

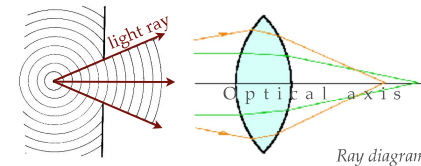
WAVE OPTICS

MIKLÓS KELLERMAYER

GEOMETRIC OPTICS AND WAVE OPTICS

Geometric optics

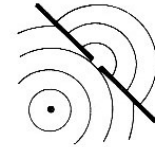
If light propagates through a slit much larger than its wavelength, then the spreading of the wavefront (phase) may be simplified into a line ("light ray").



- Optical (light) ray ("light beam"): abstraction, mathematical line.
- Arrows represent the direction of energy propagation.
- Optical axis: line connecting the midpoint of optical components (e.g., lenses).
- Principle of reversibility: the direction of energy propagation (arrows) may be reversed.

Wave optics

If light propagates through a slit comparable or smaller than its wavelength, then its wave properties must be taken into account.



Important parameters of the propagating wave:

- Period (T)
- Frequency ($f=1/T$)
- Velocity (v, c)
- Wavelength (λ): distance covered in a period:

$$\lambda = cT = \frac{c}{f}$$

Speed of propagation of light in *vacuum*: $c=2,99792458 \times 10^8 \text{ ms}^{-1}$

In *optically denser media* the speed of propagation is reduced (c_1). $n_1 = \frac{c}{c_1}$

This may be expressed with the *absolute refractive index* (n_1):

Wave: propagating oscillation

What is an oscillation?

Example:
Tacoma Narrows Bridge



- Tacoma Narrows Bridge ("Gallopin' Gertie")**
- ("Gertie the Dinosaur" (1914), cartoon, Winsor McCay)
 - Opening: July 1, 1940.
 - During wind (50-70 km/h): oscillation for hours
 - Oscillation amplitude initially 0.5 m, then, after snapping of a suspension cable, up to 9 m!
 - Collapse: November 7, 1940.

(Explanation of the effect)



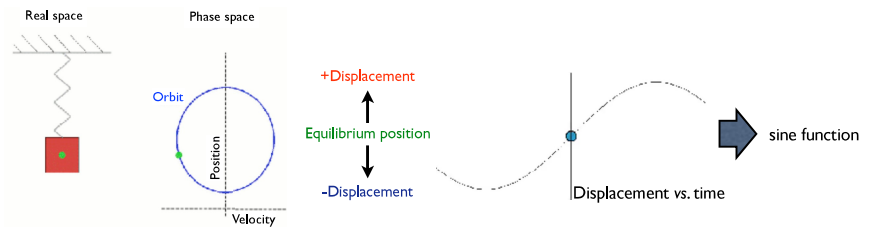
Kármán vortex street



Theodore von Kármán
1881-1963

Harmonic oscillation

Restoring force acts on a system displaced out of equilibrium (e.g., mass on a spring).



$$y = R \sin \phi$$

$$\text{Because } \phi = \omega t: y = R \sin(\omega t)$$

$$\text{If the initial phase angle } (\phi_0) \text{ differs from 0: } y = R \sin(\omega t + \phi_0)$$

$$\text{Because angular velocity } (\omega) \text{ is the full circular orbit } (2\pi) \text{ per period } (T): y = R \sin\left(\frac{2\pi}{T}t + \phi_0\right)$$

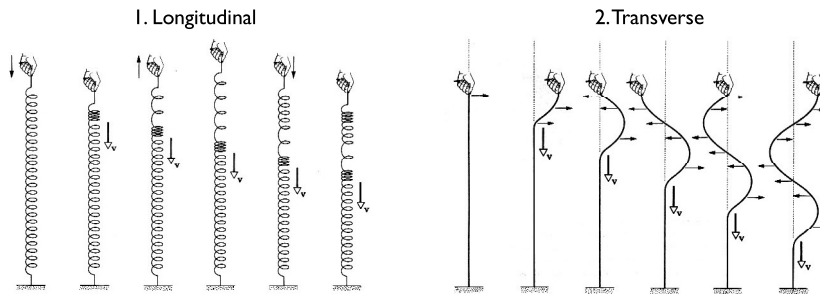
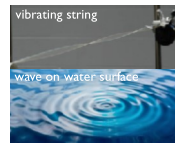
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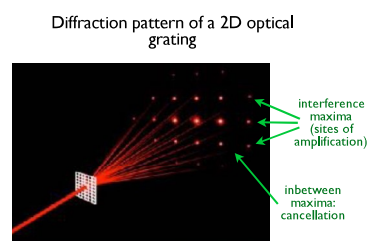
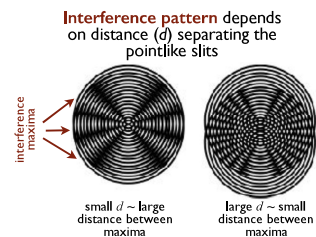
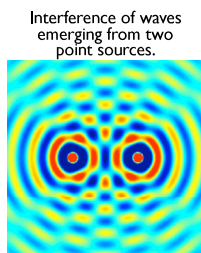
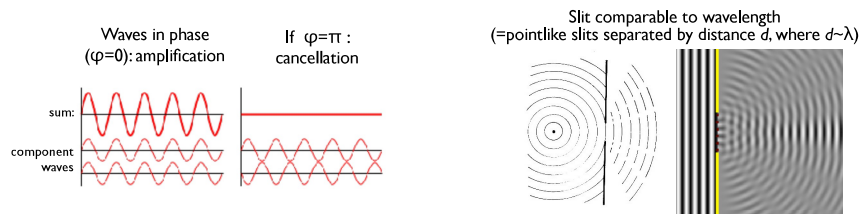
Types of waves

- According to **source**:
 1. Mechanical: elastic deformation propagating through elastic medium
 2. Electromagnetic: electric disturbance propagating through space (vacuum)
- According to **propagation dimension**:
 1. One-dimensional (rope)
 2. Surface waves (pond)
 3. Spatial waves (sound)
- According to **relative direction of oscillation and propagation**:



Wave phenomena II. interference

Principle of superposition



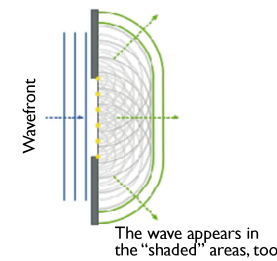
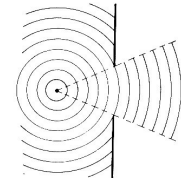
Wave phenomena I. Diffraction

Huygens-Fresnel principle:

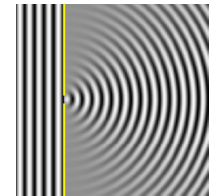
every point of a wavefront is the source of further waves



Slit much greater than the wavelength (λ)



Slit much smaller than wavelength (λ)



Wave phenomena III. Polarization

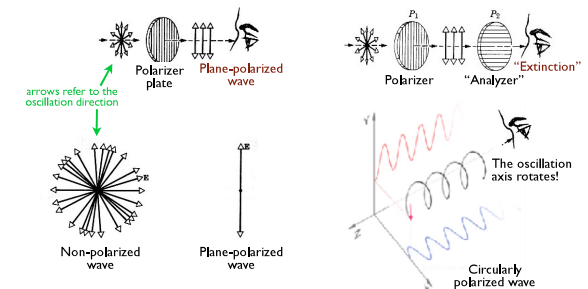
- **Polarization**: oscillation is oriented in some **preferred** direction
- **Birefringence** is related to polarization: anisotropic propagation velocity
- Only **transverse** waves can be polarized.



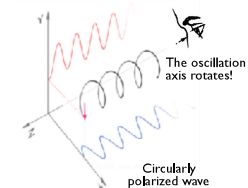
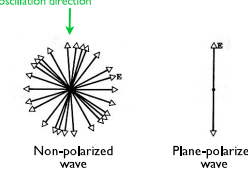
Polarization of Mechanical waves



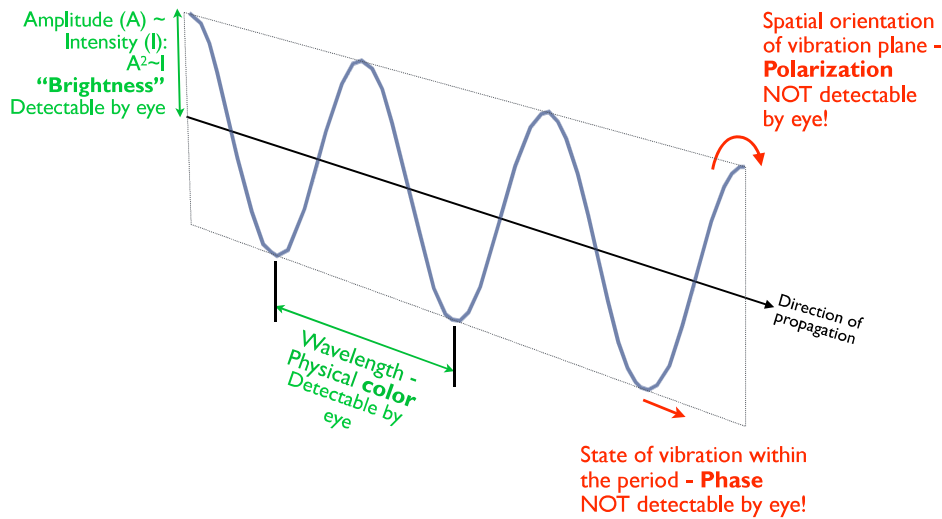
Polarization of Electromagnetic waves



Polarization can be understood by observing the **head-on** view of the wave:



Detectable parameters of the light wave



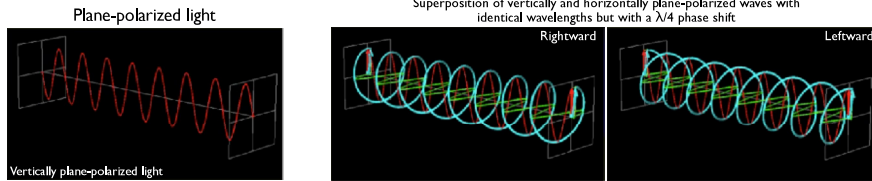
Origin and nature of wave: next week!

Polarized light and its interactions

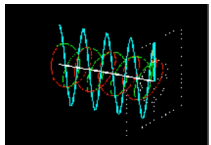
Direction of the vibration (electric or magnetic field) has preferred orientation

Circularly polarized light:

Superposition of vertically and horizontally plane-polarized waves with identical wavelengths but with a $\lambda/4$ phase shift



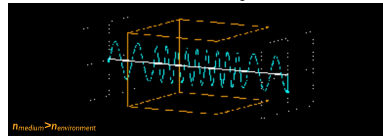
Superposition of a right- and leftward circularly polarized wave results in plane-polarized light.



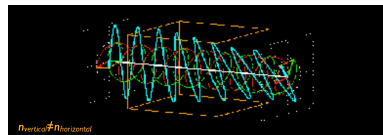
Orientation of polarization plane depends on the relative phase of the two circularly polarized waves

*Anisotropy (birefringence): refractive index (~light speed) is orientation dependent (i.e., in different directions within the sample, light propagates with different speeds).

Light decelerates in optically dense medium; because its frequency is constant, its wavelength is reduced.



In an anisotropic* medium a phase shift occurs between the circularly polarized components: the polarization plane of the emerging wave rotates.

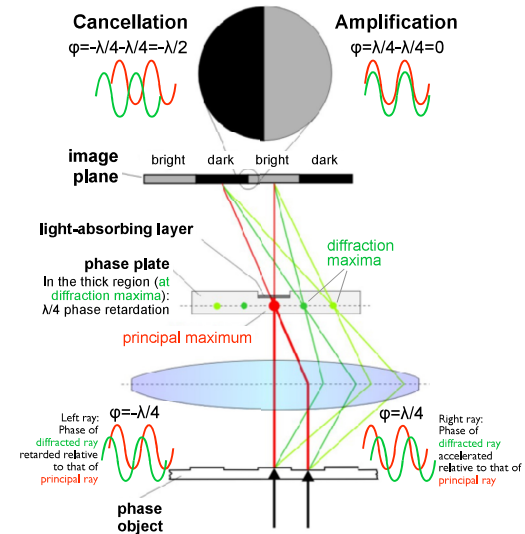


Movies - <http://www.enzim.hu/~szia/cddemo/demo0.htm>

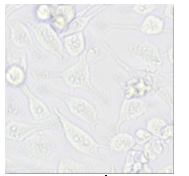
Phase, phase contrast microscopy



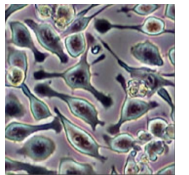
- **Phase:** shows the state of vibration within the entire period (2π).
- Expressed with the phase angle (ϕ).
- Phase difference between waves: **phase shift** (retardation or acceleration)



Live (unstained) cells



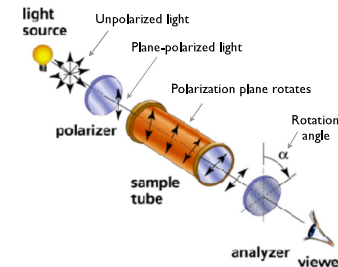
Bright-field microscopic image



Phase-contrast microscopic image

Applications of polarization

Polarimetry



Rotation angle depends on the concentration (c) of the optically active* material:

$$\alpha = [\alpha]_D^{20} \cdot c \cdot l$$

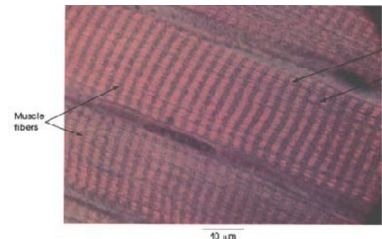
$[\alpha]$ = specific angle of rotation ("20": room temperature; "D": emission spectral line of Na $\lambda = 589$ nm)

l = length of sample tube

*Optically active material: contains *chiral* (mirror-symmetric) molecules that rotate the plane of polarization.

Polarization microscopy

Cross-striated skeletal muscle in the polarization microscope



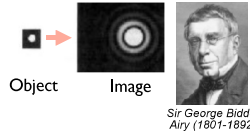
- **A-band:** anisotropic (birefringent) region (contains myosin helically ordered myosin molecules)
- **I-band:** isotropic region



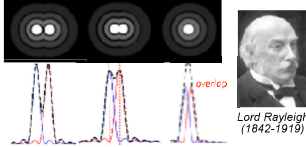
Resolution of the human eye I.

Diffraction limit

Because of diffraction: image of a point object is an Airy disk



Rayleigh criterion: objects may be resolved if their corresponding Airy disks do not overlap



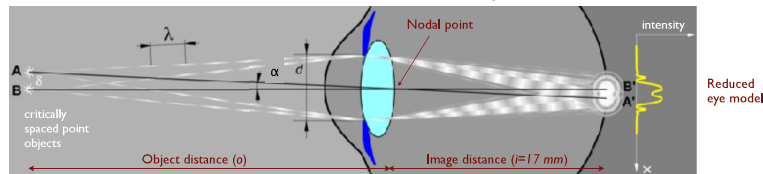
Smallest resolved distance has a limit (Abbe equation):

$$d = \frac{0.61\lambda}{n \sin \alpha}$$

λ = wavelength
 n = refractive index of medium
 α = angle between axis and outermost ray

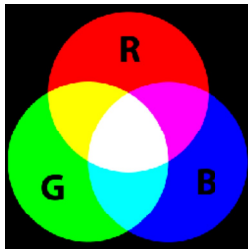


Diffraction limit of the human eye



Limiting visual angle: $\alpha_H = 1.22 \frac{\lambda}{d}$ Smallest angle of view at which two closely spaced objects may be resolved.
At average wavelength (550 nm) and pupil diameter (4 mm): **0.6'** (angular minutes)

Color coding, color vision



Additive color coding

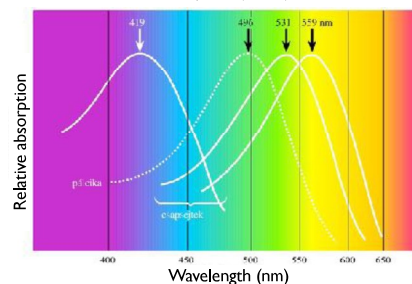
Any color may be generated by mixing three basic colors (R=red, G=green, B=blue) with varying weighing factors.

$$X = rR + gG + bB$$

In the human eye:

- 3 different color-sensitive receptors.
- Each receptor absorbs in different regions of the visible spectrum (R=64%, G=32%, B=2%).

Absorption spectra of the human color-sensitive receptors (cones)



Resolution of the human eye II.

Biological limit: receptor cell density

Object	Image on receptors	Sensed image

- Condition of resolution: at least one inactivated receptor cell falls in between two activated ones. The limiting angle of view under this condition is $(\alpha_8) \approx 0.8'$
- The diffraction and biological limits of the human eye are **comparable!**

Visual Acuity ("visus", vision):

$$visual_acuity = \frac{1'}{\alpha} 100\%$$

α = experimental (measured) visual angle

Average visual angle in healthy humans:
1' (= 100% vision)

Measurement of visual acuity

