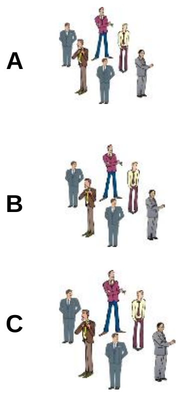
Analysis of variancia (ANOVA)



Is there a difference between the groups?

No, There is only random deviation! This is the nullhypothesis.

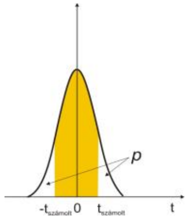
Why don't we do 2-sample t-tests?

The probability of the mistake increases rapidly with increasing number of groups!

Comparison of A – B and B – C and A – C. (not transitive)

No. of groups	No. of tests
3	3
4	6
5	10

How much is the chance of the mistake?



Suppose, that we reject the nullhypothesis in all cases.

p – probability being outside
(**1-p**) – probability being inside randomly.

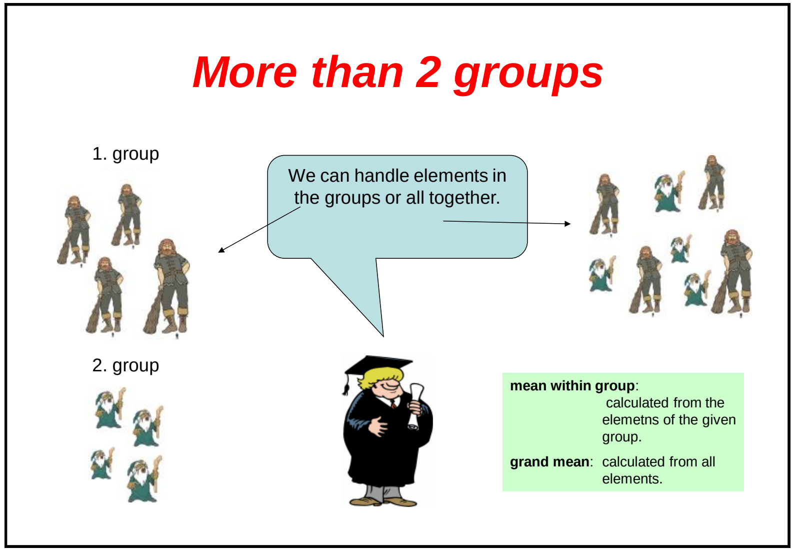
Question: How much is the probability to have mistake at least in one case?

How much is the chance that at least one is outside randomly?

1 test: ***p*** (let it be 5%).
In the case of more than 1 test, the binomial distribution may be used to calculate.

$$1-(1-p)^3$$

In the case of 3 groups is about 15%!!!



More than 2 groups

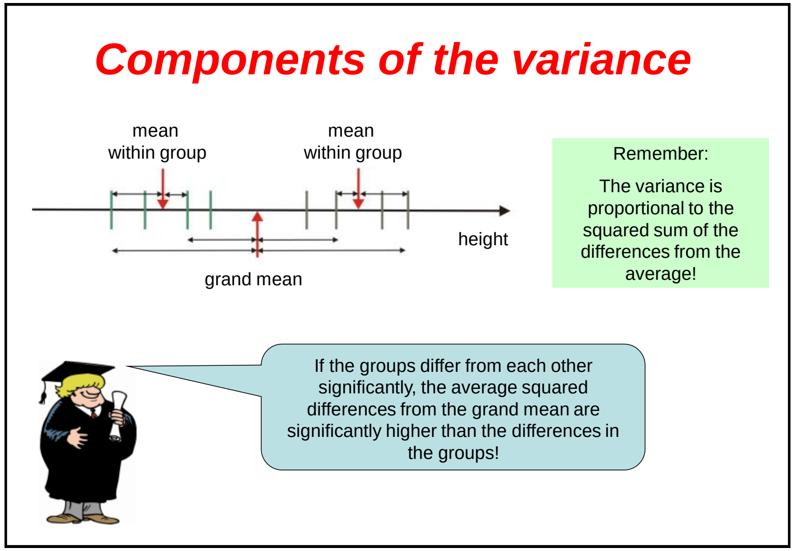
1. group

2. group

We can handle elements in the groups or all together.

mean within group: calculated from the elemetns of the given group.

grand mean: calculated from all elements.



Components of the variance

mean within group

mean within group

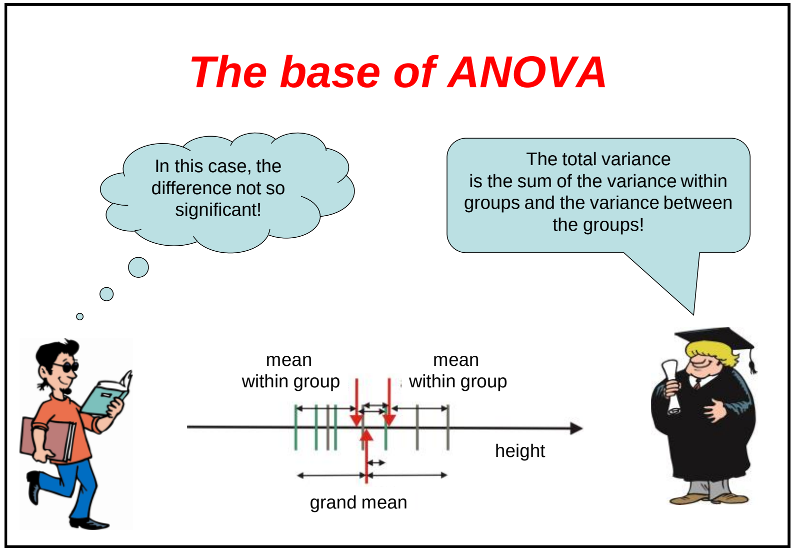
height

grand mean

Remember:

The variance is proportional to the squared sum of the differences from the average!

If the groups differ from each other significantly, the average squared differences from the grand mean are significantly higher than the differences in the groups!



The base of ANOVA

In this case, the difference not so significant!

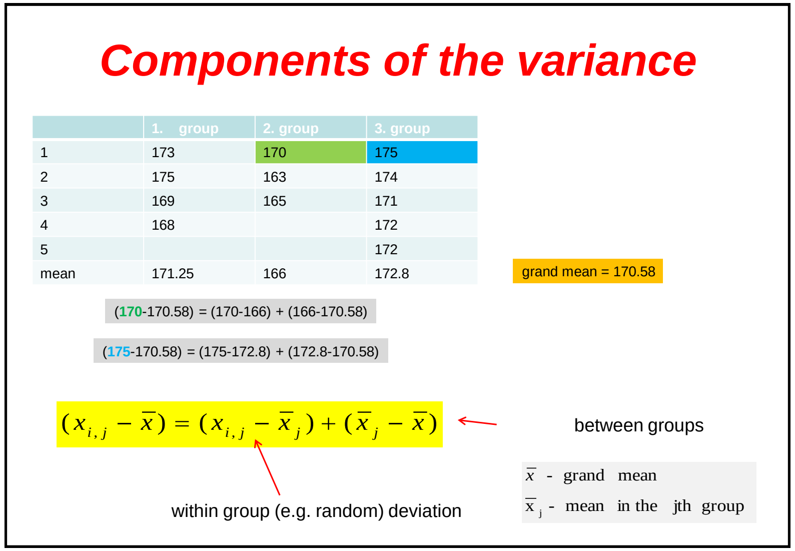
The total variance is the sum of the variance within groups and the variance between the groups!

mean within group

mean within group

height

grand mean



Components of the variance

	1. group	2. group	3. group
1	173	170	175
2	175	163	174
3	169	165	171
4	168		172
5			172
mean	171.25	166	172.8

grand mean = 170.58

(170-170.58) = (170-166) + (166-170.58)

(175-170.58) = (175-172.8) + (172.8-170.58)

$$(x_{i,j} - \bar{x}) = (x_{i,j} - \bar{x}_j) + (\bar{x}_j - \bar{x})$$

between groups

within group (e.g. random) deviation

$\bar{x}$ - grand mean

$\bar{x}_j$ - mean in the jth group

Take the square!

$$(\bar{x}-x_{ij})^2 = (\bar{x}-\bar{x}_i)^2 + (\bar{x}_i - x_{ij})^2 + 2(\bar{x}-\bar{x}_i)(\bar{x}_i - x_{ij})$$

$$\sum(\bar{x}-x_{ij})^2 = \sum(\bar{x}-\bar{x}_i)^2 + \sum(\bar{x}_i - x_{ij})^2 + \sum 2(\bar{x}-\bar{x}_i)(\bar{x}_i - x_{ij})$$

covariance = 0 (indepent events)

$$\sum 2(\bar{x}-\bar{x}_i)(\bar{x}_i - x_{ij}) = 0$$

$$\sum(\bar{x}-x_{ij})^2 = \sum(\bar{x}-\bar{x}_i)^2 + \sum(\bar{x}_i - x_{ij})^2$$

between groups

within groups

(not necessary to know!)

We can decompose the variance!

Calculation of variances

Variance	Sum of squares	d.f.		F value
between groups	$SS_A = \sum_j n_j(\bar{x}_j - \bar{x})^2$	k-1	$MS_A = \frac{SS_A}{k-1}$	$F = \frac{MS_A}{MS_E}$
Within groups	$SS_E = SS_T - SS_A$	N-k	$MS_E = \frac{SS_E}{N-k}$	
Total	$SS_T = \sum_{i,j}(x_{i,j} - \bar{x})^2$	N-1		

$\bar{x}$ grand mean

$\bar{x}_j$ mean in the jth group

N – no. of all elements
k – no. of groups

Nullhypothesis

There is no difference between the groups.

The difference between the averages of the groups due to the random deviation.

Decision: On the base of the comparison of the variance between groups and within groups!

How can we compare them?

Comparison of variances? It was already discussed!

Really, in the case of 2-sample t-test.

$$F = \frac{MS_A}{MS_E}$$

Decision

- 1. If the probability of the random deviation is small (p ≤ α) – we ***reject*** the nullhypothesis.
- 2. If the probability of the random deviation is large (p > α) – we ***accept*** the nullhypothesis.

(After decision, if it is necessary,
we can do t-tests.)

Example

Groups	n. of elements	sum	mean	Variance
1	4	685	171.25	10.92
2	3	498	166	13
3	5	864	172.8	2.7

ANOVA

Factors	SS	df	MS	F	p-value	F crit.
Between groups	89.367	2	44.68	5.782	0.02427	4.257
Within group	69.55	9	7.728			
Total	158.92	11				

α = 0.05
p = 0.024

Decision:
we reject the nullhypothesis, there is a significant difference.

Conditions for ANOVA

- Task: comparison 3 or more independent groups.
- The variable has **normal distribution**.
- sd values are the same in the groups.

Kruskal-Wallis test

Rank data without separating into groups, than sum ranks in each group!

If the variable has no normal distribution!

Ranking

	1. group	2. group	3. group
1	173	170	175
2	175	163	174
3	169	165	171
4	168		172
5			172

value	163	165	168	169	170	171	172	172	173	174	175	175
rank	1	2	3	4	5	6	7.5	7.5	9	10	11.5	11.5

group	n. of elements	sum of the ranks
1	4	27.5
2	3	8
3	5	42.5

Nullhypothesis

There is no difference between the groups.

The difference between the averages of the groups due to the random deviation.

Which distribution is suitable?

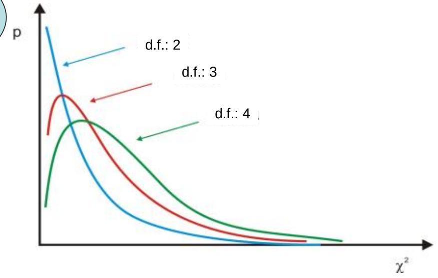
$$H = \frac{12}{N(N+1)}\sum_i \frac{R_i^2}{n_i} - 3(N+1)$$

N – no of elements
R_i – the sum of the ranks in the i-th group
n_i – no. of elements in the i-th group

χ^2-distribution

The value of $H \geq 0$!

H variable has χ^2-distribution!



Decision

- 1. If the probability of the random deviation is small ($p \leq \alpha$) – we ***reject*** the nullhypothesis.
- 2. If the probability of the random deviation is large ($p > \alpha$) – we ***accept*** the nullhypothesis.

Example

group	n_i	Sum of the ranks (R_i)
1	4	27.5
2	3	8
3	5	42.5

N = 12

$$H = \frac{12}{N(N+1)} \sum_i \frac{R_i^2}{n_i} - 3(N+1)$$

$$4.97 = \frac{12}{12(12+1)} \left(\frac{27.5^2}{4} + \frac{8^2}{3} + \frac{42.5^2}{5} \right) - 3(12+1)$$

df. = 3 – 1 = 2

$\alpha = 0.05$
$p = 0.083$

Decision:

we accept the nullhypothesis, there is no significant difference.

Compare them!

	1. group	2. group	3. group
1	173	170	175
2	175	163	174
3	169	165	171
4	168		172
5			172

ANOVA

$\alpha = 0.05$
$p = 0.024$

Decision:
we reject the nullhypothesis.

Kruskall-Wallis test

$\alpha = 0.05$
$p = 0.083$

Decision:
we accept the nullhypothesis.

!!!

Hypothesis test?

- Set up the **nullhypothesis!**
- Look for a **variable with known distribution**.
- Calculate the **probability of the random deviation** on the base of the distribution.
- If it is smaller than the significance level **reject**, in opposite case **accept the nullhypothesis**.
- That's all!