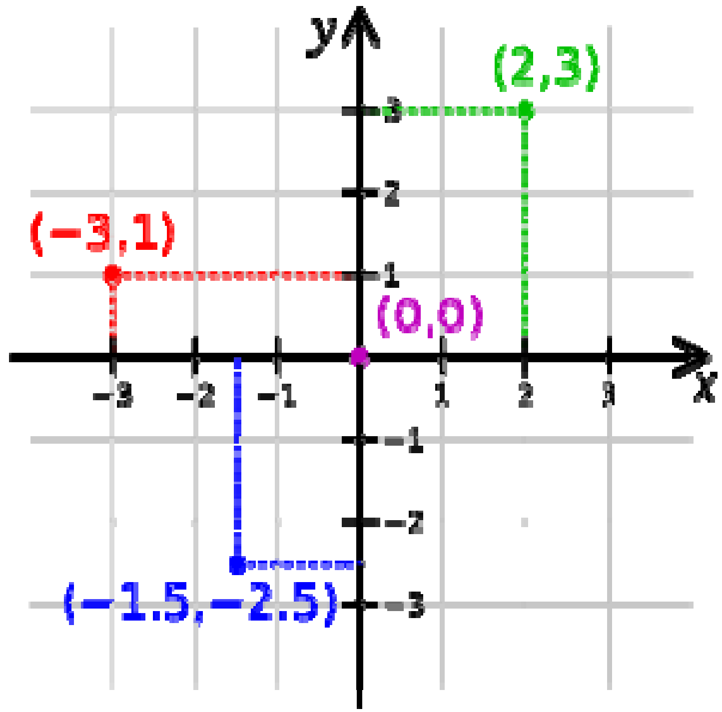
Physical bases of biophysics

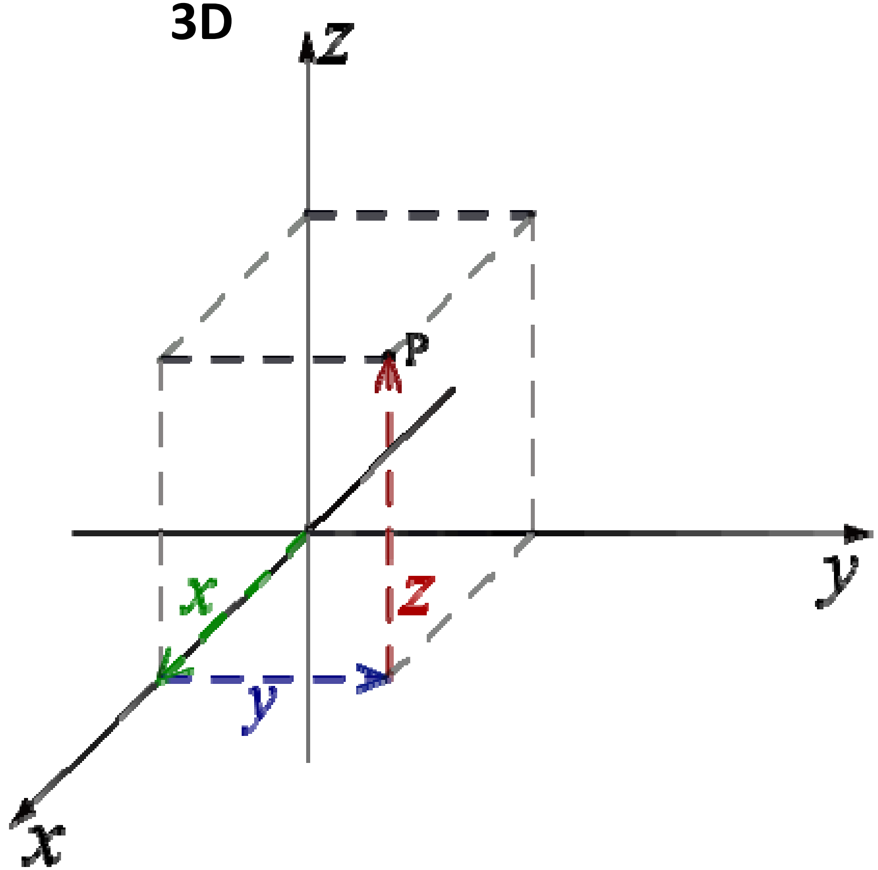
2nd Lecture
Kinematics
14th September 2018.
Ádám Zolcsák

Cartesian coordinate system

2D

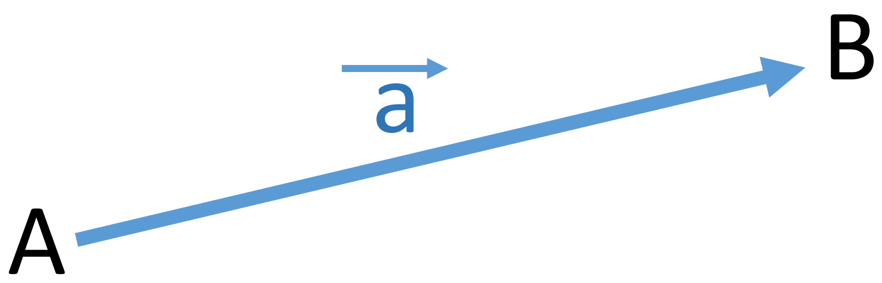


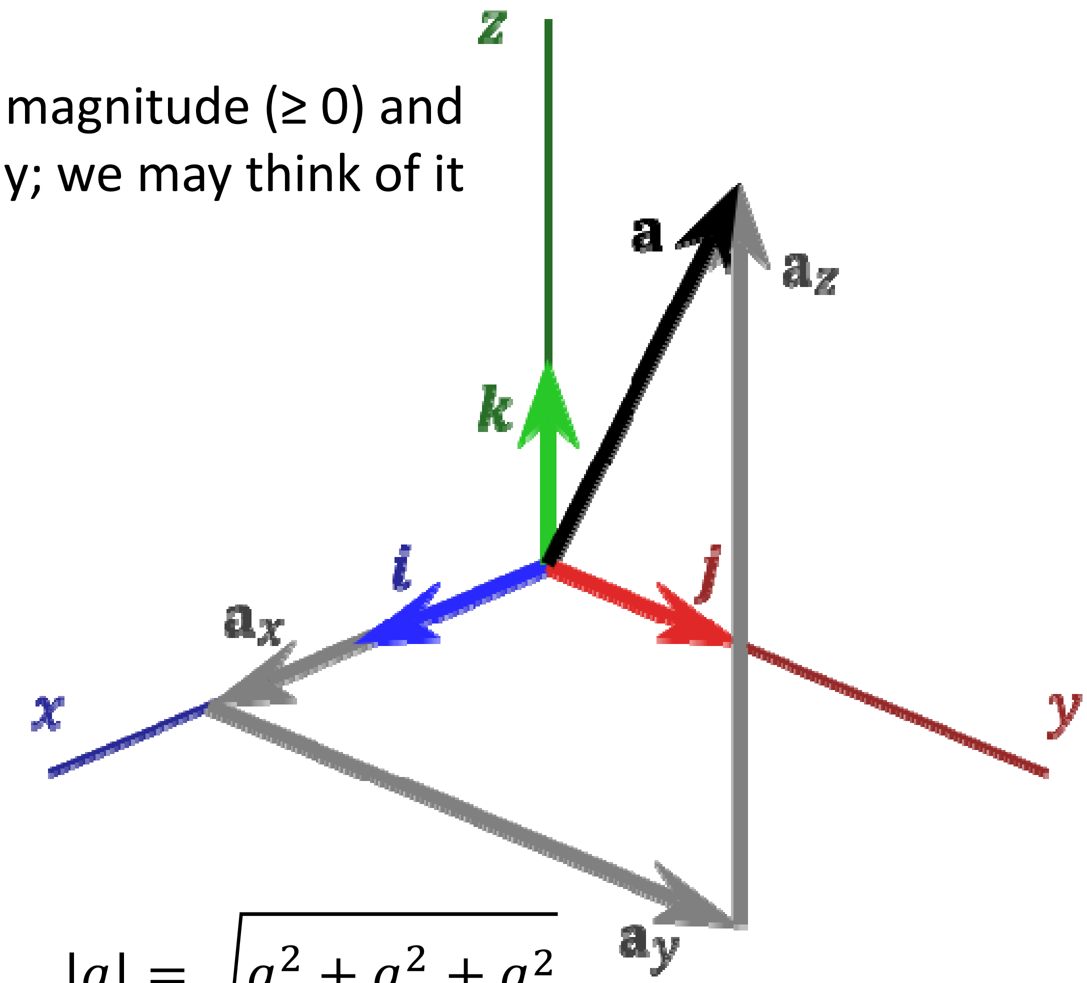
3D



Vectors

A vector is a quantity that requires both a magnitude (≥ 0) and a direction in space to specify it completely; we may think of it as an arrow in space





$$|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2}$$

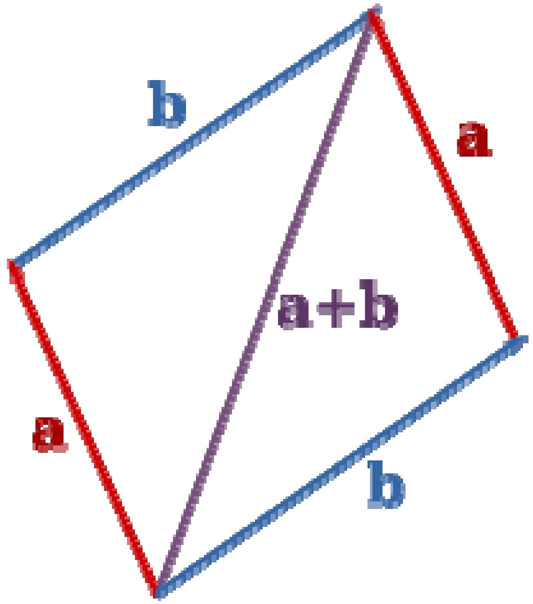
$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

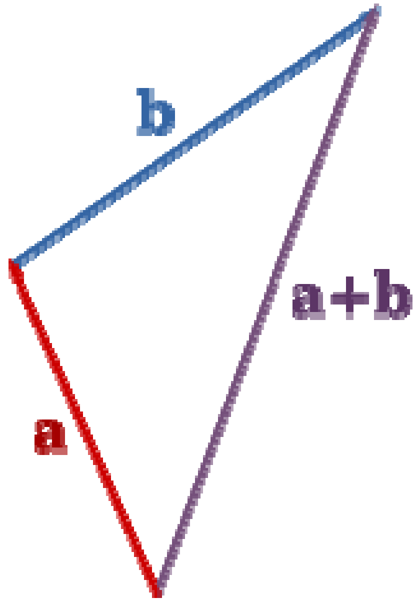
Vector algebra

Addition of vectors

$$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$$

$$(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c} = \boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$$

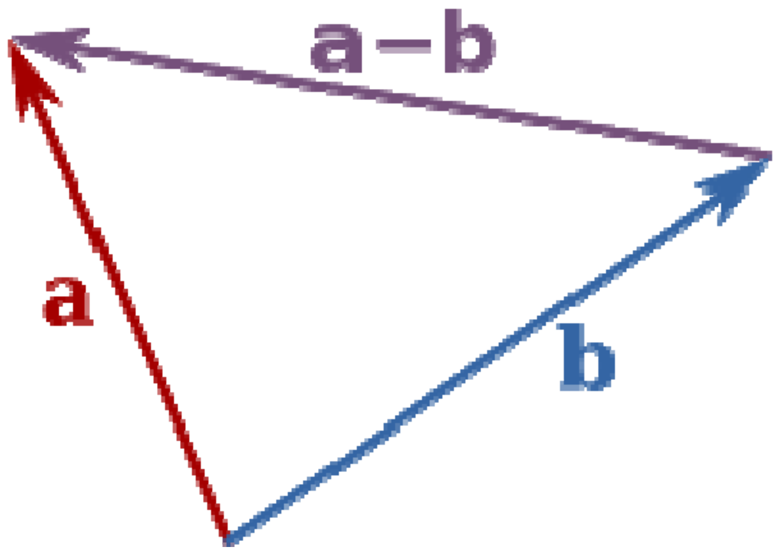




$$\boldsymbol{a} + \boldsymbol{b} = (a_1 + b_1)\boldsymbol{i} + (a_2 + b_2)\boldsymbol{j}$$

Vector algebra

Substraction of vectors



-b is a vector with the same magnitude of **b** but with opposite directions
Place the tails of the vectors **a b** together draw an arrow rom the head of the substracted arrow towards the vector from which you substracted from THIS will give the **resultant vector**

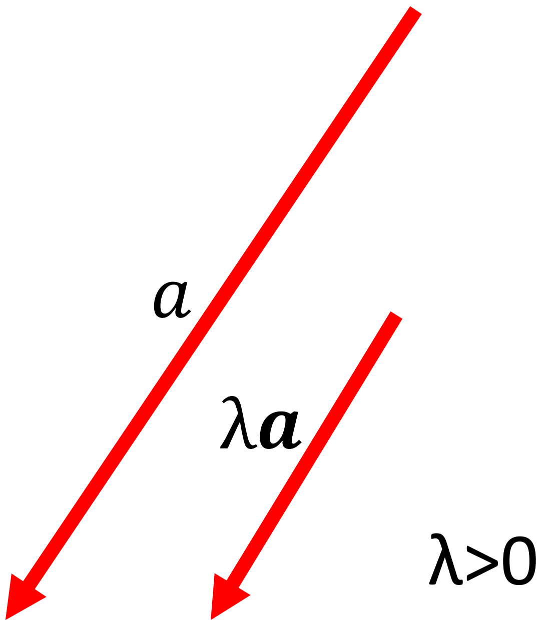
$$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$$

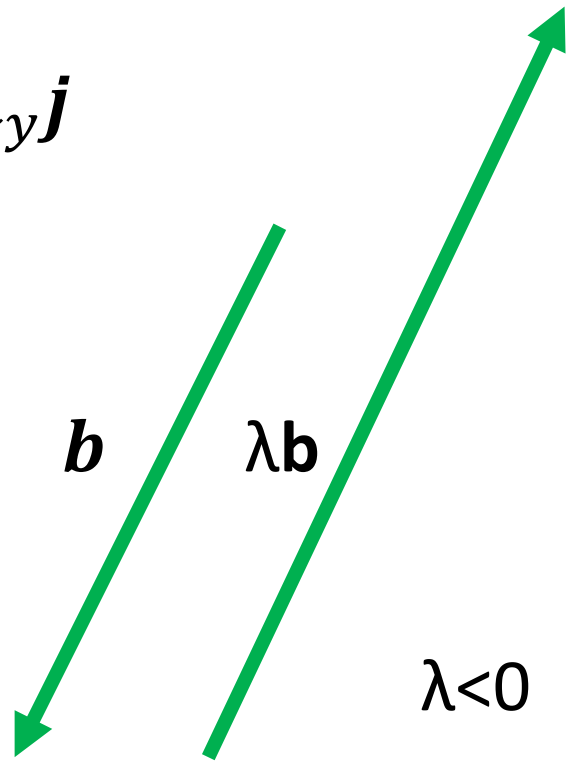
$$\boldsymbol{a} - \boldsymbol{b} = (a_x - b_x)\boldsymbol{i} + \left(a_y - b_y\right)\boldsymbol{j}$$

Vector Algebra

Multiplication by a scalar

$$\lambda \boldsymbol{a} = \lambda a_x \boldsymbol{i} + \lambda a_y \boldsymbol{j}$$



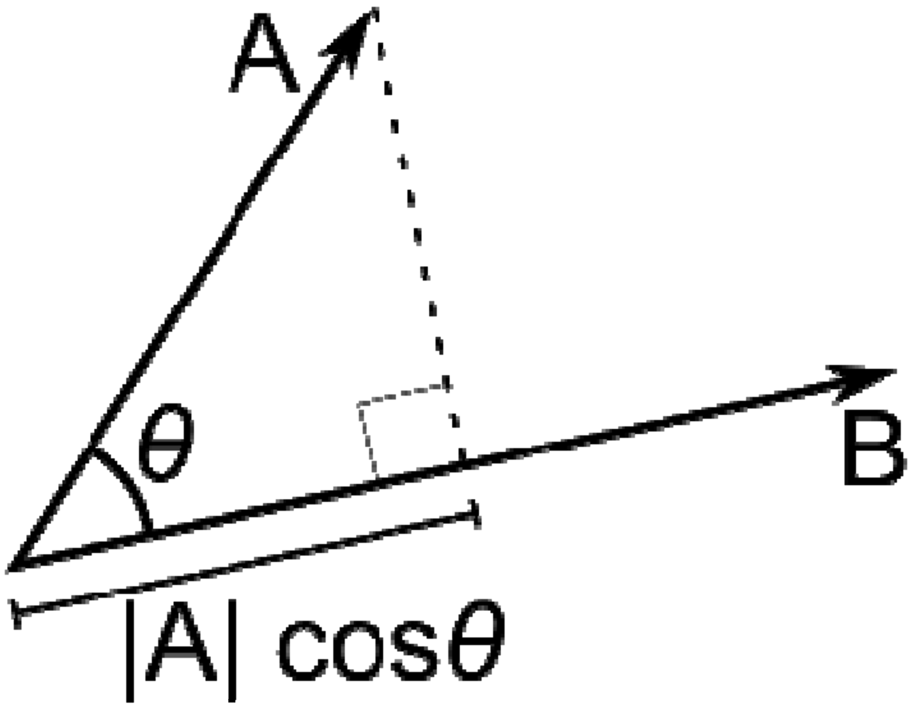


Vector algebra

Scalar product of two vectors

$$\boldsymbol{ab} = |a||b| \times cos\Theta$$

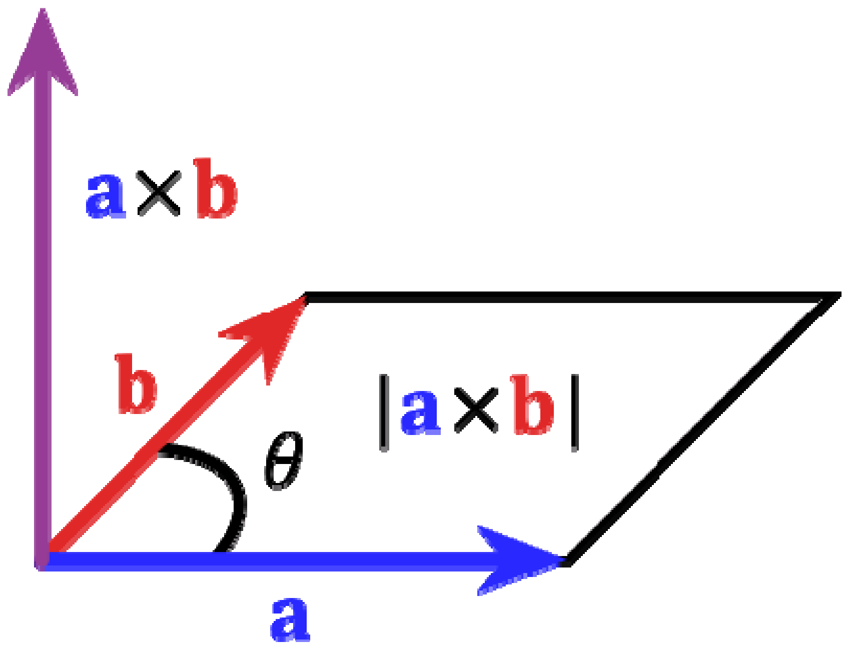
$$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y$$

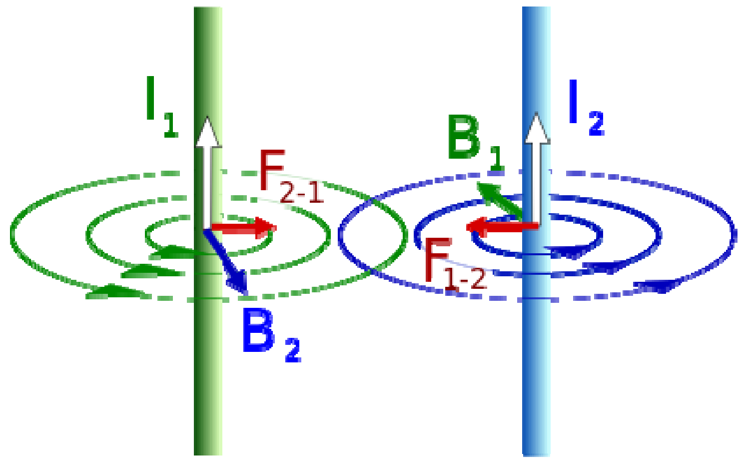


Vector algebra

Vector product

$$|a \times b| = |a||b| \sin\theta$$



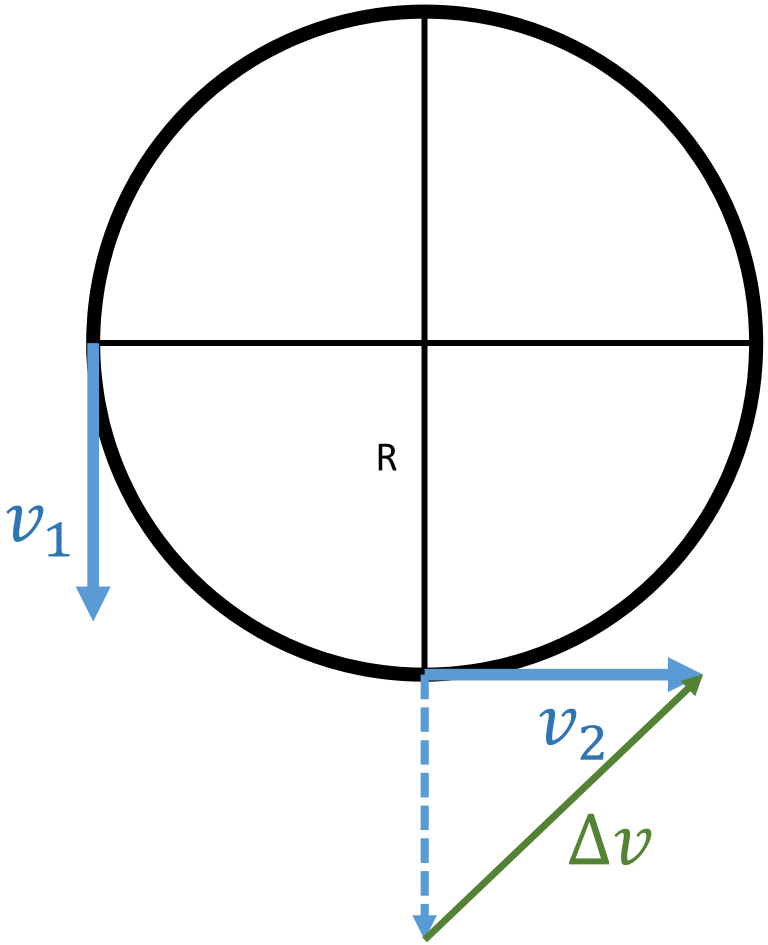


Kinematics

Kinematics is a branch of classical mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without the causes.

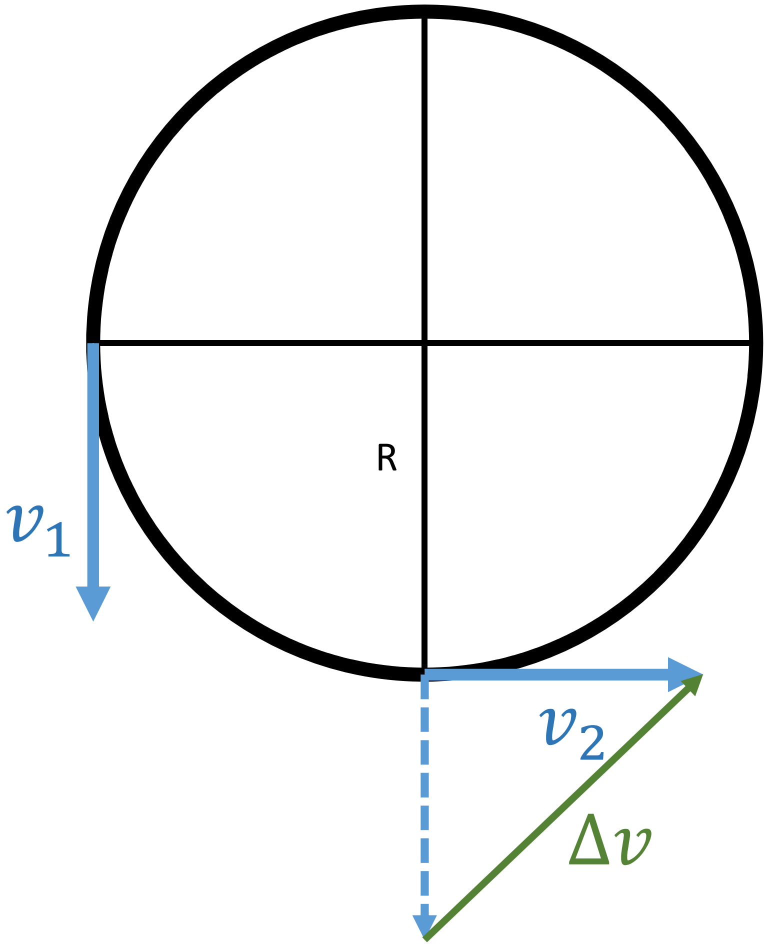
Describes the motion of a point in space and time.

Circular motion



T= **Period** The duration of one cycle in a repeating event [s]
f=1/T **Freuqency** is the number of occurences of a repeating event per unit of time [Hz=1/s]
ω= Δα/Δt = 2π/T **angular velocity** [rad/s]
B= Δω/Δt **angular acceleration** [rad/s^2]

Circular motion



Uniform circular motion

Arc quantities

$$s = \varphi R$$

$$v = \omega R$$

$$a_t = \beta R$$

$$a_{cp} = R\omega^2 = \frac{v^2}{R}$$

Angular quantities

φ [rad]

$$\omega = \frac{\Delta\varphi}{\Delta t}\left[\frac{rad}{s}\right]$$

$$\beta = \frac{\Delta\omega}{\Delta t}\left[\frac{rad}{s^2}\right]$$

Exponential function

$$f(x) = b^{x+c}$$

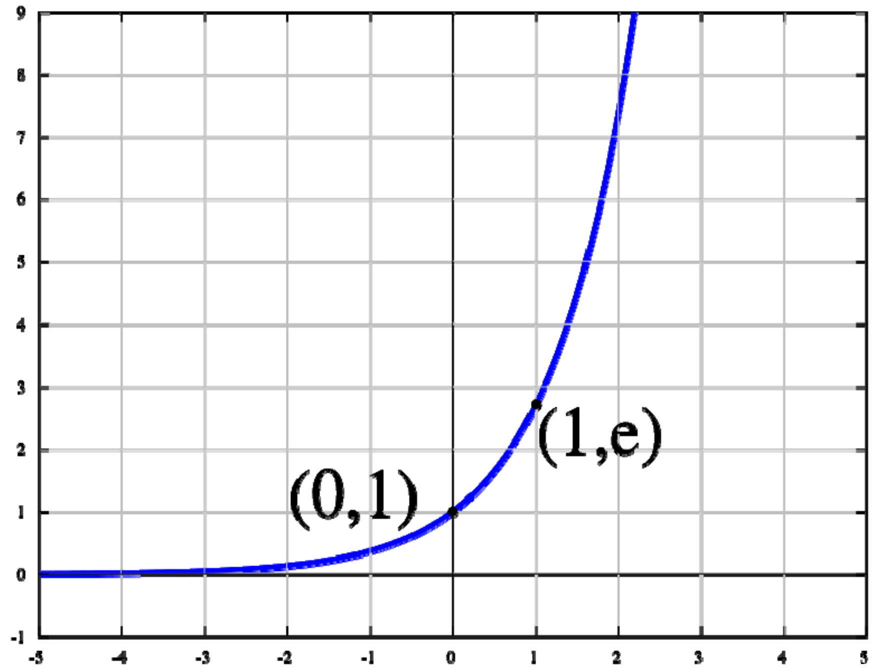
For any posotive number b>0 there is a function $f\colon \mathbb{R} \to [0, \infty]$ called an exponential function that is defined as $f(x) = b^{x+c}$

$b^x b^y = b^{x+y}$

$\frac{b^x}{b^y} = b^{x-y}$

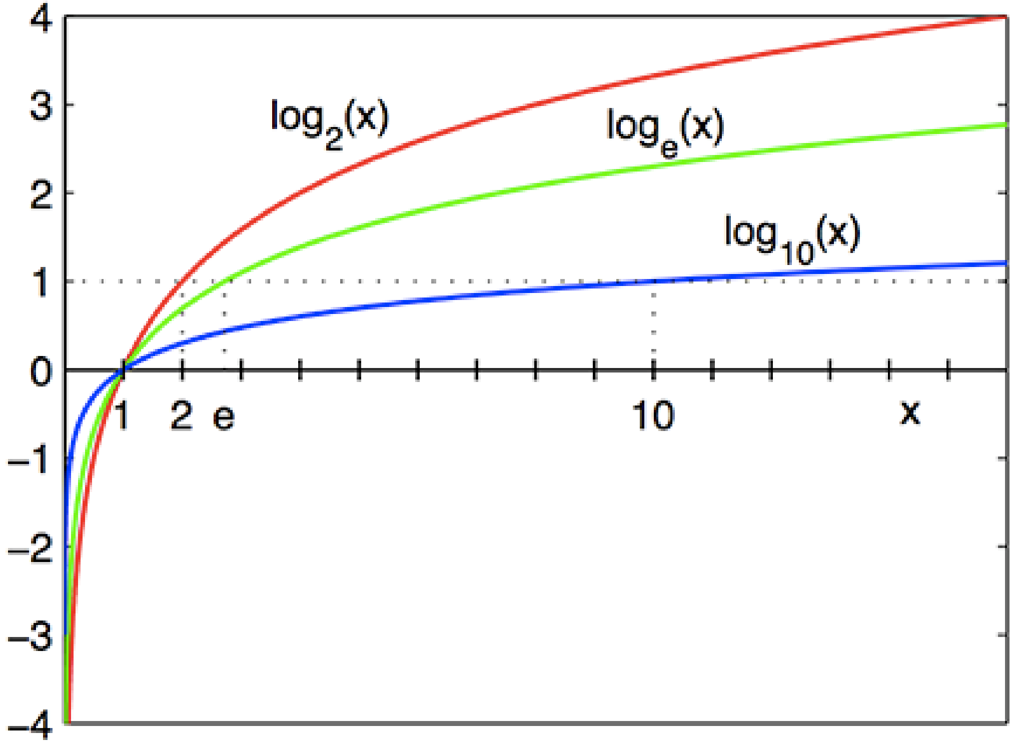
$(b^x)^y = b^{xy}$

Ha x=o
Akkor y=y_0



b>1 monotonically increases
0<b<1 monotonically increases

Logaritmic function



a>1

b is usually 2, e, or 10

$\mathbb{R}^+ \to \mathbb{R}, x \to log_a x$; a>0

The inverse function for the exponential function

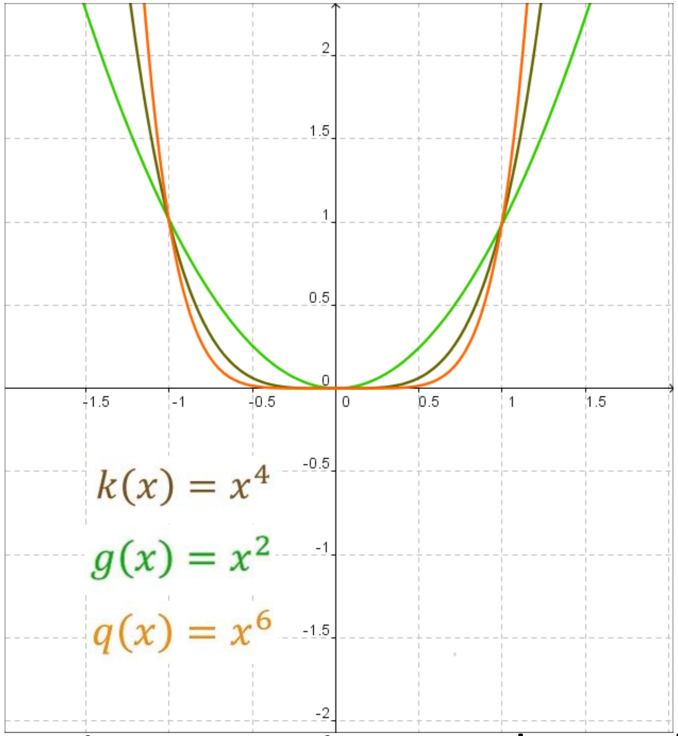
$$e^{lna} = a \Leftrightarrow ln(e^x) = x$$

$$log_b(xy) = log_b(x) + log_b(y)$$

$$log_b(x^p) = p\, log_b(x)$$

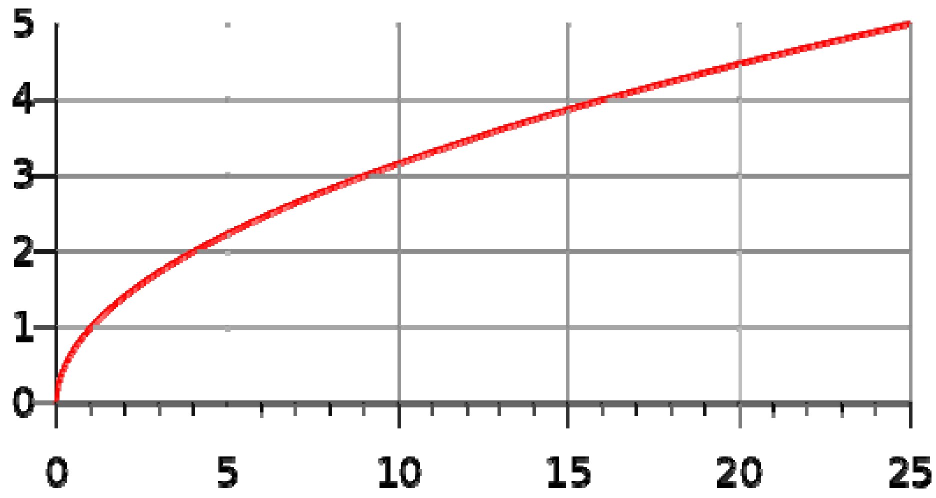
$$log_b(x) = \frac{log_k(x)}{log_k(b)}$$

Power function



$f(x) = a\,x^n$

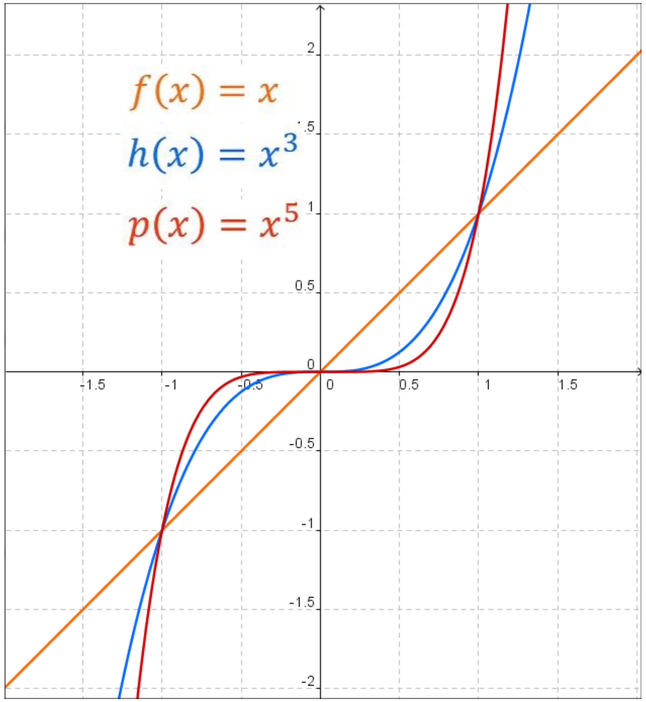
Squareroot function

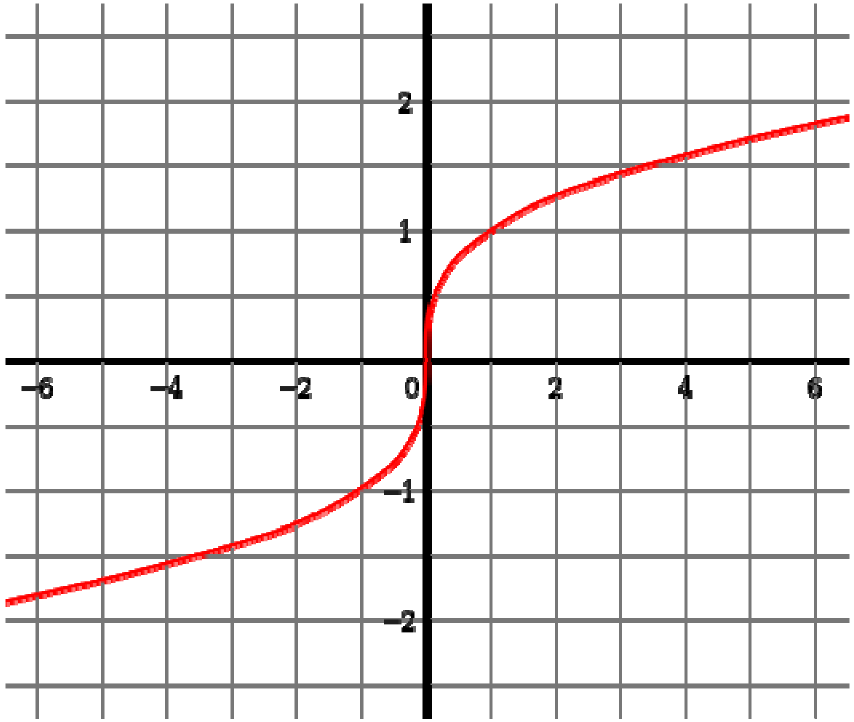


if n= is an even integer then at [0, ∞] the function is monotonically increasing so the function can be inverted. The inverse of the function is $\sqrt[n]{x}$

$[0, \infty] \rightarrow \mathbb{R},$

Power function





The f(x)= x^3
function is invert$able\ it's\ inverse\ is$: $f^{-1}(x)$= $\sqrt[3]{x}$

Linearizing a power relation

$$y = m\,x^n$$

$$\log(y) = n\,log(x) + \log(m)$$

y=ax+b

Log-log method for lineariznig power relations

Linearizing an exponential function

$$y = Ae^{kx}$$

$$\ln(y) = kx + lnA$$

y=ax+b

Semi-log method for exponential relations