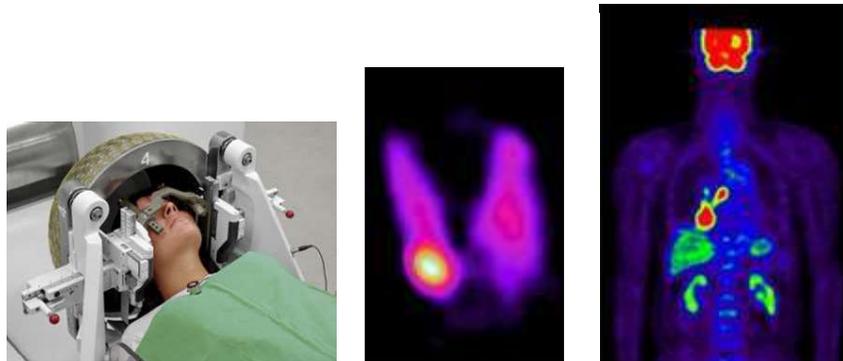


The atomic nucleus. Radioactivity. Nuclear radiations

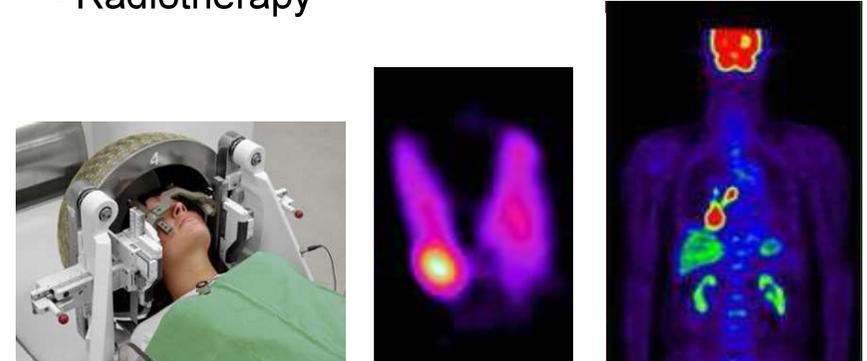
László Smeller



Why?

Medical applications of the nuclear radiation:

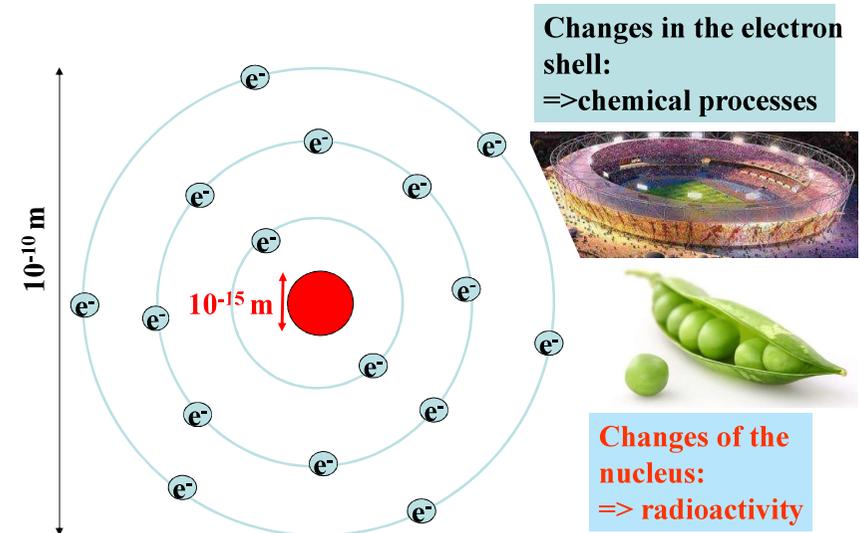
- Nuclear imaging
- Radiotherapy



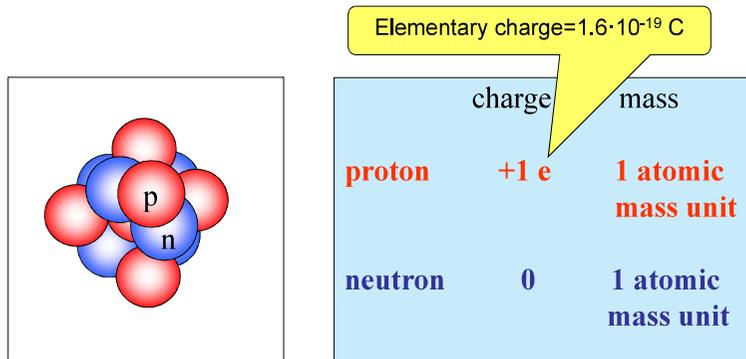
Length scale of the nature

| | | | |
|------------|------------|---|--|
| m | | | |
| 10^0 | meter | men | |
| 10^{-3} | millimeter | size of letters you can read | |
| 10^{-6} | micrometer | size of a cell (e.g. erythrocyte) | |
| | | $\varnothing 7\mu\text{m}$ | |
| 10^{-9} | nanometer | protein | |
| 10^{-10} | - angstrom | diameter of an atom, bond length | |
| | | H atom $\varnothing \approx 1$ angstrom (Å) | |
| 10^{-12} | picometer | wavelength of the X-ray | |
| 10^{-15} | femtometer | size of the nucleus | |

The electrons and the nucleus



Structure of the nucleus



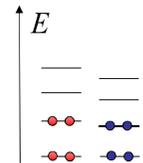
A (mass number) = number of protons + number of neutrons
 Z (atomic number) = number of protons

→ 99
 → 43 **Tc**

99 nucleon, 43 proton and 56 neutron

Stability of the nucleus

- Coulomb force: destabilization (electrostatic repulsion between the protons)
- Nuclear force: very strong attractive force acts only on short range (~fm) independent on the charge
- Quantized energy levels for the nucleus.
- Typical binding energy is in the MeV range $eV=1.6 \cdot 10^{-19} \text{ J}$



Isotopes

Variants of an element, where the

- numbers of protons are the same

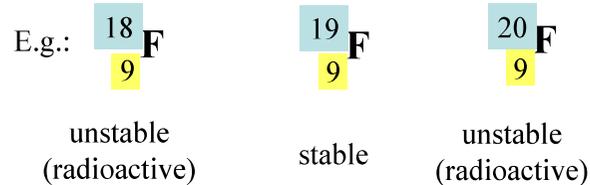
⇒ the chemical properties are identical.

- numbers of neutrons are different

⇒ the atomic numbers are different

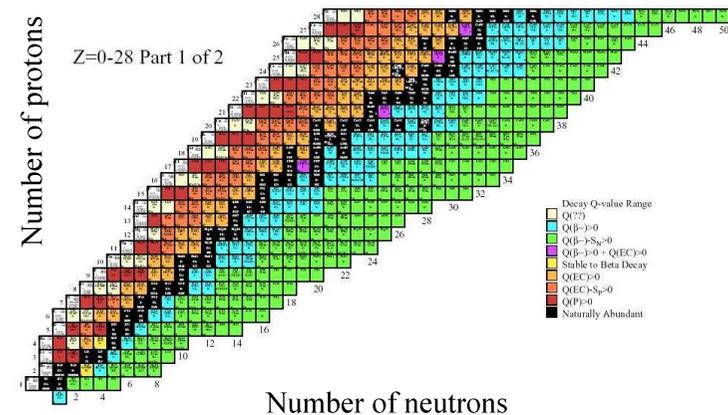


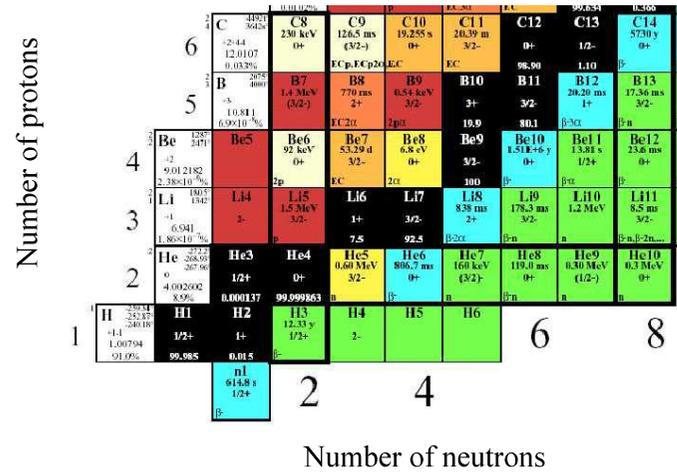
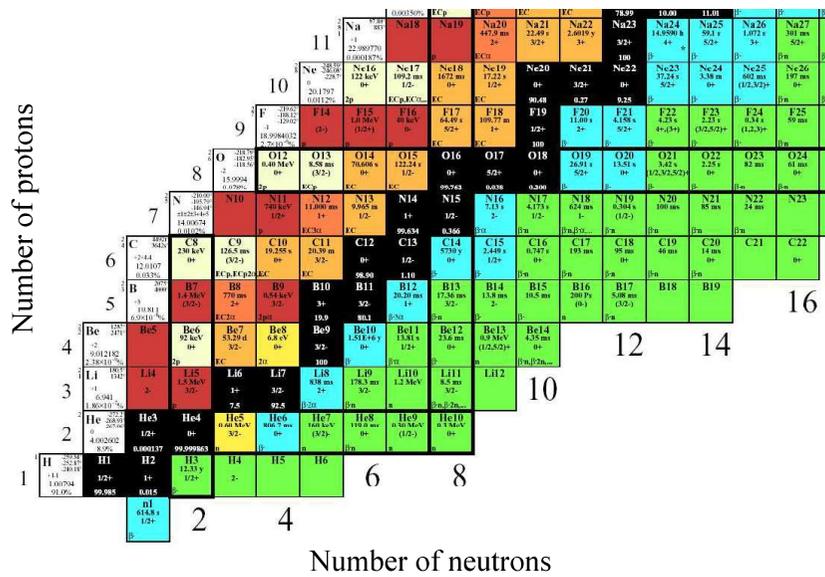
György Hevesy
Nobel prize 1943



isotope \leftrightarrow radioactive isotope

Table of isotopes





Radioactive decays and particles

α - decay

α - particle = ${}^4_2\text{He}$ nucleus

β - decay : β^-
 β^+

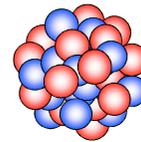
β^- particle = electron
 β^+ particle = positron

Isomeric transition

γ -ray

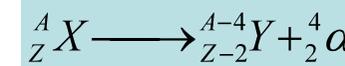
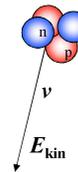
K-electron capture

characteristic x-ray photon

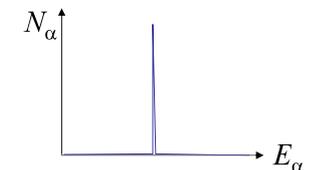


α - decay

an α particle (${}^4\text{He}$ nucleus) will be emitted
typical for the heavy atoms



Line spectrum
 $E_\alpha \sim \text{MeV}$



neutron surplus

β^- - decay

e.g.: ${}^{20}_9F$
 ${}^{32}_{15}P$
 ${}^{59}_{26}Fe$
 ${}^{131}_{53}I$

$${}^A_Z X \longrightarrow {}^A_{Z+1} Y + {}_{-1}\beta + \bar{\nu}$$

$${}^1_0 n \longrightarrow {}^1_1 p + {}_{-1}\beta + \bar{\nu}$$

remains in the nucleus leave the nucleus

e.g.: ${}^{32}_{15}P \longrightarrow {}^{32}_{16}S + {}_{-1}\beta + \bar{\nu}$

β^- -ray e^-
 $\bar{\nu}$
 $\beta^- = {}_{-1}\beta = e^-$

continuous energy spectrum

proton surplus

β^+ - decay

e.g.: ${}^{11}_6C$
 ${}^{15}_8O$
 ${}^{18}_9F$
 ${}^{52}_{26}Fe$

$${}^A_Z X \longrightarrow {}^A_{Z-1} Y + {}_{+1}\beta + \nu$$

$${}^1_1 p \longrightarrow {}^1_0 n + {}_{+1}\beta + \nu$$

remains in the nucleus leave the nucleus

e.g.: ${}^{30}_{15}P \longrightarrow {}^{30}_{14}Si + {}_{+1}\beta + \nu$

β^+ -ray e^+
 ν
 continuous energy spectrum

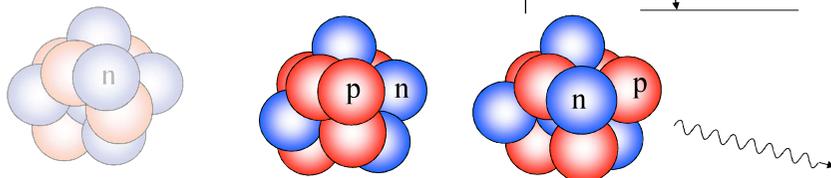
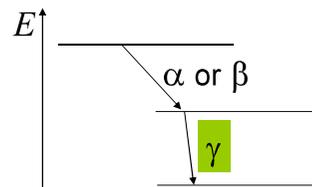
These isotopes must be produced artificially (e.g. in cyclotron)

Medical application: PET

Prompt γ -radiation

The daughter nucleus has an **energetically unfavoured** arrangement of nucleons.

The surplus energy will normally be emitted immediately (<ps) in form of the γ radiation



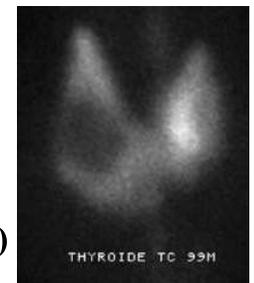
Atomic number, mass number are unchanged.

Isomeric transition

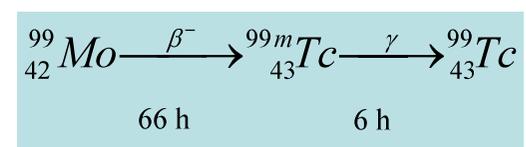
If the excited state of the daughter nucleus is metastable, the γ -radiation will be emitted later.

The parent and daughter atoms can be separated: the daughter atom emits **only γ -radiation!**

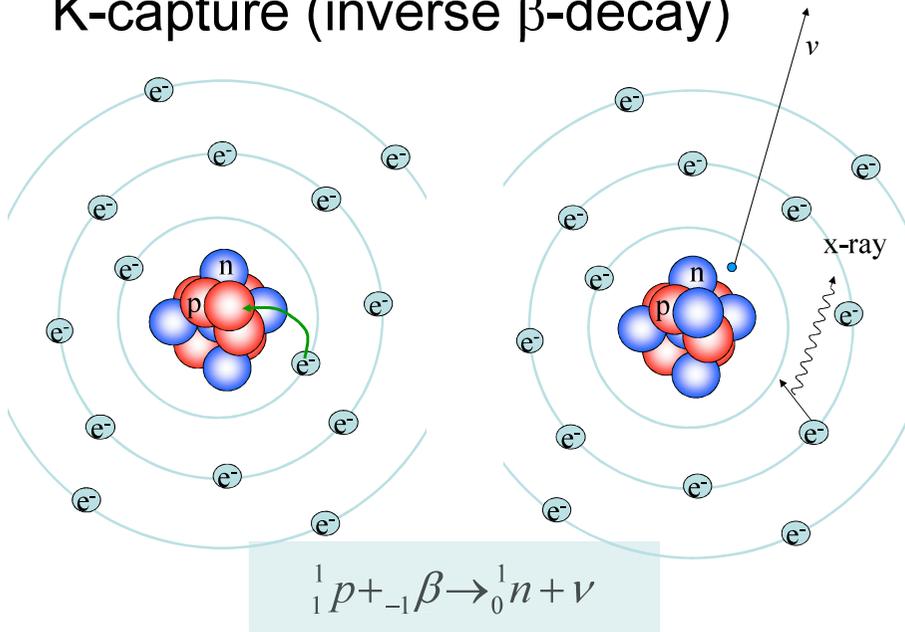
=> Isotope diagnostics (nuclear imaging)



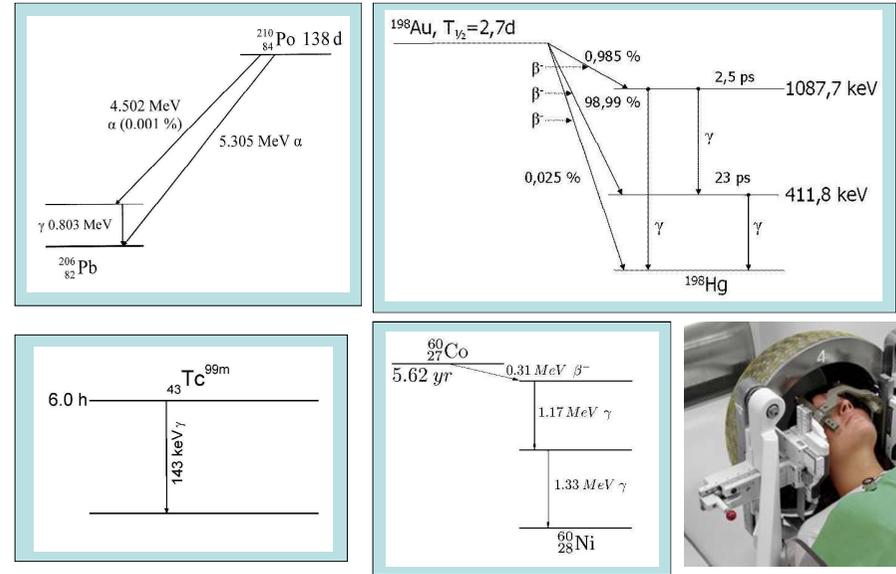
E.g.: ${}^{99m}_{42}Tc$



K-capture (inverse β -decay)



Some examples of the decay paths



Characteristics of radioactive decays in general

- activity characterizes the source
- half life time characterizes the speed of the decay*
- particle energy characterizes the radiation*

*depends on the type of the isotope

Activity (Λ)

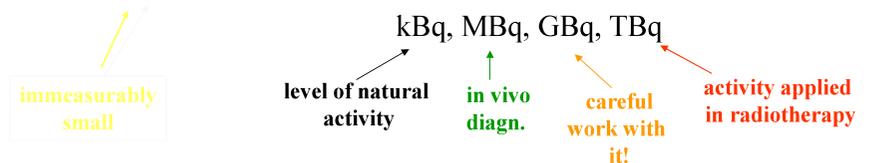
$$\Lambda = \left| \frac{dN}{dt} \right| \quad \left(= \left| \frac{\Delta N}{\Delta t} \right| \right)$$

N = Number of undecayed atoms
 t = time
 $-\Delta N$ = Number of decays during Δt time

Activity = number of decays in a unit time

unit: becquerel Bq
 1 Bq = 1 decay/sec

old unit: Ci = 37GBq



Law of radioactive decay

$$\Delta N = -\lambda N \Delta t$$

N: Number of undecayed nuclei

$$\frac{dN}{dt} = -\lambda N$$

λ : decay constant (probability of the decay [1/s])
 $1/\lambda = \tau$ average lifetime

Differential equation

solution:

$$N(t) = N_0 e^{-\lambda t}$$

Exponential decrease

number of undecayed atoms at $t=0$

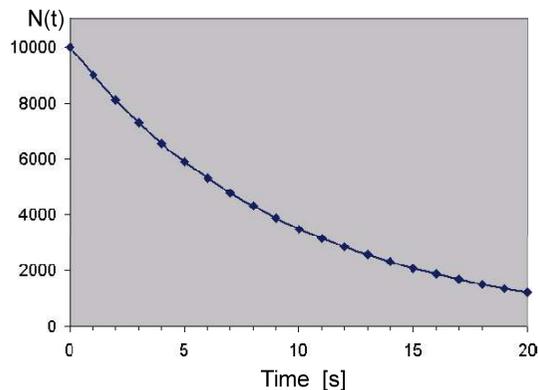
An example

- At start: $N_0=10000$ (number of atoms)
 $\lambda=0.1 \text{ 1/s}$ (decay probability)
 $\Delta N = -\lambda N \Delta t$
- after 1 sec: 9000 (10000x0.1=1000 decayed)
- after 2 sec: 8100 (9000x0.1=900 decayed)
- after 3 sec: 7290 (8100x0.1=810 decayed)
- after 4 sec: 6561 (7290x0.1=729 decayed)
-

An example

- At start: $N_0=10000$ (number of atoms)
 $\lambda=0.1 \text{ 1/s}$ (decay probability)

- after 1 s: 9000
- after 2 s: 8100
- after 3 s: 7290
- after 4 s: 6561
-



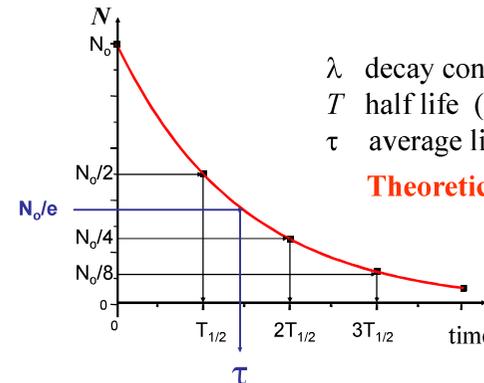
Law of radioactive decay

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}}$$

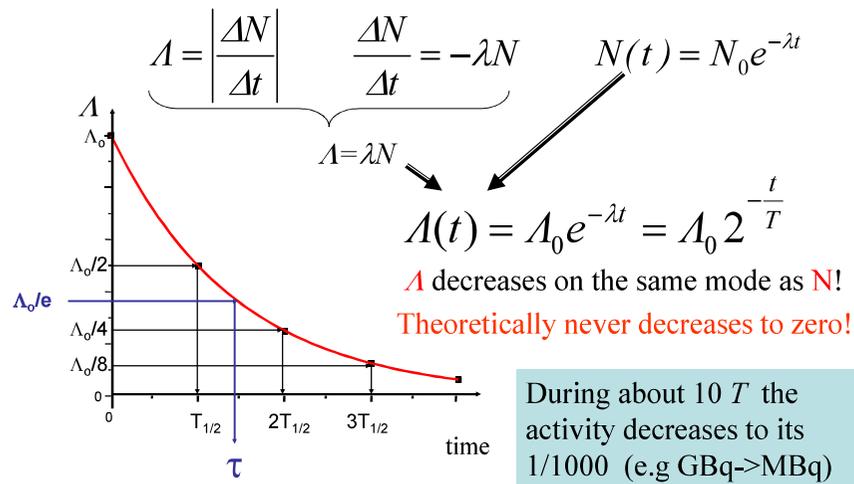
$$T = \frac{\ln 2}{\lambda} = \frac{0,693}{\lambda}$$

λ decay constant (probability of the decay)
 T half life ($T_{1/2}$)
 τ average lifetime

Theoretically never decreases to zero !

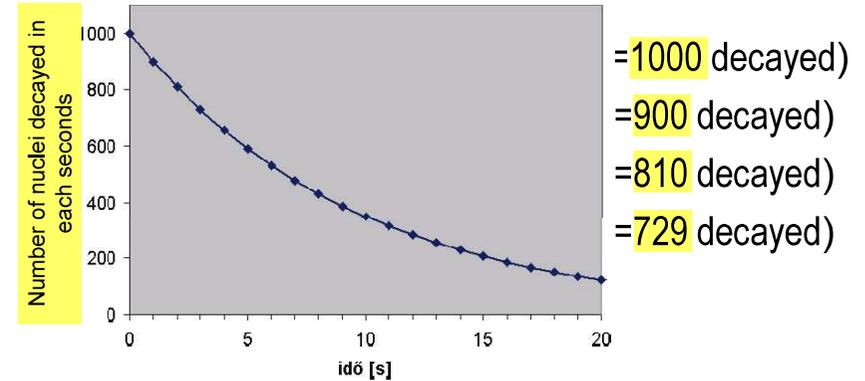


Decrease of the activity as a function of time



An example

- At start: $N_0=10000$ (number of atoms)
 $\lambda=0.1 \text{ 1/s}$ (decay probability)



Few examples for half life

| | | | |
|-------------------|-------------------------------|--------------------------|------------|
| ^{232}Th | $1.4 \cdot 10^{10} \text{ y}$ | ^{60}Co | 5.3 y |
| ^{238}U | $4.5 \cdot 10^9 \text{ y}$ | ^{59}Fe | 1.5 m |
| ^{40}K | $1.3 \cdot 10^9 \text{ y}$ | ^{56}Cr | 1 m (28 d) |
| ^{14}C | 5736 y | ^{131}I | 8 d |
| ^{137}Cs | 30 y | $^{99\text{m}}\text{Tc}$ | 6 h |
| ^3H | 12.3 y | ^{18}F | 110 min |
| | | ^{11}C | 20 min |
| | | ^{15}O | 2 min |
| | | ^{222}Th | 2.8 ms |

Don't learn these numbers!

Typical energy levels in the microworld

Excitation of the outer electrons

eV (aJ)

light



Electron transition between inner electrons

keV (fJ)

X-ray



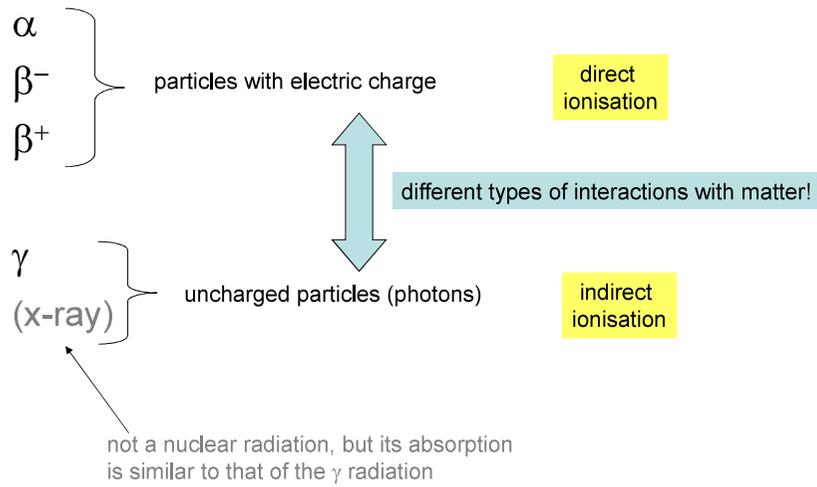
Transformation of the nucleus (decay)

MeV (pJ)

Nuclear radiation
 α, β, γ



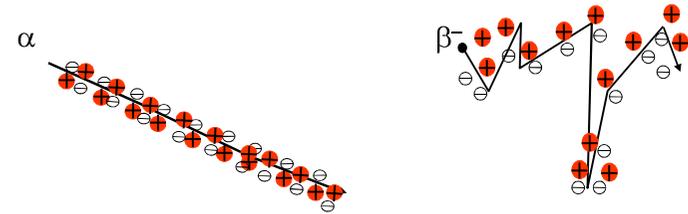
Absorption of the nuclear radiation



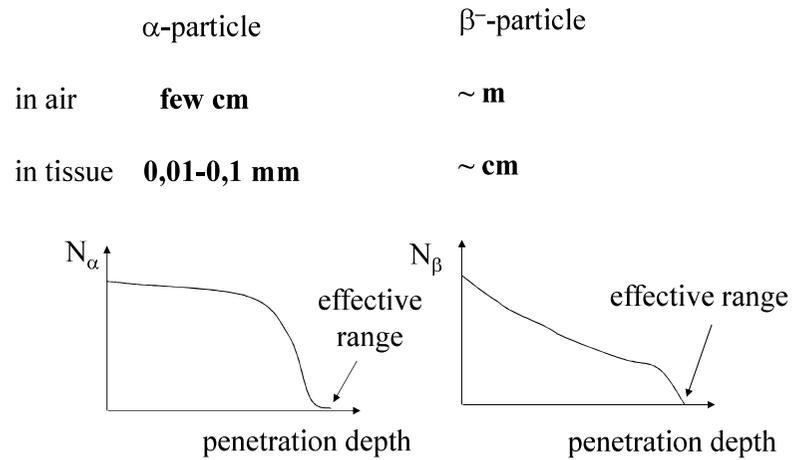
Absorption of the charged particles

Ionizing during the path => continuous decrease of the particle energy
 The energy after a given path length decreases to the thermal value

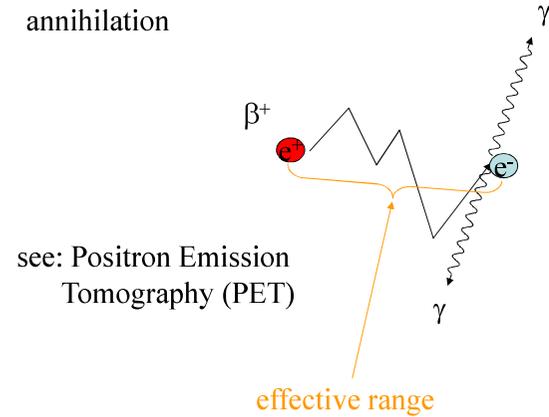
effective range



Effective range

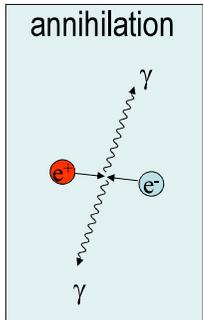


β^+ -radiation

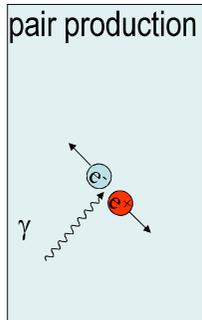


Electron and positron

- particle - antiparticle
- same mass,
- charge: same value, but different



and

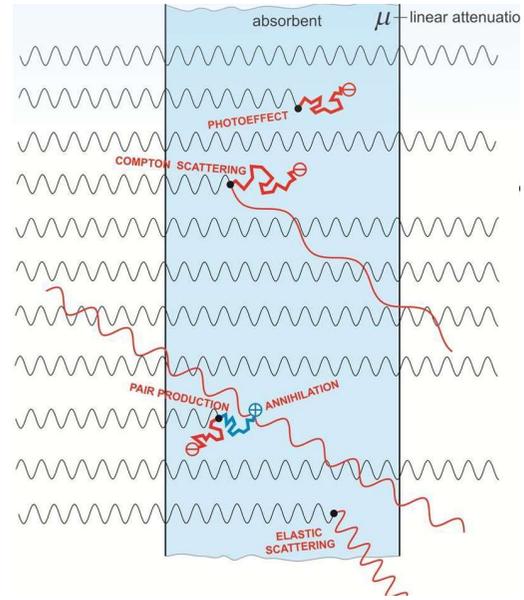


Einstein:
mass-energy
equivalence

$$E=mc^2$$

$$m_e c^2 = 511 \text{ keV} \approx 0.5 \text{ MeV}$$

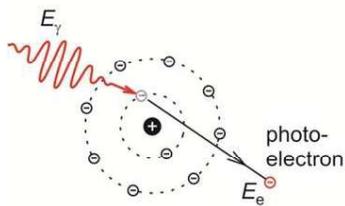
Absorption of the γ -radiation (and x-ray)



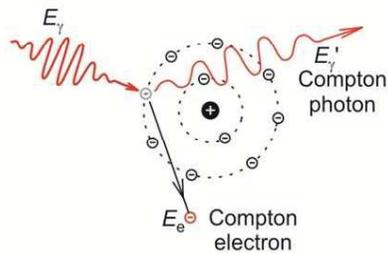
Absorption processes happen accidentally:

Photoeffect,
Compton-effect,
Pair production,
(elastic scattering)

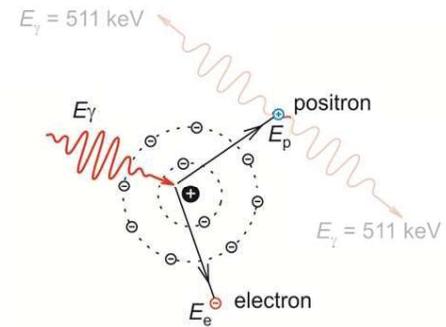
Photoeffect



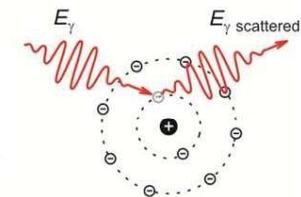
Compton effect
Compton-scattering



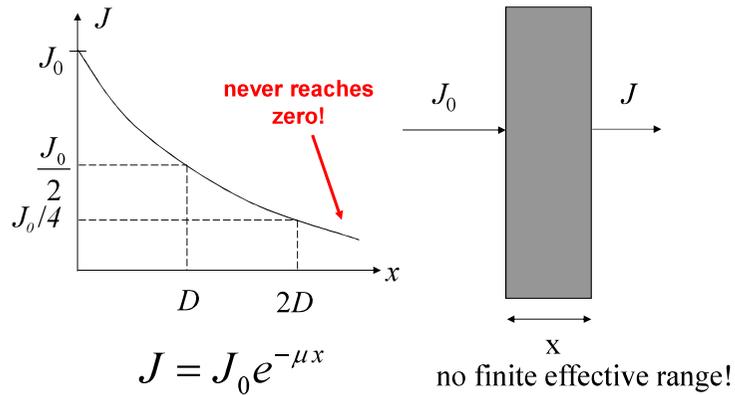
Pair production



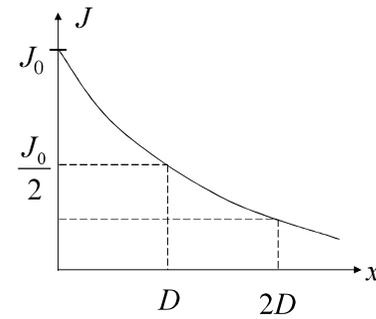
elastic scattering



Attenuation of the γ -radiation and x-ray



few rules of thumb: $x_{1/10} = 3,33 D$ $x_{1/1000} = 10 D$



$$J = J_0 e^{-\mu x}$$

μ : (linear) attenuation coefficient
its units are: 1/m, 1/cm

$\delta = \frac{1}{\mu}$ „penetration depth”
Intensity decreases to the e-th part (c.a. 37%)

μ (material, number of absorbing centers, energy of the radiation)
 $= \mu(\text{material}, \rho, E_{\text{photon}}) \sim \rho$

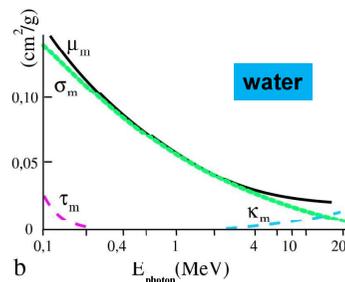
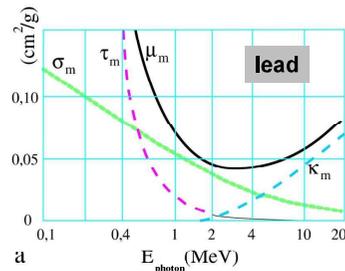
$$\mu_m = \frac{\mu}{\rho} \quad \text{mass attenuation coefficient}$$

mass attenuation coeff.

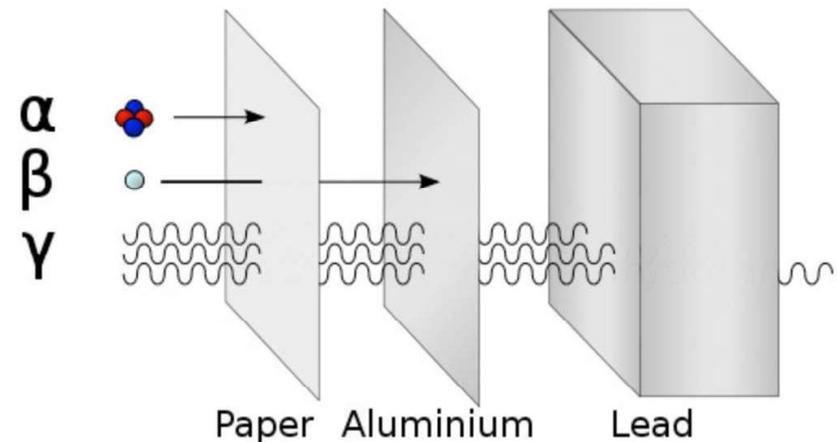
$$\mu_m = \frac{\mu}{\rho}$$

$$\mu_m = \tau_m + \sigma_m + \kappa_m$$

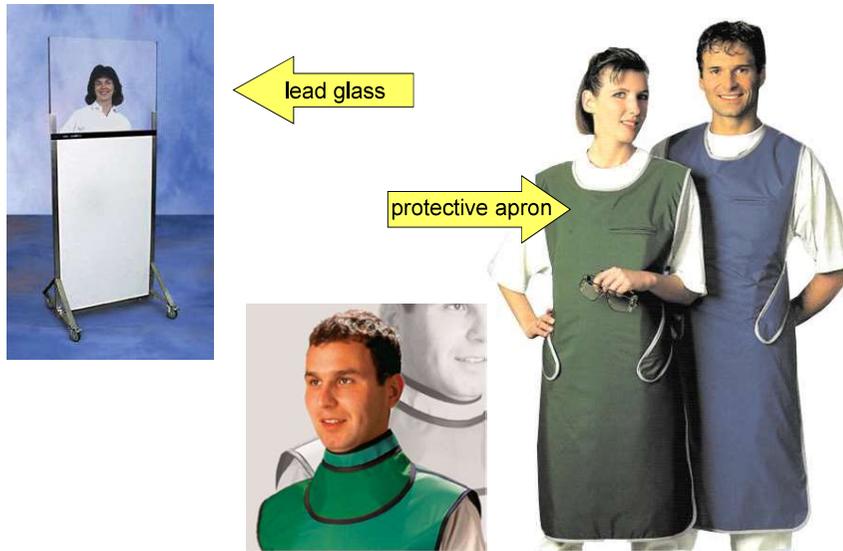
$$\tau_m = c \lambda^3 Z^3$$



Summary of the absorption of α , β and γ radiation



Applications (attenuation)



Applications: isotopes and nuclear radiation

