



## Medical signal processing



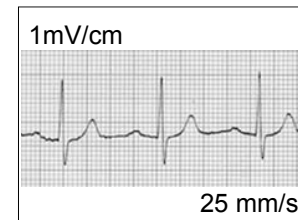
KAD 2018.12.12

A **signal** is any kind of physical quantity that conveys/transmits/stores information

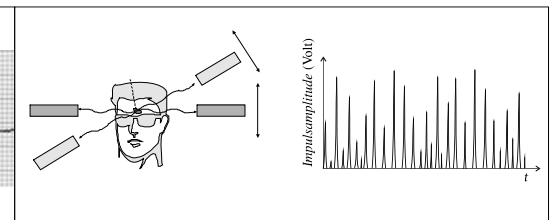
e.g. (1)  
electrical voltage, that can be measured on the surface of the skin/head as a result of the heart-/muscle-/brain activities (ECG/EMG/EEG)

e.g. (2)  
gamma photon detection in radioisotope diagnostics

(1)



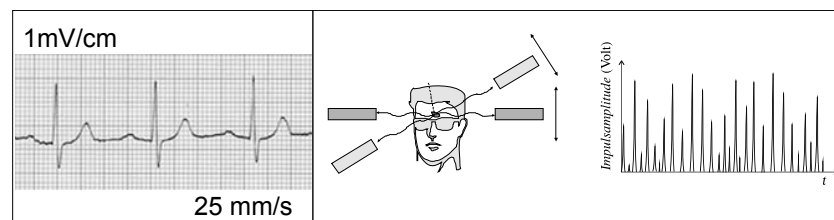
(2)



2

## Classification of signals

static	—	time-dependent
periodic	—	non-periodic
random	—	deterministic
pulsed	—	continuous
electric	—	non-electric
analog	—	digital



3

in a very special role

## electric signals

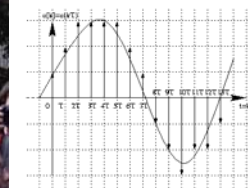
non-electric signals are transferred to electric ones

advantages of **electric** signals:  
they are easy to transform, amplify, transmit

## digital signals

analog signals are transferred to digital ones

advantages of **digital** signals:  
they are easy to store, the noise can be engineered and influence can be reduced



4

quantity that compares the magnitudes of two signals:

**Signal level or Bel-number (or Decibel-number):  $n$**

(named after A. Bell)

unit of  $n$  : Bel (B) or decibel (dB)

$$n = \lg \frac{P_2}{P_1} \text{ B} = \lg \frac{J_2}{J_1} \text{ B} = \lg \frac{E_2}{E_1} \text{ B}$$

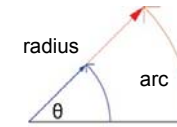
decimal logarithm of ratio of two powers (intensities, energies)

5

cf. **radian**

$$\Theta = \frac{\text{arc}}{\text{radius}}$$

$$[\Theta] = \frac{\text{m}}{\text{m}} = \text{rad} = 1$$



cf. **pH** (power of Hydrogen)

$$\text{pH} = -\lg \frac{[\text{H}^+]}{1\text{M}}$$

$$\text{e.g.: } [\text{H}^+] = 10^{-7}\text{M}$$

$$\Rightarrow \text{pH} = -\lg 10^{-7} = -1 \cdot (-7) = 7$$

instead of Bel number we are using **decibel-number**

$$n = 10 \cdot \lg \frac{P_2}{P_1} \text{ dB}$$

$$(10\text{d} = 1)$$

6

the **characteristic** unit: **power** (or intensity/energy),  
the **practical** unit: (electric) **voltage**

the relation between power and voltage:

$$P = U \cdot I = \frac{U^2}{R} \quad (\text{Ohm: } U = R \cdot I)$$

signal level with voltages:

$$n = 10 \cdot \lg \frac{P_2}{P_1} \text{ dB} = 10 \cdot \lg \frac{\frac{U_2^2}{R_2}}{\frac{U_1^2}{R_1}} \text{ dB} =$$

$$= 10 \cdot \lg \frac{U_2^2}{U_1^2} \text{ dB} = 20 \cdot \lg \frac{U_2}{U_1} \text{ dB}$$

7

$$\frac{P_2}{P_1} = 2 \Leftrightarrow 10 \lg 2 \text{ dB} =$$

$$= 10 \cdot 0,3 \text{ dB} = 3 \text{ dB}$$

$$\frac{P_2}{P_1} = \frac{1}{2} \Leftrightarrow -3 \text{ dB}$$

cf. half life,  
half value thickness

$$\frac{P_2}{P_1} = 10 \Leftrightarrow 10 \cdot \lg 10 \text{ dB} =$$

$$= 10 \cdot 1 \text{ dB} = 10 \text{ dB}$$

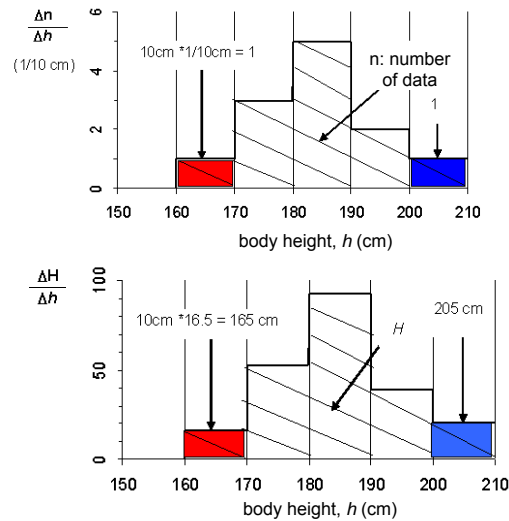
$$\frac{P_2}{P_1} = 100 \Leftrightarrow 10 \lg 100 \text{ dB} =$$

$$= 10 \cdot 2 \text{ dB} = 20 \text{ dB}$$

$U_2/U_1$	$P_2/P_1$	dB
1,414	2	3
2	4	6
	8	9
3,16	10	10
	20	13
10	100	20
	$1000=10^3$	30
$100=10^2$	$10000=10^4$	40
$1000=10^3$	$10^6$	60

8

## empirical density function



$H$

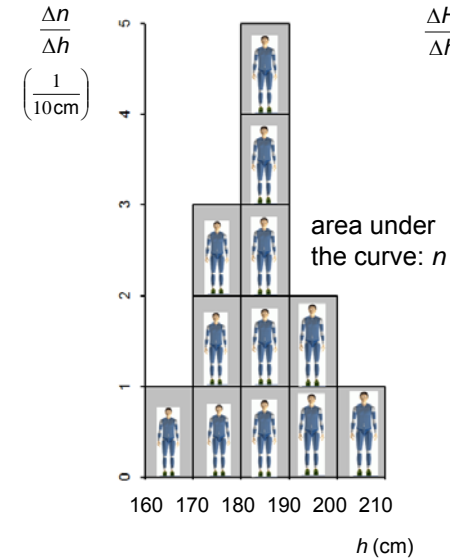


$h$

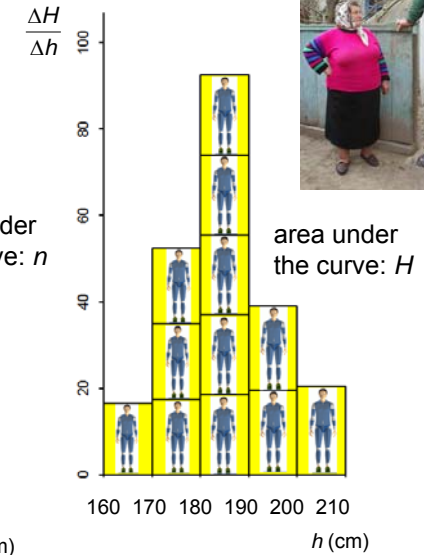
## spectrum, as a special density function

9

## Density function



## Spectrum



10

## Fourier's theorem for periodic functions (signals)

all (usual) periodic functions can be expressed as a sum of sine (and cosine) functions from the fundamental frequency and the overtones

periodic function:  
there is a period,  $T$



$\frac{1}{T} = f$ , where  $f$  is the frequency

the sine function, which has the same frequency as the periodic function:

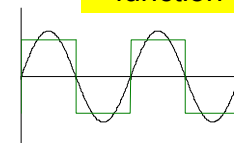
**fundamental frequency**

$2f, 3f, 4f, \dots$ : **overtones**

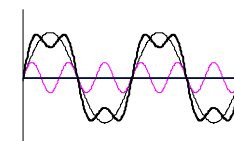
(line spectrum)

11

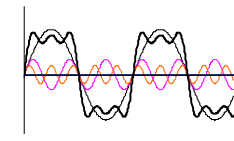
## function



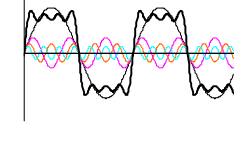
square pulse train  
fundamental  
fr(equency)



fundamental fr.+  
3rd overtone

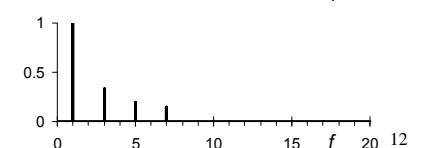
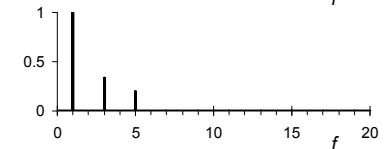
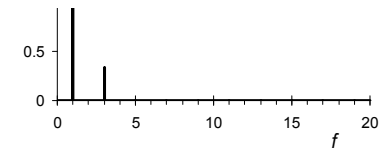
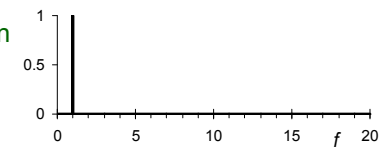


fundamental fr.+  
3rd overtone +  
5th overtone



fundamental fr.+  
3rd overtone +  
5th overtone +  
7th overtone

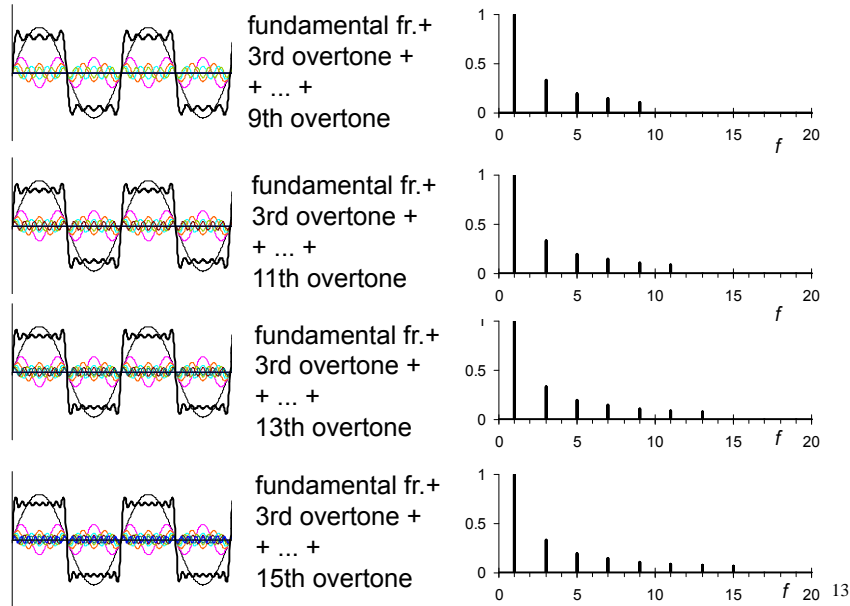
## spectrum



12

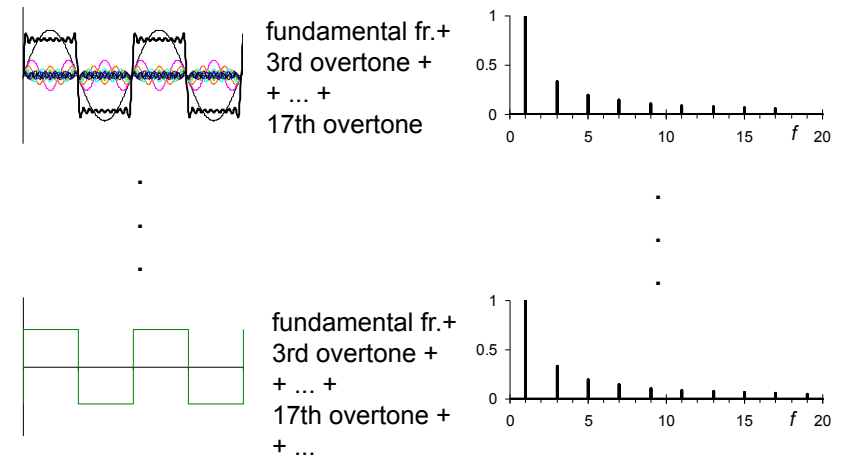
## function

## spectrum



## function

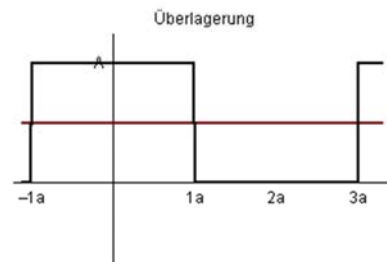
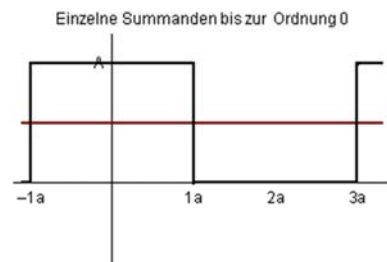
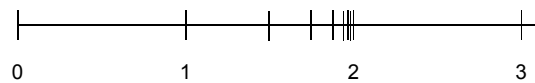
## spectrum



14

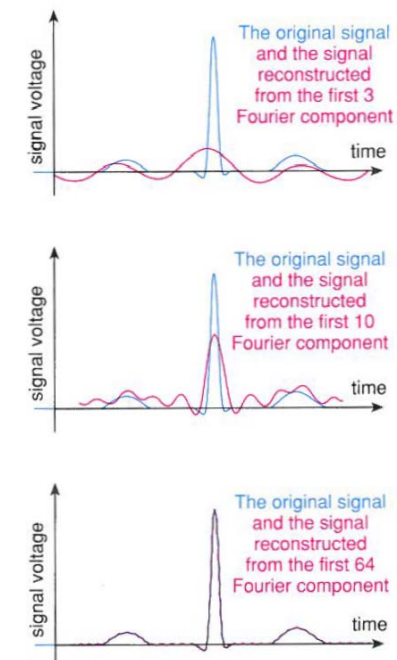
cf. infinite **series**

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \dots = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2$$



15

Creating an  
ECG signal  
from sine  
functions



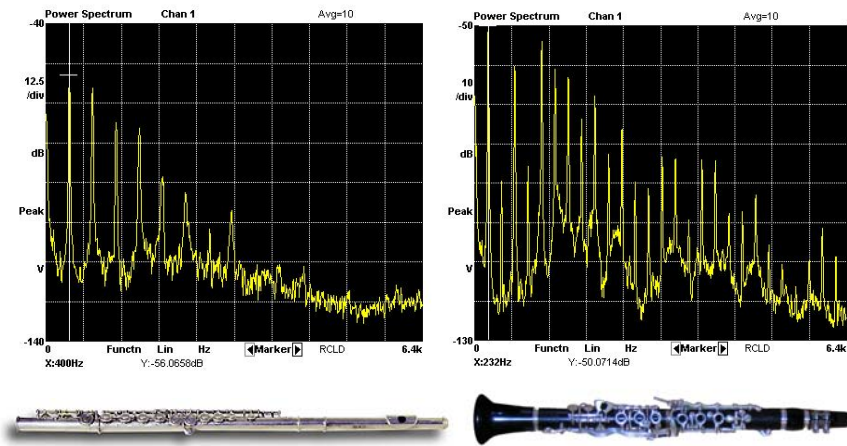
Textbook, Figure VII.3.

16

## Measured spectra

flute

clarinet



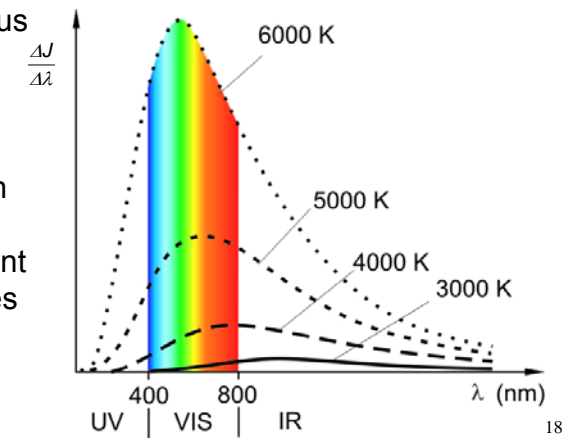
17

## Fourier's theorem for non-periodic functions (signals)

all (usual) functions can be expressed as a sum of sine (and cosine) functions

spectrum: continuous

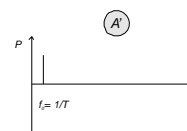
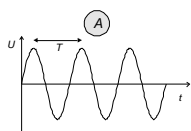
cf. emission spectra of incandescent light sources



18

## function

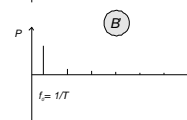
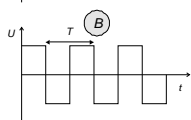
sine function



## spectrum

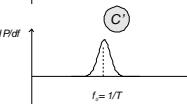
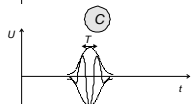
line spectrum (1 line)

periodic function



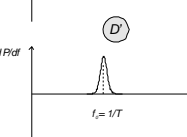
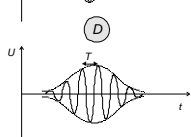
line spectrum

a few periods



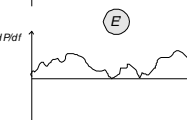
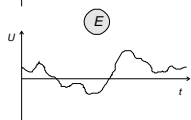
band spectrum

more periods



band spectrum

non-periodic function



continuous spectrum

19

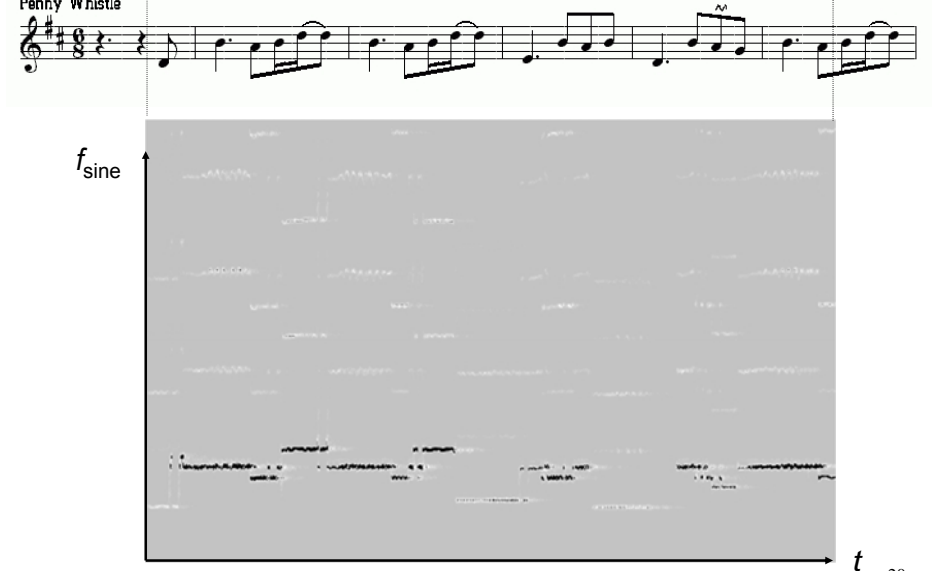
Inisheer

Penny Whistle

## Music in time-frequency representation

Traditional

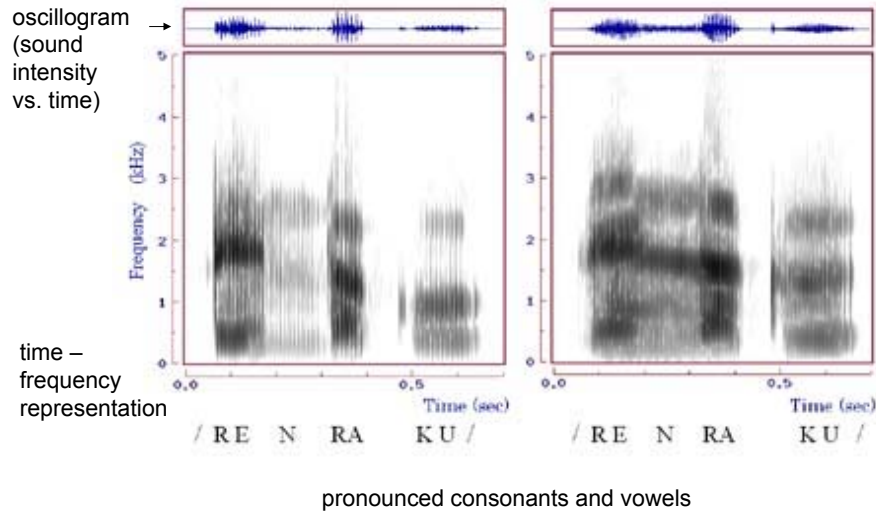
Air



20



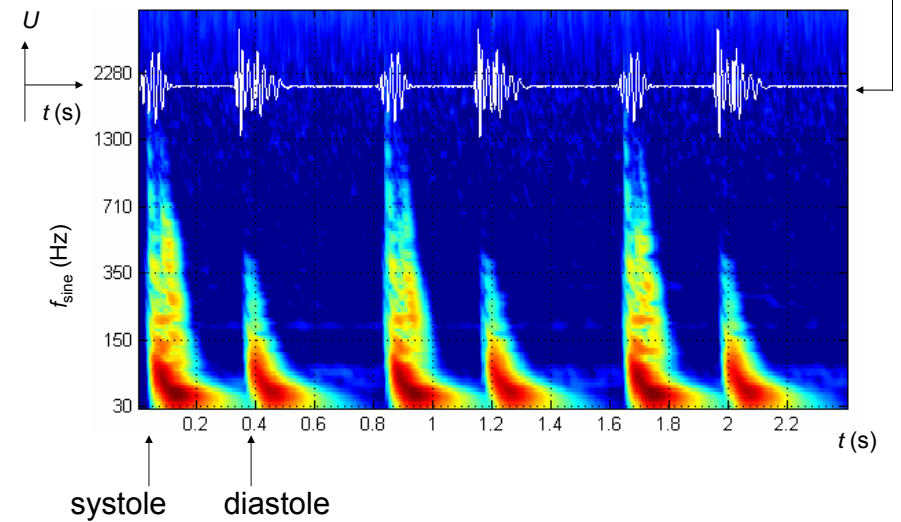
## Voiceprint



<http://www.nrips.go.jp/org/fourth/info3/index-e.html>

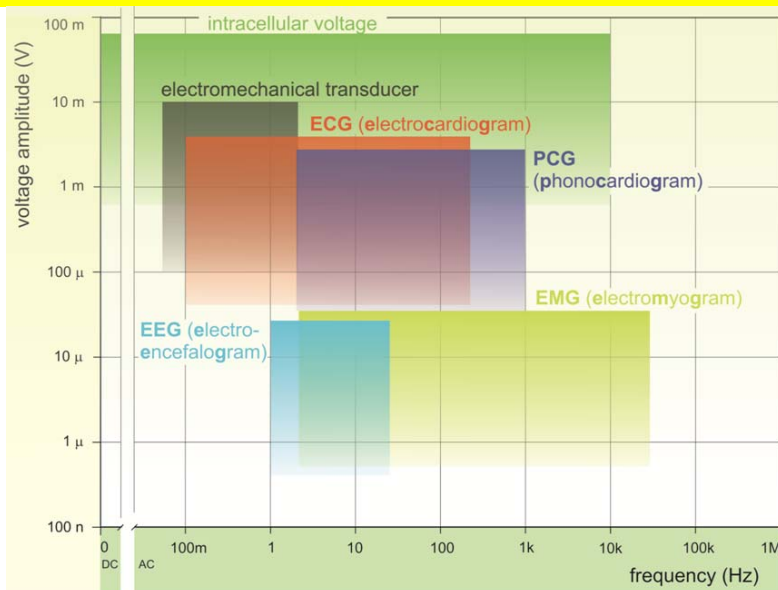
21

## Heart beats in time-frequency representation (+ oscillogram)



22

## Frequency and amplitude ranges of biological signals



Practical manual, titel page of meas. 17

23

## Frequency dependent unit: Electronic amplifier

- (1)  $P_{in} < P_{out}$
- (2)  $P_{in}$  and  $P_{out}$  : same functions

same: „fundamentalist“ requirement  
similar: realistic requirement

$$(1) + (2) \quad A_p \cdot P_{in}(t) \equiv P_{out}(t), \text{ where } A_p > 1$$

$$A_p = \frac{P_{out}}{P_{in}},$$

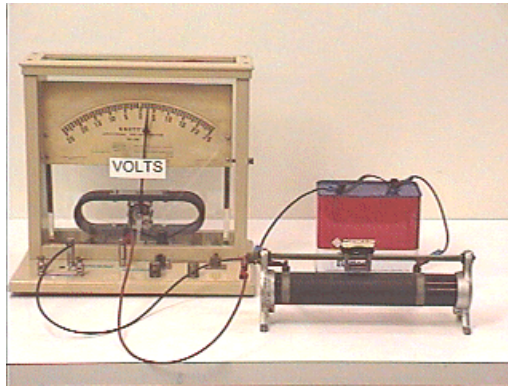
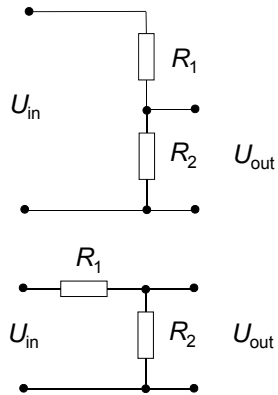
**power gain** (amplification)

$$A_U = \frac{U_{out}}{U_{in}},$$

**voltage gain** (amplification)

24

### (frequency independent) voltage-divider



$$U_{\text{out}} = \frac{R_2}{R_1 + R_2} U_{\text{in}}$$

frequency dependent voltage-divider: with capacitor

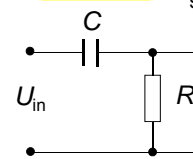
25

supplementary material

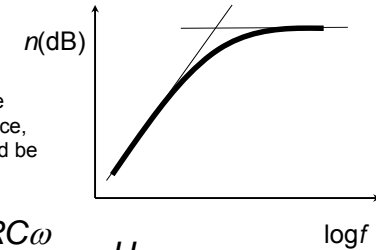
### High-pass/low-cut filter

$$R_C = \frac{1}{C\omega}$$

at high frequencies the capacitor is a shortcut



because of the phase difference, the sum should be calculated as vectors



$$U_{\text{out}} = \frac{R}{\sqrt{\frac{1}{C^2\omega^2} + R^2}} U_{\text{in}} = \frac{RC\omega}{\sqrt{1 + R^2C^2\omega^2}} U_{\text{in}}$$

at very low frequencies: if  $\omega \ll \omega_0$  ( $\omega \approx 0$ ),  $U_{\text{out}} = 0$

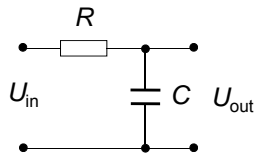
at low frequencies: if  $\omega \ll \omega_0$ ,  $U_{\text{out}} = RC\omega U_{\text{in}} \leftrightarrow 6 \text{ dB/octave}$

at high frequencies: if  $\omega \approx \infty$ ,  $U_{\text{out}} = U_{\text{in}}$

26

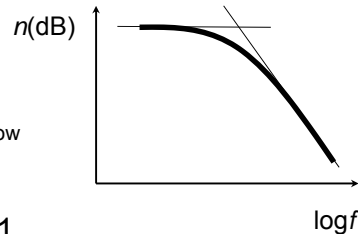
supplementary material

### Low-pass/high-cut filter



$$R_C = \frac{1}{C\omega}$$

the capacitor at low frequencies is a discontinuity



$$U_{\text{out}} = \frac{\frac{1}{C\omega}}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} U_{\text{in}} = \frac{1}{\sqrt{R^2C^2\omega^2 + 1}} U_{\text{in}}$$

at low frequencies: if  $\omega \ll \omega_0$  ( $\omega \approx 0$ ),  $U_{\text{out}} = U_{\text{in}}$

at high frequencies: if  $\omega \gg \omega_0$ ,  $U_{\text{out}} = \frac{1}{RC\omega} U_{\text{in}} \leftrightarrow -6 \text{ dB/octave}$

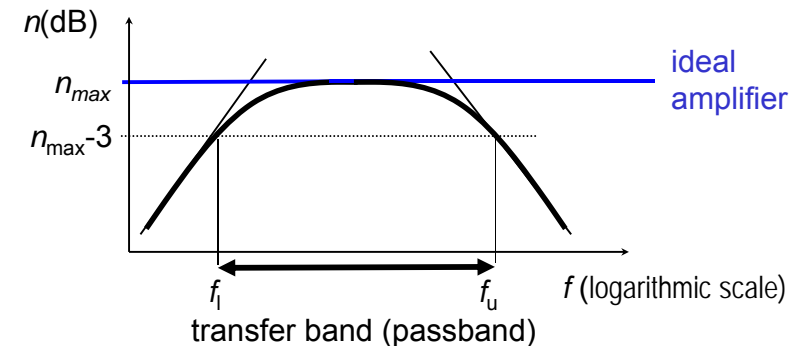
at very high frequencies: if  $\omega \gg \omega_0$  ( $\omega \approx \infty$ ),  $U_{\text{out}} = 0$

27

for (1 [on page 24]):  $A_P > 1$ ,

$$n = 10 \lg A_P = 20 \lg A_U > 0 \text{ dB}$$

for (2 [on page 24]): **frequency characteristics**

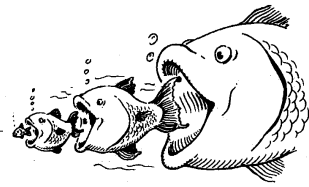
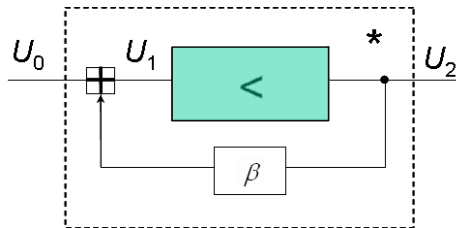


$f_l$ : lower frequency limit

$f_u$ : upper frequency limit

28

## Amplifier with feedback



Without appropriate perspective we may not recognize our real position.

$$(a) \quad U_1 = U_0 + \beta U_2 \quad (b) \quad A_U = \frac{U_2}{U_1}$$

$$(c) \quad A_U^* = \frac{U_2}{U_0} = \frac{U_1 A_U}{U_0} = \frac{(U_0 + \beta U_2) A_U}{U_0} = A_U + \beta \frac{U_2}{U_0} A_U = A_U + \beta A_U^* A_U$$

$$A_U^* - \beta A_U^* A_U = A_U \quad \boxed{A_U^* = \frac{A_U}{1 - \beta A_U}}$$

29

$$A_U^* = \frac{A_U}{1 - \beta A_U}$$

$A_U^*$  : voltage gain with feedback

$A_U$  : voltage gain without feedback

$\beta > 0$ , **positiv feedback** (same phase),  $A_U^* > A_U$  (advantage)

$\beta < 0$ , **negativ feedback** (in opposite phase),  $A_U^* < A_U$  (disadv.)

**positiv feedback:**

(a)  $\beta A_U = 1$ , amplification: „infinite“

– sine wave oscillator

e.g.: ultrasound generator,  
heat therapy

(b)  $\beta A_U \leq 1$ , amplification: very big

– regenerative amplifier

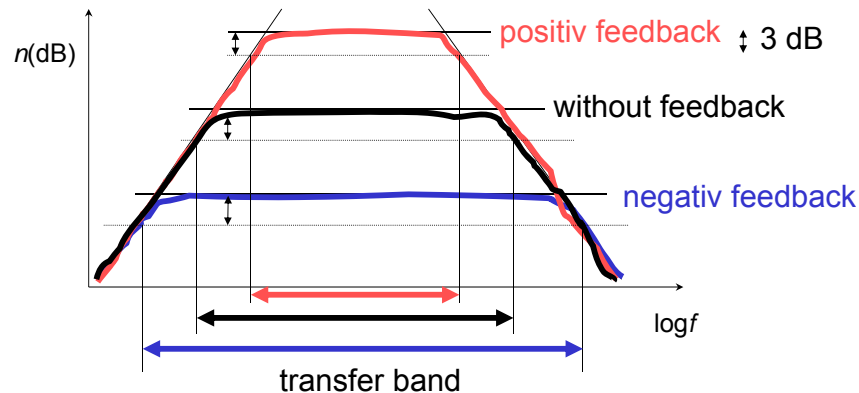
e.g.: hearing, outer haircells



„... and whose idea was it to ask for employee feedback?“

**negativ feedback:** „all“ amplifier

30

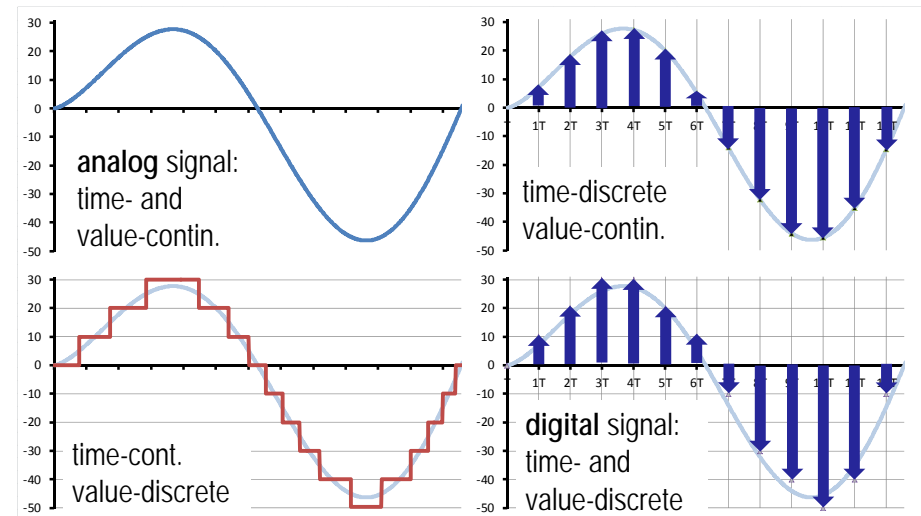


**positiv feedback:** transfer band – narrower (big disadvantage)  
higher gain (advantage)

**negativ feedback:** transfer band – broader (advantage)  
less gain (small disadvantage)

31

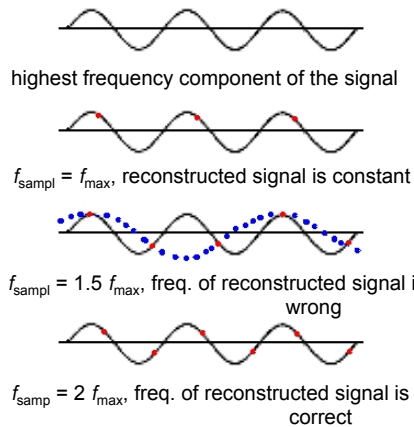
## Analog signal – digital signal



32



time-discrete: the value of the signal is not known for all moments in time



### Nyquist-Shannon sampling theorem:

for complete reconstruction the minimum sampling frequency should be twice the frequency of the highest overtone of the signal

e.g.: hifi,  $f_{\text{max}} = 20 \text{ kHz}$

$f_{\text{samp}} = 44.1 \text{ kHz} > 2 \cdot 20 \text{ kHz}$

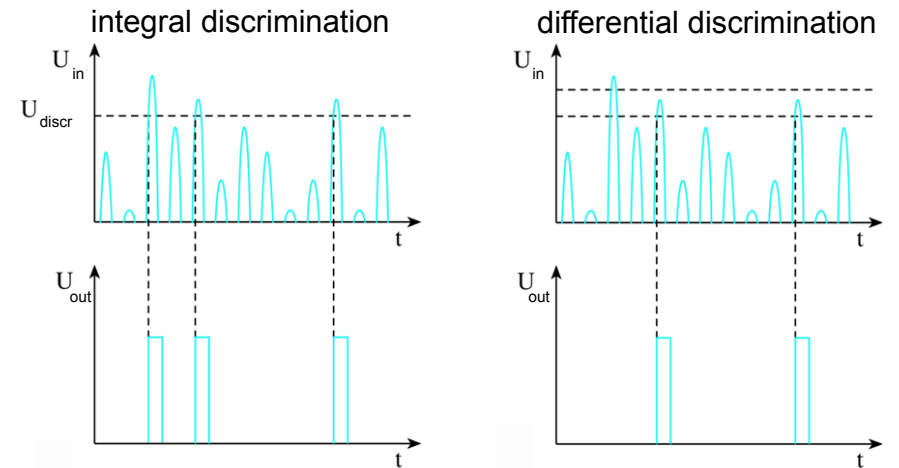
value discrete: the value of the signal can not be arbitrary

e.g.: hifi, 16 bit =  $2^{16} = 65\,536$  (CD standard)

24 bit =  $2^{24} = 16\,777\,216$  ("best" audio card)

33

### Pulse processing



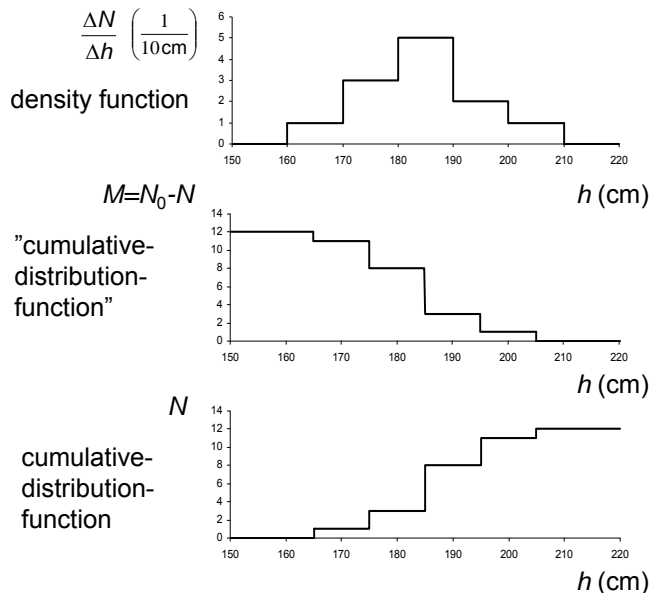
to select only those pulses that are larger than a preset amplitude

to select only those pulses whose amplitudes lie within a preset window

Textbook, Figure VII.32.

34

### Distribution functions and ID/DD "spectra"



DD-"spectrum"

ID-"spectrum"

how many pulses are larger than  $h$ ?

how many pulses are smaller than  $h$ ?

35

### Concentration of white blood cells

Coulter counter

