Selection

1-sample t-test

Mann-Whitney U-test

2-sample t-test

Analysis of variance

Important: on the base of the quantity and the question!

Which is the best?

According to the variable

parametric

Testing the hypothesis on the parameter(s) of a known distribution.
(most frequently the normal distribution).

non-parametric

Testing the hypothesis on the parameter(s) or types of an unknown distribution.

Non-parametric tests

Distribution free methods.

advantage : independent from the distribution.
disadvantage: normally it's power is less.

Ranking tests:
Instead of original values we use the so-called **ranks**.

Ranks

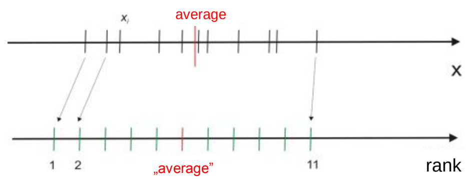
Rank: numerical or ordinal data belonging to a value in data series sorted according to a certain rule.

tied ranks:
In the case of same values every value replaced by the average of the ranks.

e.g.:
•lieutenant
•major
colonel
etc.

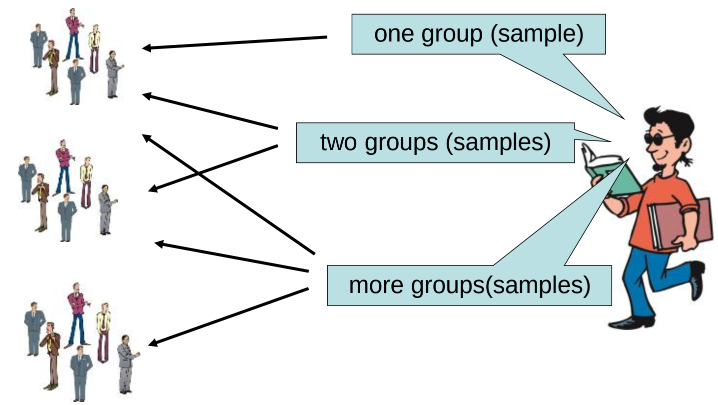
value:	1.2	2	2	3.5	4
rank:	1	2,5	2,5	4	5

the „average” of the ranks is the median



The median plays the role of the average.

according to the question



Summary table

	parametric	non-parametric
one group	1-sample t-test,	Wilcoxon’s signed-rank test, sign-test
two groups	2-sample t-test	Mann-Whitney U-test
more groups	ANOVA	Kruskall-Wallis test

Examination in one group

Question: On the base of the sample the parameter of the population may be a given value?

parametric

μ = ?

Nullhypothesis: $\overline{x} = \mu_0$

1-sample t-test

non-parametric

median = ?

Nullhypothesis: the median is a given value

Wilcoxon’s signed-rank

1-sample t-test

example: The medicine effective or not?

↓

Nullhypothesis: not! $\mu_0 = 0$. But the average is not 0!

sample	Average
1.	-0.2 °C
2.	-1 °C
3.	-1.5 °C

If the difference is bigger, it seems to be more probable being non random.

What does it mean big difference?

What is the measure of the difference?

Standard error: the average devations of the averages from the μ.

$(\bar{x} \pm s_{\bar{x}})$ ~ 68% - confidence interval.

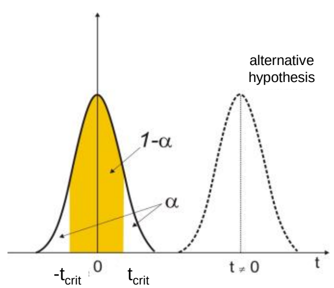
t-value

$$t = \frac{x - \mu_0}{s_x}$$

Compare the difference to the standard error!
(μ_0 very frequently = 0)

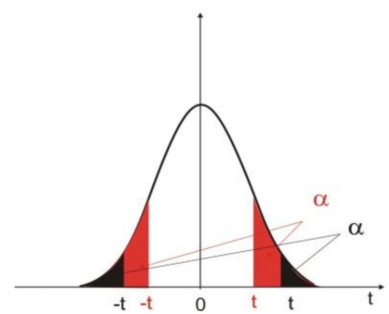
The averages fluctuate around the μ_0 so the *t*-values deviate around the 0.

(providing, that the nullhypothesis is true!)



Why is the t-value is better?

We are able to calculate the probabilities on the base of this distribution!!! (Student- or *t*-distribution)



It describes only the **random deviations** of the *t*-values!

The shape of the distribution depends on the no. of elements.

Why has it t-distribution?

$t = \frac{x-\mu_0}{s_x}$ $\longrightarrow$ $\bar{x}$

The fluctuation of the average has normal distribution. The numerator is variable having normal distribution!

$s_{\bar{x}} = \frac{s}{\sqrt{n}}$ $\longrightarrow$ $s = \sqrt{\frac{\sum_i (x_i-\bar{x})^2}{n-1}}$

variance is the sum of squared probability variables.

remember!

t-distribution

$$\xi_n = \frac{\sqrt{n}\cdot\xi}{\sqrt{\sum_i \zeta_i^2}}$$

(Quad erat demonstrandum)

the ***t*** variable has *t*-distribution.

Degree of freedom (d.f.)

I think 3 numbers! (sample)

The average of them: 8! (information!)

they must be!

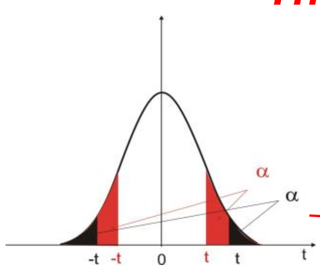
3, 12, 8 or 5, 7, 11 etc.

d.f. = n

3, 12, **9** or 5, 7, **12** etc.

d.f. = n-1

The t-table



Different t_{crit} values belong to different sigficance level.

t-table

significance level

d.f.	0.1	0.05	0.02	0.01
	probability (p)			
1	6.31	12.7	31.8	63.7
2	2.92	4.3	6.96	9.92
3	2.35	3.18	4.54	5.84
4	2.13	2.78	3.75	4.6
5	2.02	2.57	3.37	4.03

d.f.: n-1

Decision the base of t-table

t-table

significance level

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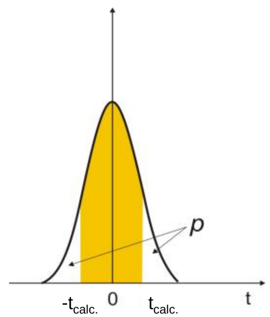
d.f.: n-1

Select an approriate significance level!

If *t*≥2.78 **reject**, if smaller **accept** the nullhypothesis.

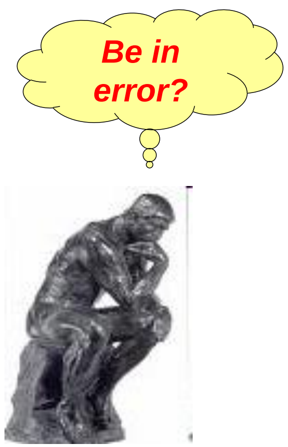
Decision using computer

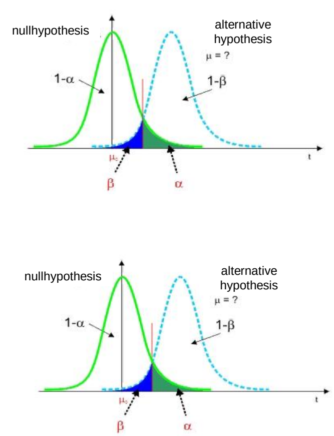


p: probability, that the $t_{calculated}$, is so large randomly.

Decision

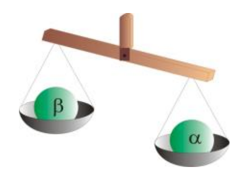
- 1. If the probalitiy of the random deviation is small ($p(|t|\geq t_{crit}) \leq 5\%$) – ***reject*** the nullhypothesis.
- 2. If the probalitiy of the random deviation is large ($p(|t|\geq t_{crit}) > 5\%$) – ***accept*** the nullhypothesis.





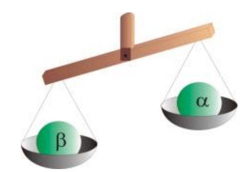
The error

Reject the nullhypothesis



α is the measure of the error. Smaller p is better.

Accept the nullhypothesis



β is the meaure of the error. Larger p is better.

Condition for 1-sample t-test

- Task: Decision about the μ on the base of one sample.
- The variable must have **normal distribution**.

What can we do if it isn't true?

Wilcoxon's signed-rank test

Example: Is there an effect of an entertaining movie on the patients? (The numbers are scores)

n	before	after	Diff.
1	2	2	0
2	0	1	1
3	3	2	-1
4	2	4	2
5	1	3	2
6	3	3	0
7	1	4	3
8	1	5	4
9	5	2	-3
10	4	4	0

Normal distribution?

Ranking

Sort the absolute values of the differences (without 0-s)! Let the sign of ranks be same then the differences!
Calculate the averages and sd of signed-ranks!

Diff.	absolute value	rank	Signed-rank
0	0		
1	1	1.5	1.5
-1	1	1.5	-1.5
2	2	3.5	3.5
2	2	3.5	3.5
0	0		
3	3	5.5	5.5
4	4	7	7
-3	3	5.5	-5.5
0	0		

The nullhypothesis

The median = 0!
The deviation is random!

There is no effect of the movie!

$\mathbf{H_0}$: $\mu_0 = 0$

H_1: $\mu_0 \neq 0$

known distribution

$$t = \frac{\overline{R} - 0}{s/\sqrt{n}}$$

If n is enough large!

$\overline{R}$ - ranks	the average of the signed-
s –	the standard deviation

Rememer!
„average" of the ranks = median

Decision

This is known!!!

Of course! This is similar to the 1-sample t-test!!!

Paired t-test

If the data may be paired according to a rule!

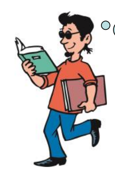
Observation on the same person, paired organ (e.g. kidney).

Rare, on the base of viewpoints (age, profession, etc.).

Look at: decreasing the fewer.

Experimental design

Experimetal design



test from previously collected data?

practical order: Question → experimental design → calculation.

Many problems: e.g. a few data are suitable only.

„real" 1-sample t-test

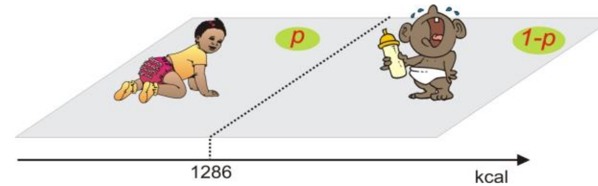
Is it possible that the μ is equal to a value?

$$t = \frac{x - \mu_0}{s_x}$$

Rare case.

sign-test

Example: Energy uptake in the population of 2-year old children.
Question: May be the median (This derives from an another test) is 1286 kcal?
Nullhypothesis: median = 1286 kcal (deviation is random).



Test

Small sample

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

binomial distribution

Large sample

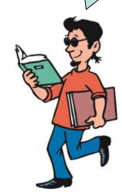
$$z = \frac{|x - np| - 1/2}{\sqrt{np(1-p)}}$$

standard normal distribution

x – no. of children below 1286 kcal.
n – no. of children in test.
p – probabilty, that randomly smaller (look at: binomial distribution)

Decision

Calculate the probability of the random deviation. (binomial, or standard normal distribution)



End of this part!