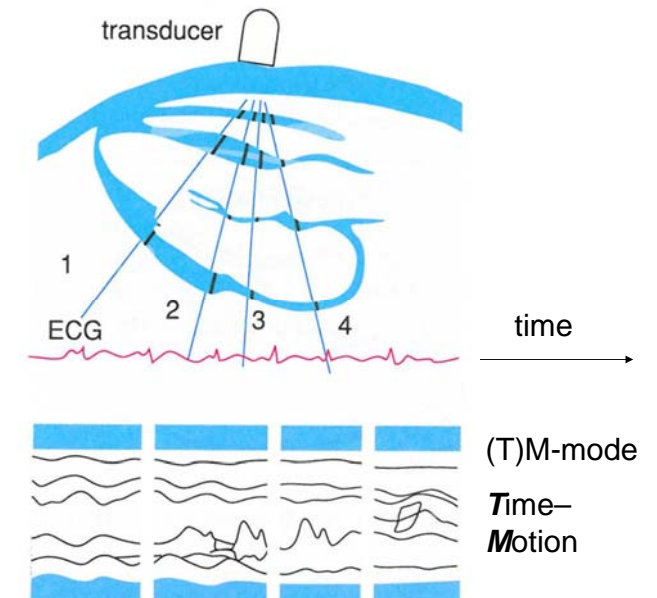
2-dimensional B-mode and A-mode
(used in opthamology)
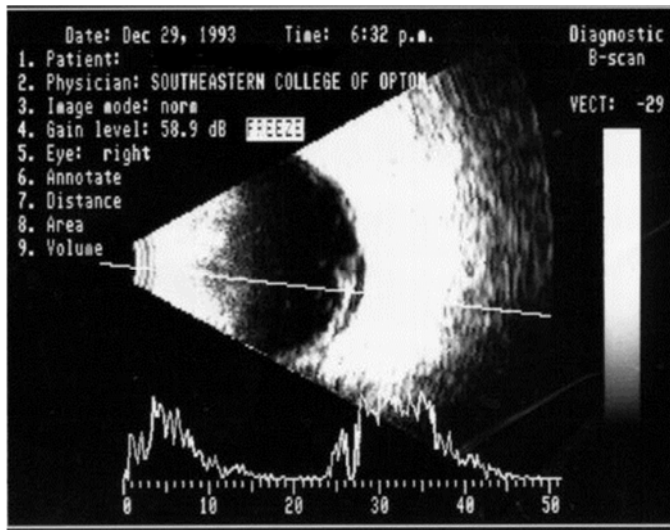
real speed of propagation for the accurate determination of distances:
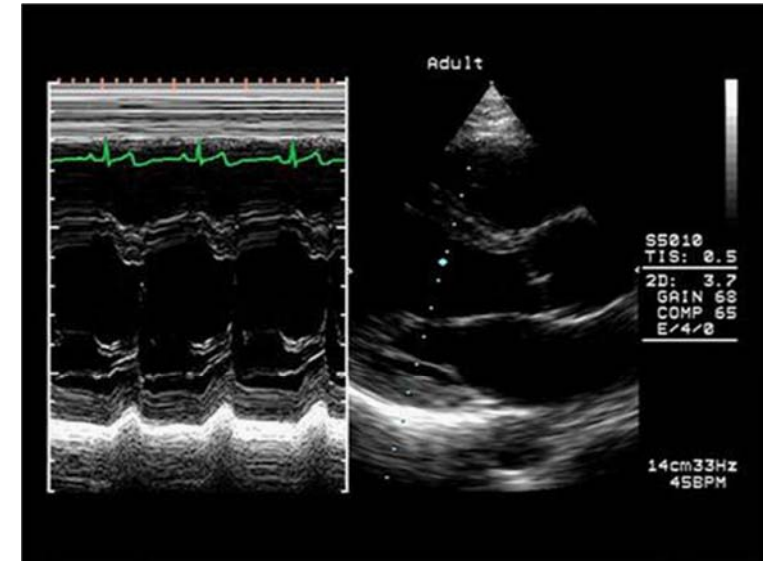
cornea: 1641 m/s

aqueos humour: 1532 m/s
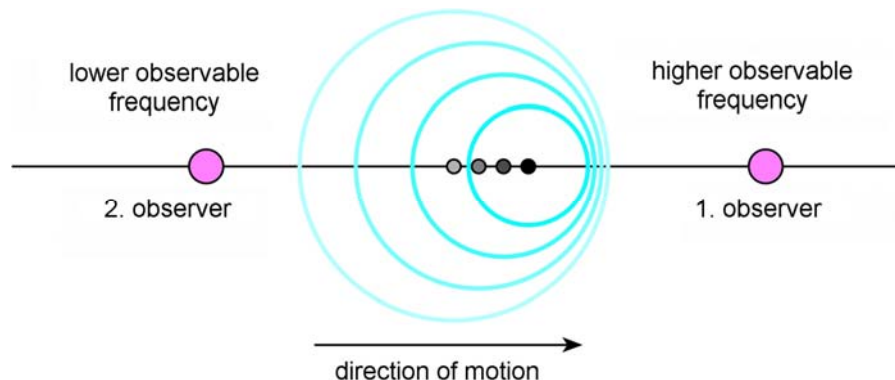
crystalline lens: 1641 m/s

vitreous body: 1532 m/s

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Slide 38

TM-mode   B-mode

38

Slide 39

Doppler phenomenon

„The pitch of a train whistle seems to get higher as it approaches, then seems to lower as the train whistle moves away." (C. Doppler, 1842)

lower observable frequency — 2. observer

higher observable frequency — 1. observer

direction of motion

Teextbook Fig. VIII.39

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Slide 40

f': **observed frequency**, f: original frequency

(a) standing source and moving observer (v_O)
+: observer approches the source
−: observer moves away from the source

$$f' = f\left(1 \pm \frac{v_O}{c}\right)$$

(b) moving source and standing observer
(if $v_S \ll c$, then „same" as (a))

$$f' = \frac{f}{1 \mp \dfrac{v_S}{c}}$$

(c) moving source and moving observer

$$f' = f\,\frac{1 \pm \dfrac{v_O}{c}}{1 \mp \dfrac{v_S}{c}}$$

(d) moving reflecting object (surface),
(if $v_R \ll c$)

$$f' = f\left(1 \pm \frac{2v_R}{c}\right)$$

40

Doppler frequency = frequency change = frequency shift

if v_i, $v_R << c$ (i= S or O)

rearranging equation (a)
moving source or observer:

$$\Delta f = f_D = \pm \frac{v_i}{c} f$$

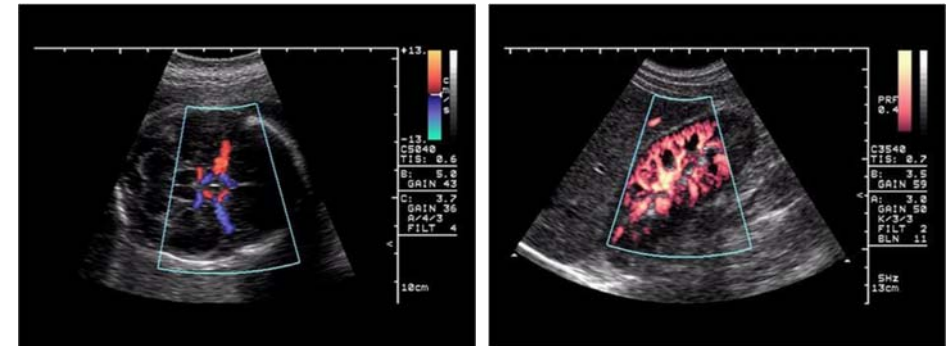rearranging equation (d)
moving reflecting object or surface:

$$\Delta f = f_D = \pm 2 \frac{v_R}{c} f$$

if v and c are not parallel, then $v \cos \Theta$ should be used instead of v (remark: if $\Theta = 90°$, $f_D = 0$)

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Colour coding

towards the transducer: warm colours
away from the transducer: cold colours

BART: Blue Away Red Towards   power Doppler

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1-dimensional CW Doppler apparatus for measuring average flow velocity. Red blood cells as sound scatterers

CW: continuous wave

source and detector are separated

$$|f_D| = 2 \frac{v_R \cos \theta}{c} f$$
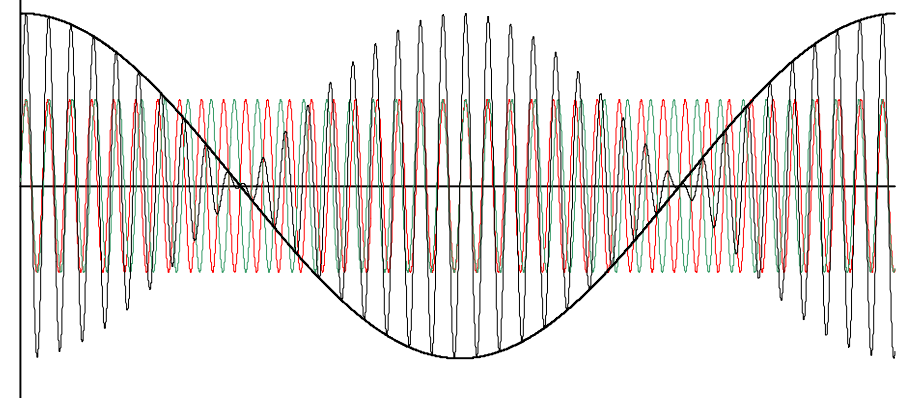
e.g. f=8000 kHz
v=12 cm/s
c=1600 m/s
$\Theta = 37^0$

$\Rightarrow f_D$=1 kHz
(beating phenomenon)

difference signal 1 kHz

sinusoidal oscillator

8000 kHz   8001 kHz

v=12 cm/s Θ= 37°

Textbook Fig. VIII.41
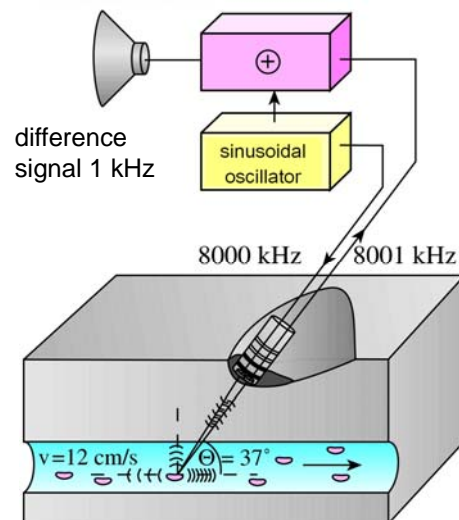
43

Beating phenomenon

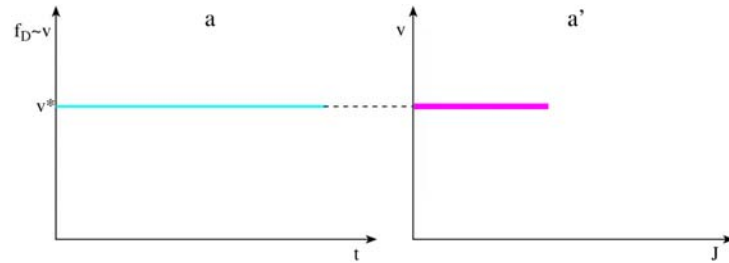$f_{red} \geq f_{green}$ the beating frequency equals to the difference of the two interfering frequency

reminder: $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$
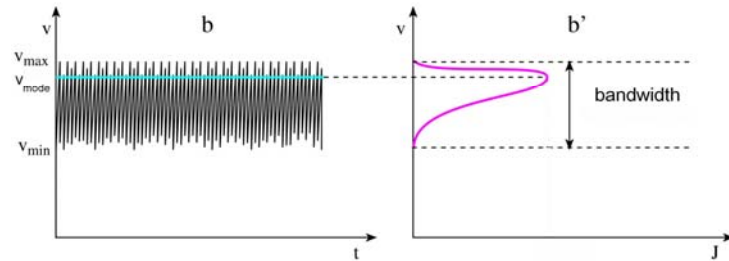
44

one constant velocity (v*)

$f_D \sim v$

a

a'

v

v^*

t

J

frequency distribution (with v_{mode})

v

b

v_{max}
v_{mode}

v_{min}

t

v

b'

bandwidth

J

velocity distribution in TM-mode
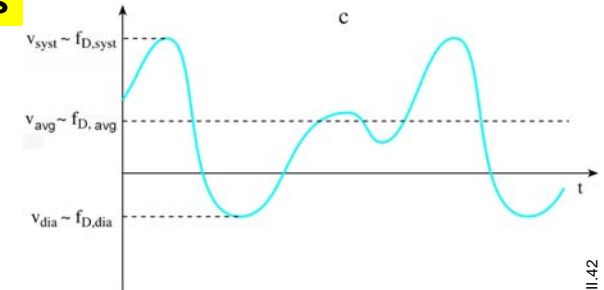
velocity distribution at a certain time

Textbook Fig. VIII.42

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flow can be represented by one velocity in each moment

c

$v_{syst} \sim f_{D,syst}$

$v_{avg} \sim f_{D, avg}$

$v_{dia} \sim f_{D,dia}$

t

Textbook Fig. VIII.42

flow can be represented by a velocity distribution in each moment

d

v

t
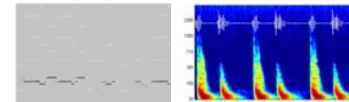
cf. voiceprint, music/heart beats in time-frequency representation

velocity distribution in TM-mode

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PIG 695
University Hospital

LAM

C8–5 PVasc/Ven

27 Jun 00   TIs 0.2   MI 0.2
5:25:42 pm   F# 73   3.0 cm

Map 8
DynRg 50dB
Persist Med
Fr Rate Med
2D Opt:Res

SV Angle –46°
Dep 1.5 cm
Size 4.0 mm
Freq 5.0 MHz
WF Low
Dop 68% Map 2
PRF 10000Hz

120
80
40
cm/s
–40

0.21cm
0.03cm²
TAM 65.8cm/s
VF   118.4ml/min

Normal portal vein flow

PRF
0.6

C3540
TIS: 1.0
B: 3.5
GAIN 54
A: 3.0
PH: 3.0
GAIN 0
ANG 0°
F 100Hz
0 6.9cm

5Hz
14cm

PRF
1.4

CS040
TIS: 0.3
B: 5.0
GAIN 49
A: 3.7
PH: 3.7
GAIN 50
ANG 0°
F 100Hz
0 4.2cm

12cm
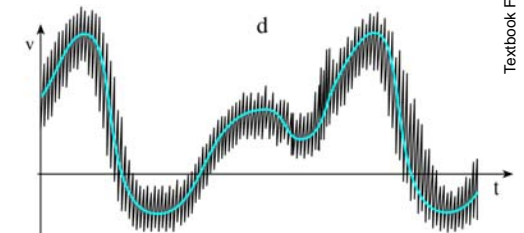
Error of measuring the angle between the US-beam and axis of
blood vessel influences the error of the flow velocity

error of Φ

relative error of flow velocity, %

Φ (= angle between the US-beam and axis of blood vessel)
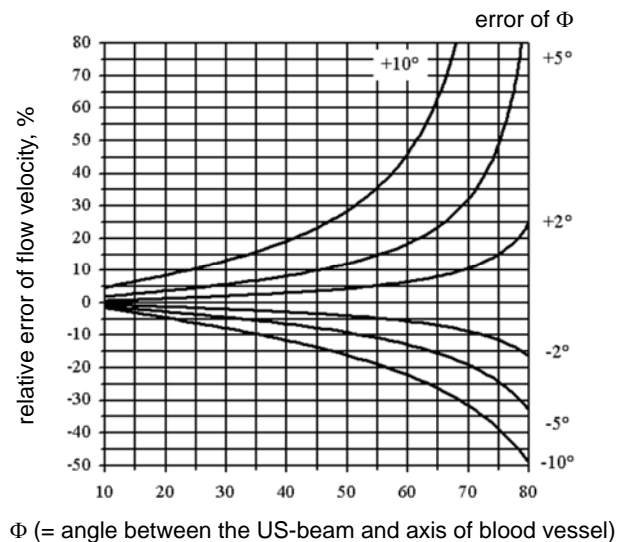
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Time domain correlation method
(CVI = color velocity imaging)

If the reflecting surface and/
or the scatterer are moving
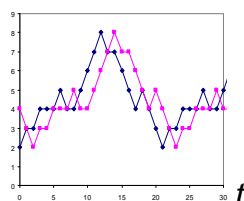then the US signal at a fixed
position depends on time.

Similar US pattern can be
measured at a certain
distance from the earlier
position (where the reflecting
surface/scatterer moved).

How can be compared the
similarity of different
functions?
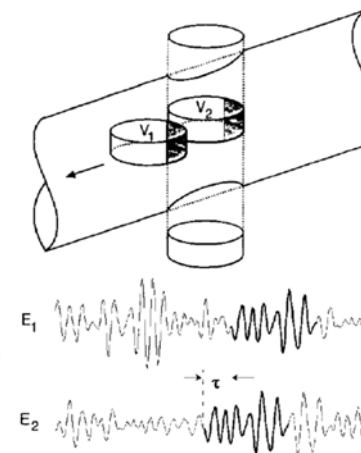
50

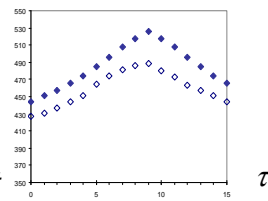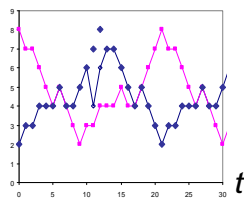
time domain   correlation function
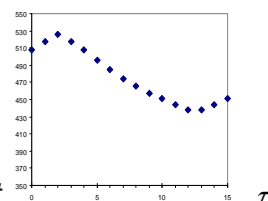
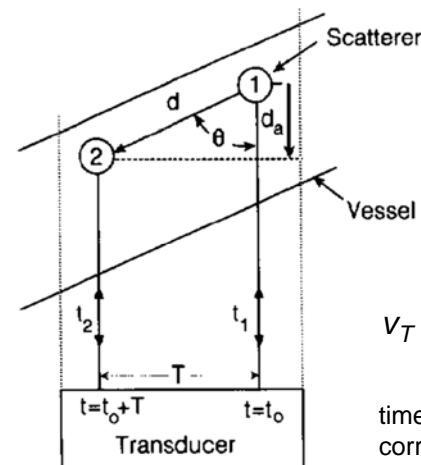$$f_{\text{blue}}(t) = f_{\text{pink}}(t + \tau^*)$$

$$\tau^* = 2\,u$$
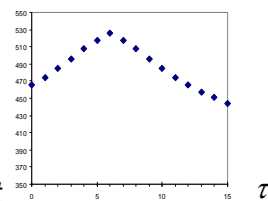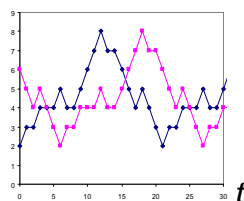
$$\tau^* = 6\,u$$

$$\tau^* = 9\,u$$

$$f_{\text{empty}}(t) \cong f_{\text{blue}}(t)$$

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$$d_a = \frac{(t_1 - t_2)c}{2}$$

$$d = \frac{(t_1 - t_2)c}{2\cos\theta}$$

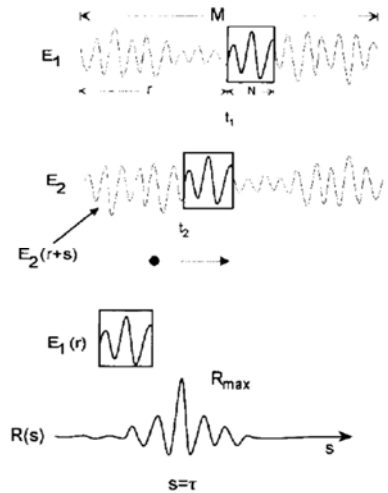$$v_T = \frac{(t_1 - t_2)c}{2T\cos\theta} \qquad v_D = \frac{f_D c}{2f\cos\theta}$$

time domain   cf.: Doppler
correlation method   method
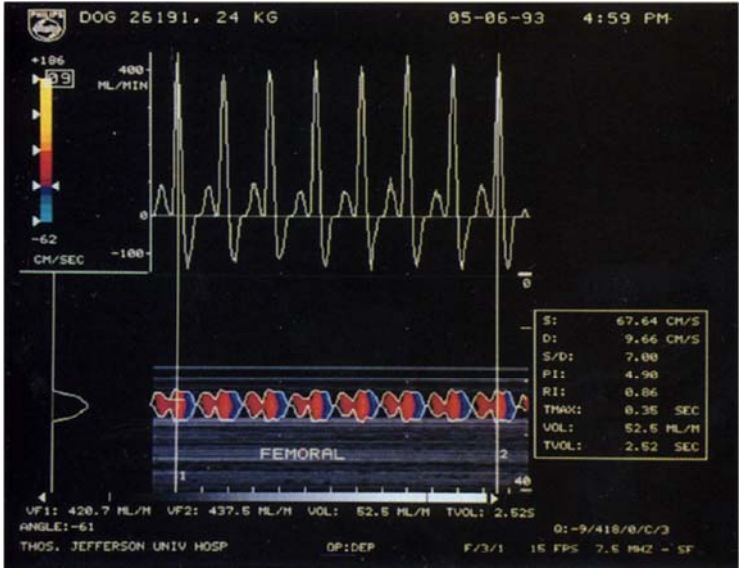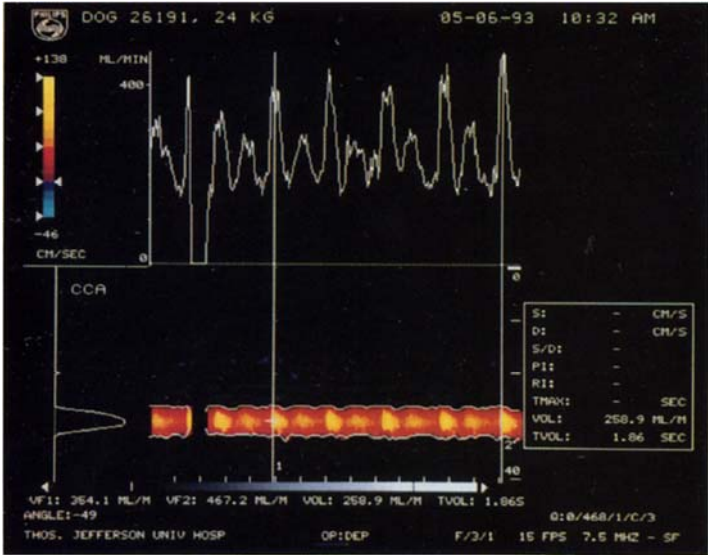
$T:$ pulse repetition $f:$ frequency of US
time

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correlation procedure consists of removing a window of width N at desired range from on echo signal E_1

E_1 is correlated at different locations along another echo signal E_2

the value of s producing the maximum corresponds to s = τ

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Sono-CT

image reconstruction from several multidirectional B-images

SonoCT yields better results than conventional B-mode sonography in terms of delineation, visualization of borders and artefact-free representation
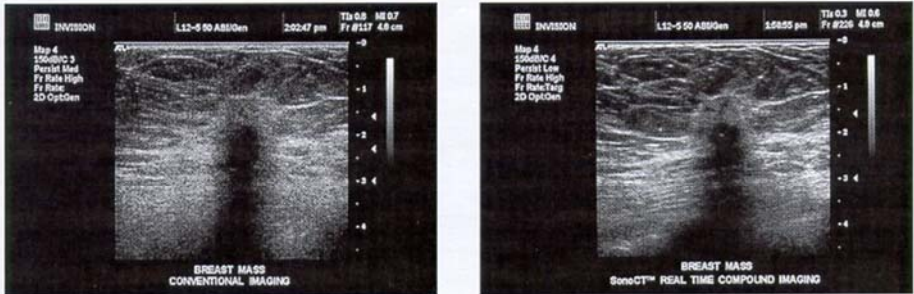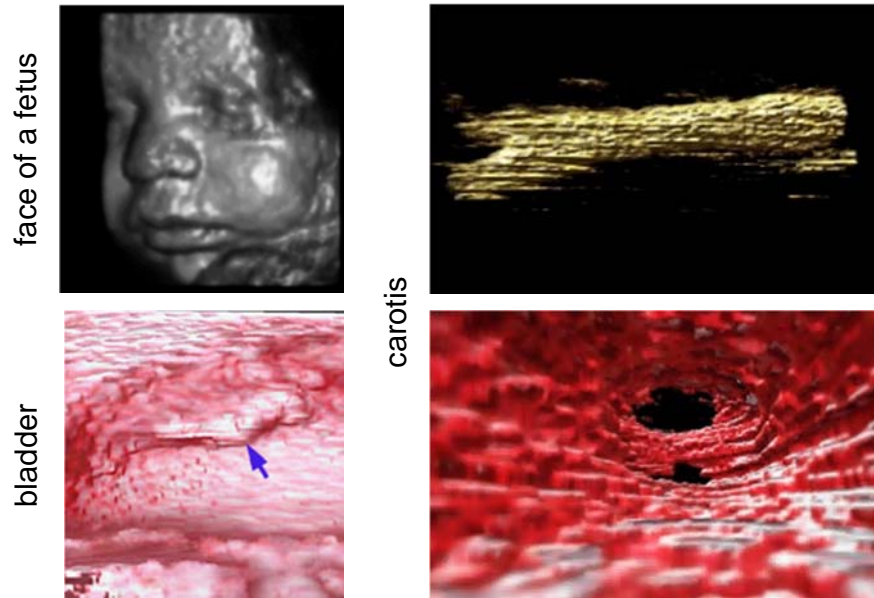

Figure 1. Breast fibroadenoma as shown on conventional ultrasound (left) and SonoCT ultrasound imaging (right).
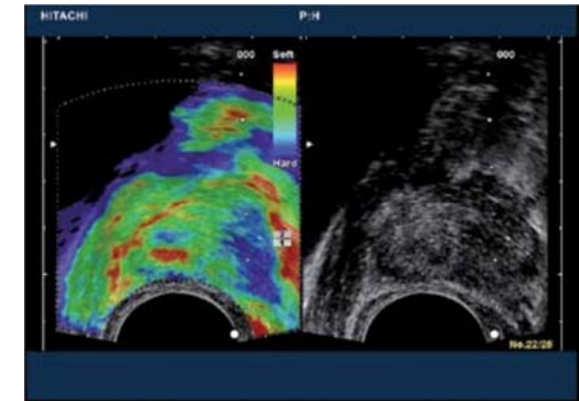
3D reconstruction

face of a fetus

bladder

carotis

7

palpation:
one of the oldest
clinical skills,
tissue elasticity

strain:
a region of tissue is
compressed and the
**degree to which it
distorts** is assessed

Prostate cancer. The right panel shows the B-mode frame of a prostate ultrasound
scan with the corresponding elastography frame on the left.
The colour codes (here) blue for hard and red for soft.
This hard lesion, which is not apparent on the B-mode, was a carcinoma on biopsy

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Cytoskeletal filaments as rigid bodies

In the first approximation, cytoskeletal filaments may be described as rigid
bodies, which are deformed upon the action of external force. The defor-
mation occurs in the direction of the applied force. The elasticity of rigid
bodies is described by Hooke's law:

$$\frac{F}{A} = E\,\frac{\Delta L}{L}, \qquad (V.1)$$

where F is force, A is cross-sectional area, L is resting length, and ΔL is
extension. The ratio F/A is stress (σ), E is the Young modulus and $\Delta L/L$ is
relative extension or strain (ε). The ratio of stress and strain is known as the
Young modulus (elastic modulus), the dimension of which is pressure (Pa).
In the case of homogenous bodies the Young modulus is characteristic of the
material properties only and is independent of shape and size.

Biophysics Textbook, pg.348

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Supplementary material
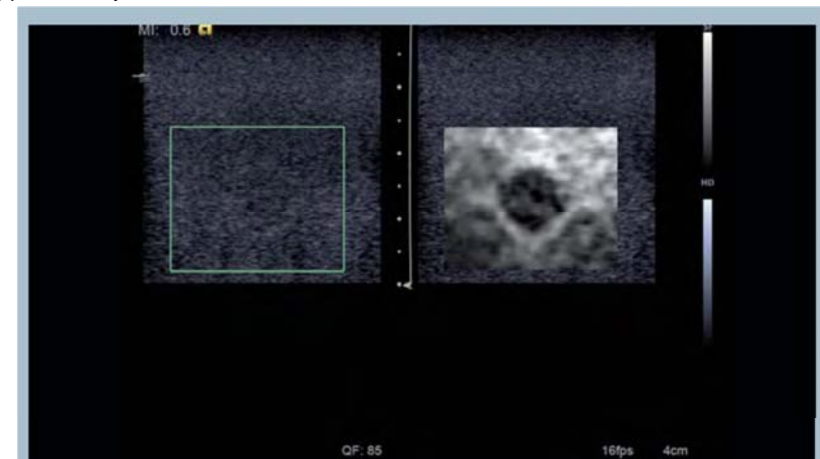
Grayscale eSie Touch Elasticity Imaging demonstrates a lesion
which is more stiff (black) than the surrounding tissue. The
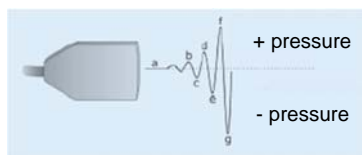conventional B-mode ultrasound is the same as displayed in
the previous image.
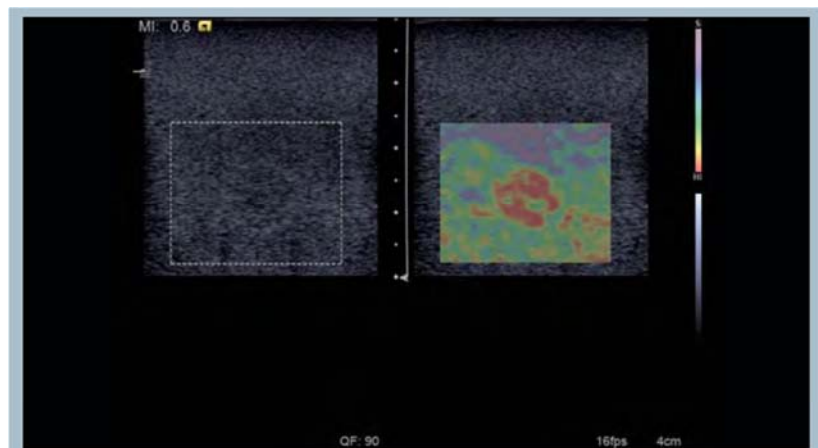
Color scale eSie Touch Elasticity Imaging demonstrates
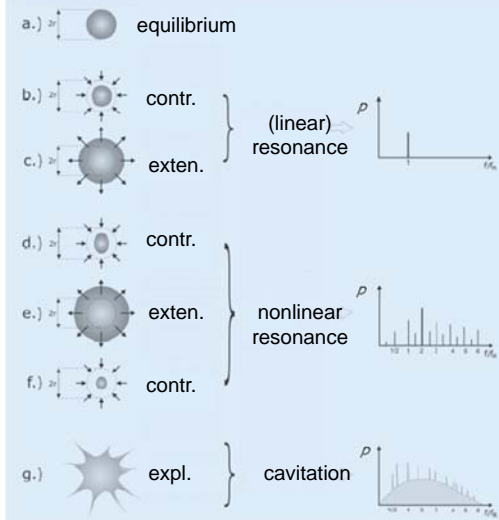a lesion which is more stiff (red) than the surrounding tissue.

Contrast agents
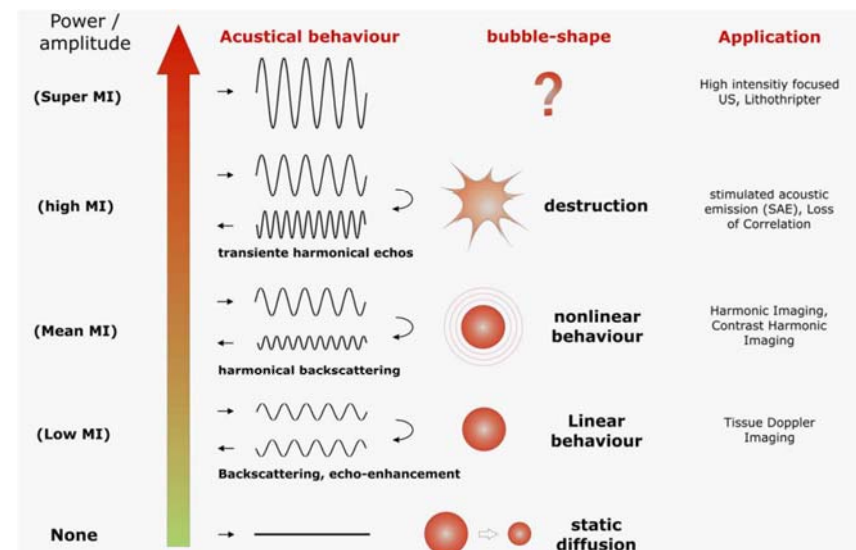
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Contrast harmonic imaging = CHI

C. Kollmann · M. Putzer
**Ultraschallkontrastmittel –
physikalische Grundlagen**
Radiologe 2005 · 45:503–
512

63

Tissue harmonic imaging = THI

Principle:
- Simultaneous transmission of 2 pulses at different frequencies
- Reception of signals at harmonic and differential frequencies
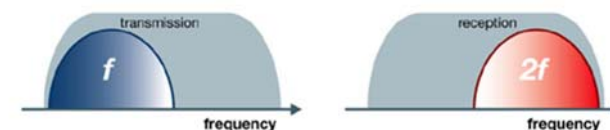- Cancellation of fundamental signals using Pulse Subtraction

Toshiba

Philips

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Intravascular ultrasound (IVUS)

20-40 MHz, frame rate: 30 Hz

Schematic of an intravascular ultrasound (IVUS) catheter within a blood vessel. Adapted from Kimura et al., Am Heart J 1995; 130:386-96.
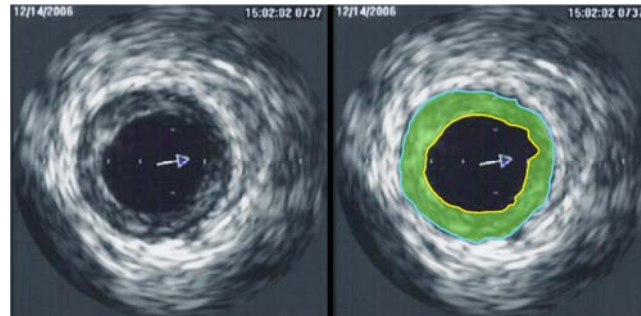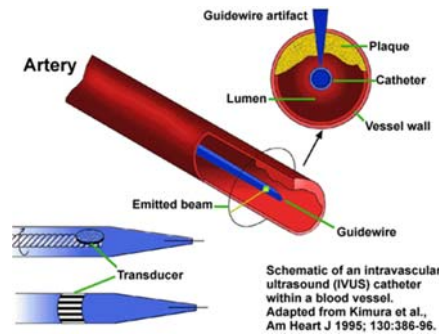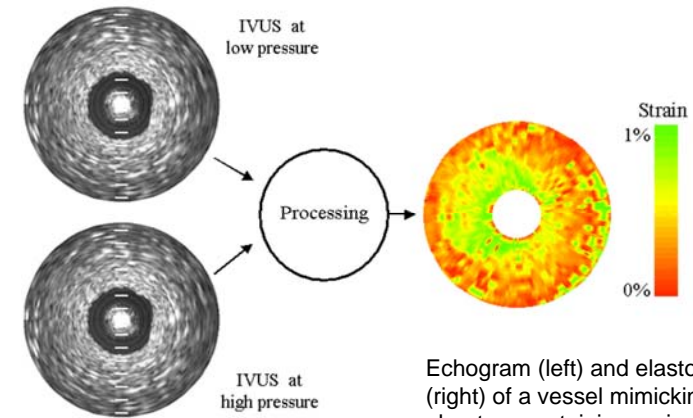
lumen of the coronary artery: yellow

outer elastic membrane: blue

Intravascular sonoelastography

Echogram (left) and elastogram (right) of a vessel mimicking phantom containing an isoechoic soft lesion between 7 and 11 o'clock. The lesion is invisible in the echogram, while it is clearly depicted in the echogram
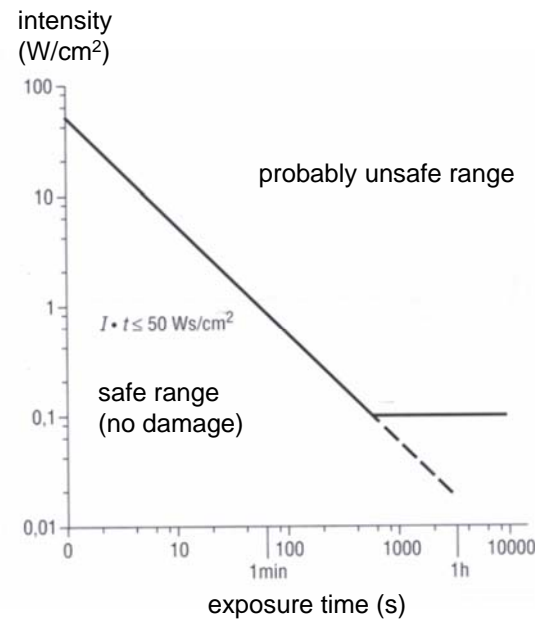
Safety

in the diagnostics:

10 mW/cm^2 = 100 W/m^2

cf. pain threshold: 10 W/m^2

in the therapy: 1 W/cm^2

spatial average temporal average (SATA) intensity;
spatial peak temporal peak (SPTP) intensity;
spatial peak temporal average (SPTA) intensity;
spatial peak pulse average (SPPA) intensity
spatial average pulse average (SAPA) intensity

intensity (W/cm^2)

probably unsafe range

$I \cdot t \leq 50$ Ws/cm^2

safe range (no damage)

exposure time (s)

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more:

in practice

water droplets, aerosol

concave reflecting surface   water   ultrasound source

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