

# Sedimentation and electrophoresis methods

Schay G.

## Physical basis of sedimentation methods

Goal: we would like to measure the mass of tiny particles

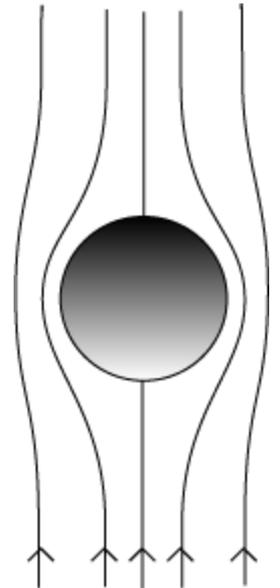
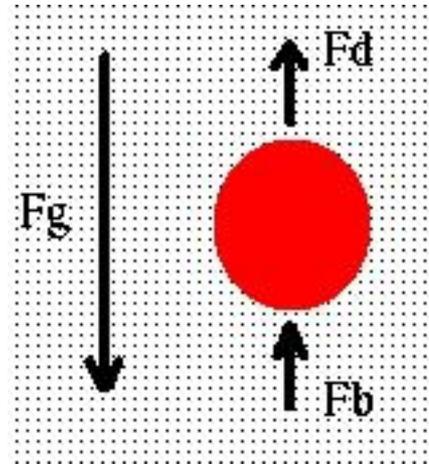
(this method originates long before the AFM or resonance methods, but is still in use)

Put the particle into a solvent/liquid, and see what happens:

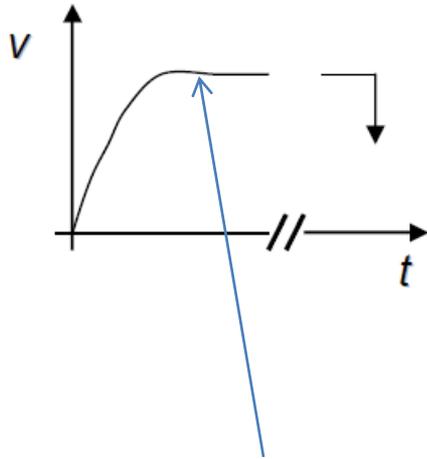
If it's density is higher than that of the liquid, it will sink, or settle down.

This is called **sedimentation**.

$F_g$  : gravity force,  $F_d$ : drag force,  $F_b$ : buoyant force.



Drag: a force ( $F_d$ ) acting on a moving object (usually in a fluid of given viscosity), working against the movement.  
 $F_d \sim v, \eta, \text{size}$



**The particle will accelerate until the force equilibrium is reached.**

(or until the bottom of the holder tube is reached)

Here we have the force equilibrium

Newton-II. Law :  $\Sigma F = m \cdot a$

and

$$\frac{\Delta v}{\Delta t} = a$$

At force equilibrium  $a=0$ .

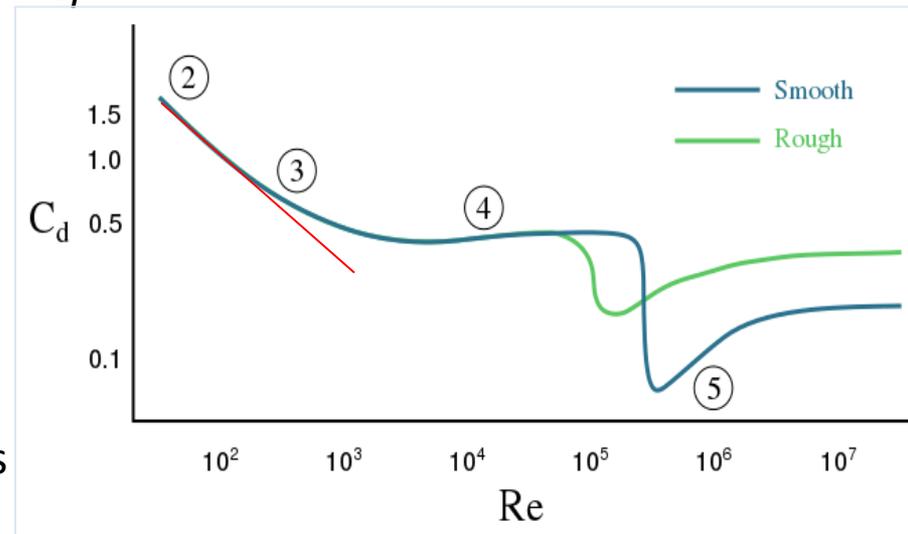
The drag force:

General equation:  $F_d = \frac{1}{2} \rho v^2 \cdot C_d \cdot A$ , where A is the cross-sectional area, and  $C_d$  is the **drag coefficient**.

At low speeds  $C_d \sim 1/Re$ , which means  $F_d$  is linearly proportional to the speed.

$$Re = \frac{v \cdot L}{\eta / \rho} = \frac{v \cdot L \cdot \rho}{\eta}$$

If we substitute this into the Eq. of  $F_d$ , then one can see that  $F_d$  at low speeds depends on the viscosity, and the diameter. (L is the characteristic length, in case of a sphere it is the diameter, but A also depends on L)



Double-log, and slope=-1

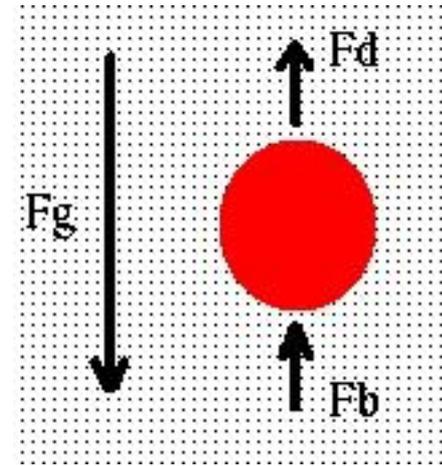
N.B.:  $Re$  is the Reynolds number

Fg : gravity force, Fd: drag force, Fb: buoyant force.

Newton-II. Law :  $\Sigma F = m \cdot a$

and

$$\frac{\Delta v}{\Delta t} = a$$



$$F_g = m \cdot g$$

$$F_d = f \cdot v \quad (Re < 0.1)$$

where  $f$  is the shape constant, embedding  $C_d$ .

From Archimedes's law we get:  $F_b = g \cdot \rho_{fluid} \cdot V_{particle}$ , but  $V_{particle} = m / \rho_{particle}$

$$\text{so } F_b = m \frac{\rho_{fluid}}{\rho_{particle}}$$

From the force equilibrium we get:

$$\Sigma F = 0, \text{ which means } F_g - F_b - F_d = 0,$$

(or  $F_d = F_g - F_b$ ) thus

$$f \cdot v = m \cdot g \cdot \left( 1 - \frac{\rho_{fluid}}{\rho_{particle}} \right)$$

There is one problem with this:

If the particles are really small, then the **Brownian motion** will prohibit settling, and thus the method will not work.

**Solution: Centrifuge!**



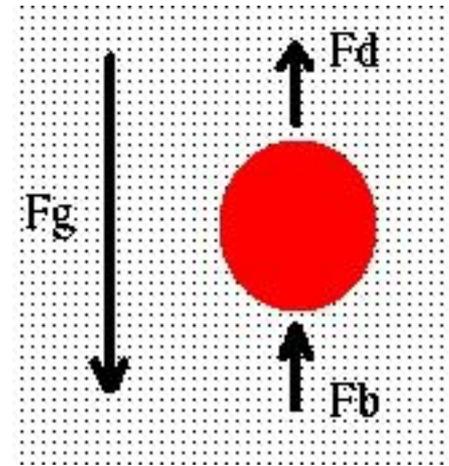
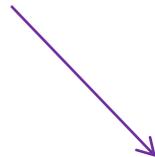
Let's centrifuge the fluid+particles:

**Fg** : gravity force, Fd: drag force, Fb: buoyant force.



$g = 9.8 \text{ m/s}^2$  would be the gravity force, but in the centrifuge  
 $a = r\omega^2$  is the acceleration felt by the particle. ( $\omega$  is the angular velocity)

$$f \cdot v = m \cdot \mathbf{g} \cdot \left( 1 - \frac{\rho_{fluid}}{\rho_{particle}} \right)$$



$$f \cdot v = m \cdot r \cdot \omega^2 \cdot \left( 1 - \frac{\rho_{fluid}}{\rho_{particle}} \right)$$

We can rearrange such as:

$$S \equiv \frac{v}{r \cdot \omega^2} = \frac{m}{f} \cdot \left( 1 - \frac{\rho_{fluid}}{\rho_{particle}} \right)$$

here S is the sedimentation coefficient. Unit is Svedberg, 1Sv = 10<sup>-13</sup> s

(Theodor Svedberg , Nobel prize 1926)

This shows, that mass and density play a crucial role.  
If the density is identical, then the bigger particle will sediment faster.

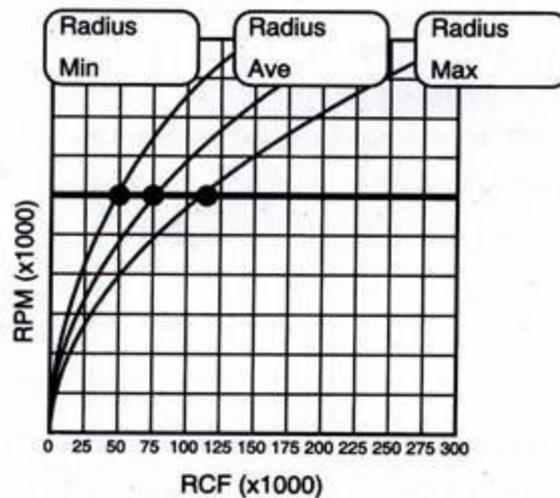


## Useful equations

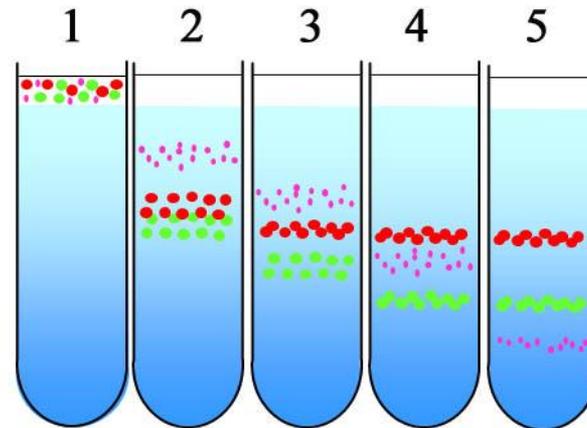
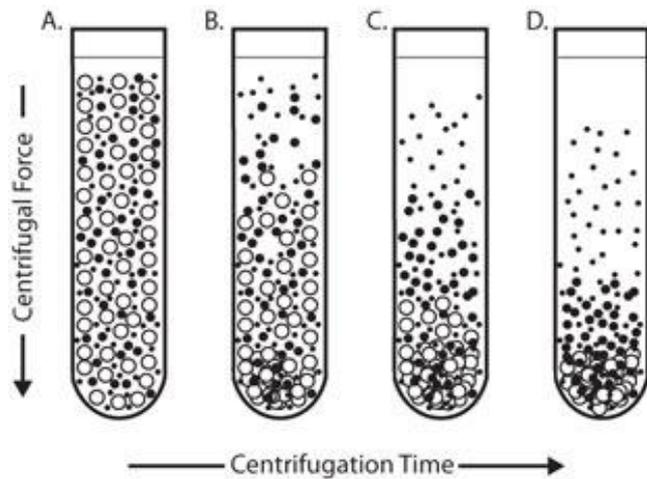
$$\omega = 2\pi \left( \frac{rpm}{60} \right), \text{ rpm} = \text{revolutions per minute}$$

RCF: relative centrifugal field

$$RCF = a = r\omega^2 = 4\pi^2 \text{ rpm}^2 / 3600$$



Since the terminal velocities are different, the particles segregate/separate by mass during the process

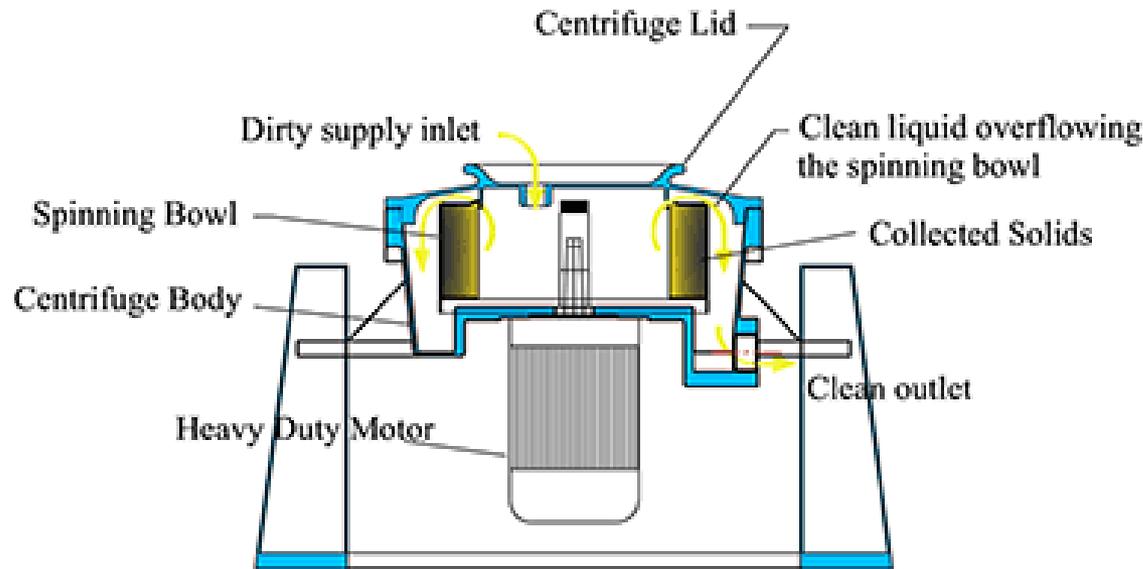


There is always an optimal centrifugation time!

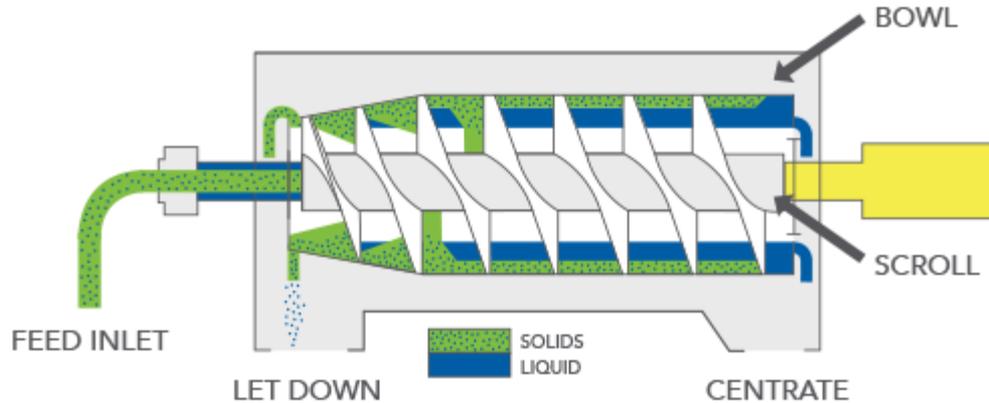
Too short: no separation

Too long: every size reaches the bottom, also no separation.

## The centrifugal separator



The centrifugal force can be generated by many ways.



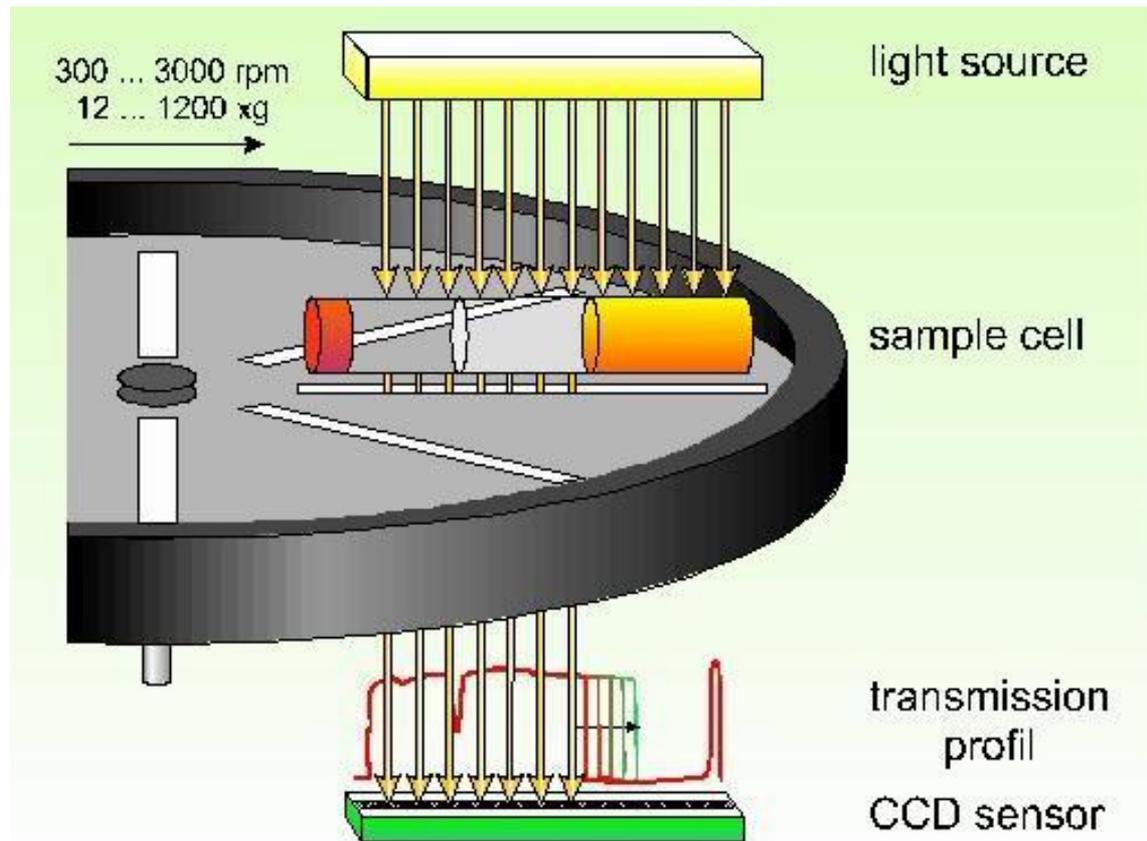
Everyday life:  
Fruit centrifuge.



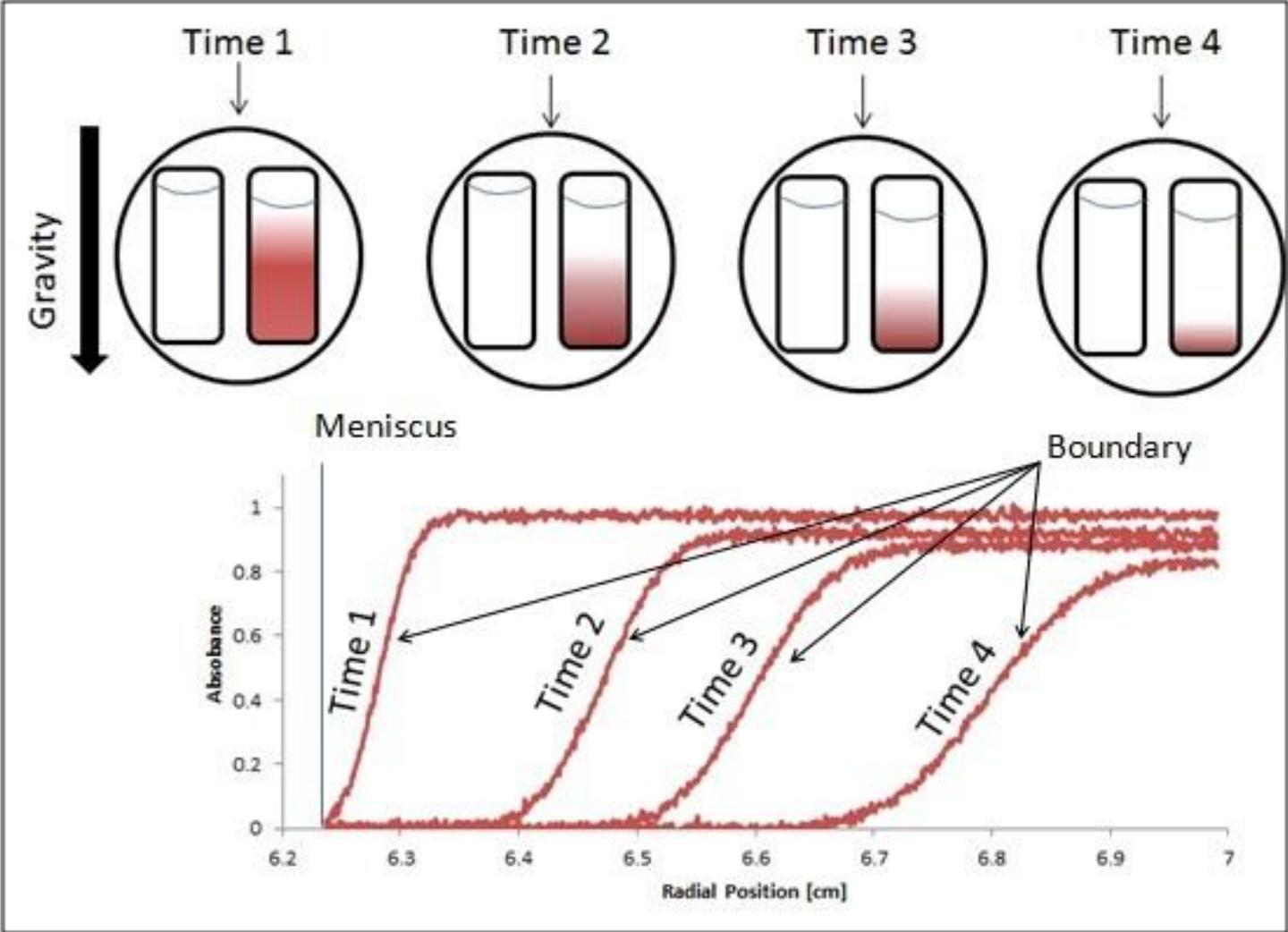
Rotating the mixture, or rotating the container, both, etc..

## Quantitative methods

We want to measure during centrifugation



# Centrifuging a mono-component system +fluid

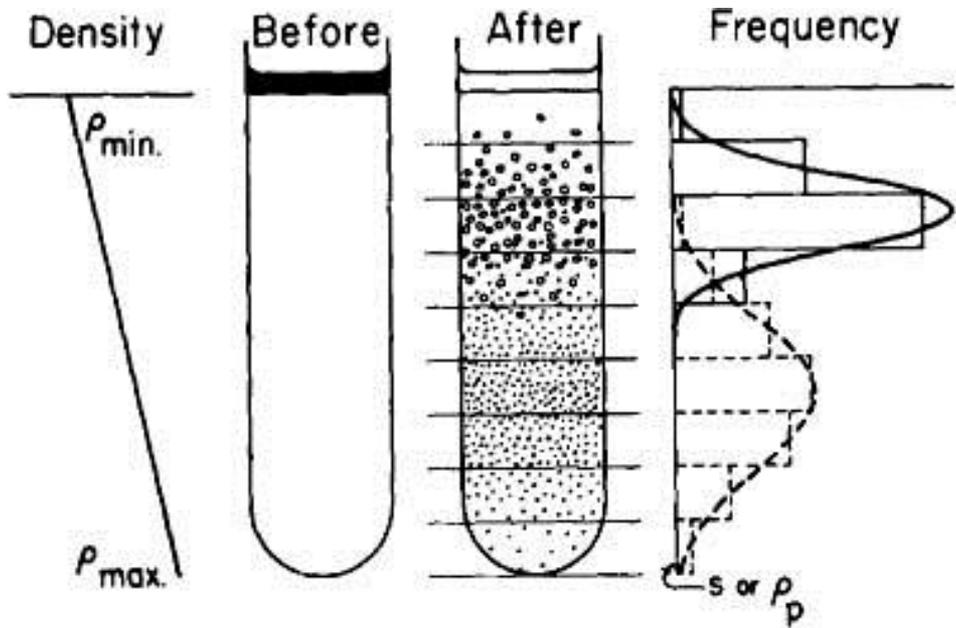


The only remaining unknown is the  $f$ : **form-factor**

But this is also in the diffusion:

$$f = \frac{kT}{D} \text{ where } D \text{ is the diffusion coefficient.}$$

**So we need to measure diffusion, in order to get the particle size.**



It is possible to use a density gradient in the sedimentation medium.

In this case during centrifugation different density particles will stop settling at different positions.

This can be used also for separation: **Preparative or analytical ultracentrifugation methods.**

1. Differential sedimentation

Gradient: *Shallow stabilizing,  $\rho_{max} < \rho_{p min}$*   
 Centrifugation:  $\rightarrow$  *Incomplete sedimentation*  
 Abscissa of frequency distribution: *Sedimentation coefficient*

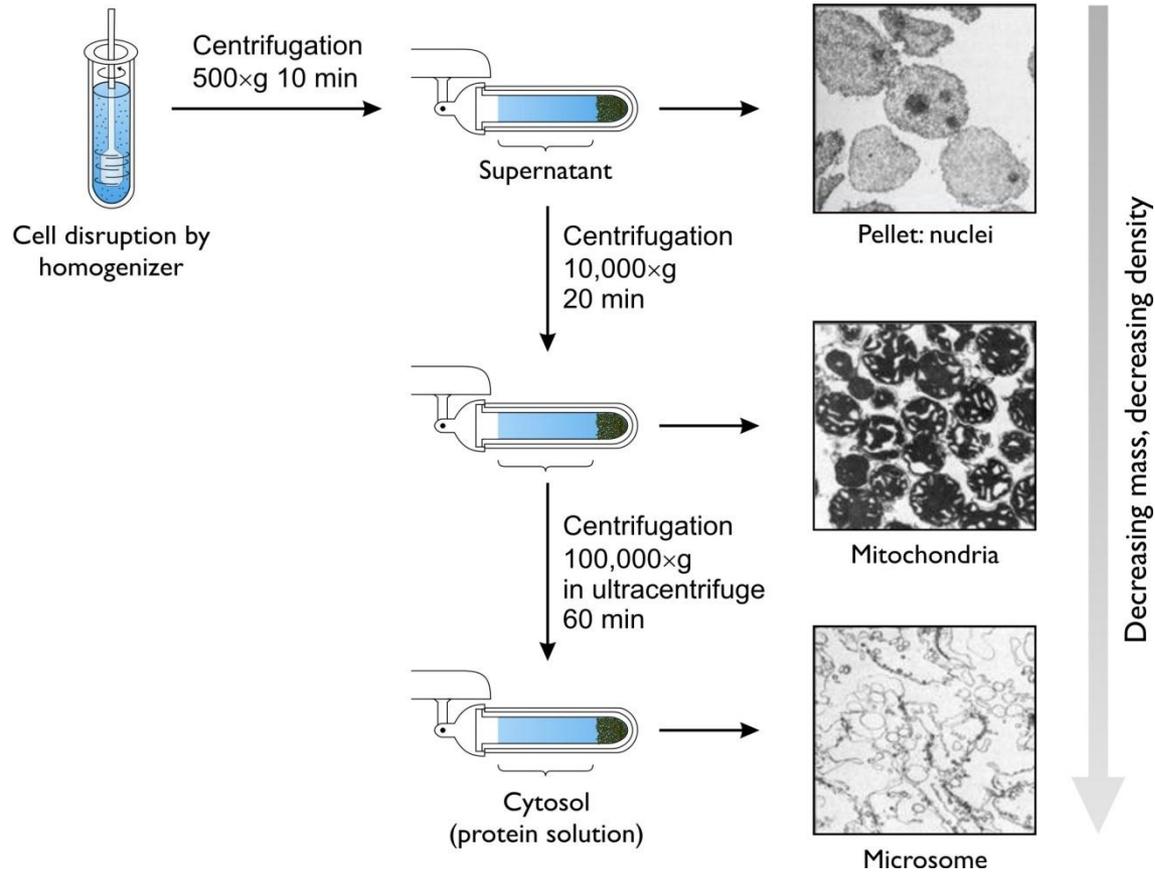
2. Density equilibration

Gradient: *Steep,  $\rho_{max} > \rho_{p max}$*   
 Centrifugation: *Prolonged, high speed*  
 Abscissa of frequency distribution: *Equilibrium density*

## Differential centrifugation:

Separation based on the size of the particles.

*This is not an equilibrium method.*



## Sedimentation equilibrium method

Here we wait, until the sedimentation and the Brownian motion reach an equilibrium. (so there will be a concentration profile in the tube)  
We spin with medium speed, so there is a sedimentation, but not a complete pellet formation

This means, in equilibrium the net drag force is 0.

In thermal equilibrium, the **Boltzmann distribution** will describe the position of the particles in any force field:

$$\frac{n_1}{n_2} = e^{-\frac{\Delta E}{kT}}$$

In the energy term, we take into account the work of the forces.  
If 1 and 2 denote distances  $r_1$  and  $r_2$  from the center of rotation, then

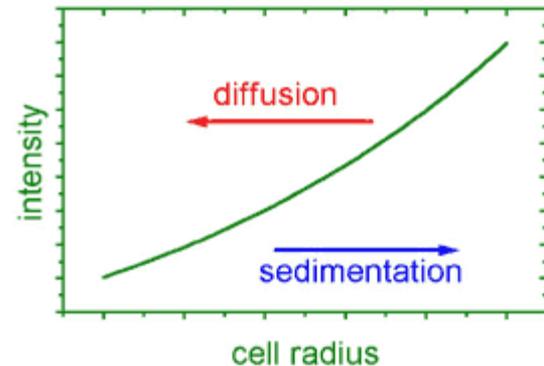
$$\Delta E = \frac{m}{2} (r_1^2 - r_2^2) \omega^2 \left( 1 - \frac{\rho_{fluid}}{\rho_{particle}} \right)$$

Substituting into the Boltzmann formula and taking the logarithm yields:

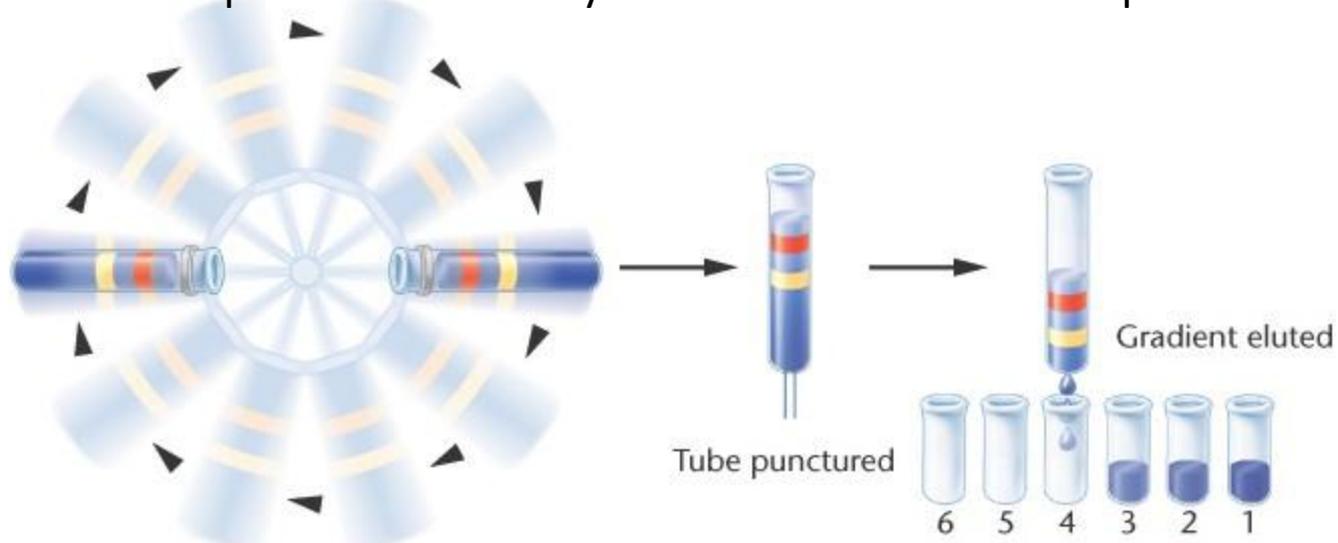
$$\ln\left(\frac{n_1}{n_2}\right) = \frac{m}{2kT} (r_1^2 - r_2^2) \omega^2 \left(1 - \frac{\rho_{fluid}}{\rho_{particle}}\right)$$

We can measure the concentrations ( $n_1, n_2$ ) the densities, and we know the radii, so the mass can be calculated.

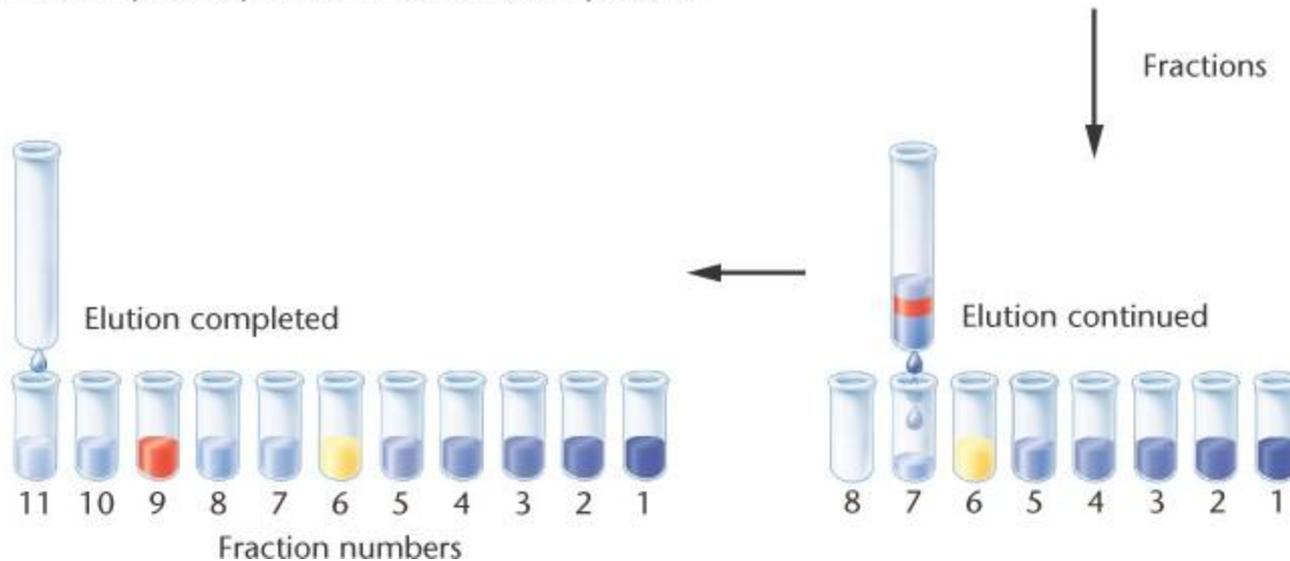
We do need the density, if that is unknown then at least 2 different solvents have to be used, so 2 independent equations can yield the 2 unknowns (m and density)

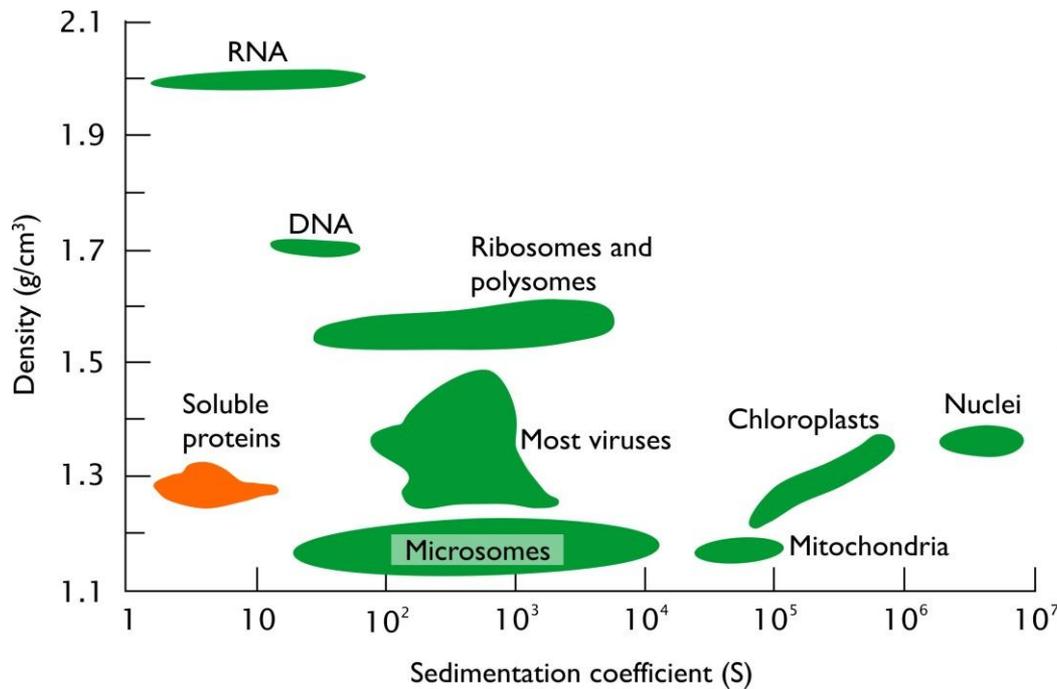


Sedimentation separation is a widely used method. After the Eq. is reached:



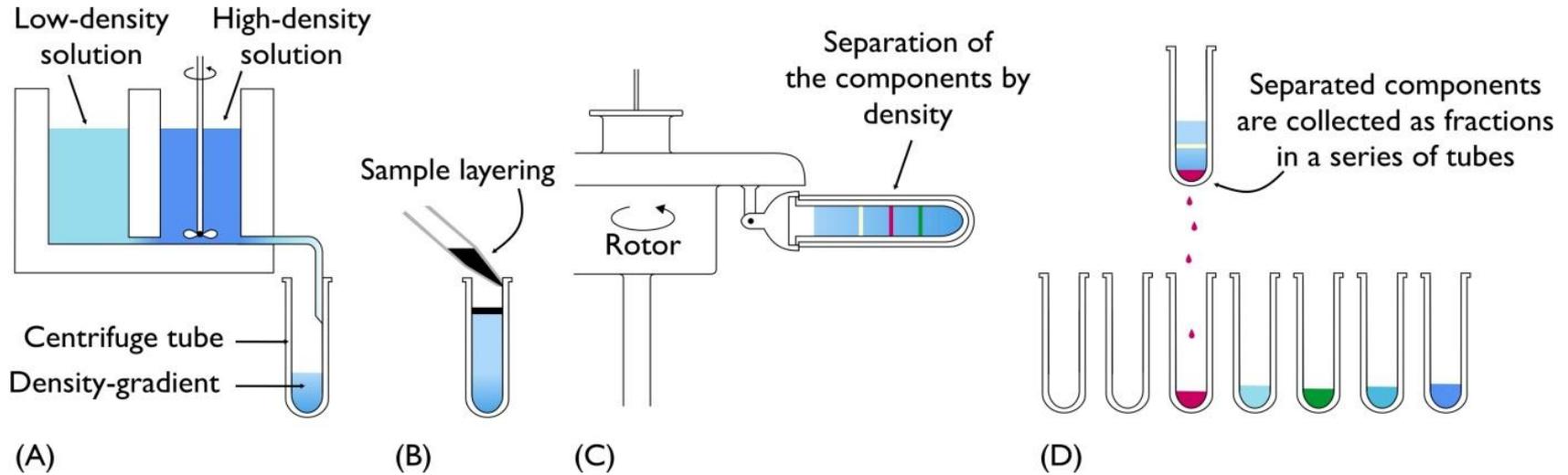
Tubes placed in ultracentrifuge and rotated at high speed; Sample is separated into its two components





Based on S:  
differential centrifugation

Then based on density:  
gradient centrifugation



Equilibrium separation method is based on the usage of a density-gradient

## Electrophoretic methods

If a molecule is charged, and placed into an electric field, then a force will act on it.

This force (analogous to the sedimentation analysis) will cause a separation of the particles/molecules.

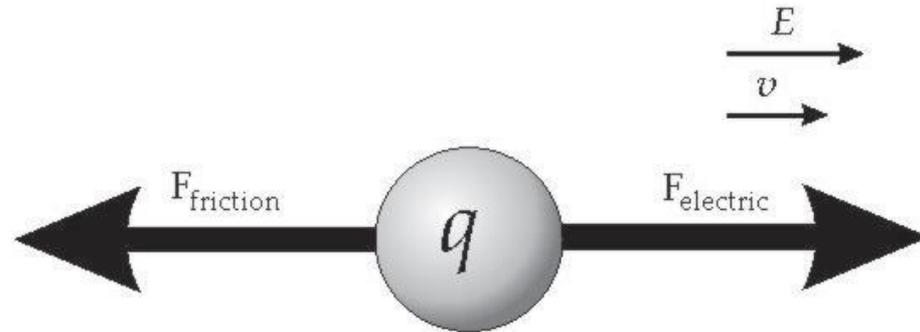
*This is not an equilibrium method.*

$$\mu_e = \frac{v}{E}$$

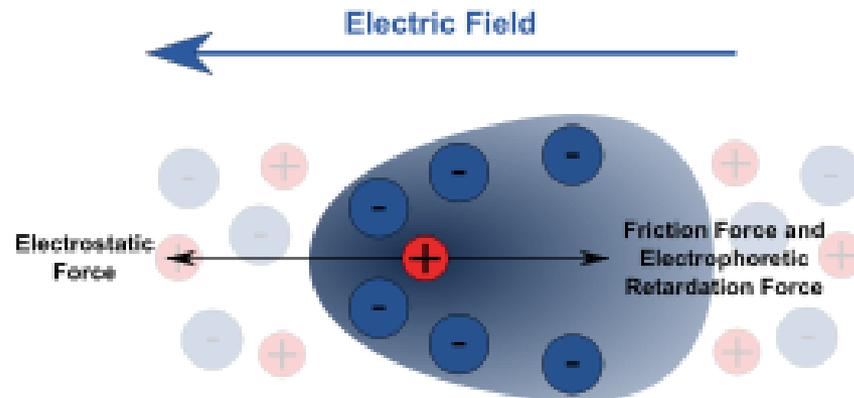
The **electrophoretic mobility** is defined by the velocity and the electric field creating that velocity.

This is specific for a given particle.

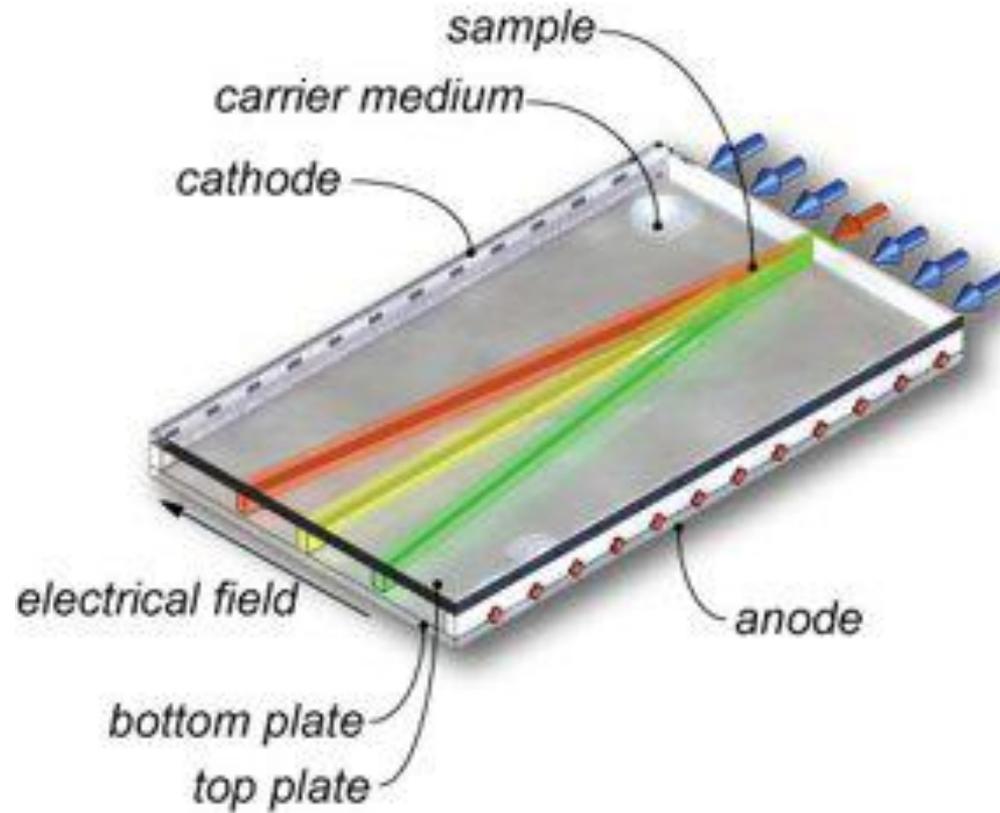
## Forces in an uncharged solvent

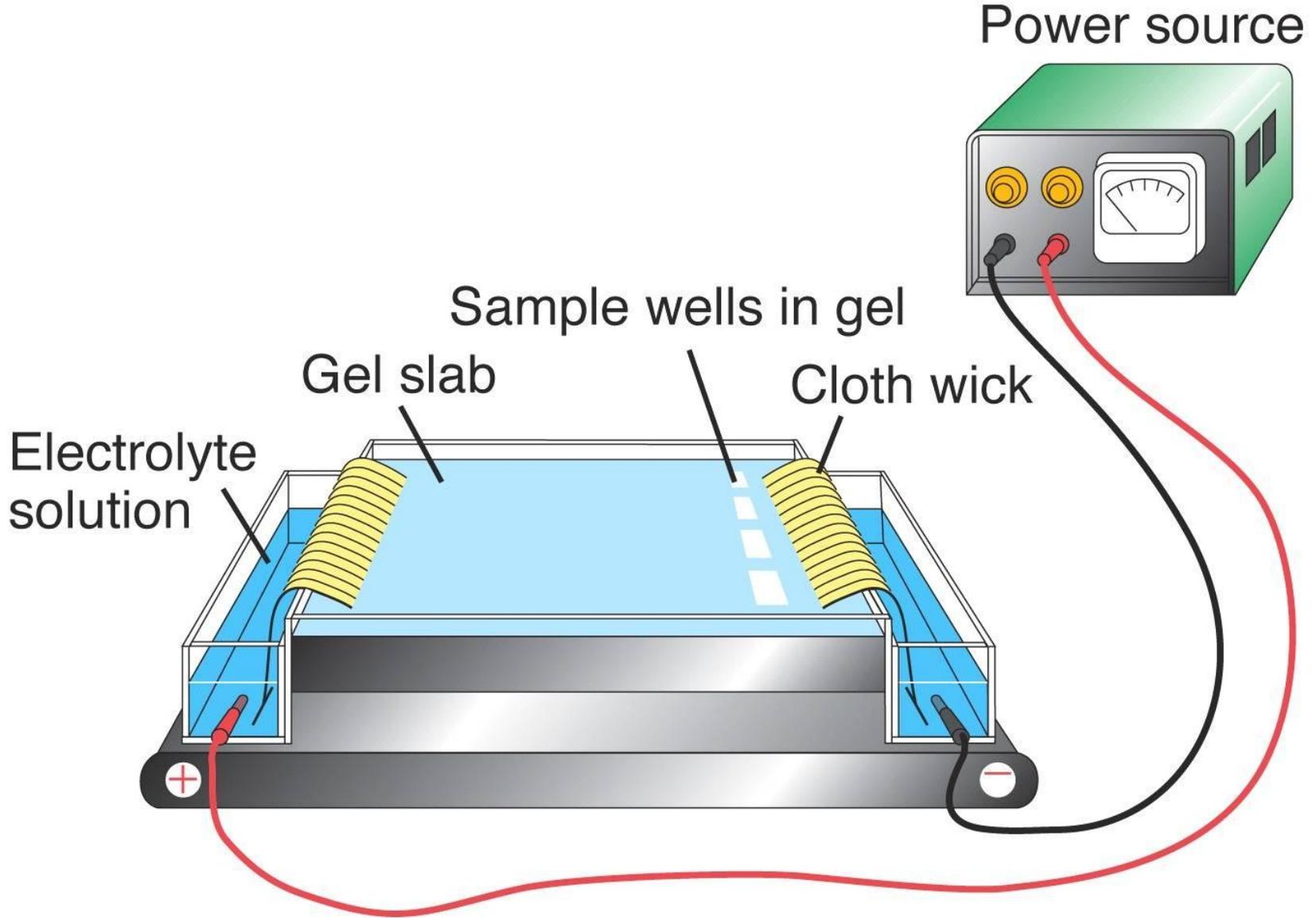


In a charged solvent there is retardation too:

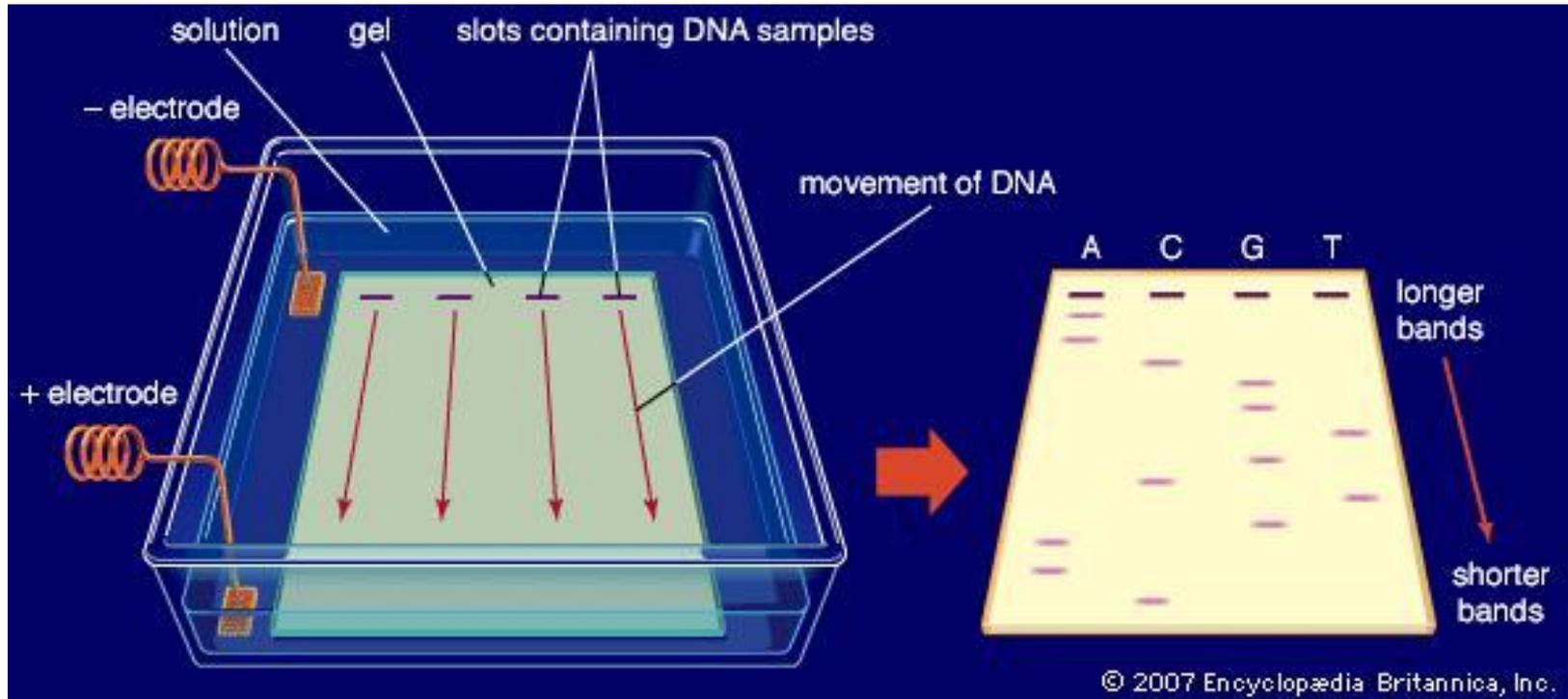


## Free flow electrophoresis



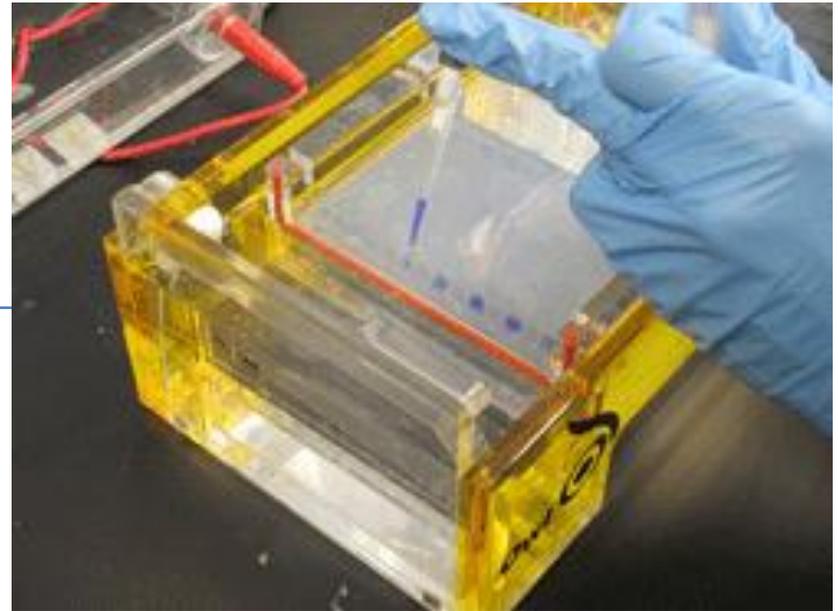


# Gel electrophoresis

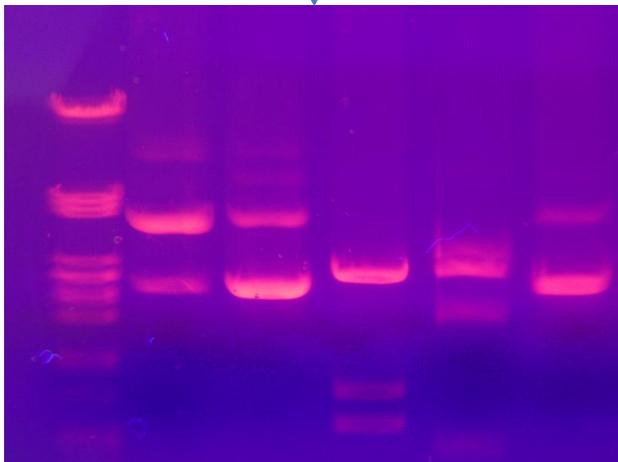
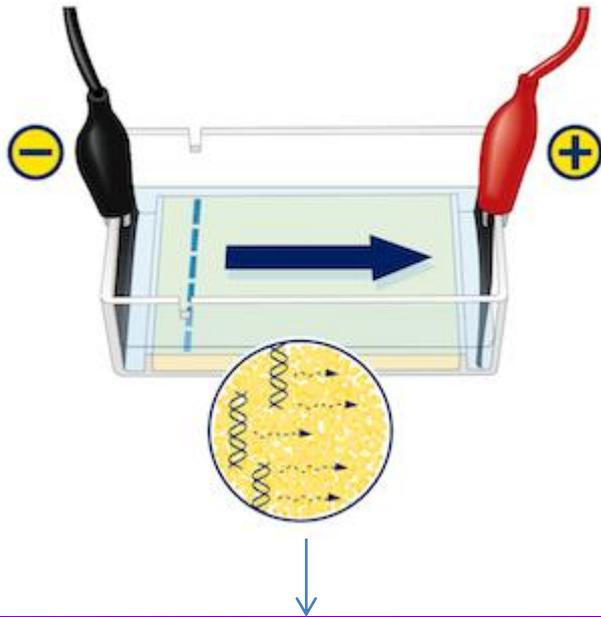


# Gel electrophoresis

Load the gel



Run the gel



Label (here e.g. in-situ) and evaluate.

Labeling in a gel is not easy.

Blotting: one can transfer (and fix) the stripes on the gel onto a vinyl, or other membrane.

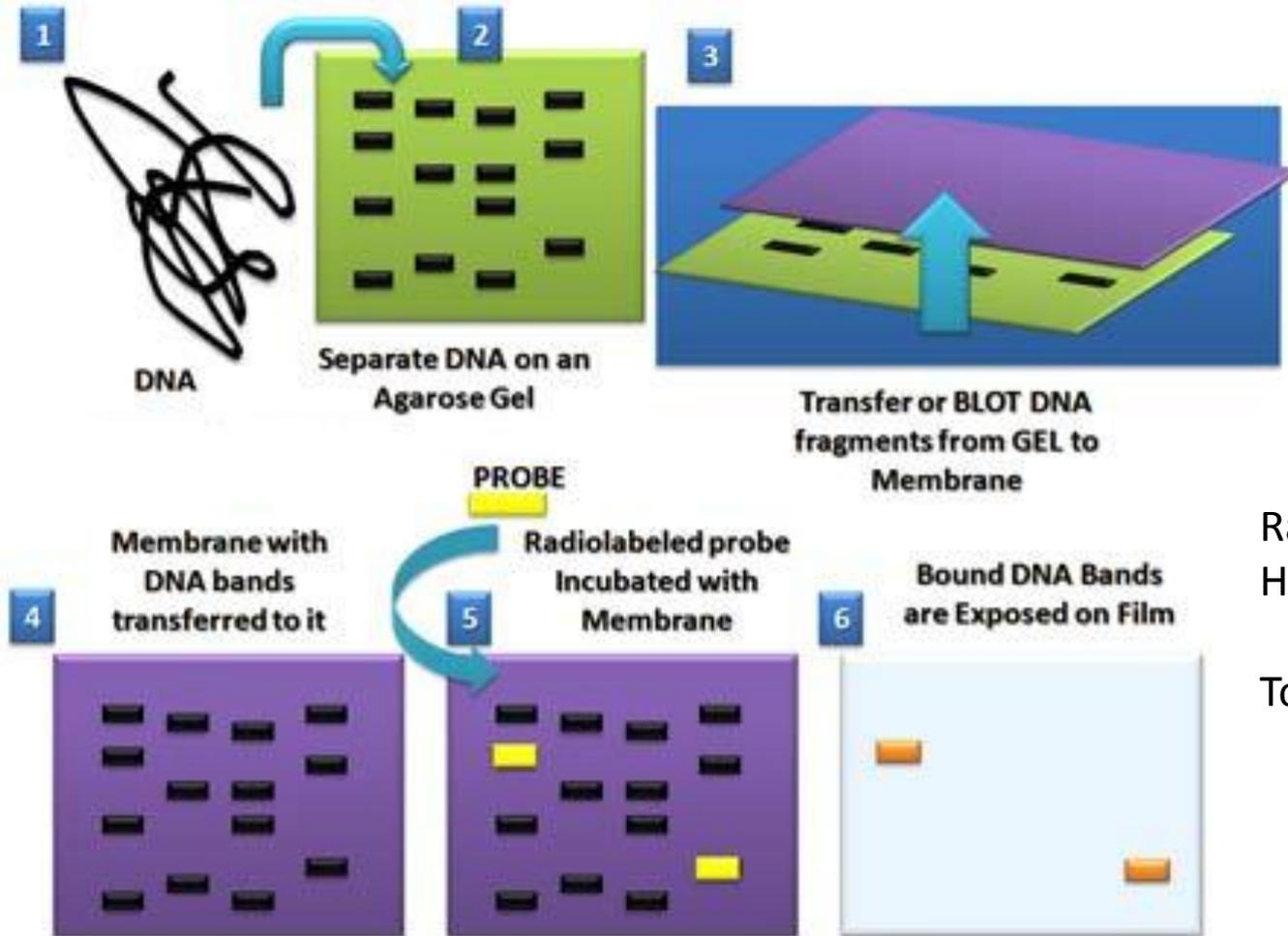
Visualization by staining is then done on the membrane.

This enables the use of complicated chemical/biochemical reactions.

Since the membrane has a higher density and viscosity than a gel, the diffusion is much less, so during the chemistry the bands will not “smear” as much.

Blotting is almost always done if the labeling takes considerable time (more than 1-2min)

## Southern blot (Edwin Southern)

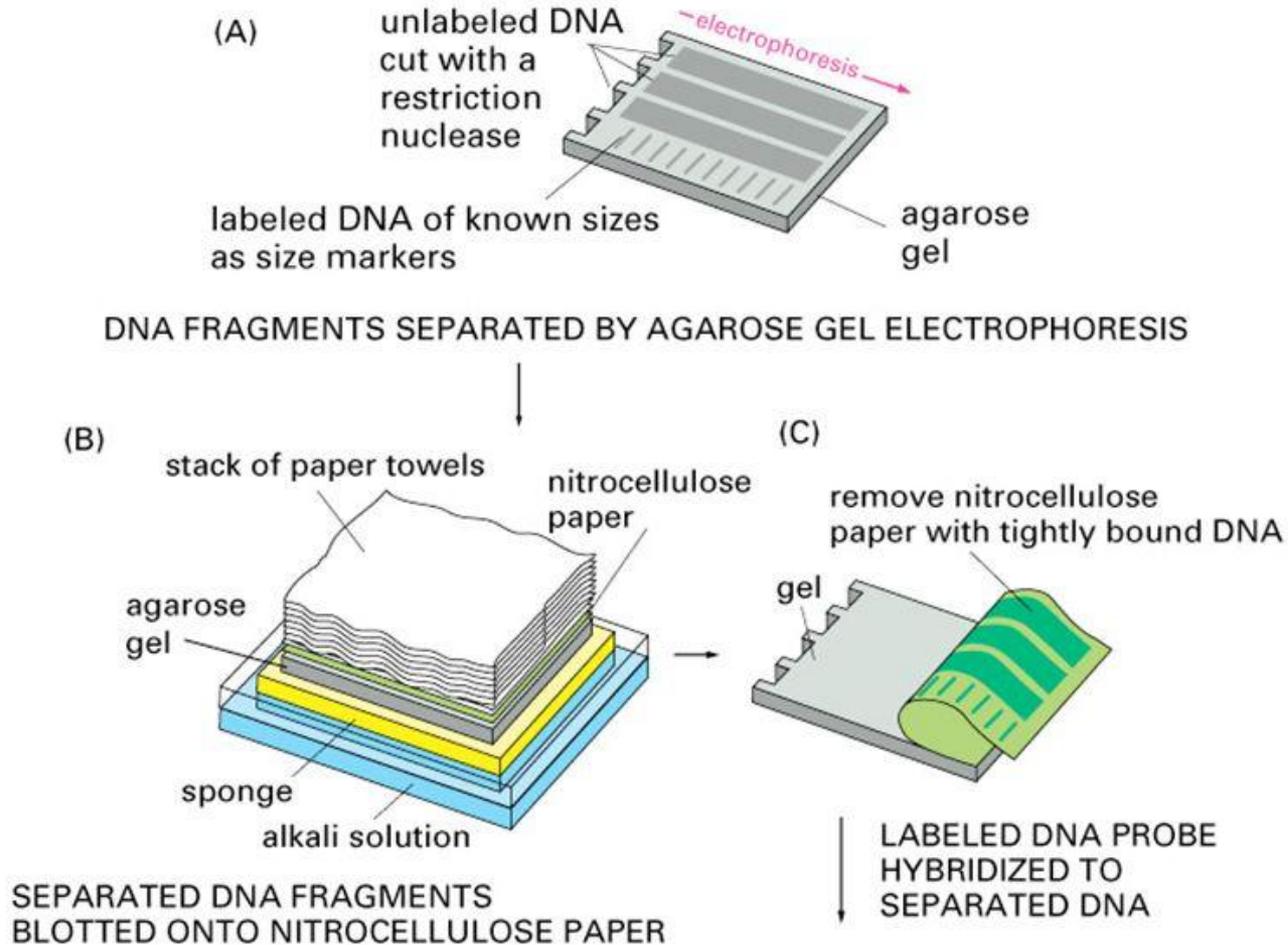


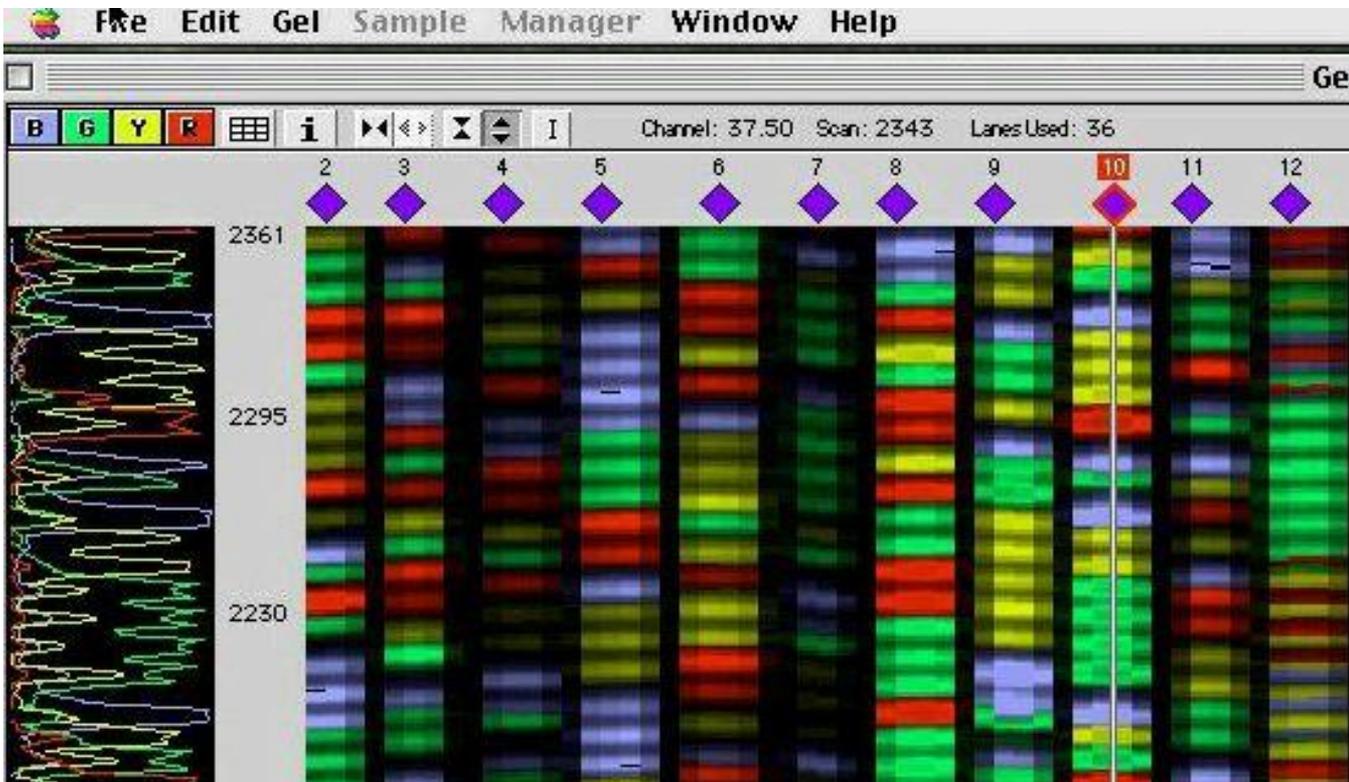
Radiolabeling:  
High sensitivity!

Today: fluorescence  
versions

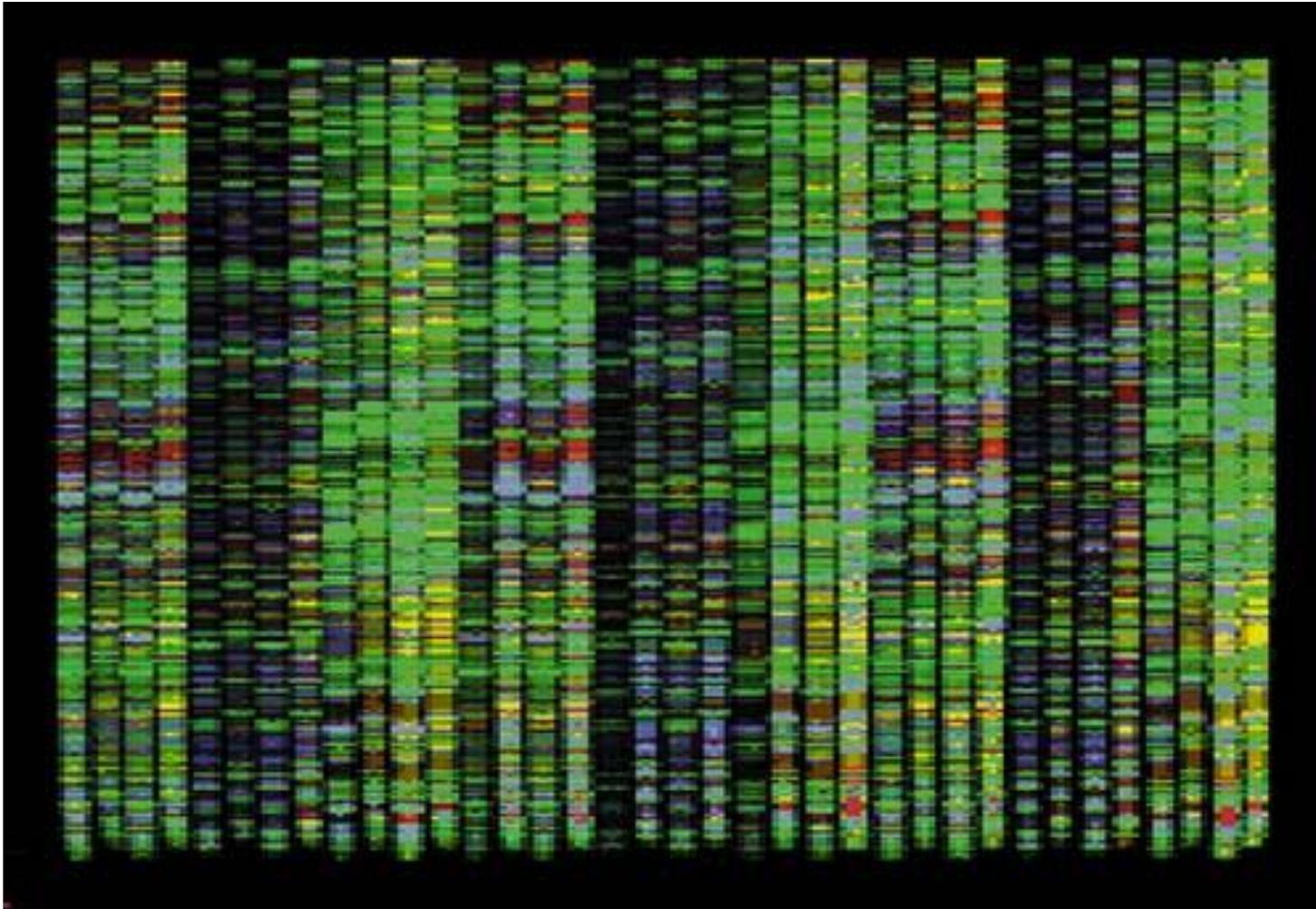
Some blotting details...

# Southern Blot (DNA)



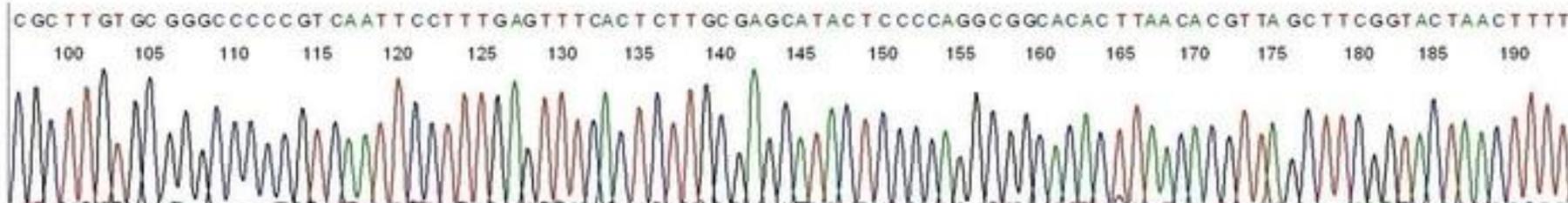
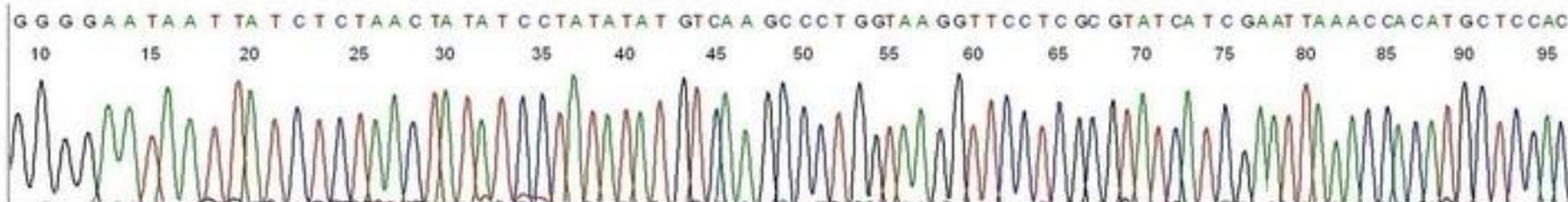


Fluorescence is much easier, and can be automated...



On a good quality gel a LOT of stripes can be differentiated...

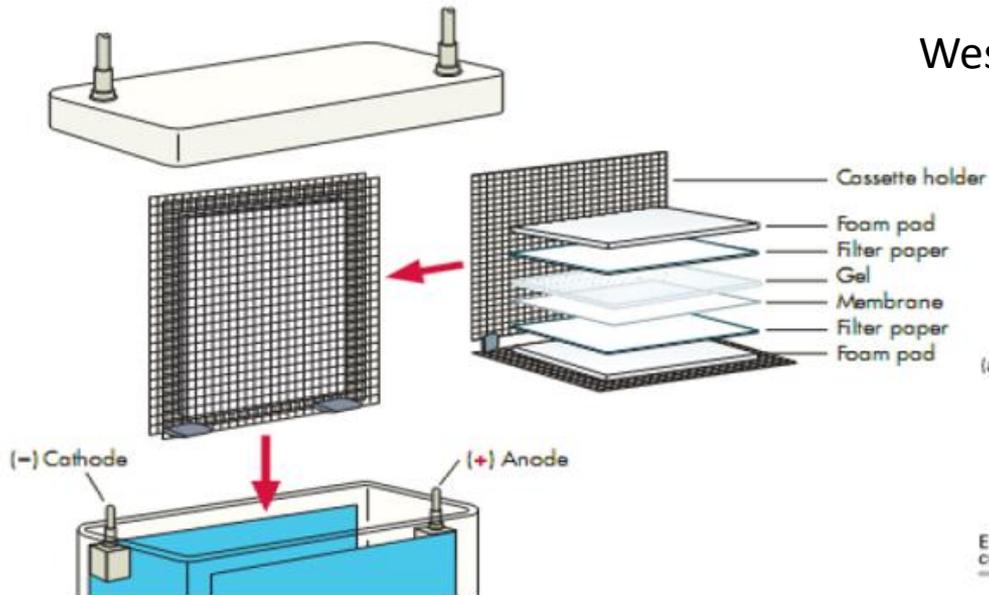
Used extensively in sequencing (getting the base sequence of DNA/RNA)



*Borrelia burgdorferi* CA382, complete genome  
 Sequence ID: [gb|CP005925.1|](#) Length: 910736 Number of Matches: 1  
 Range 1: 445107 to 445291 [GenBankGraphics](#)

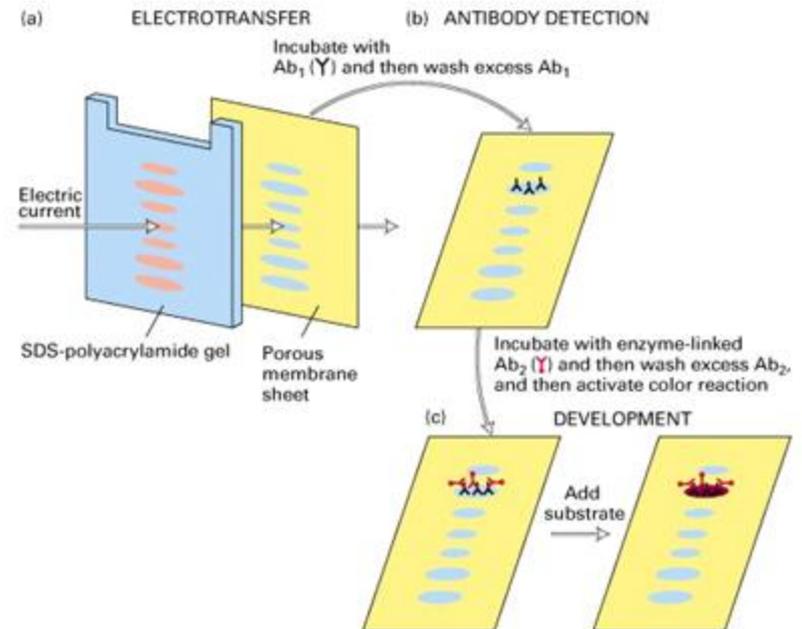
Score	Expect	Identities	Gaps	Strand
342 bits(185)	5e-91	185/185(100%)	0/185(0%)	Plus/Plus
Query 1	GGGGAATAATTATCTCTAACTATACTCTATATATGTCAAAGCCCTGGTAAGGTTCCCTCGCG			60
Sbjct 445107	GGGGAATAATTATCTCTAACTATACTCTATATATGTCAAAGCCCTGGTAAGGTTCCCTCGCG			445166
Query 61	TATCATCGAATTAACCACATGCTCCACCCTTGTGCGGGCCCCGTCAATTCCTTTGAG			120
Sbjct 445167	TATCATCGAATTAACCACATGCTCCACCCTTGTGCGGGCCCCGTCAATTCCTTTGAG			445226
Query 121	TTTCACTCTTGCAGGATACTCCCAGGCGGCACACTTAACACGTTAGCTTCGGTACTAA			180
Sbjct 445227	TTTCACTCTTGCAGGATACTCCCAGGCGGCACACTTAACACGTTAGCTTCGGTACTAA			445286
Query 181	CTTTT 185			
Sbjct 445287	CTTTT 445291			

Western and Northern blotting: the names come from the play with the original name of the southern blot.

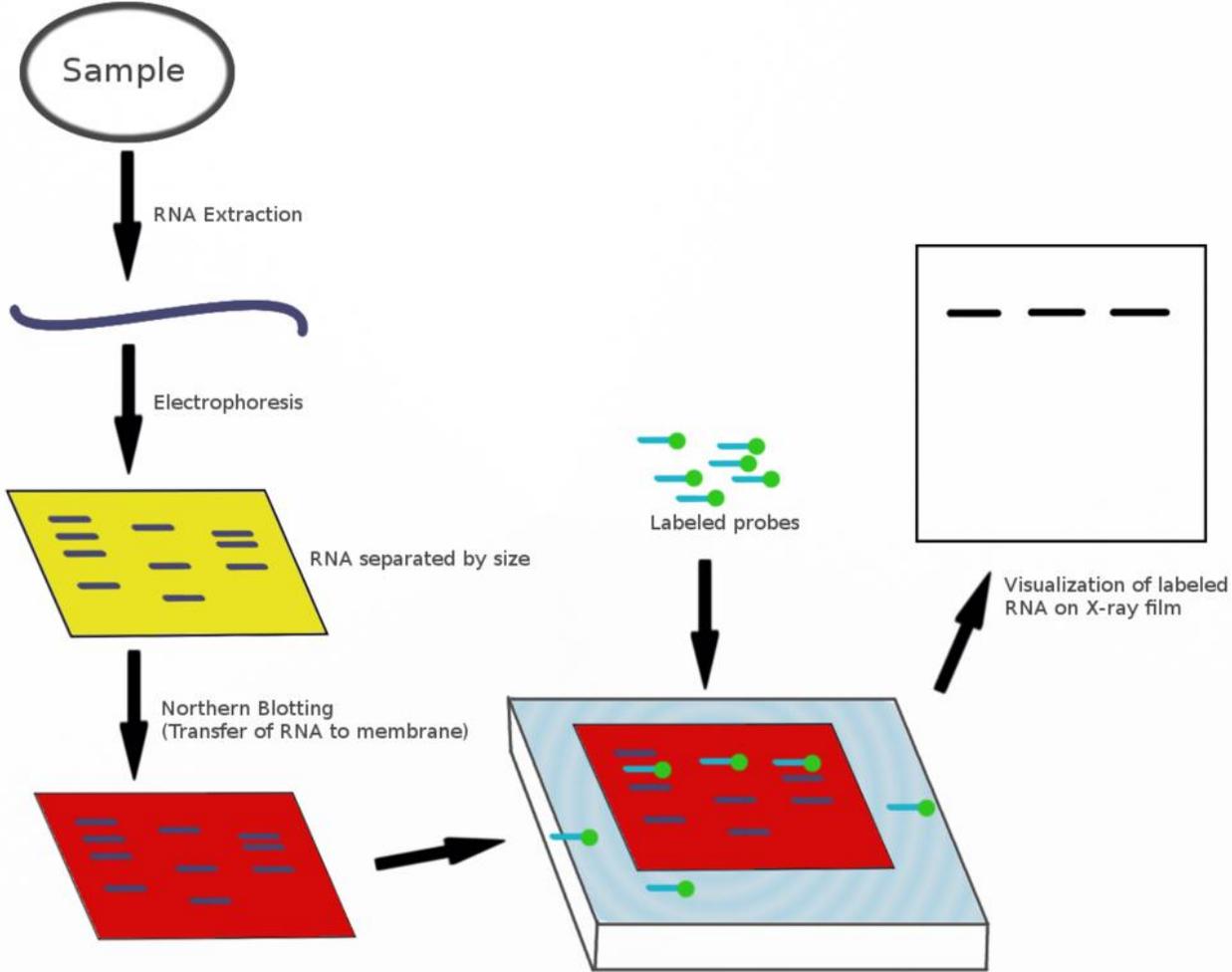


Western blot: protein detection

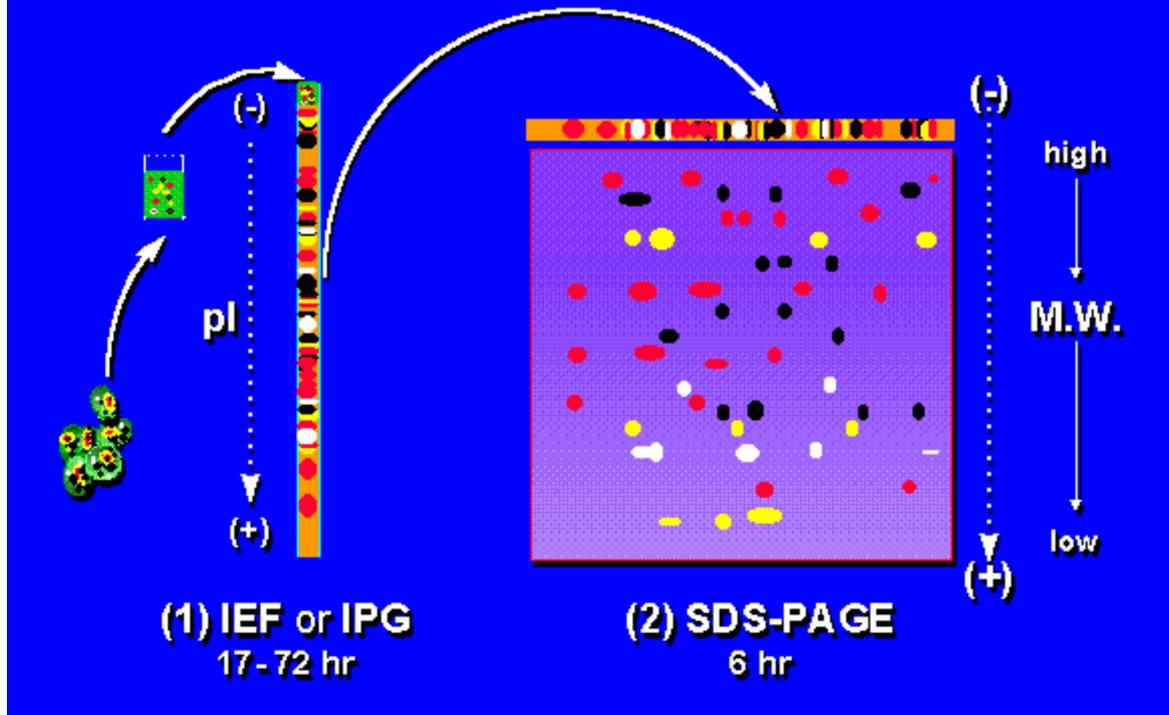
Proteins do not pair like DNA, but antibodies can be used for selective detection.



# Northern blotting: detect RNA / expression levels



# Two Dimensional Electrophoresis



Here we make TWO runs, the second goes 90 deg to the first one.  
The chemical/physical conditions are different

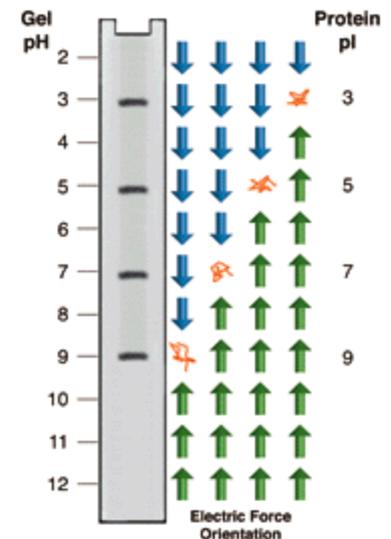
## Isoelectric focusing

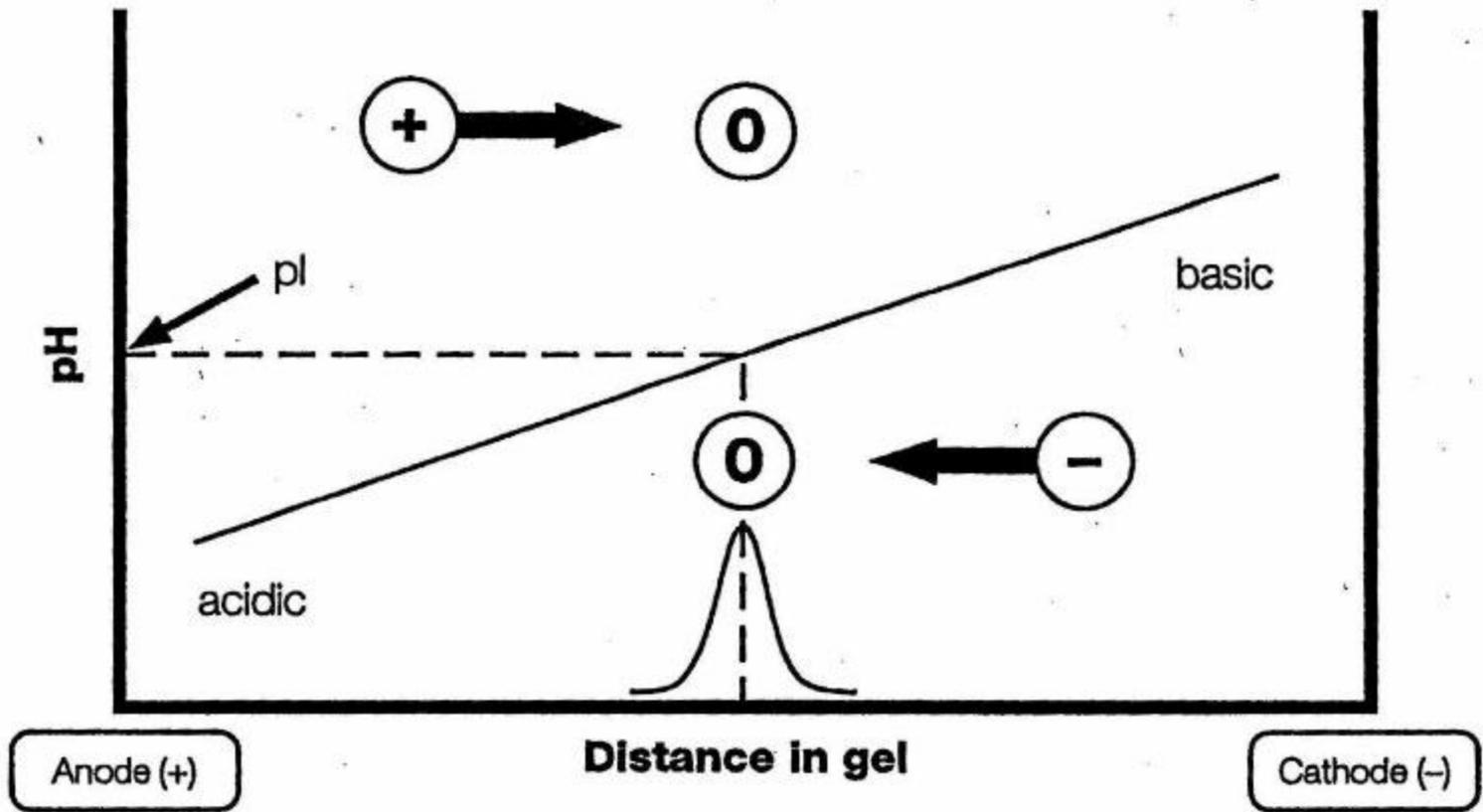
### We use a gel, which has a pH gradient.

Due to the electrophoresis, the molecules will move towards the point in the gel, where the pH is equal to their *isoelectric point*.

At this point the molecules don't move any more, and are instead focused into sharp stripes.

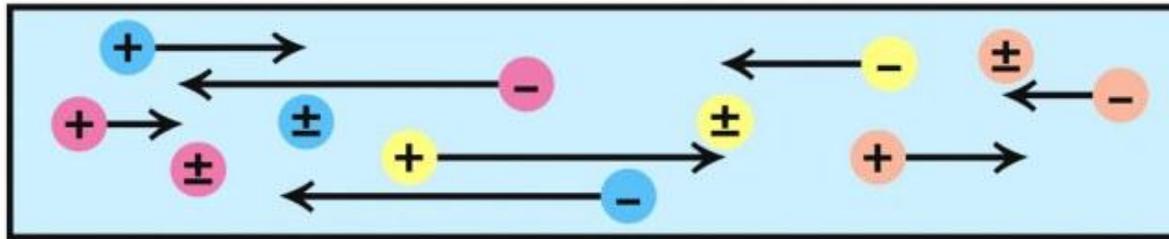
The technique is capable of separating proteins differing in a single elementary charge.





(A)

Low pH  
(+)



High pH  
(-)

(B)

Low pH  
(+)



High pH  
(-)

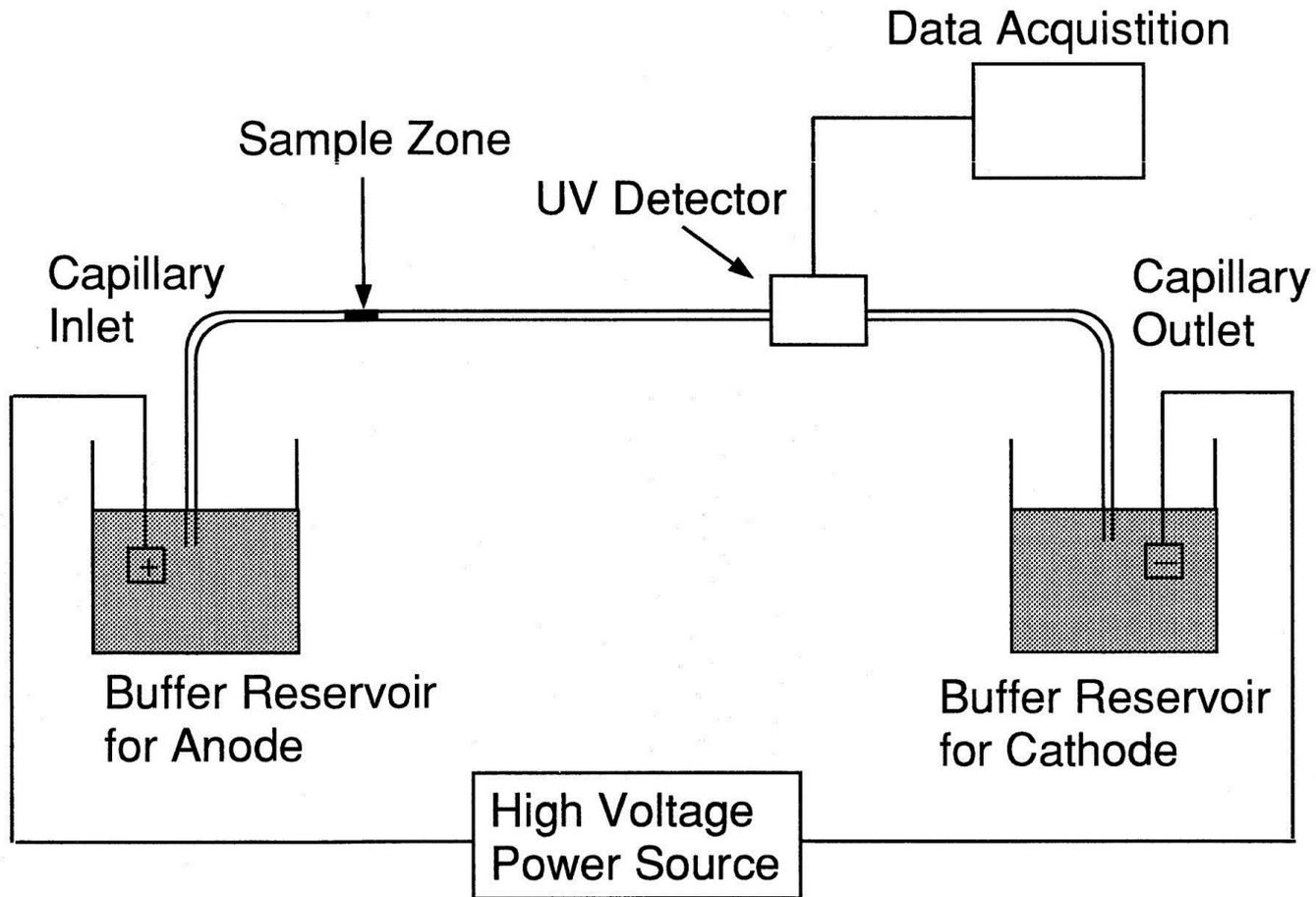
**Figure 3.11**

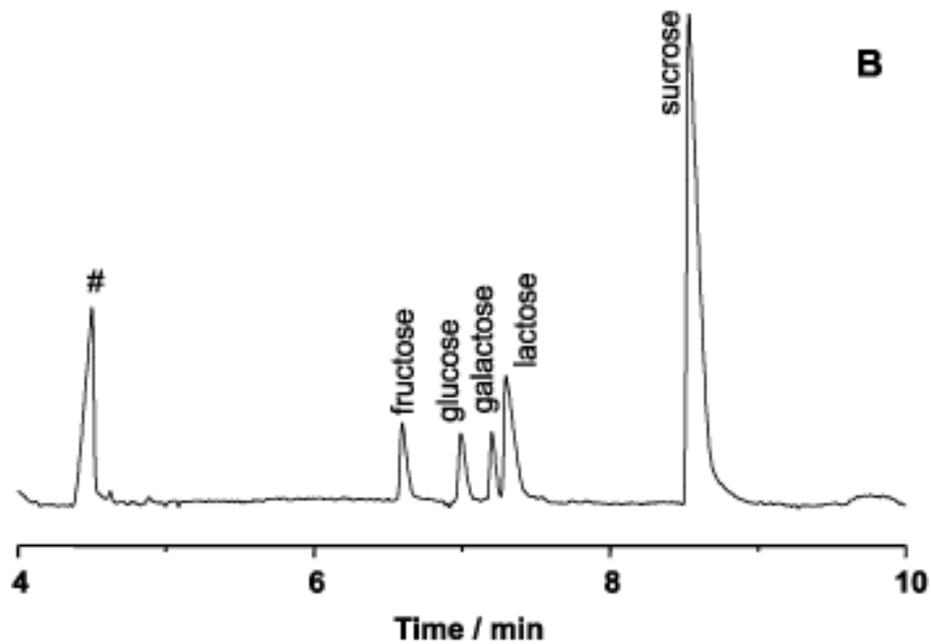
*Biochemistry, Seventh Edition*

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# Capillary electrophoresis:

A chromatography technique, also useful for separation





Automated parallel analysis

**Figure 7.** Nutritional Applications of Capillary Electrophoresis: analysis of carboxylic acids in wine (A) and carbohydrates in yogurt (B). (A) 10 mmol L<sup>-1</sup> 3,5-dinitrobenzoic acid with 0.2 mmol L<sup>-1</sup> CTAB, pH 3.6; 254 nm. (B) 15 mmol L<sup>-1</sup> sorbate, 0.5 mmol L<sup>-1</sup> CTAB and 35 mol L<sup>-1</sup> NaOH; injection 3.4 kPa/15 s, 30 °C, -18 kV and 254 nm. (#) is a non-identified peak.

