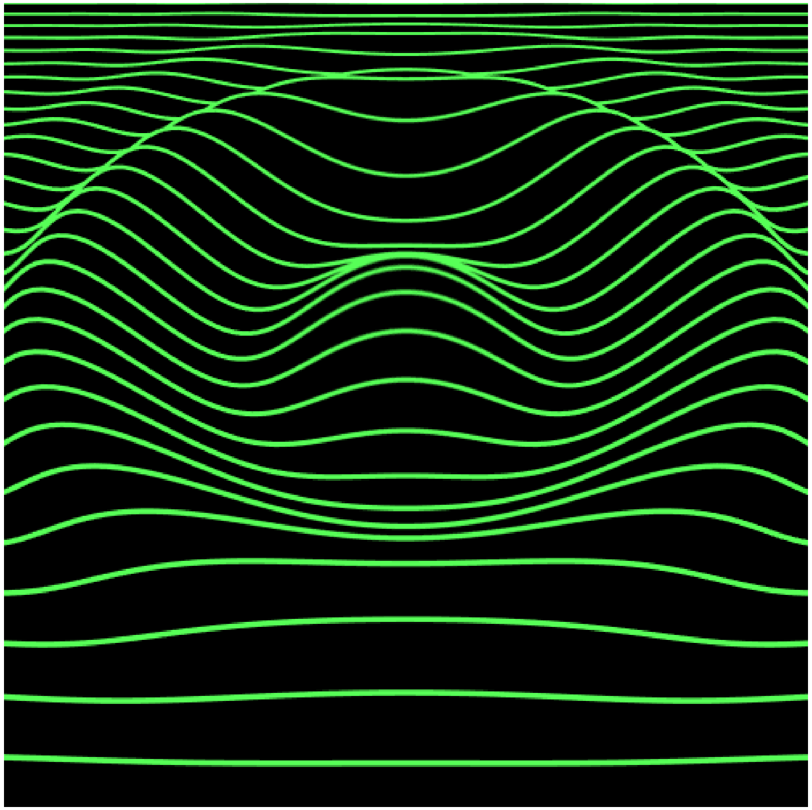
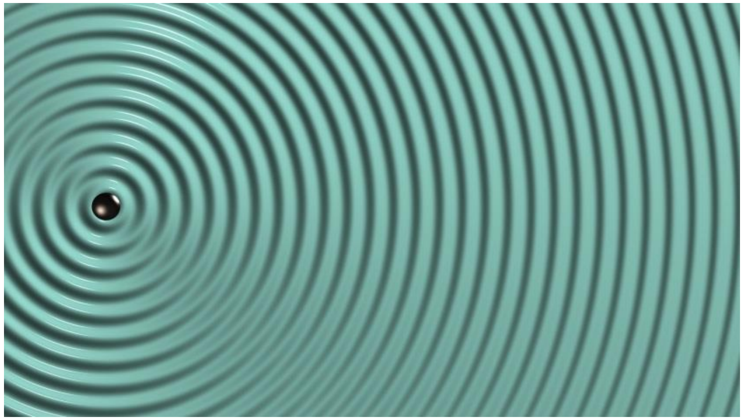
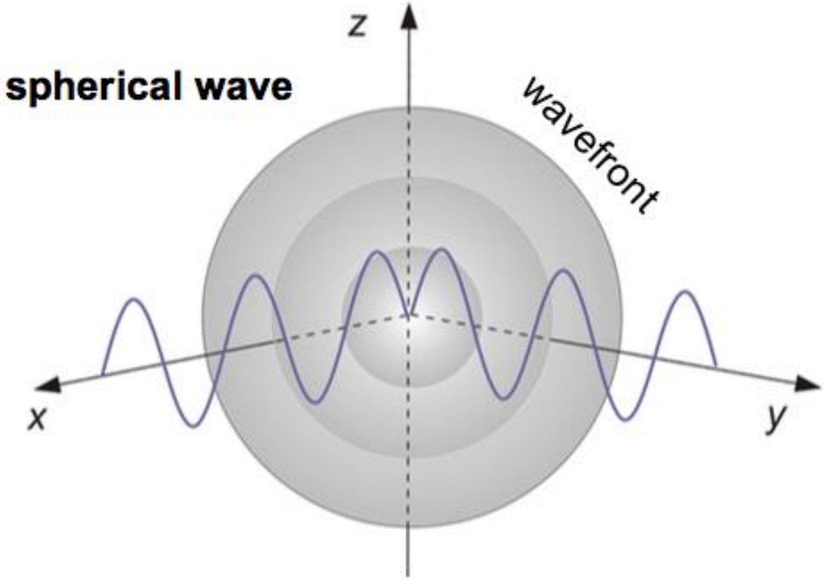
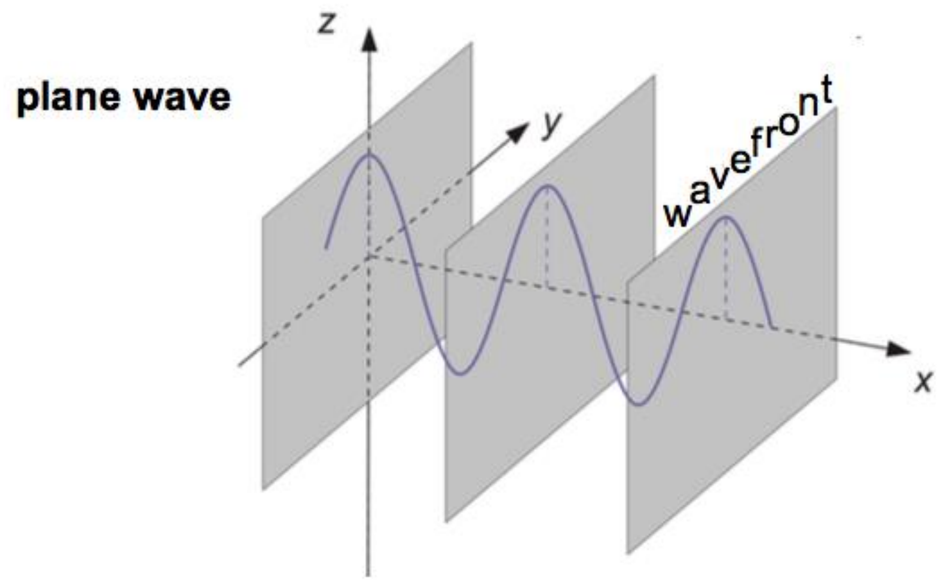
Waves

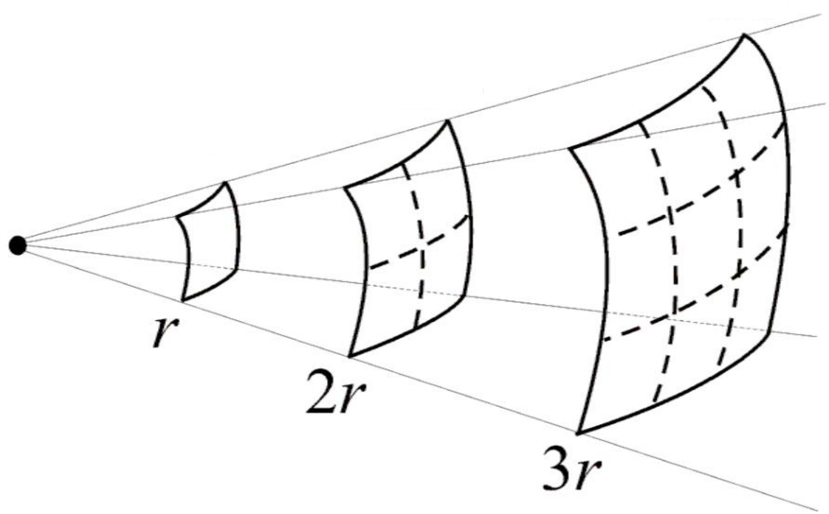




G.Schay



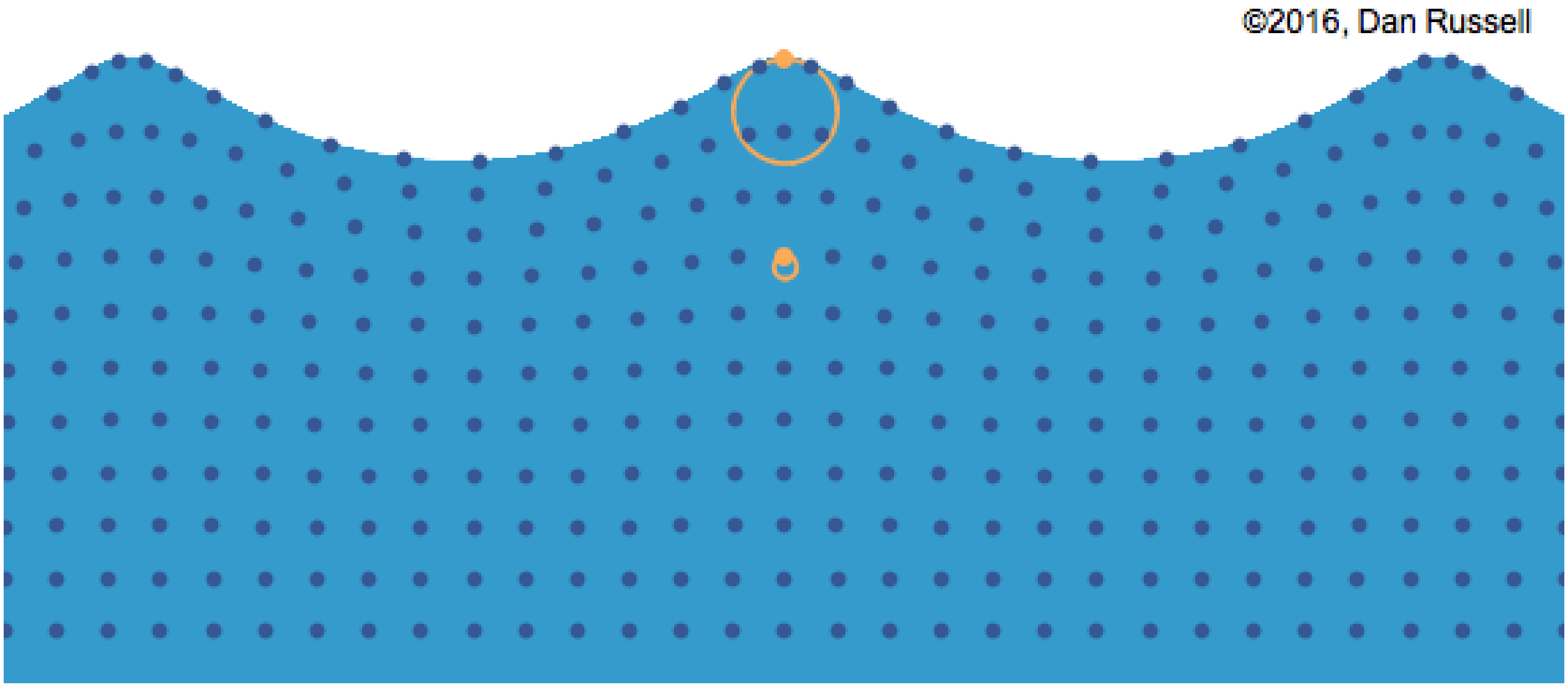




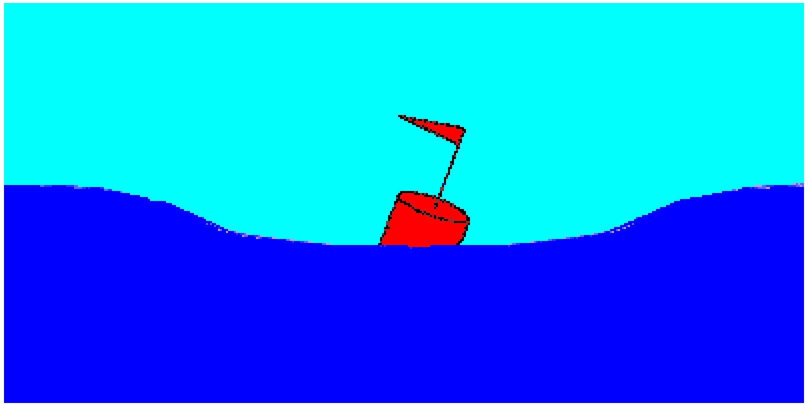
$\frac{\Delta E}{\Delta A}$ stays the same with increasing distance from source

$\frac{\Delta E}{\Delta A}$ decreases with increasing distance from source

Waves are seen most often in water:

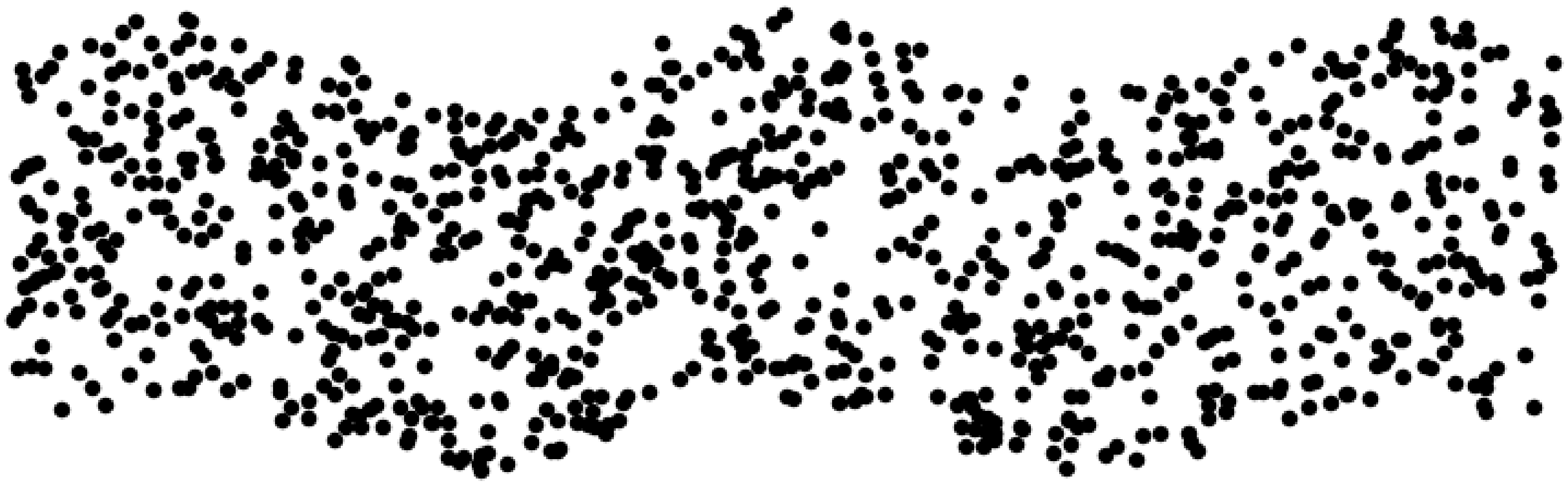


Waves can be described by the wave equation, which relates
the motion of individual parts of the medium to the observed wave.



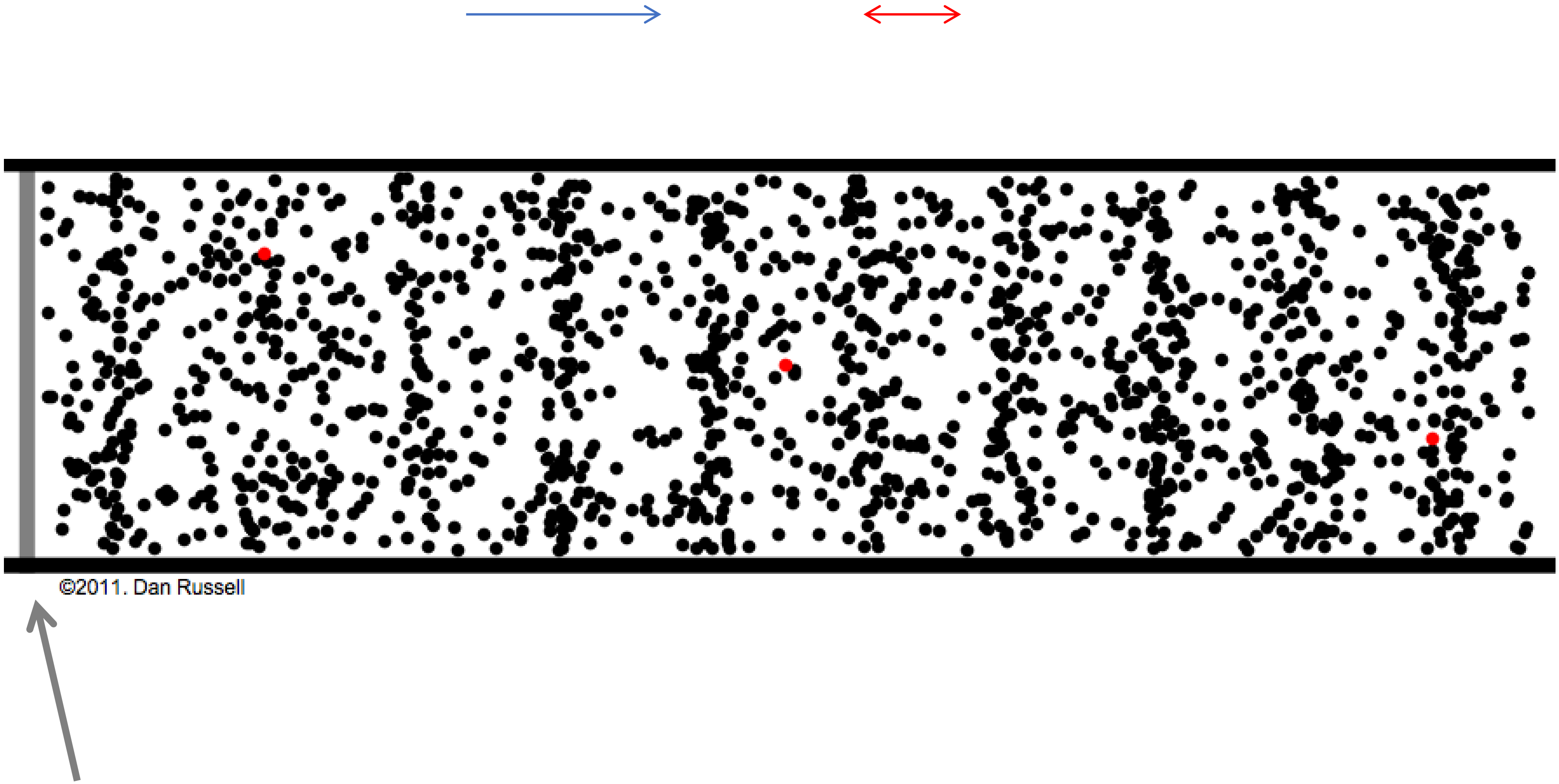
It is important to note, that as the waves propagate, the parts
of the medium (here the water molecules) stay “in place”, which means
there is no net transport of material.

Transversal wave – such as light, or sound in some cases in solids

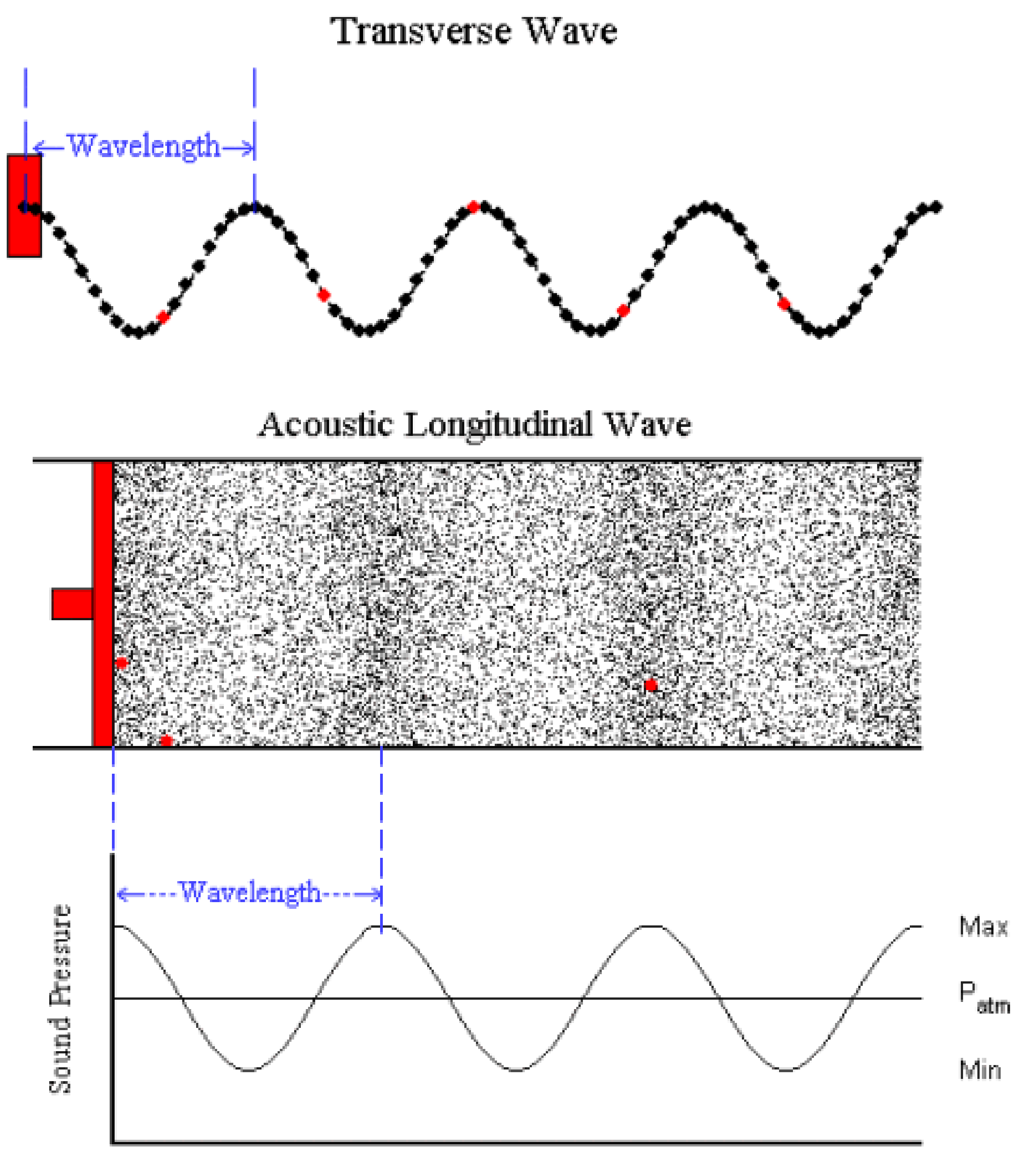


Transversal: wave propagation is perpendicular to the “motion”

Longitudinal waves:
propagation direction is parallel to the “motion”

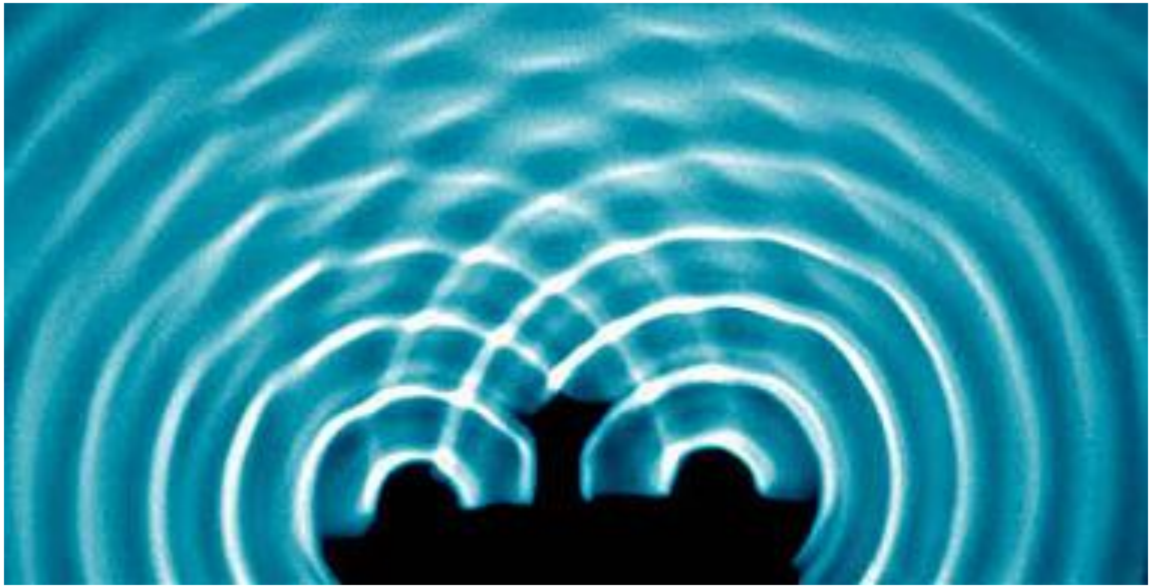


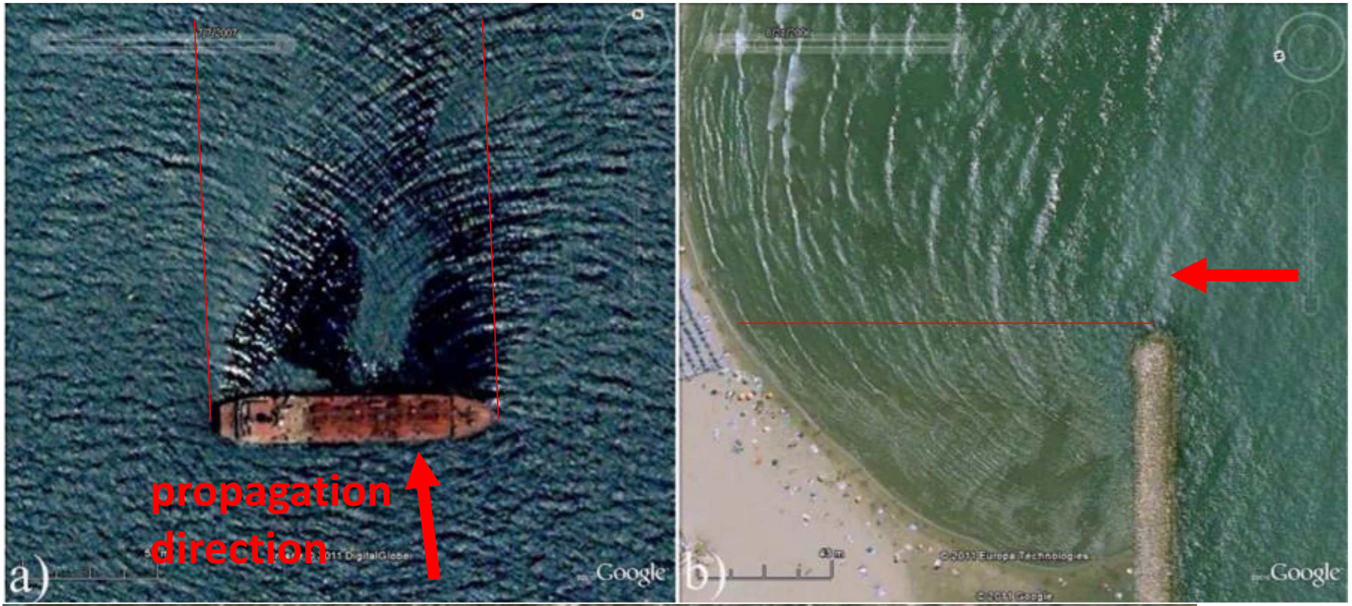
Moving surface (wave “source”)

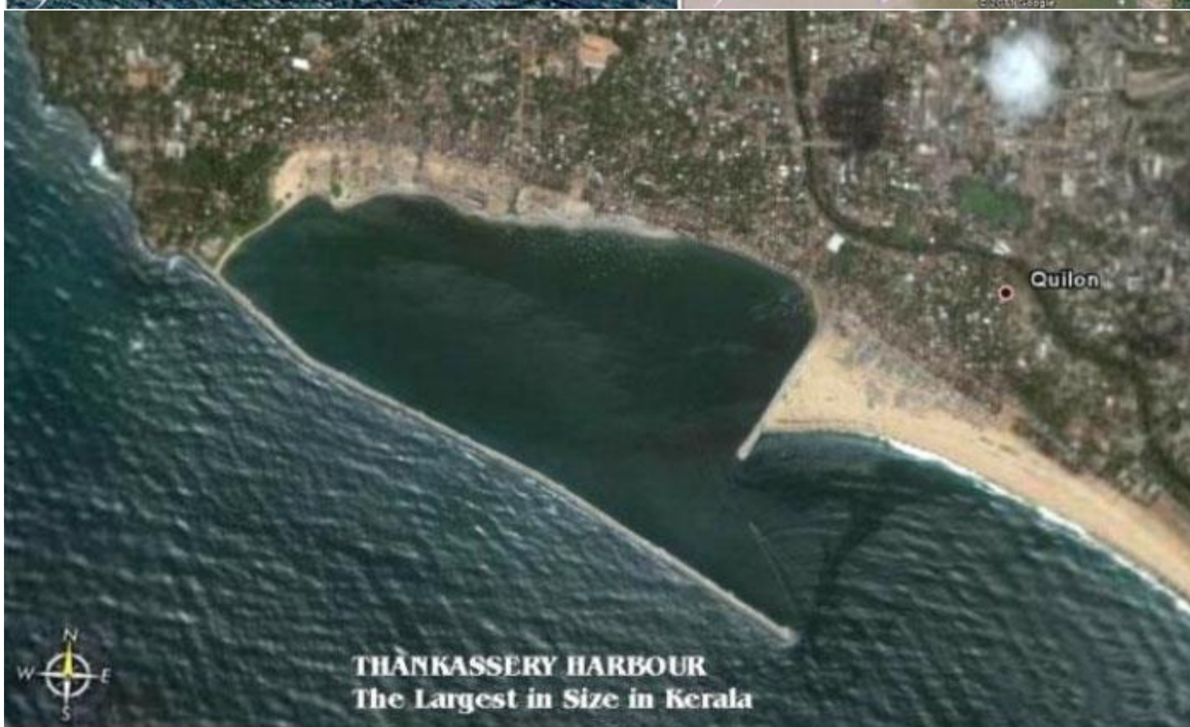

Transverse Wave
Wavelength
Acoustic Longitudinal Wave
Wavelength
Sound Pressure
Max
P atm
Min
isvr

Indications for a wave nature:

- diffraction
- superposition / interference
- polarisation







Diffraction of waves on water surface

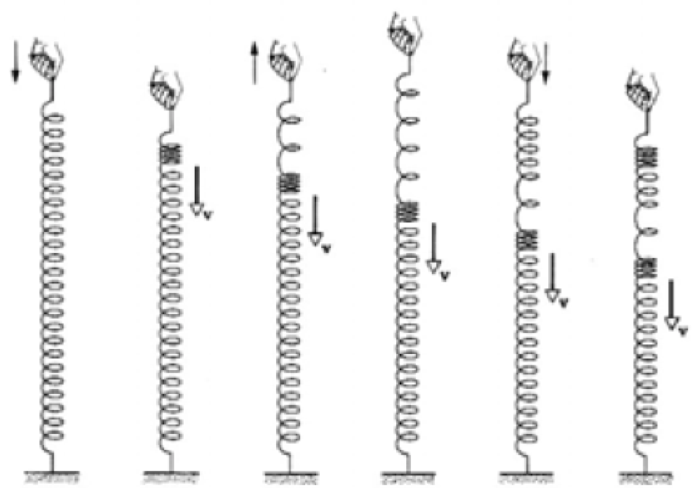
Different types of waves

- According to source:
  1. Mechanical: elastic deformation propagating through elastic medium
  2. Electromagnetic: electric disturbance propagating through space (vacuum)

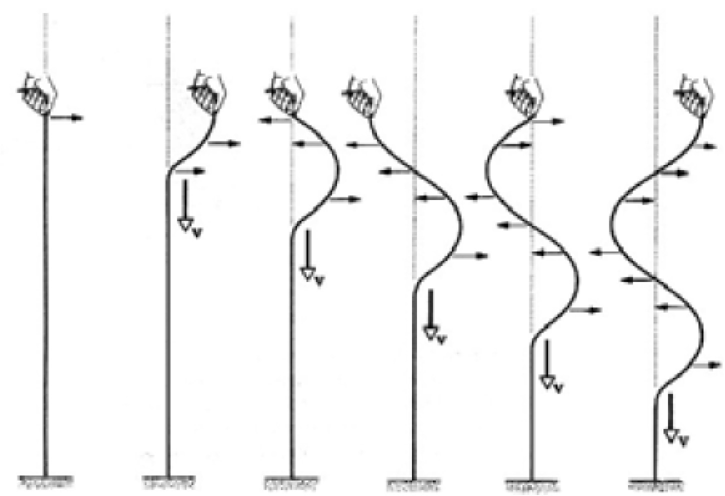
- According to propagation dimension:
  1. One-dimensional (rope)
  2. Surface waves (pond)
  3. Spatial waves (sound)

- According to relative direction of oscillation and propagation:

1. Longitudinal



2. Transverse



The wave equation is a bit complicated:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

We take the change of any property (here “u”) in time (du/dt) and also in space (du/dx), but we need to take the change of the change (d^2u/du^2), and these are linked by the propagation speed (or other named phase velocity) (here as “c”).

A simple solution for u(x,t) is:

$u(x,t) = A * \sin(k*x + \omega*t + \phi)$

where

A is the amplitude of the wave, k is the wavenumber, and ω is the angular frequency

$\omega = 2\pi f$, where f = 1/T [Hz], while T is the period time.

$\omega = c*k$ defines the wavenumber, which can be written as $k = 2\pi/\lambda$.

here λ is the wavelength.

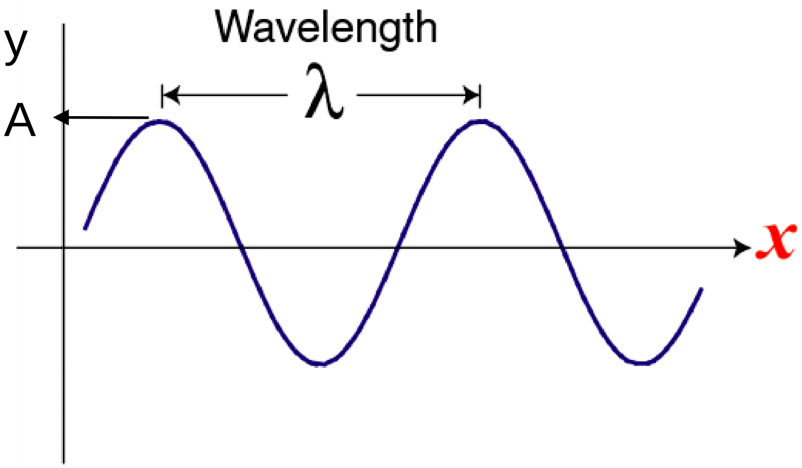
Characteristic values

Period in space – *wavelength*

λ [m] or [nm]

displacement – *amplitude*

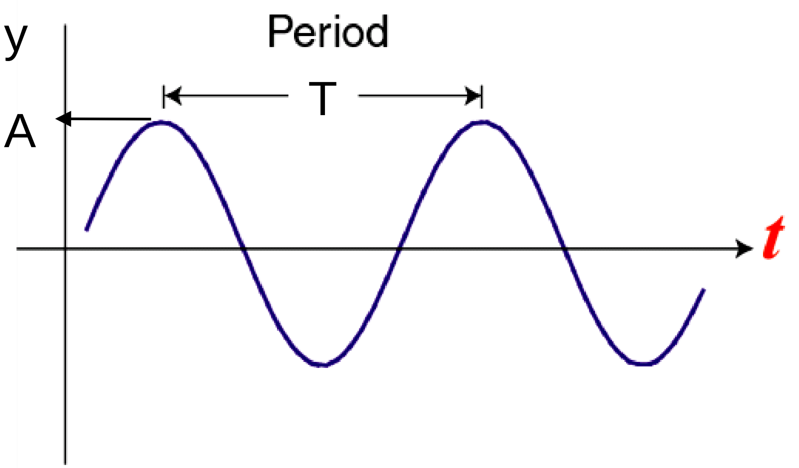
$$E \sim A^2$$



Period in time – *period, T*

– *frequency, f*

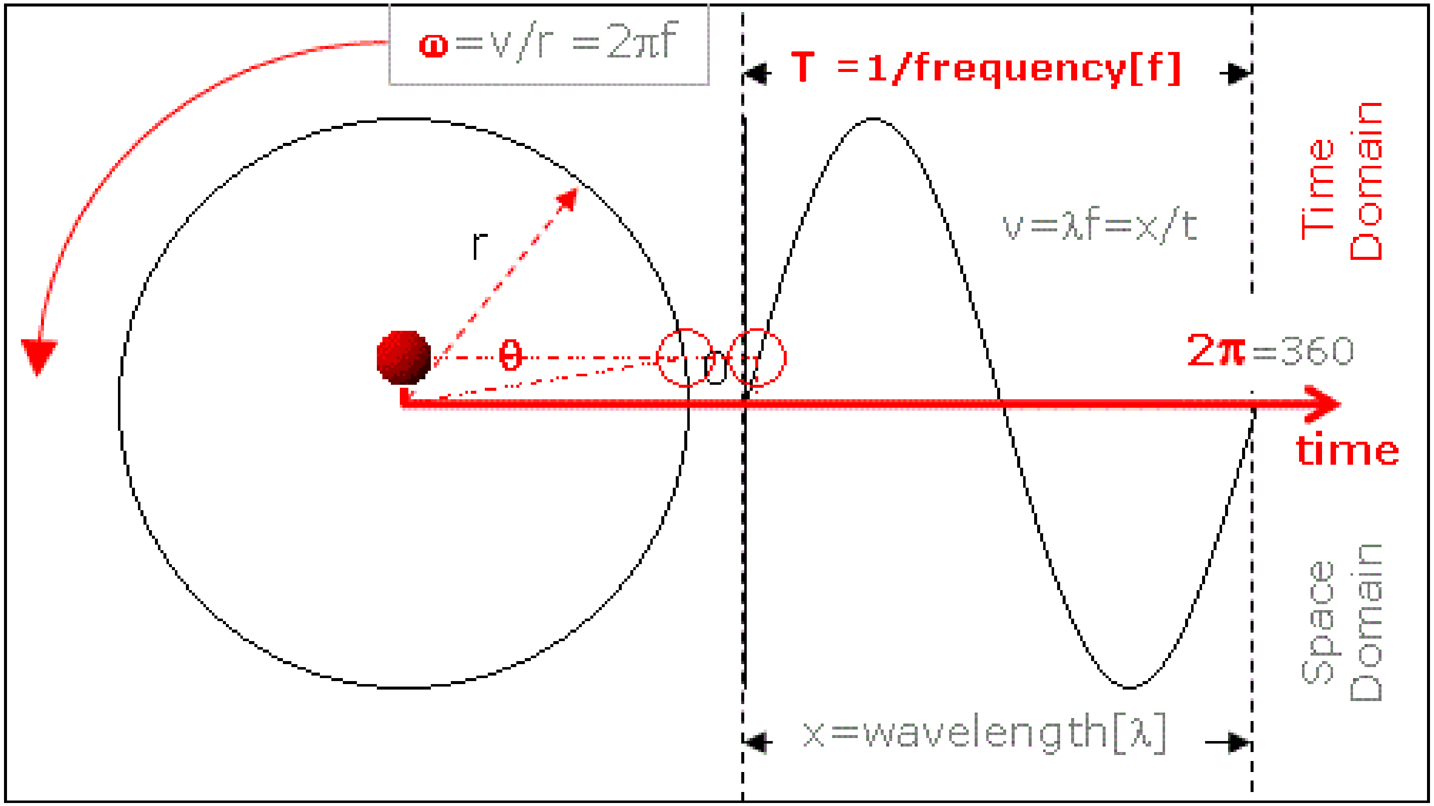
$$f = \frac{1}{T}\left[\frac{1}{s}\right]$$



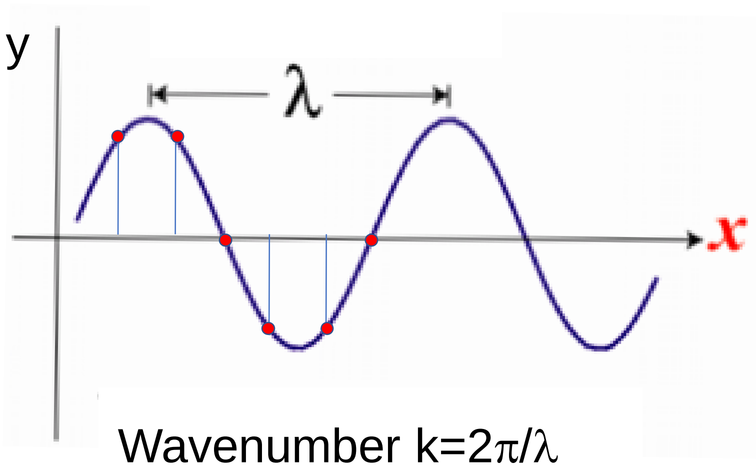
velocity of waves: $c = \lambda/T = \lambda f$

The wave is moving forward (propagating) with a speed of “v”, any point’s position is dependent on both space and time.

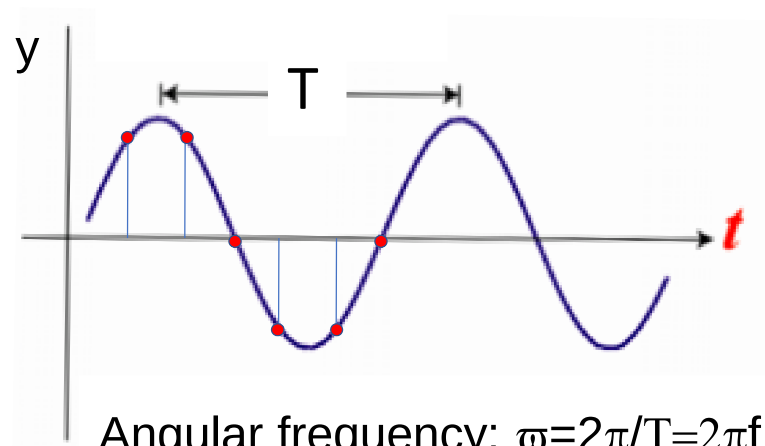
Most important motion type is the harmonic motion. (produced by a harmonic oscillator)



Phase: the location or timing of a point within a period



Wavenumber $k=2\pi/\lambda$



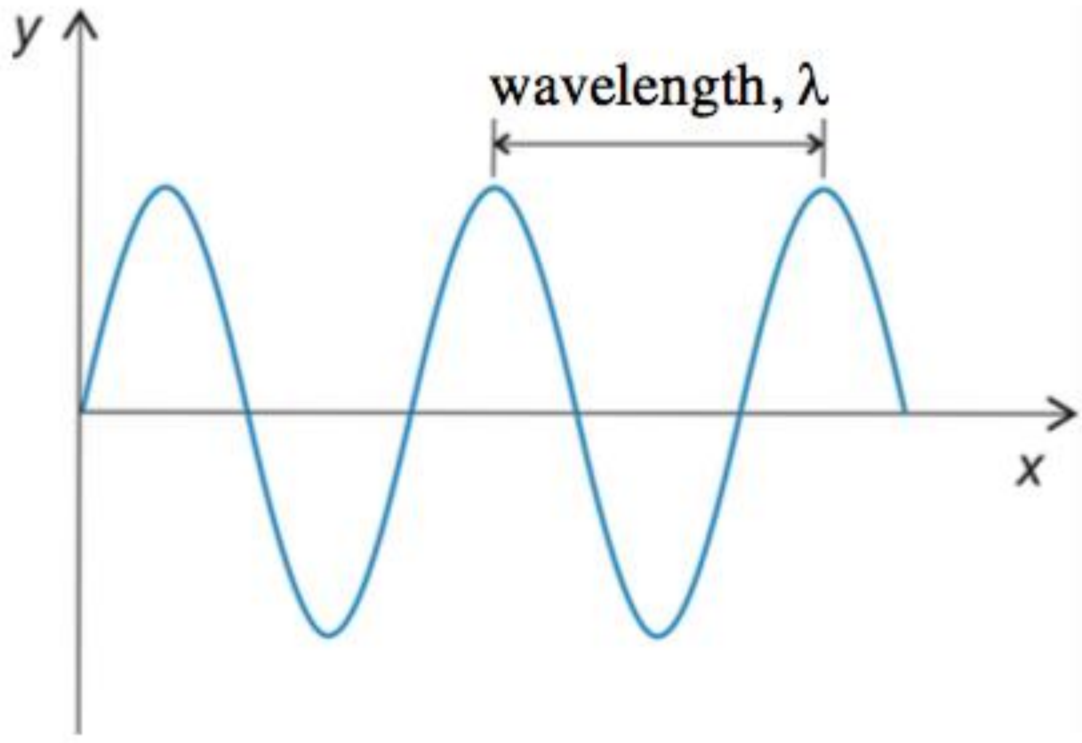
Angular frequency: $\varpi=2\pi/T=2\pi f$

$$\phi(x)=kx+\phi_o$$

$$\phi(t)=\varpi t+\phi_o$$

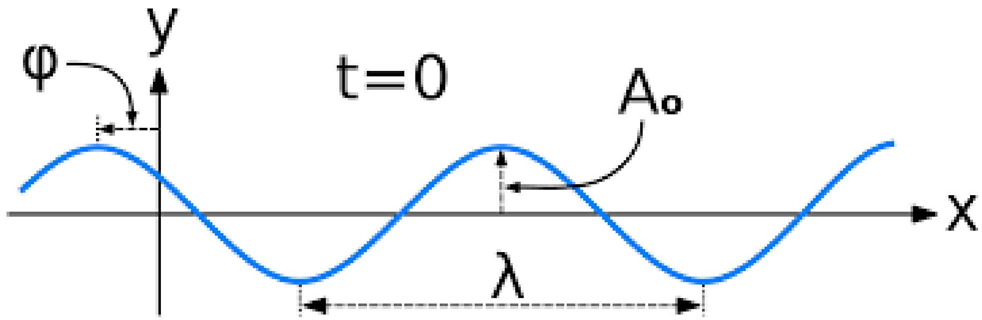
$$\phi=\varpi t+kx+\phi_o$$

Phase: fraction of the wave cycle that has elapsed relative to the origin

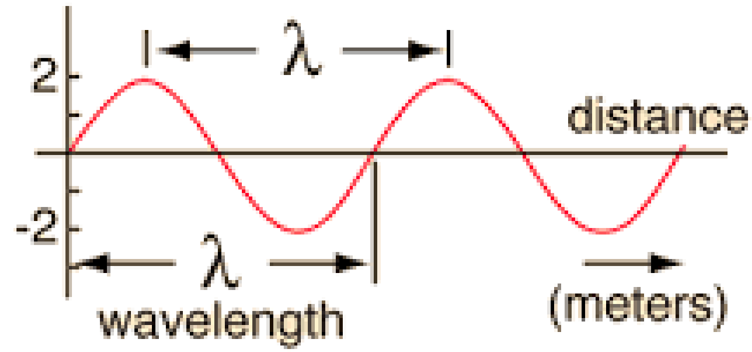


$$c = \frac{\lambda}{T} = \lambda \cdot \frac{1}{T} = \lambda \cdot f$$

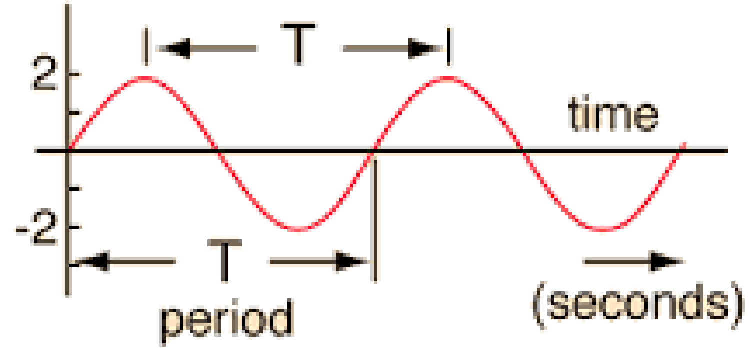
Graphical representation of the solution



Snapshot at a given time

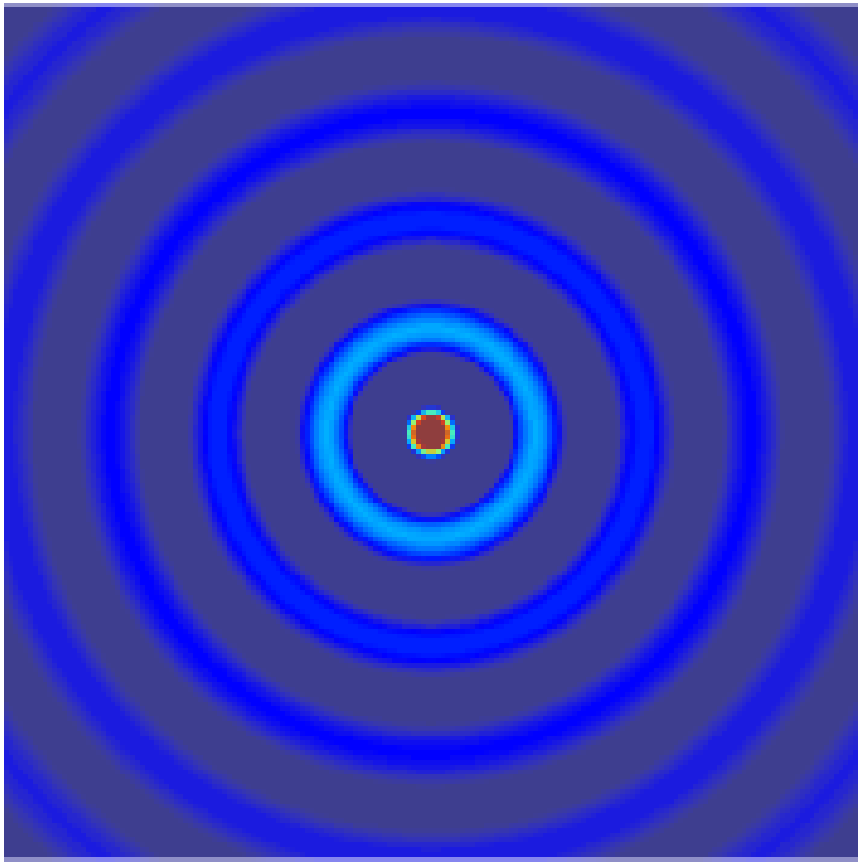


Time evolution at a given point

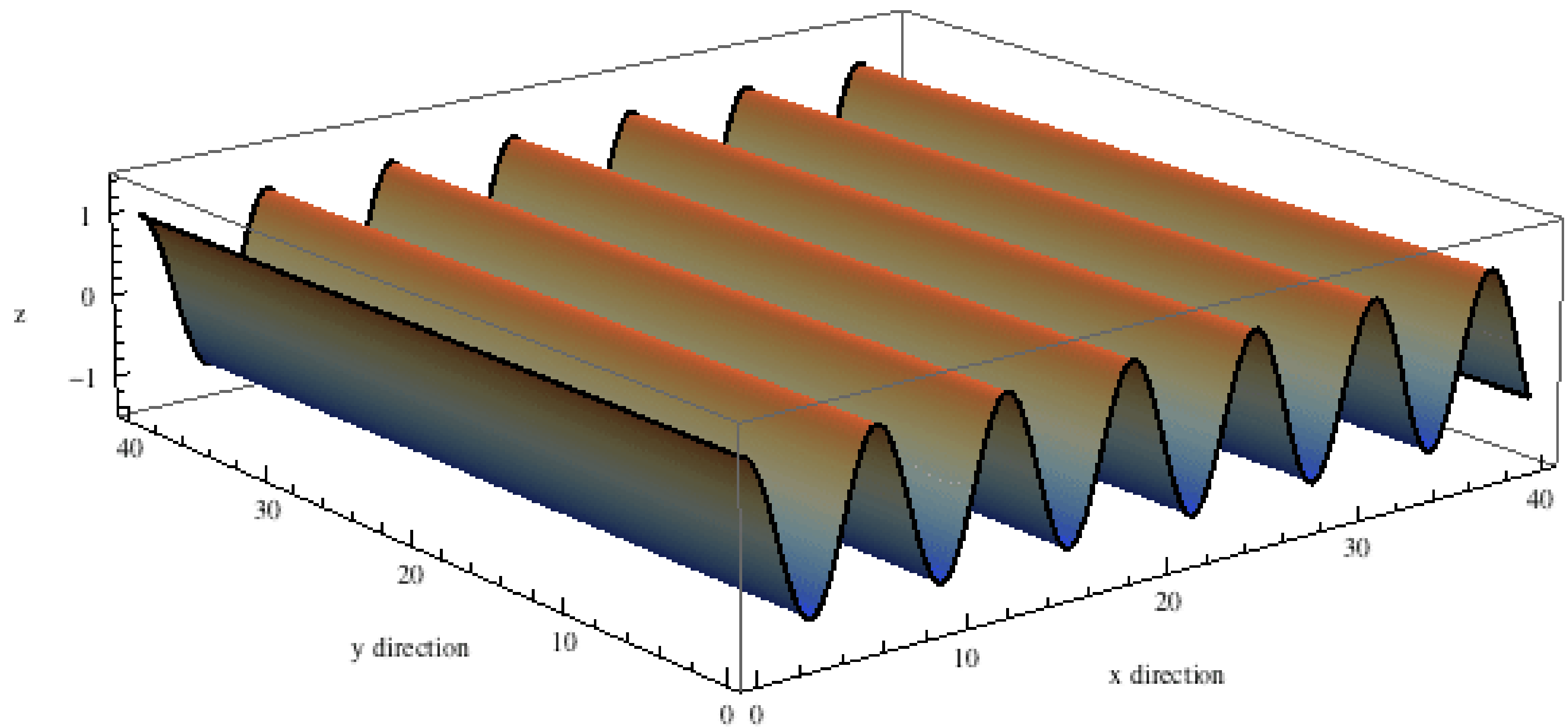


Point source: radiating in all directions along a sphere.

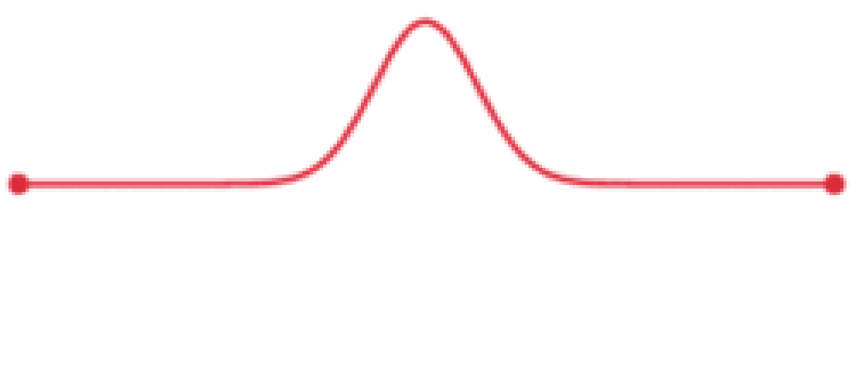
This is a transverse wave example, longitudinal is also possible.



Plane wave (again transversal)

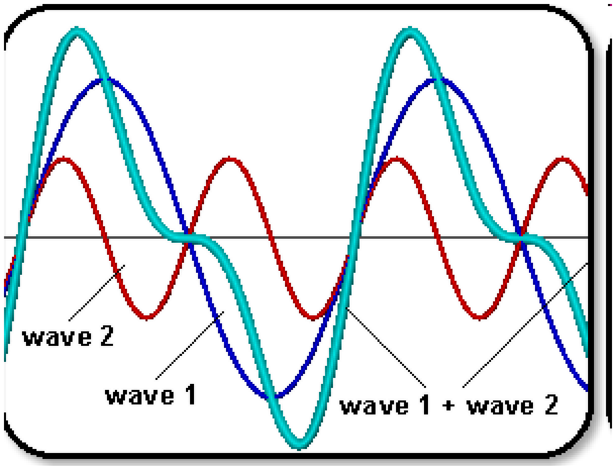


As an observation we can say that as waves travel or **propagate** the change in the state of the parts of the medium is moving.

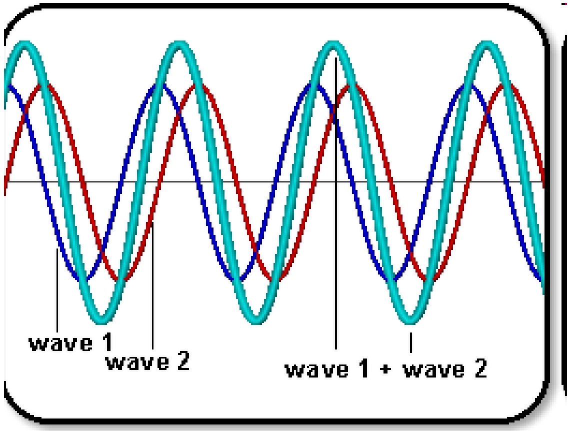


Here the “state of the parts” simply means the deflection of the string at a given point.

Superposition: The principle of superposition applies to waves whenever two (or more) waves traveling through the same medium at the same time. The net amplitude of the waves at any point in space or time, is simply the sum of the individual wave amplitudes.

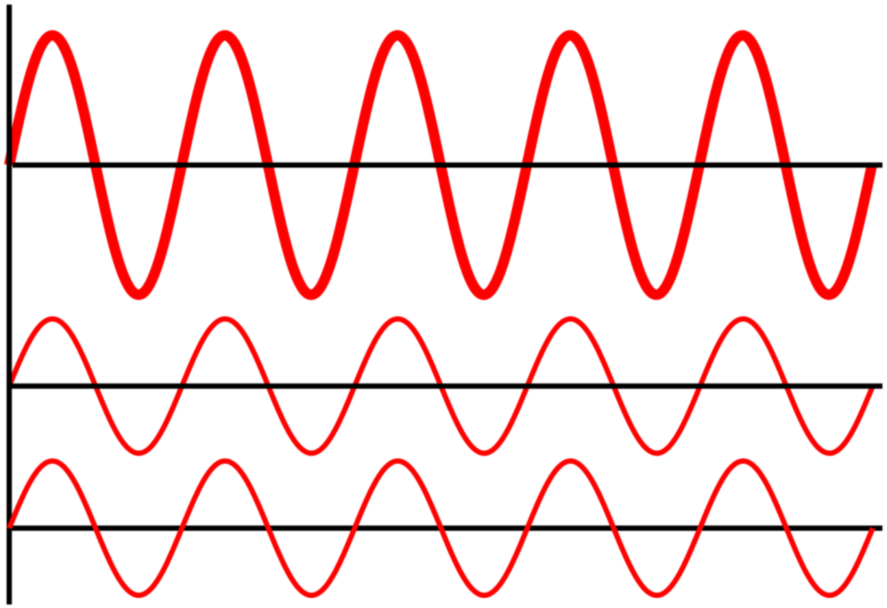


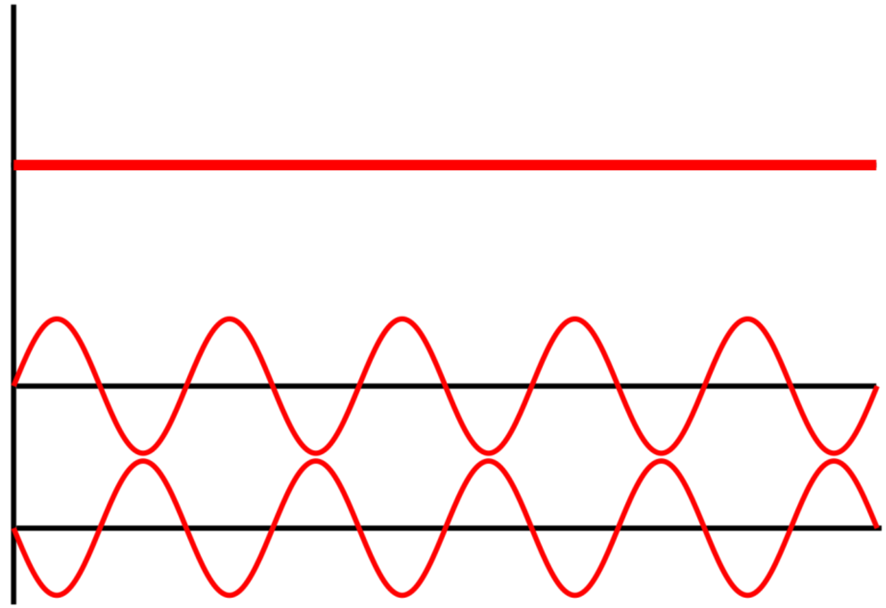
Un-equal frequencies



Equal frequencies

Interference: superposition of coherent waves





Similar phases

Constructive interference

$$\Delta\Phi = 0^\circ$$

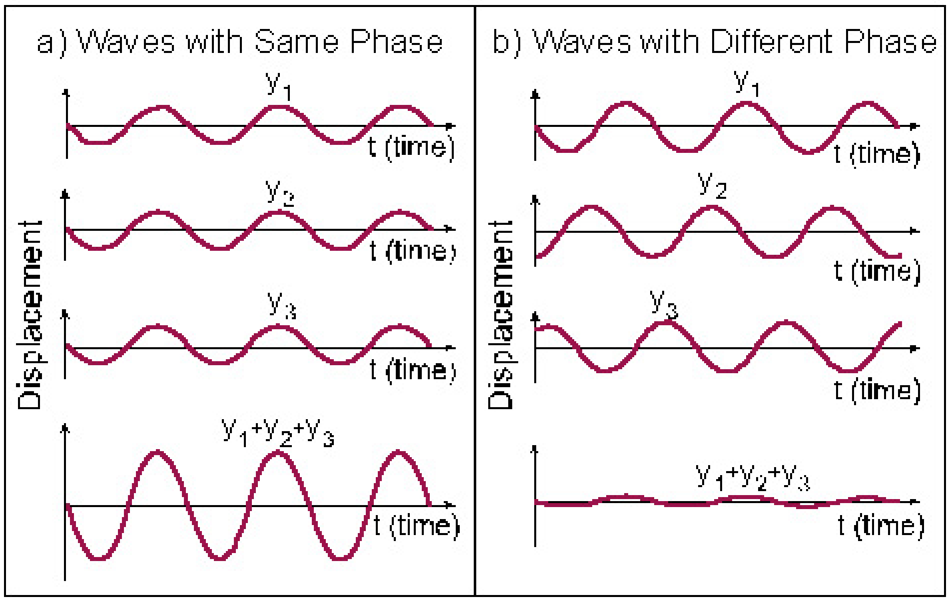
Opposite phases

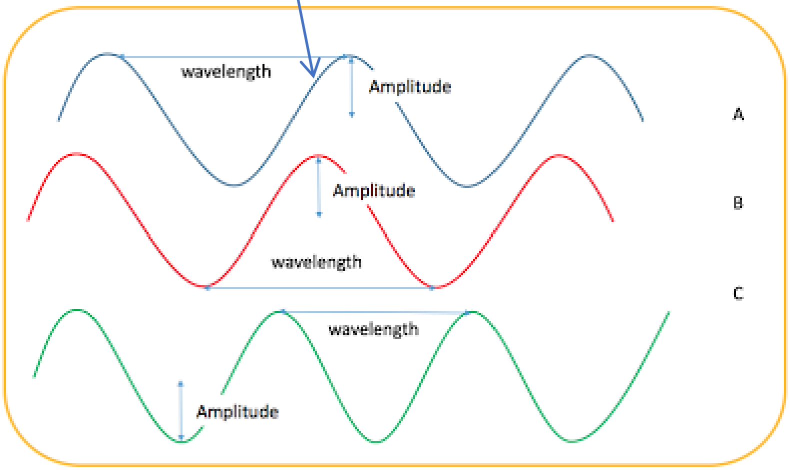
Destructive interference

$$\Delta\Phi = 180^\circ$$

Waves can be coherent or incoherent.
Coherent waves have the same frequency and a constant phase difference

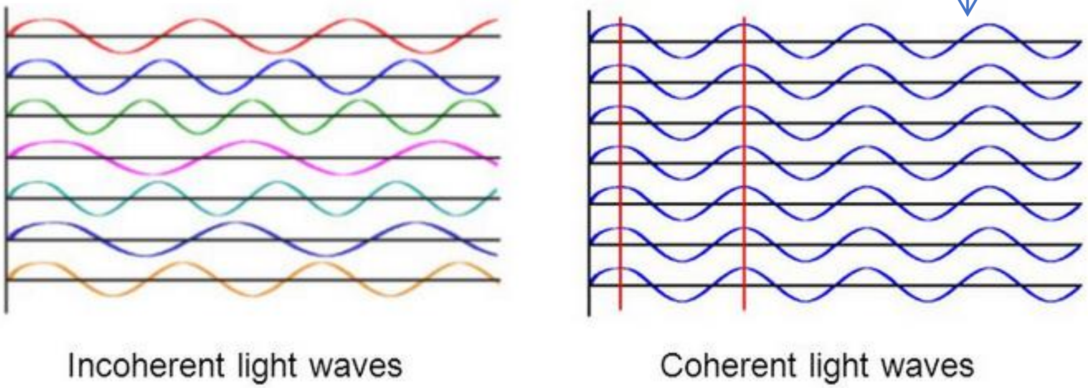
Only coherent waves can have a stable interference pattern,
incoherent waves on average sum up close to 0.





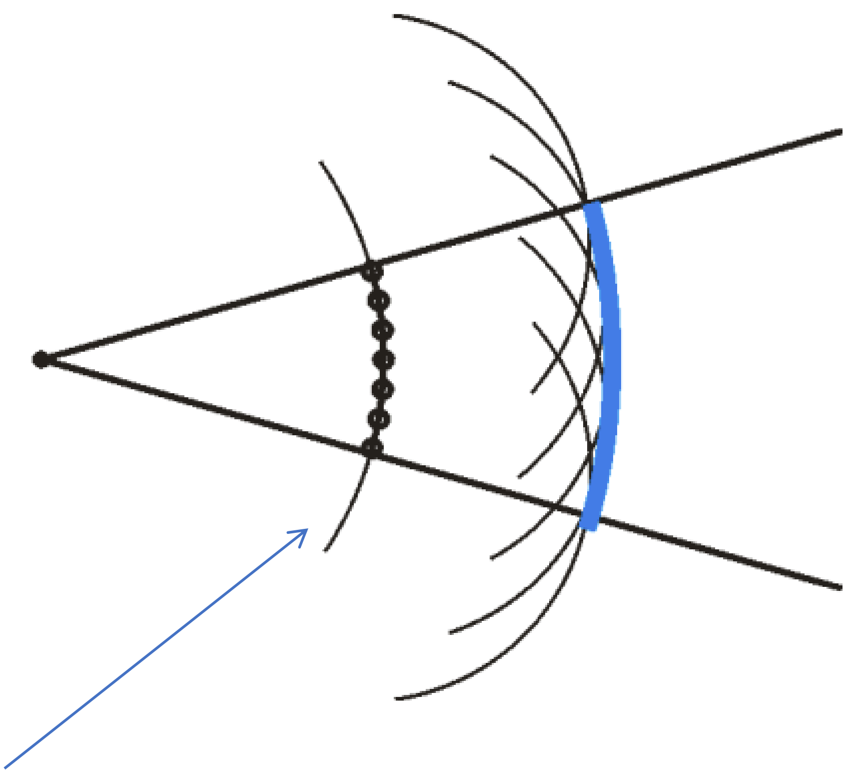
If the phase difference is 0 (or 1,2,3.. *2π),
we have constructive interference.

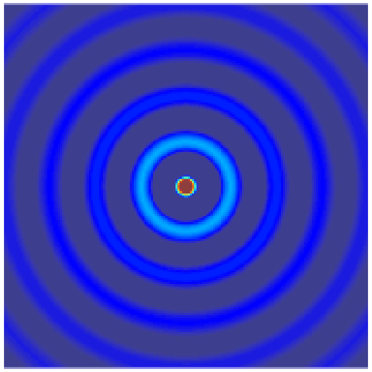
Constructive interference:
the resulting amplitude (A_{sum})
after summation is maximal.
Destructive: $A_{sum} = 0$



The Huygens-Fresnel principle describes the propagation of waves, it can be used to explain most of the experimental results (but not the quantum-mechanical ones!)

In short: every point at the wavefront acts as a new sperical point source, and the resulting new wavefront can be computed as a superposition of all of the waves generated in this way.



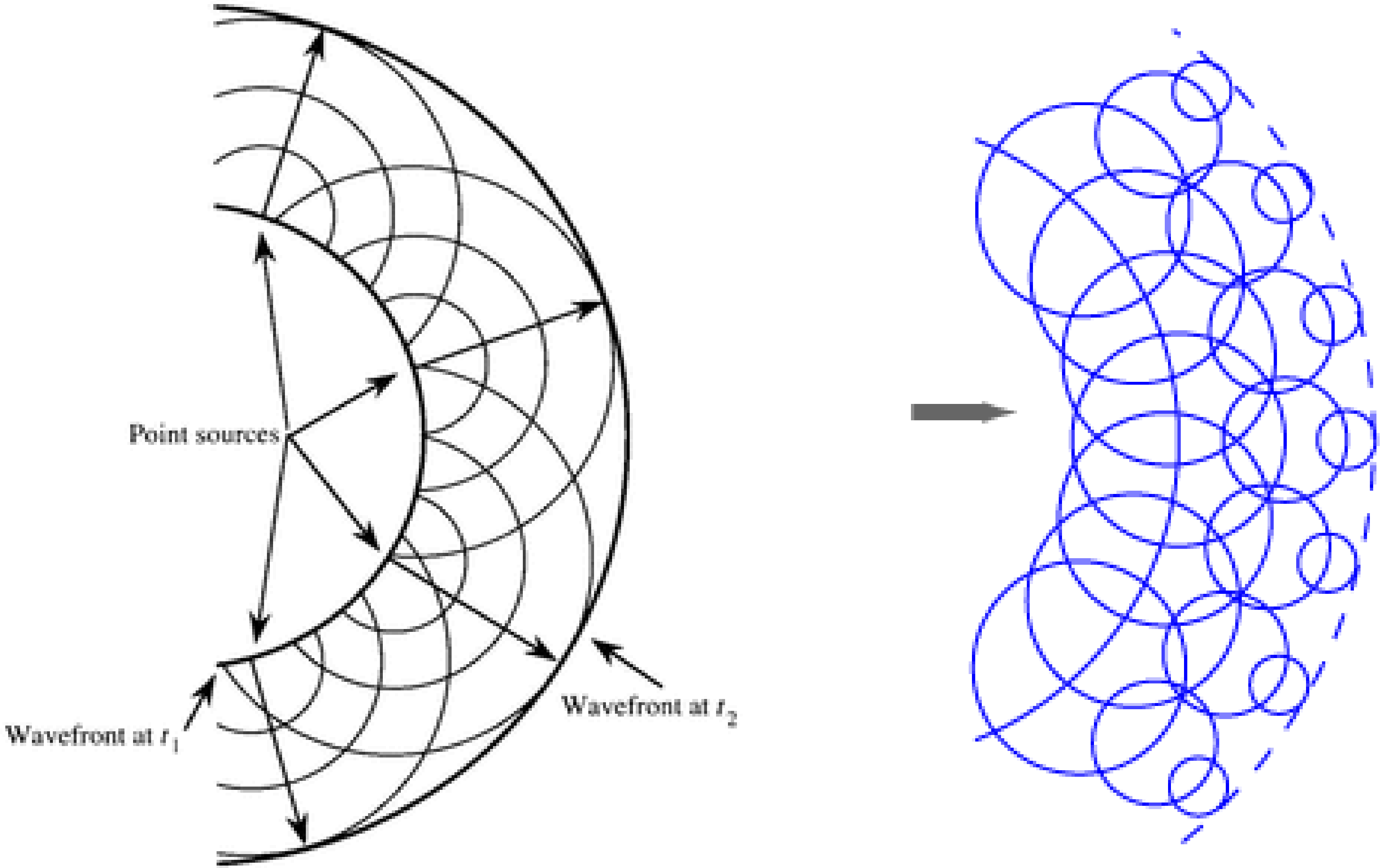


Christiaan **Huygens** (1629-1695)

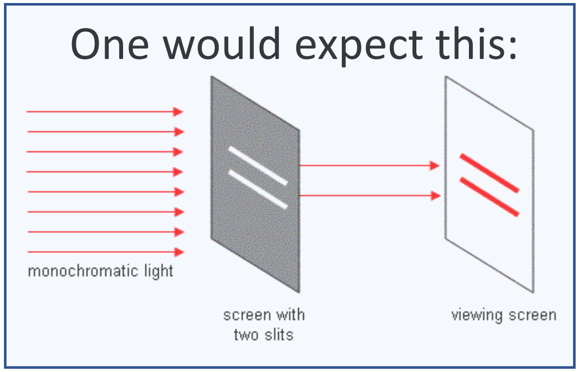
Augustin-Jean **Fresnel** (1788–1827)

Wavefront: a surface containing points affected in the same way by a wave at a given time.

Huygens principle



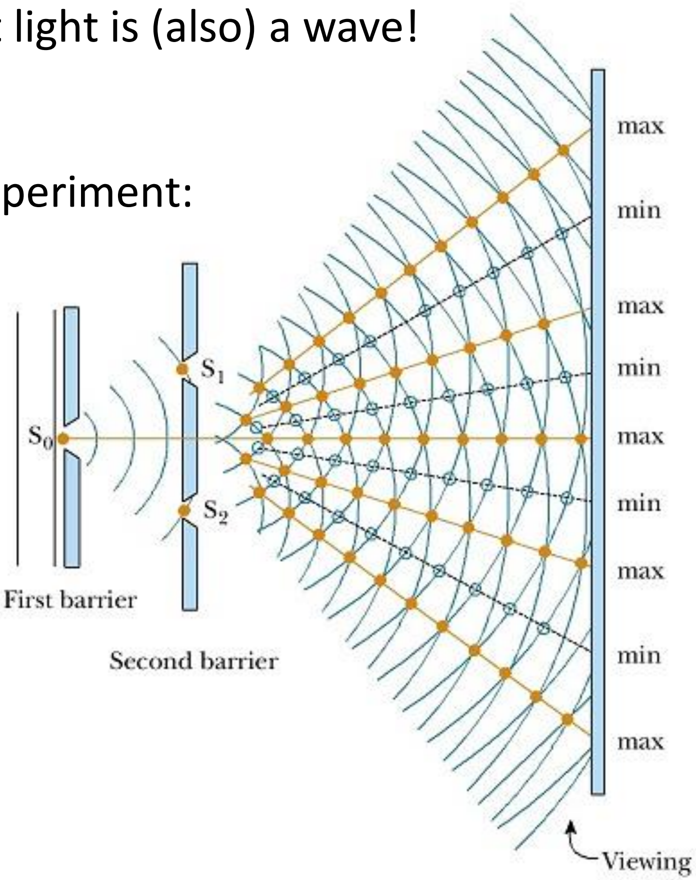
Some experiments, observations, which can only be understood with the help of wave theory.



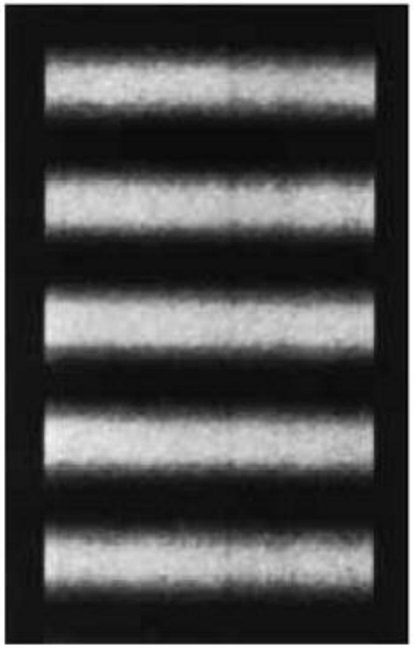
Young's two-slit experiment
Diffraction on grating
Interference patterns

this proves that light is (also) a wave!

The two-slit experiment:

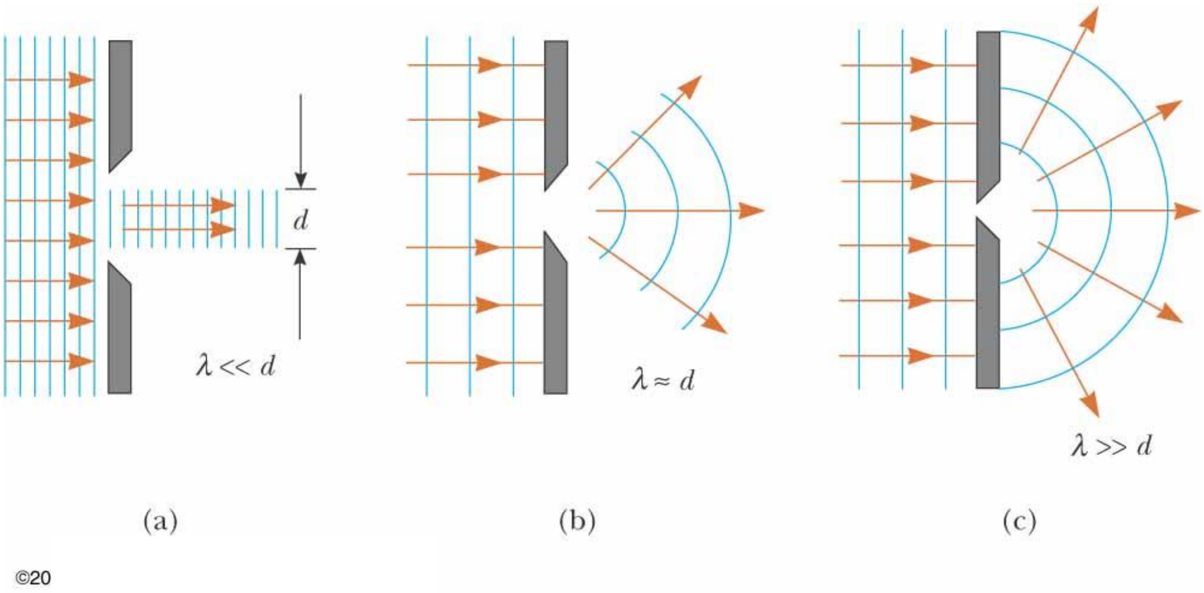


(a)

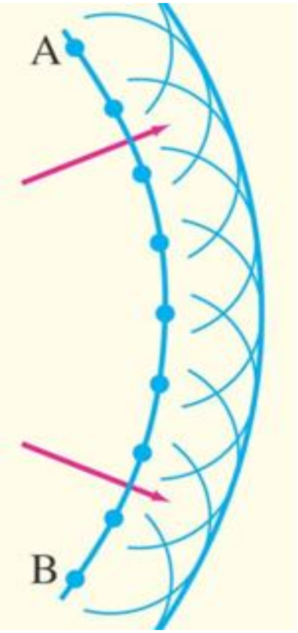

(b)


Thomas Young

Diffraction



Huygens' principle: every point in a propagating wavefront serves as the source of spherical secondary wavelets, such that the wavefront at some later time is the envelope of these wavelets.



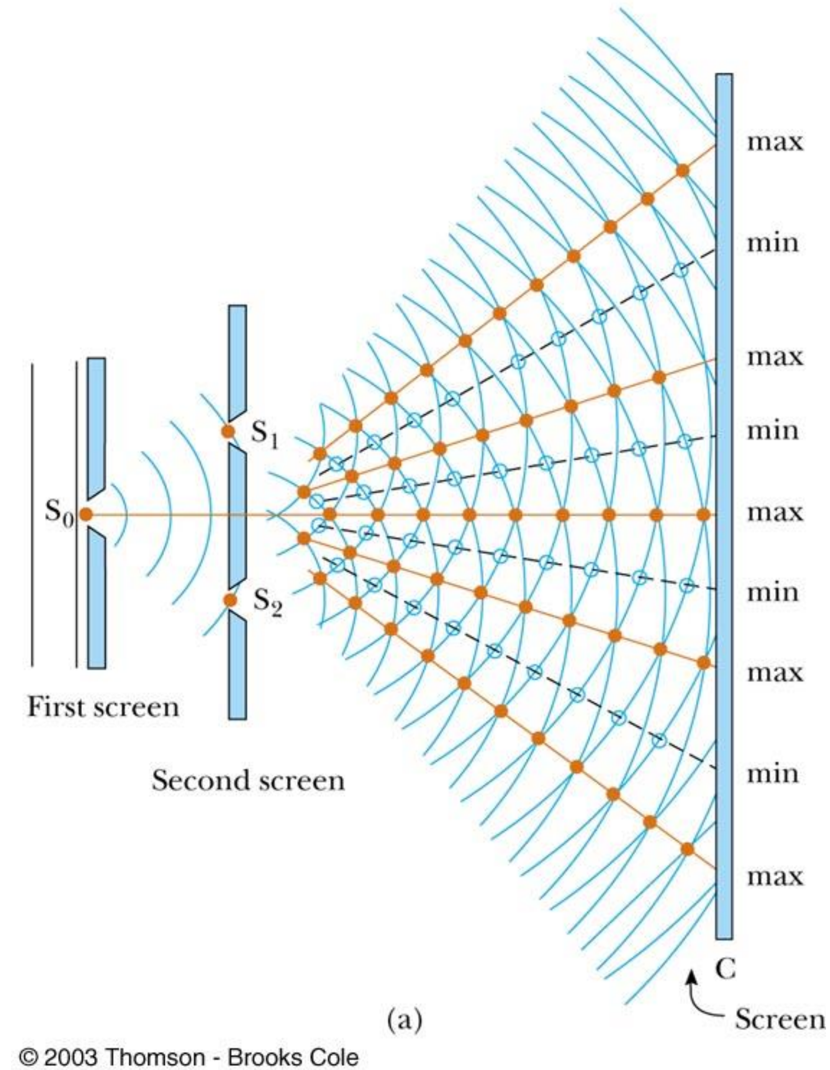
Interpretation of Thomas Young's double-slit experiment

S_1 and S_2 slits are wavesources

Two waves from S_1 and S_2 originates from the same wavefront, that is they are in the same phase.

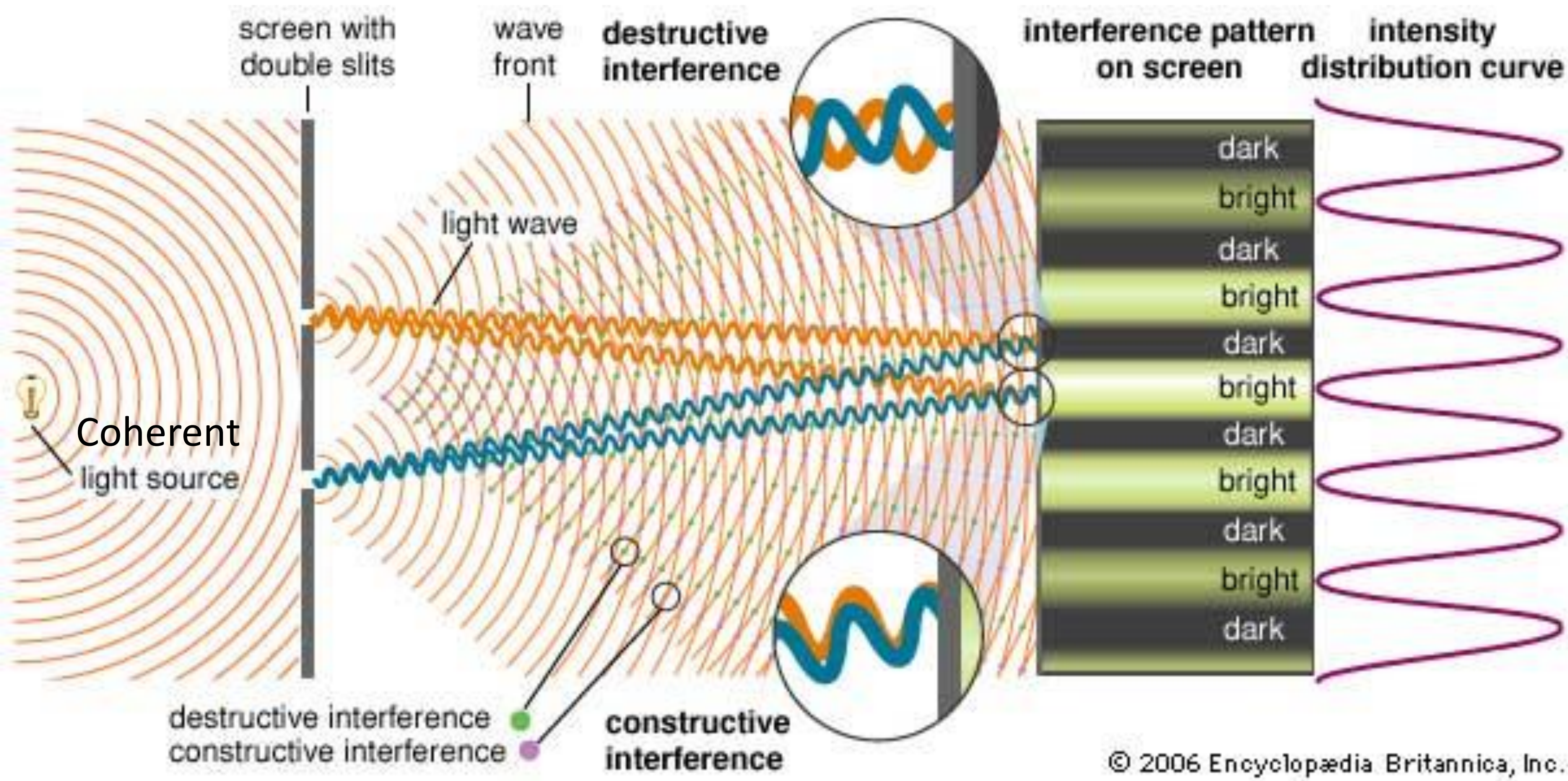
⇊

interference

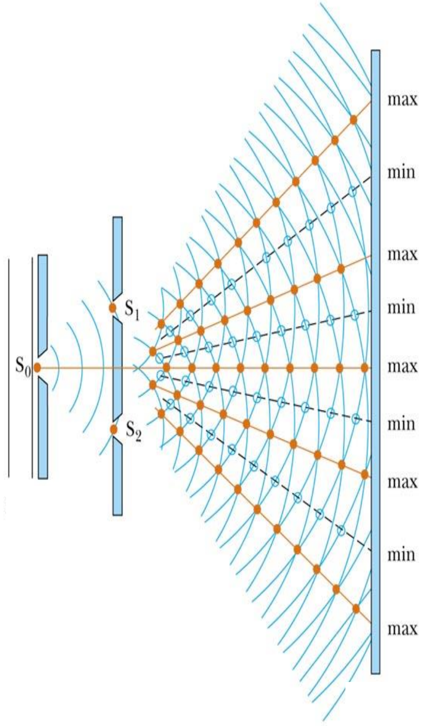


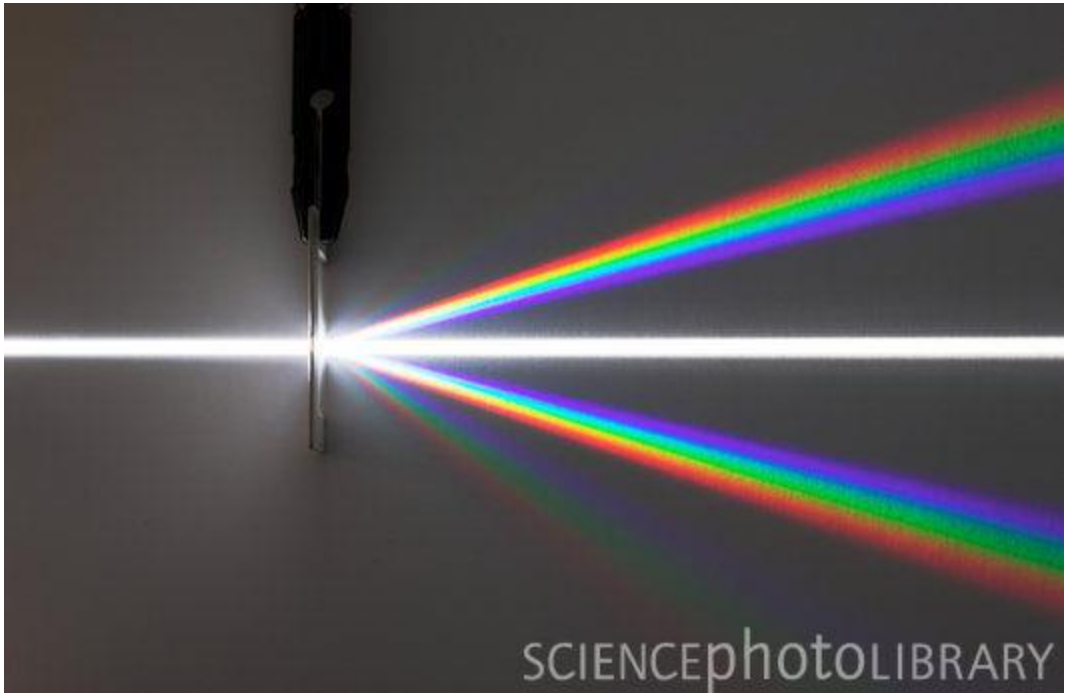
Interference fringes on a screen

The explanation of the periodic profile : interference of waves, Huygens principle.

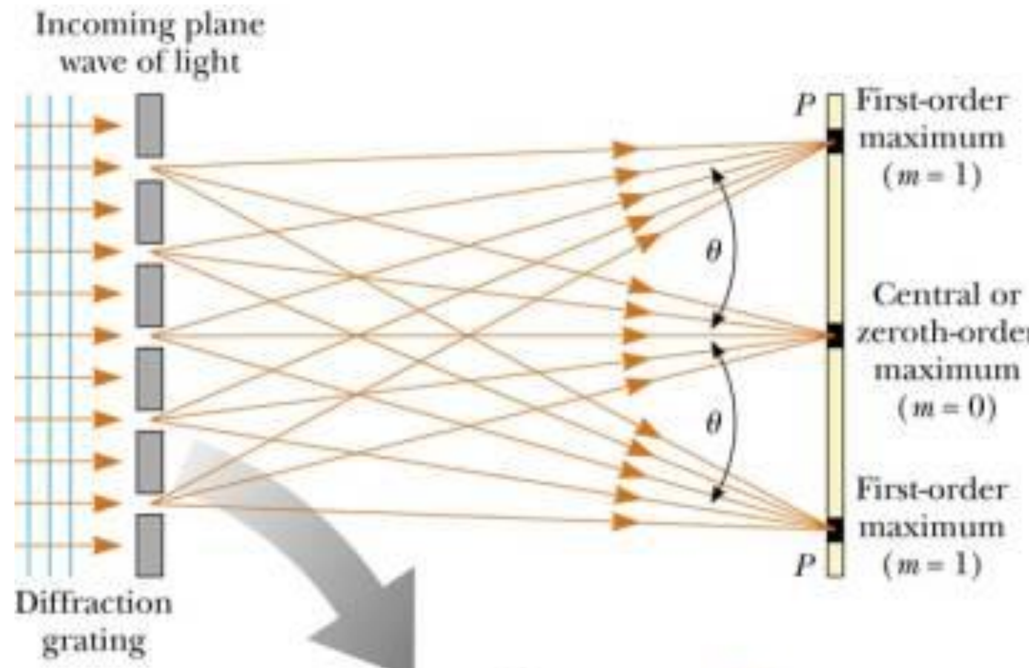


Dispersion of light by a diffraction grating



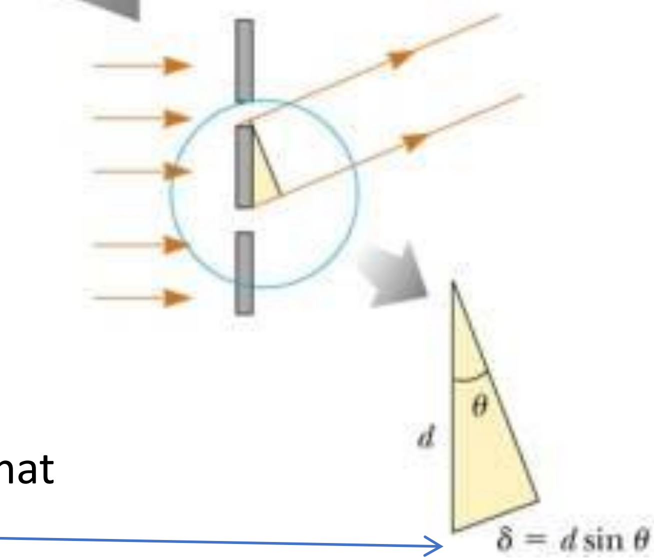


Diffraction on optical grating:

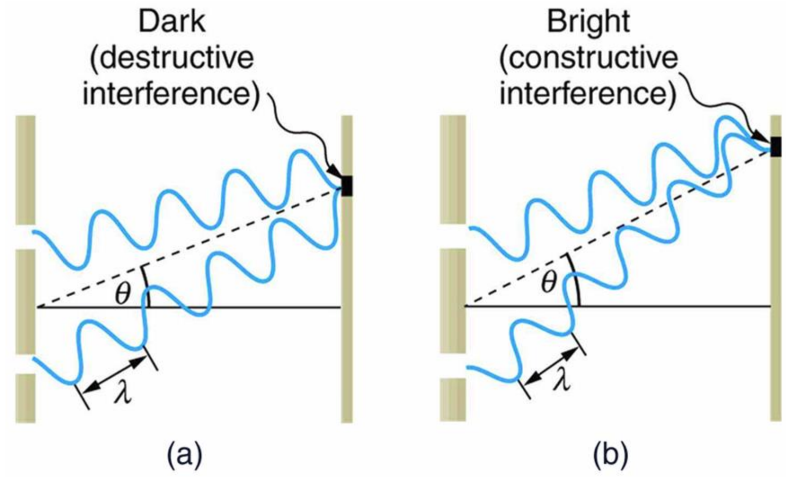


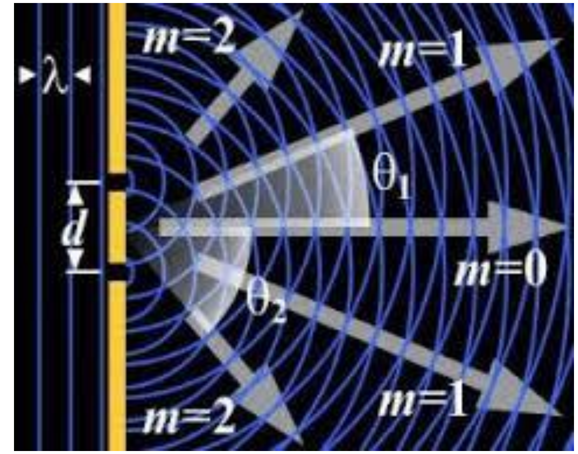
Due to the path difference of δ a phase difference of $2\pi*\delta/\lambda$ arises between waves.

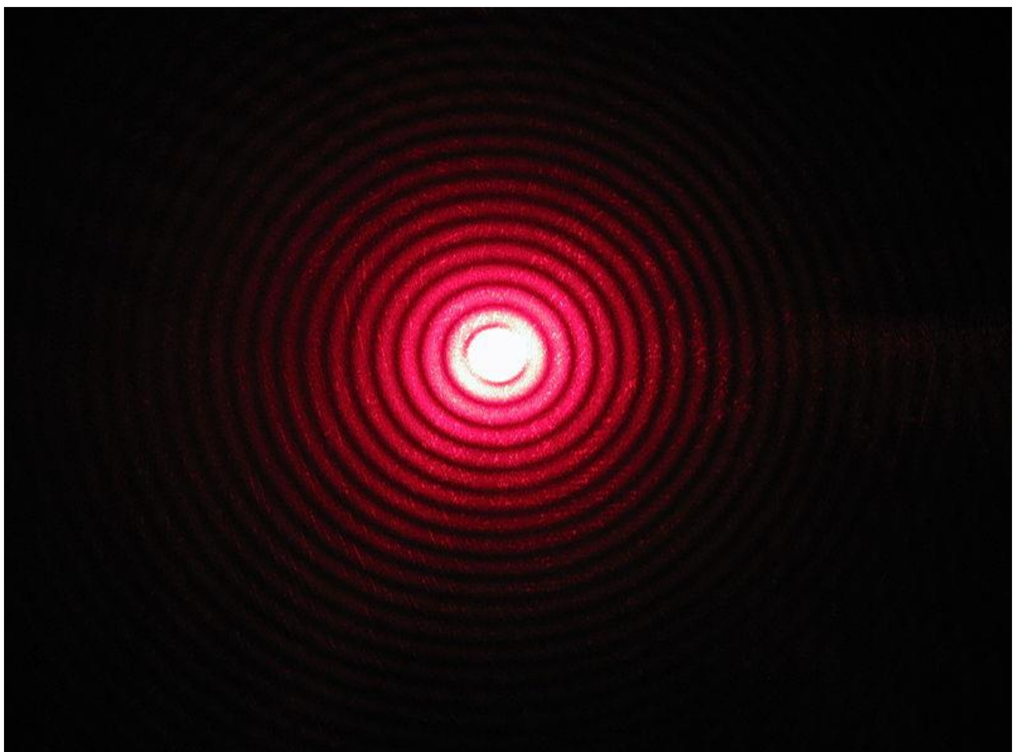
(n.B.: during a length of λ, the time it takes for the light to travel is T, under which the change in the phase is exactly 2π)

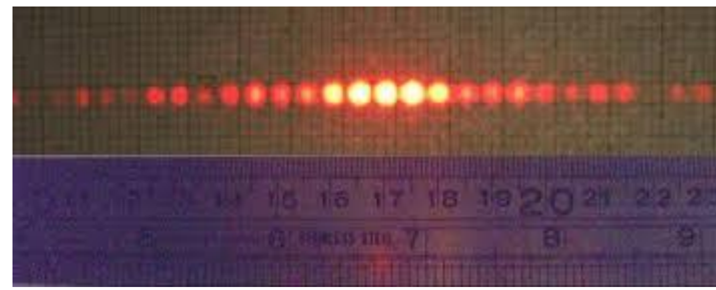


Constructive interference is only possible if the phase difference is 0,1,2,3,… * 2π. This means that δ has to be 0,1,2,3,… * λ

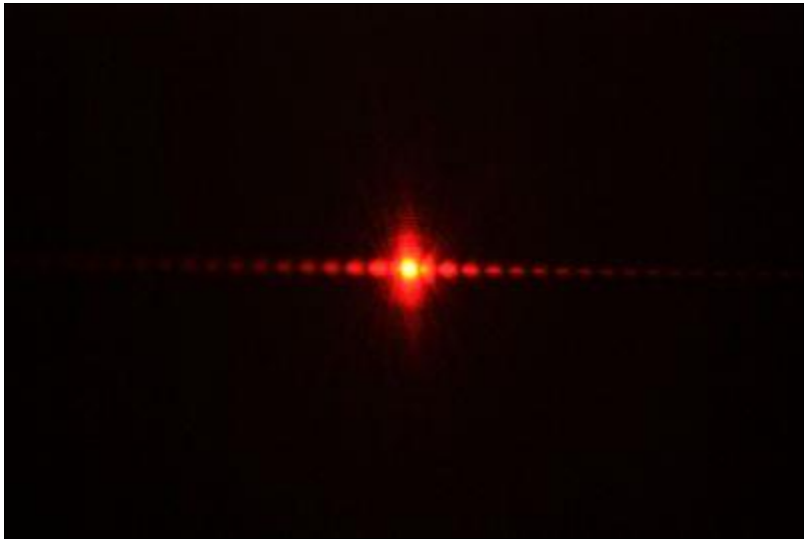

Dark
(destructive
interference)
Bright
(constructive
interference)
θ
λ
(a)
(b)

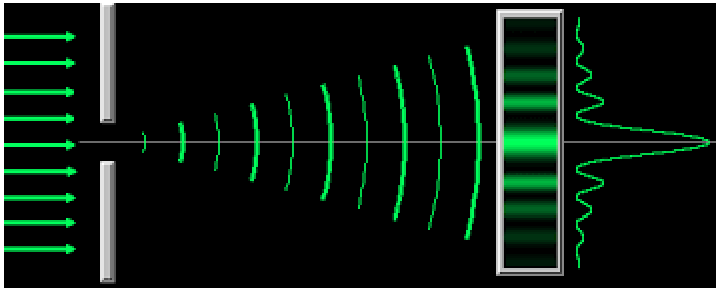

λ
d
$m=2$
$m=1$
θ_1
$m=0$
θ_2
$m=1$
$m=2$

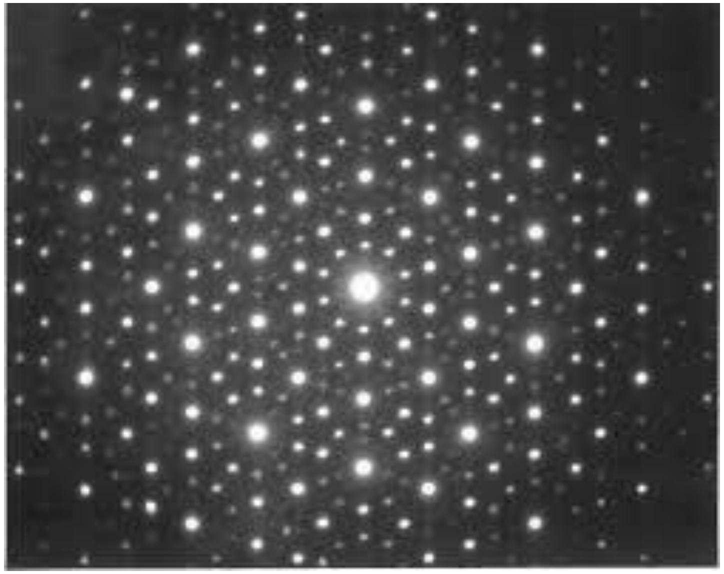


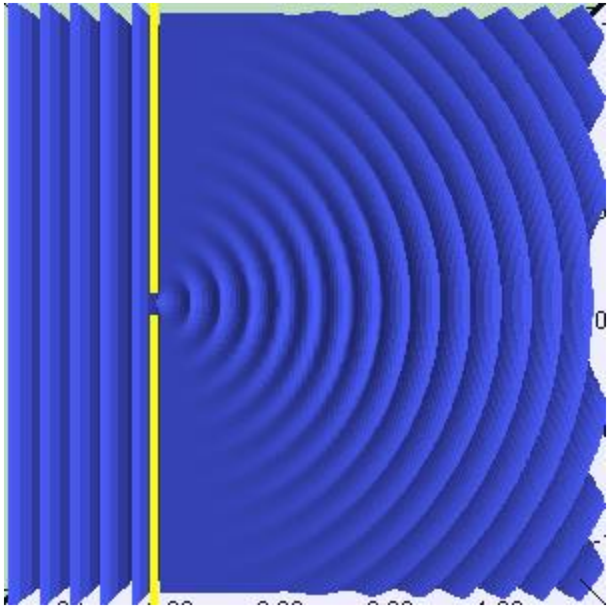


Diffraction and interference patterns with coherent light

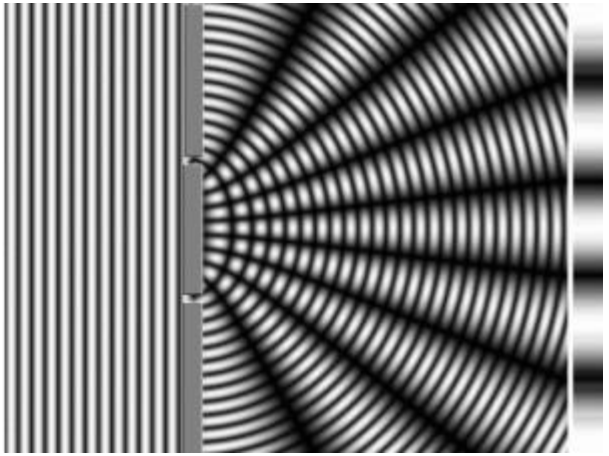


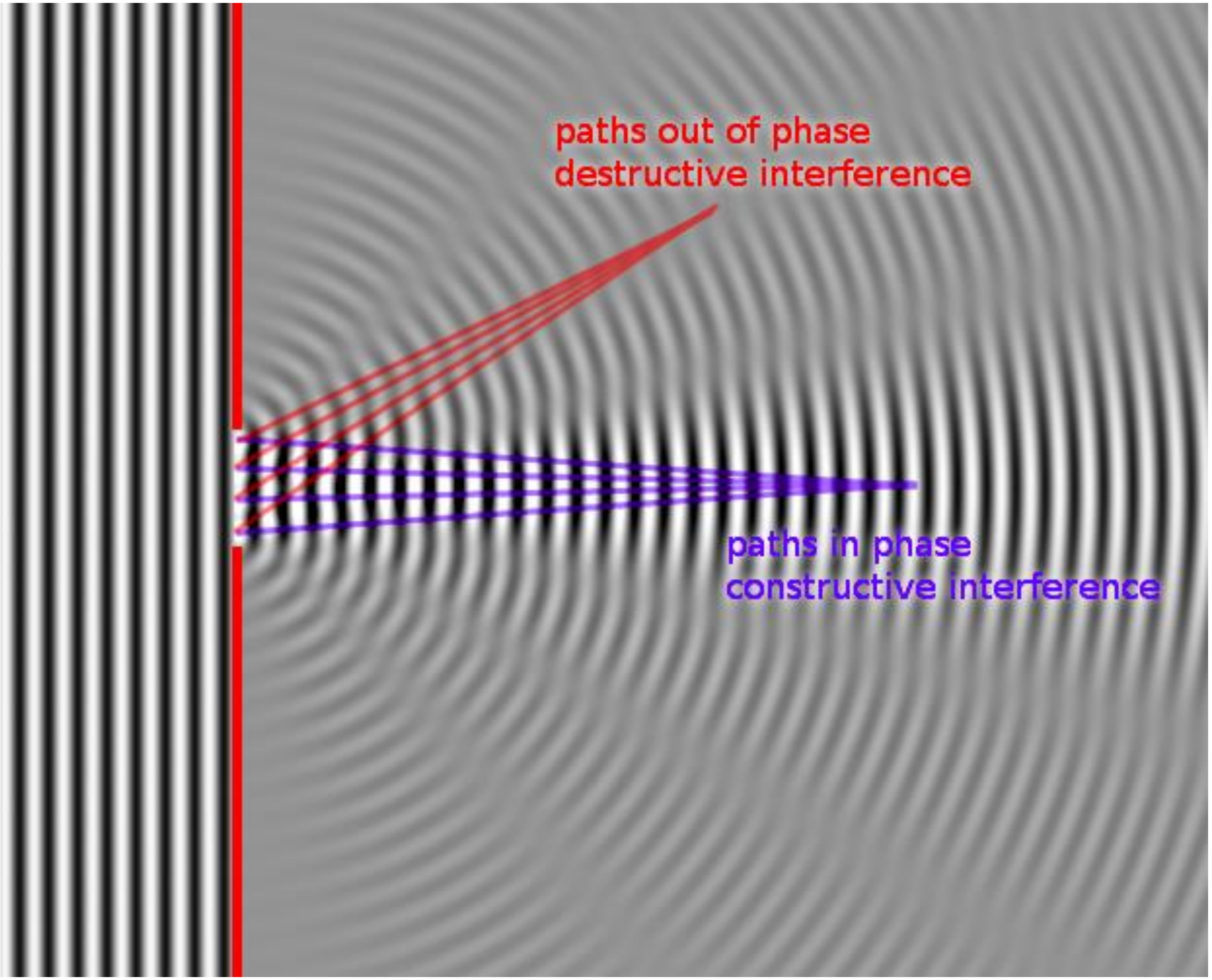






To understand the patterns we need to calculate the phase for each wave at the screen.

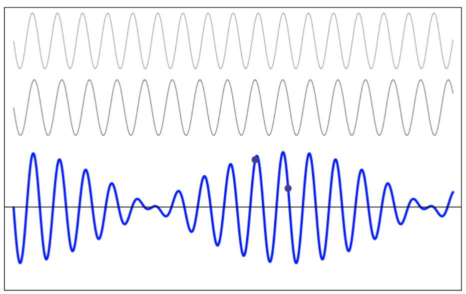


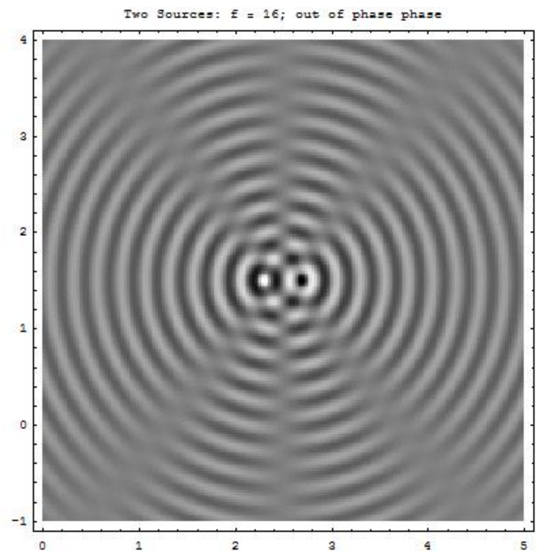


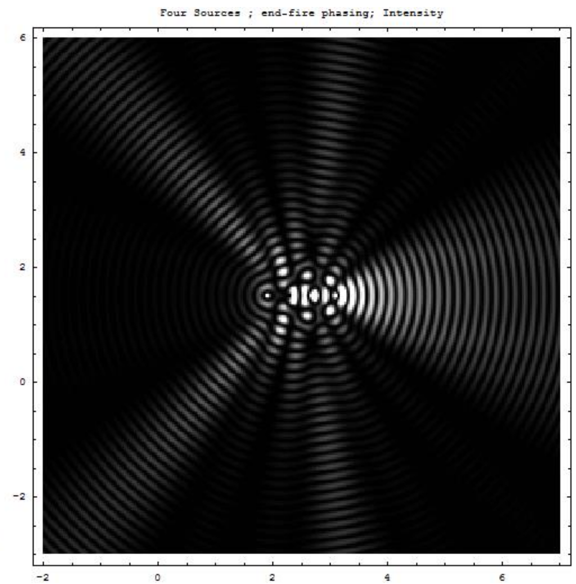
Superposition of waves:

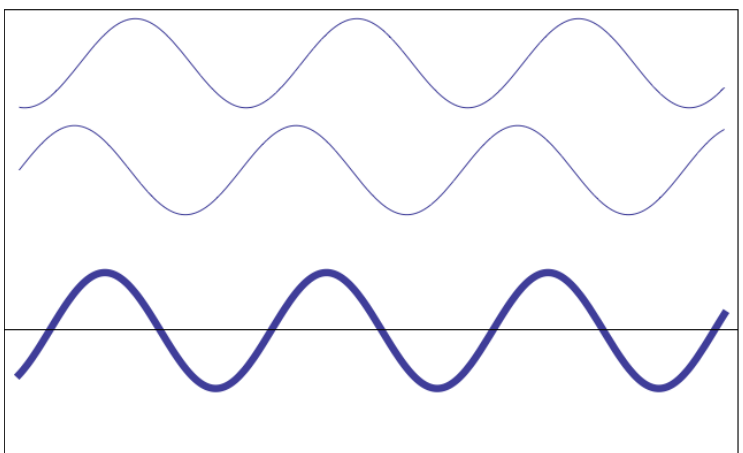
the “displacements” caused by each individual waves add up at every point

$$u(x,t) = A_1 * \sin(k_1*x + \omega_1*t + \phi_1) + A_2 * \sin(k_2*x + \omega_2*t + \phi_2) + \ldots$$



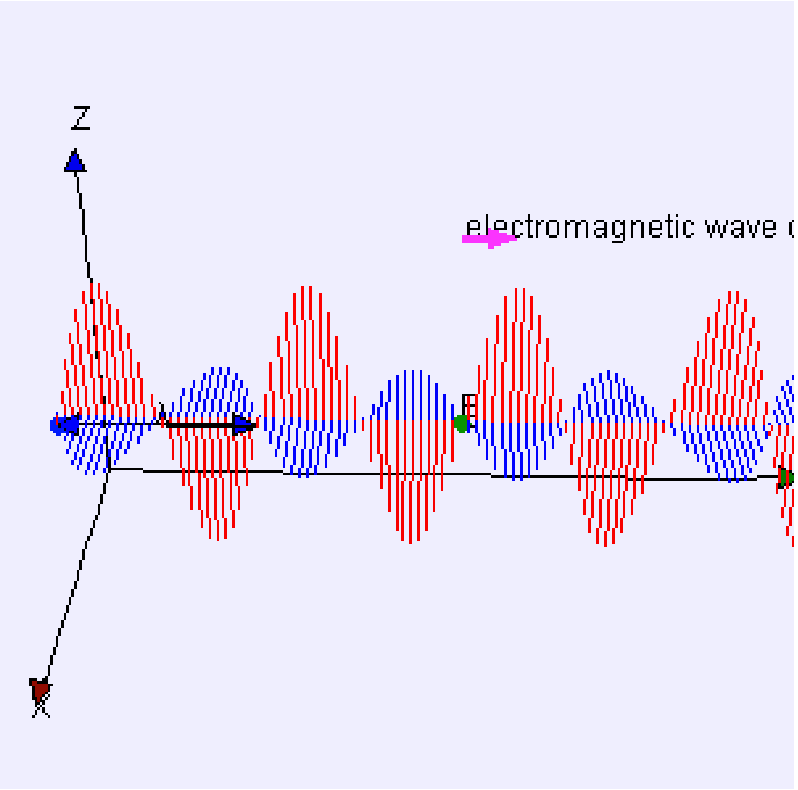


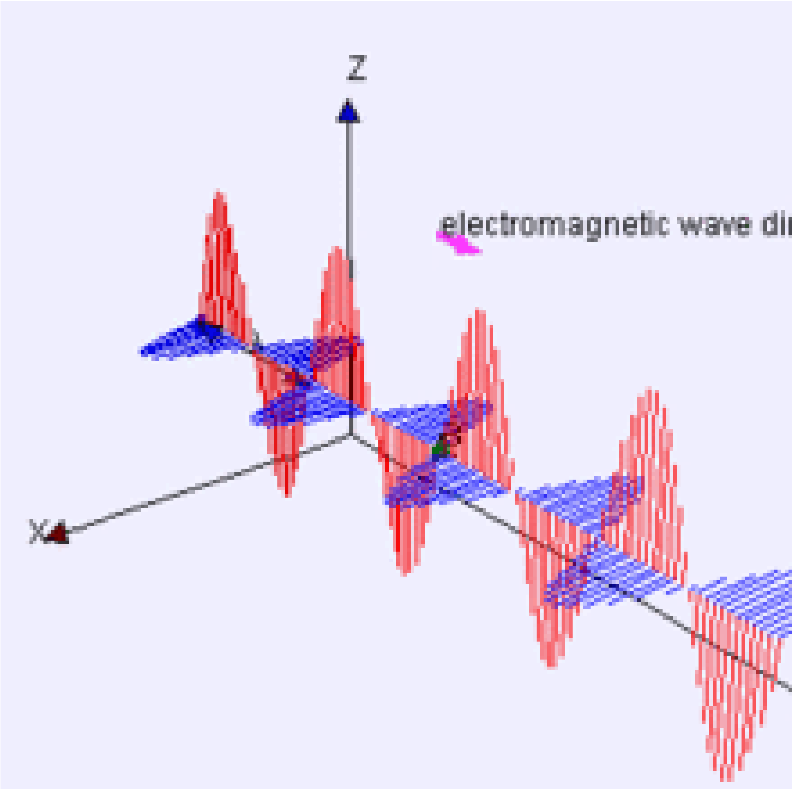




https://www.acs.psu.edu/drussell/demos/superposition/superposition.html

Since we have interference, we must assume light is a wave.
If so, then we have a wave equation for it. Since it is electro-magnetic, we have two oscillating quantities: electric field strength (**E**) and magnetic field strength (**B**).





For EM waves we have two equations, and the wave can travel to x,y,z directions, so the equations are a bit even more complicated:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$
$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

Here the ∇ means the $d^2/d...^2$ in all directions

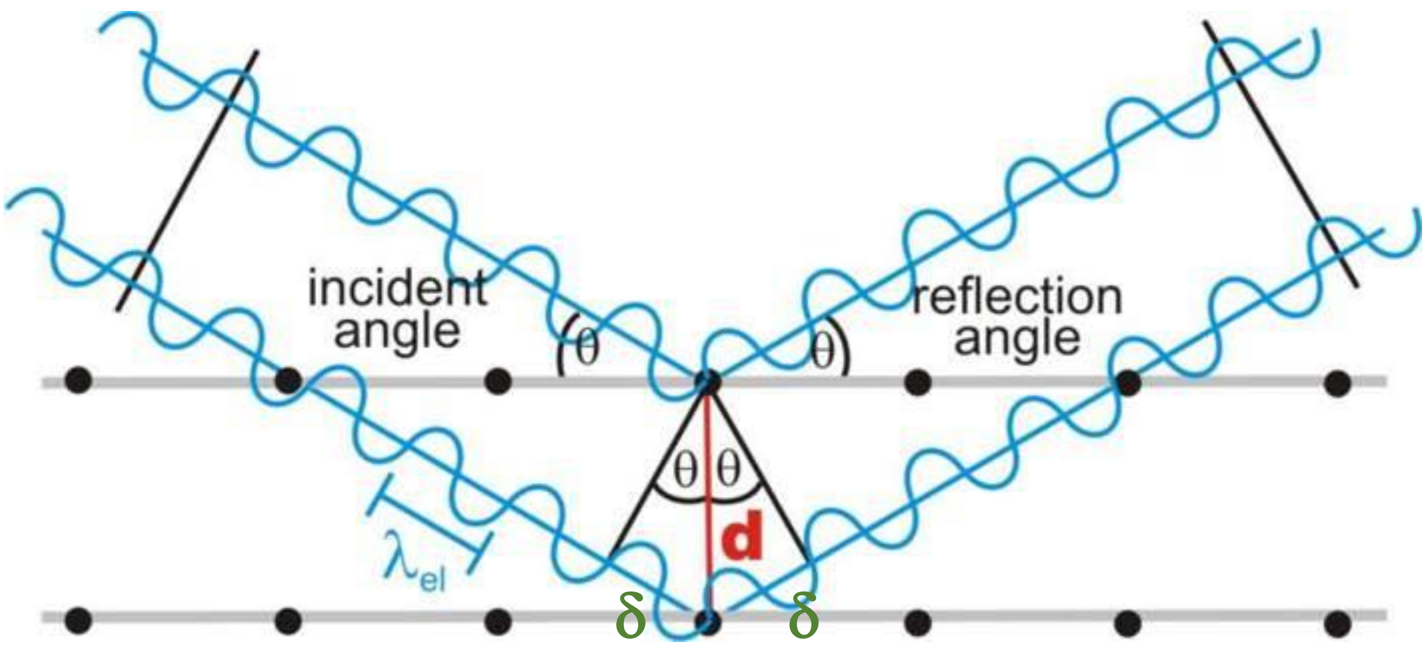
The solution is again a sine or cosine wave:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$
$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$

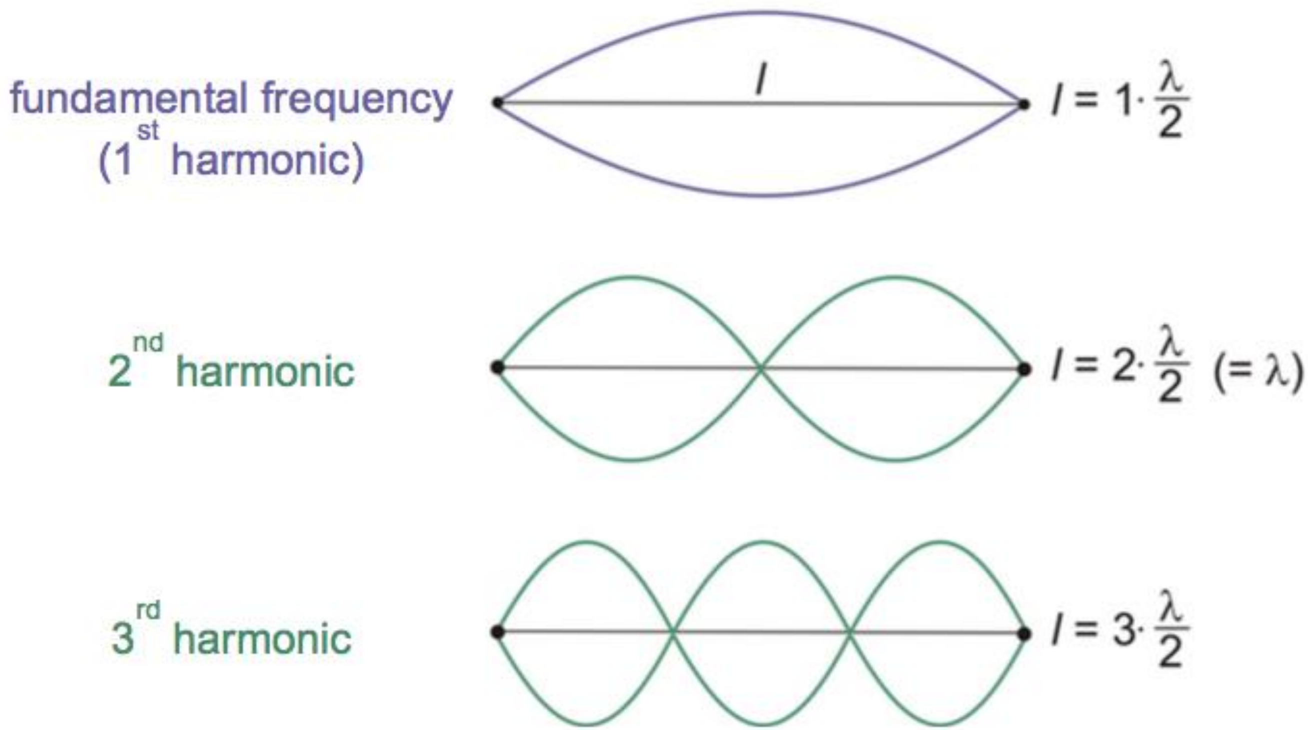
At any point of the observation we have to add all of the incoming sine waves, and that gives the net value of **E** and **B**.

Remember: incoherent waves add up to practically 0, while coherent ones can add up from 0 to a maximum, depending on the phase difference.

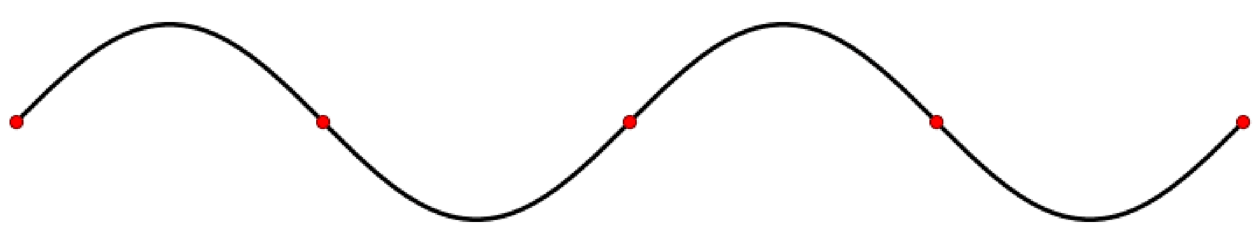
On a reflection grating the concept is the same, but the phase difference (due to **path difference 2*δ**) is twice as much as on a transmission grating.

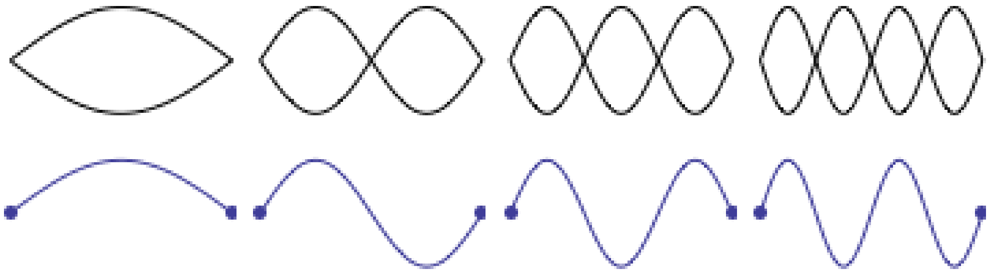


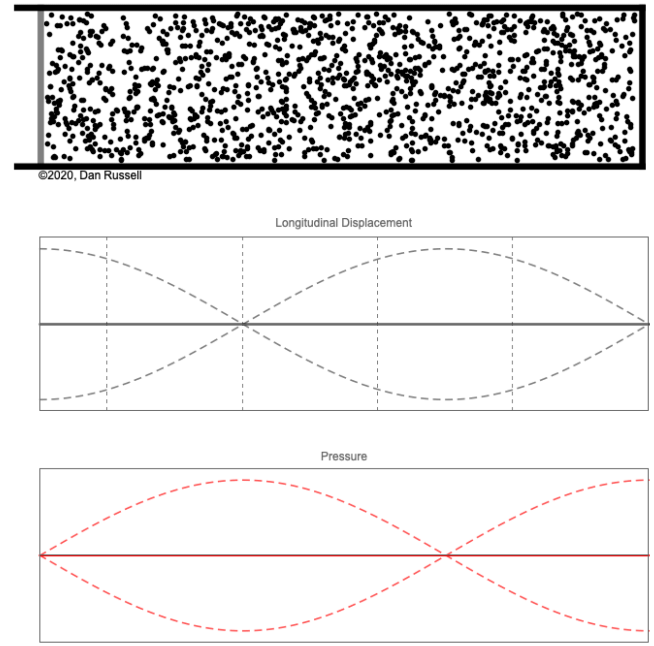
standing waves

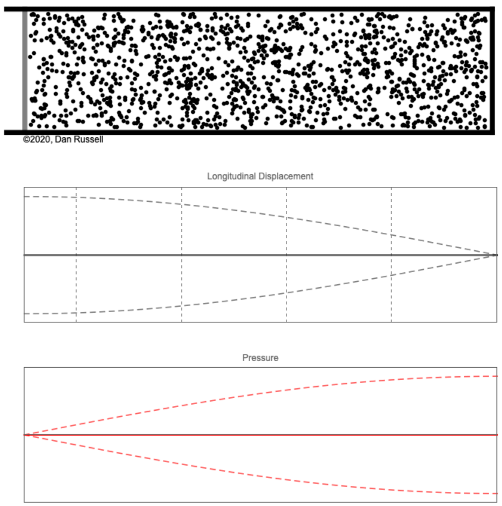


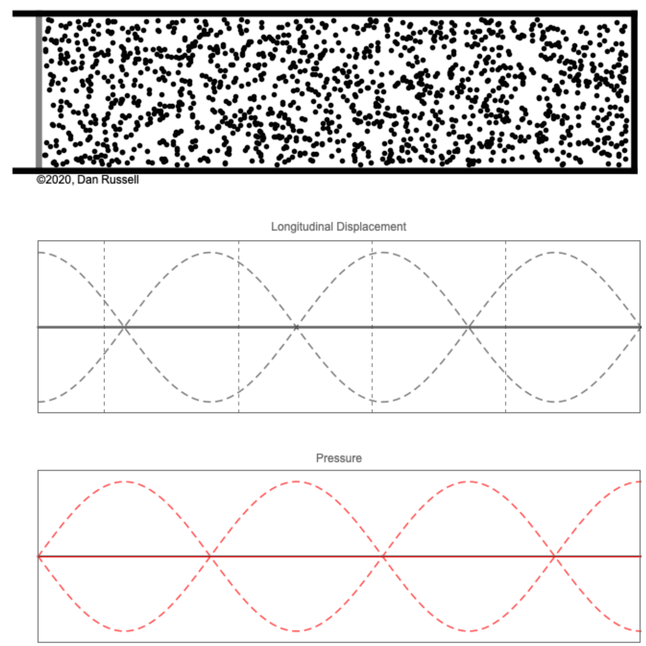
$$l = k \cdot \frac{\lambda}{2} \quad (k = 1, 2, 3, \ldots)$$






©2020, Dan Russell
Longitudinal Displacement
Pressure


©2020, Dan Russell
Longitudinal Displacement
Pressure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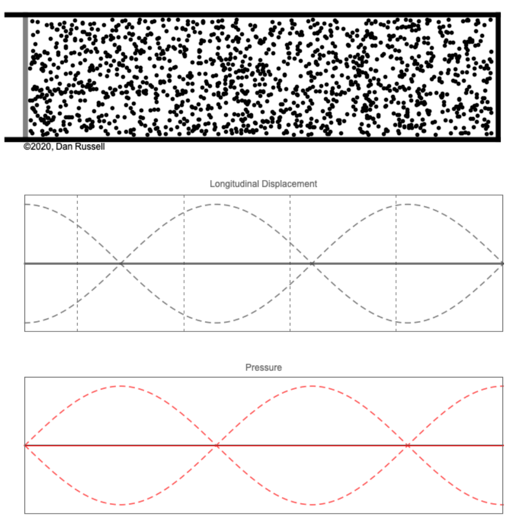
Longitudinal Displacement
Pressure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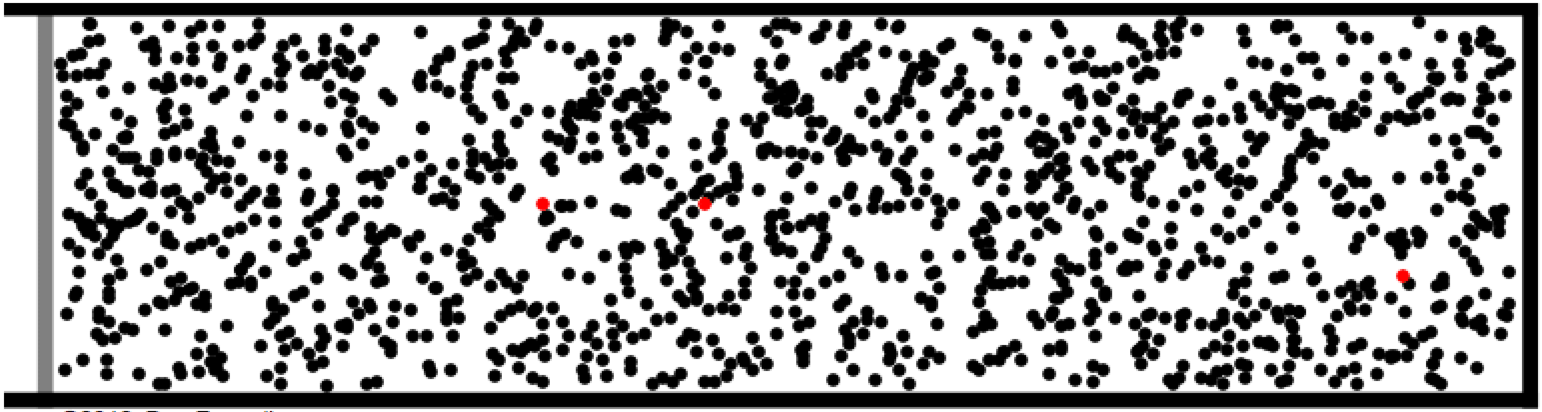
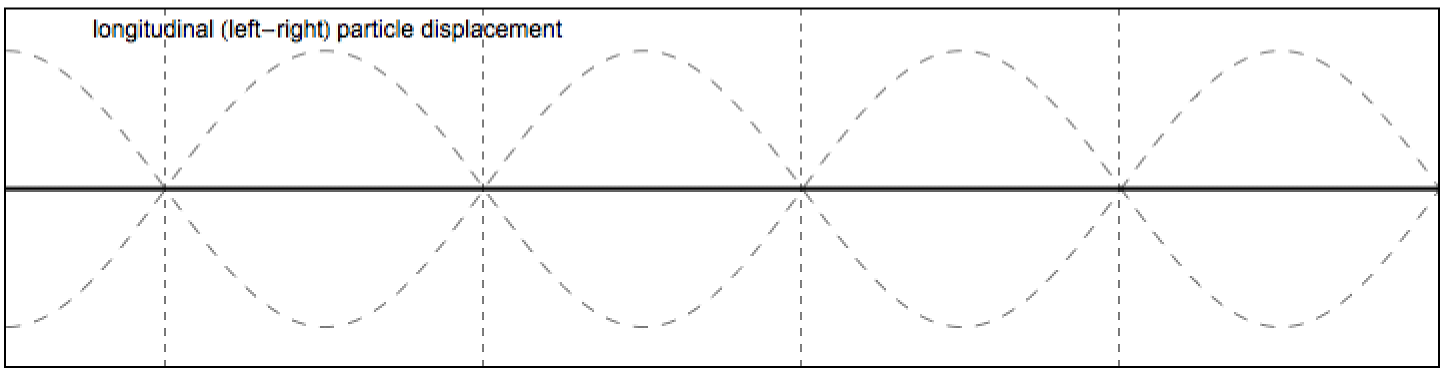
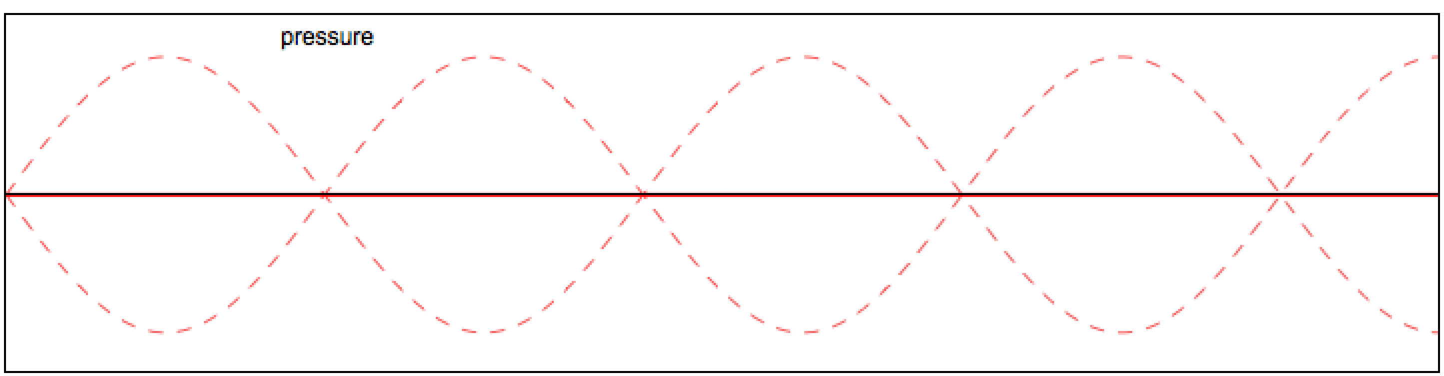
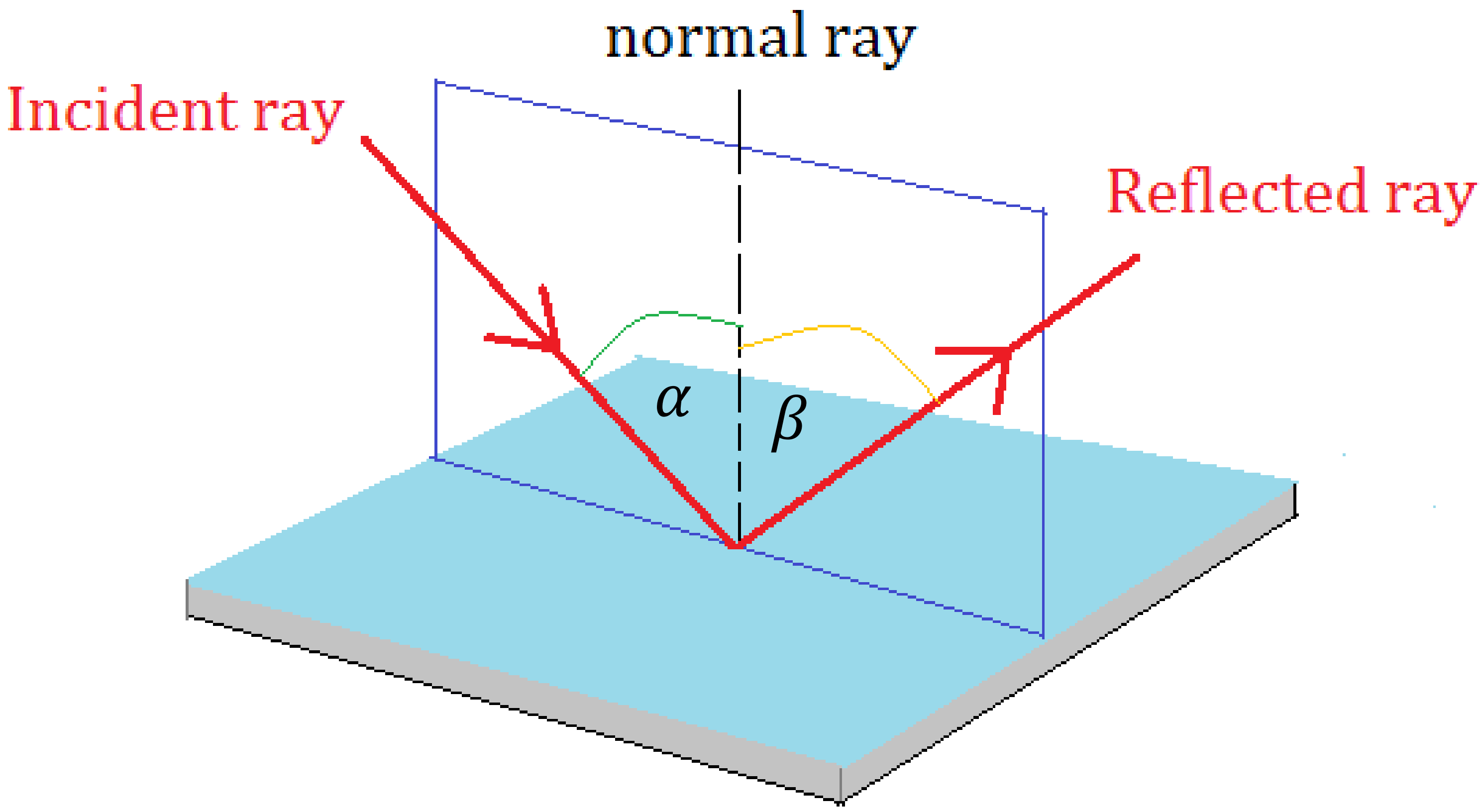
Longitudinal Displacement
Pressure



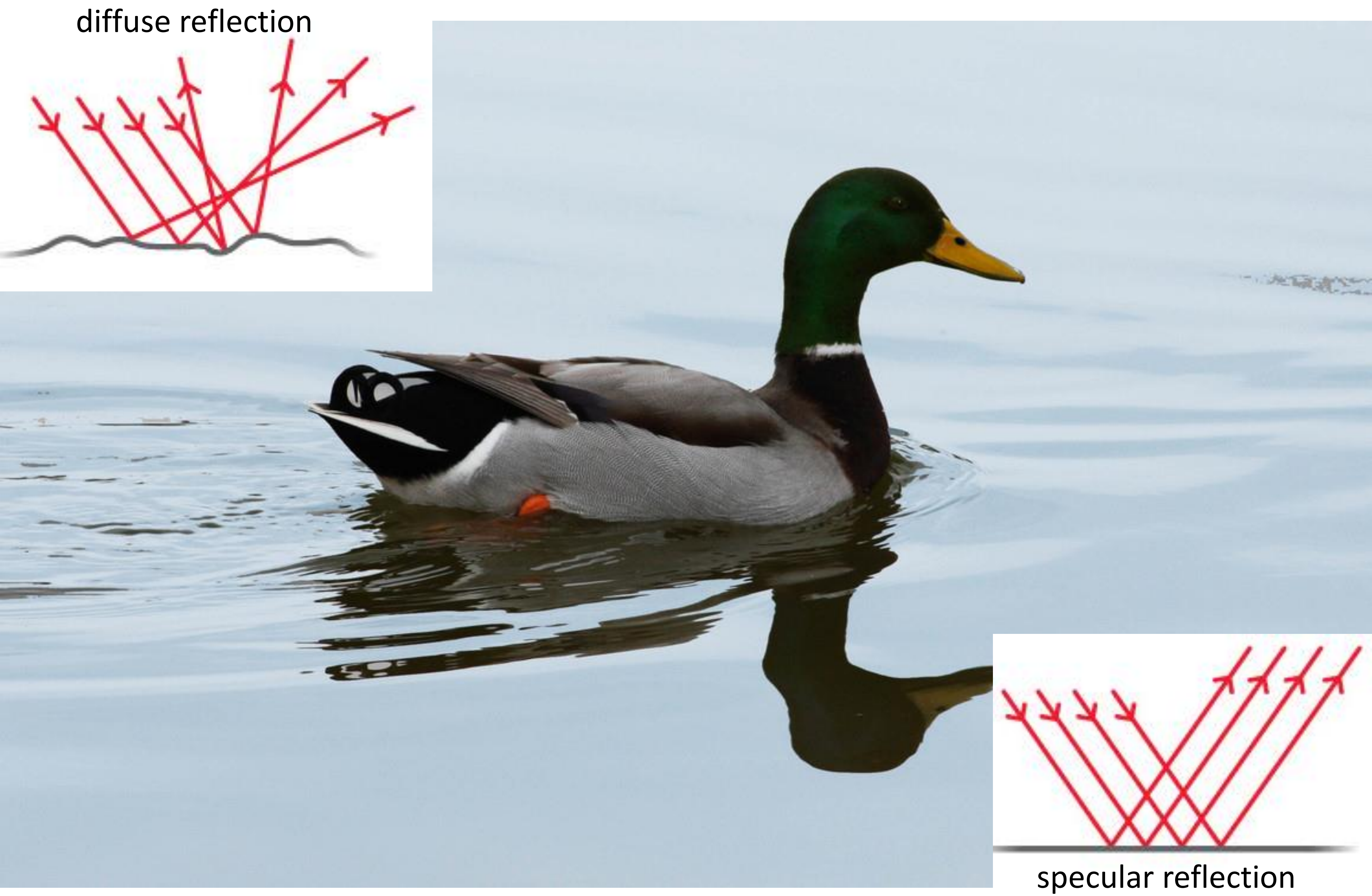

longitudinal (left–right) particle displacement


pressure

Reflection



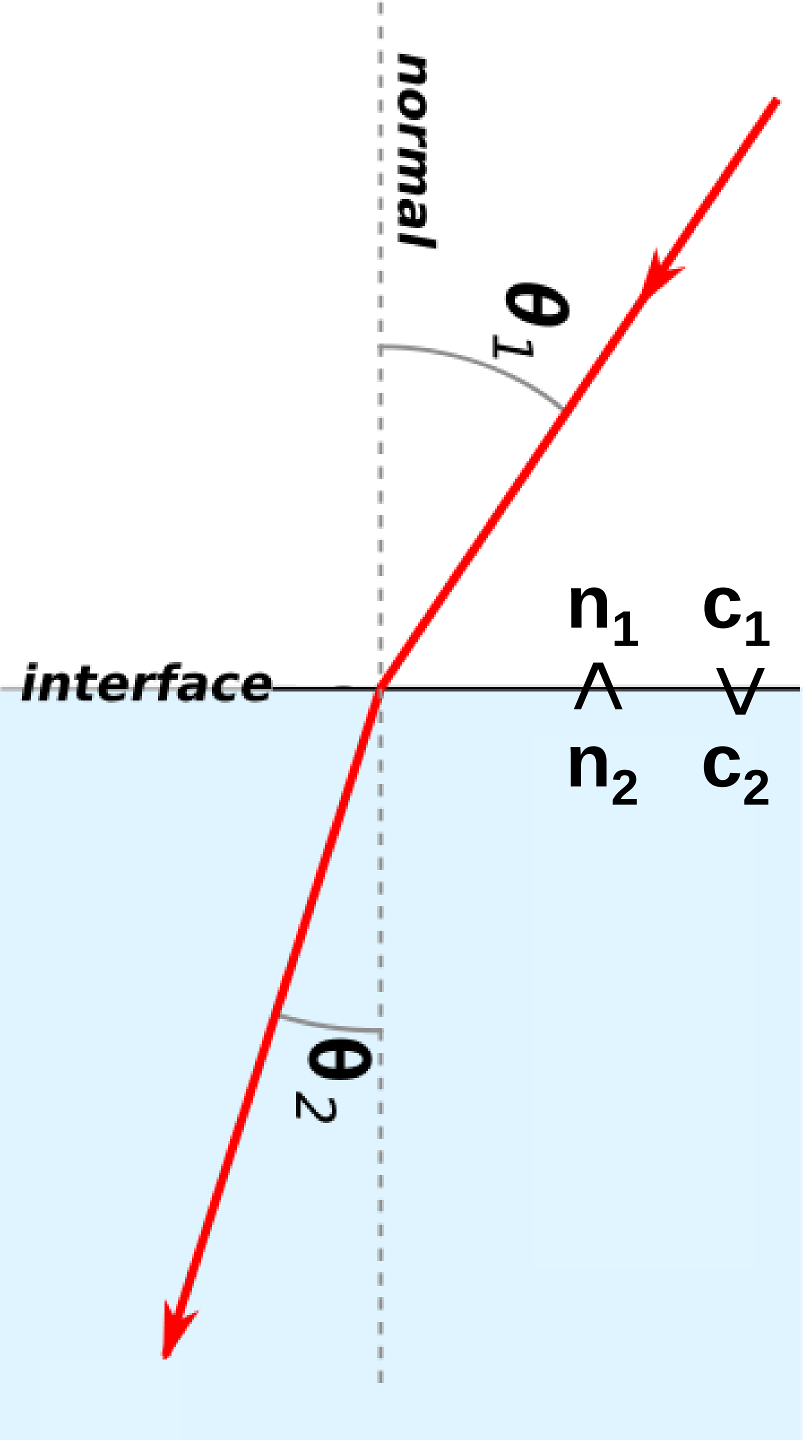
$$\alpha = \beta$$


diffuse reflection
specular reflection

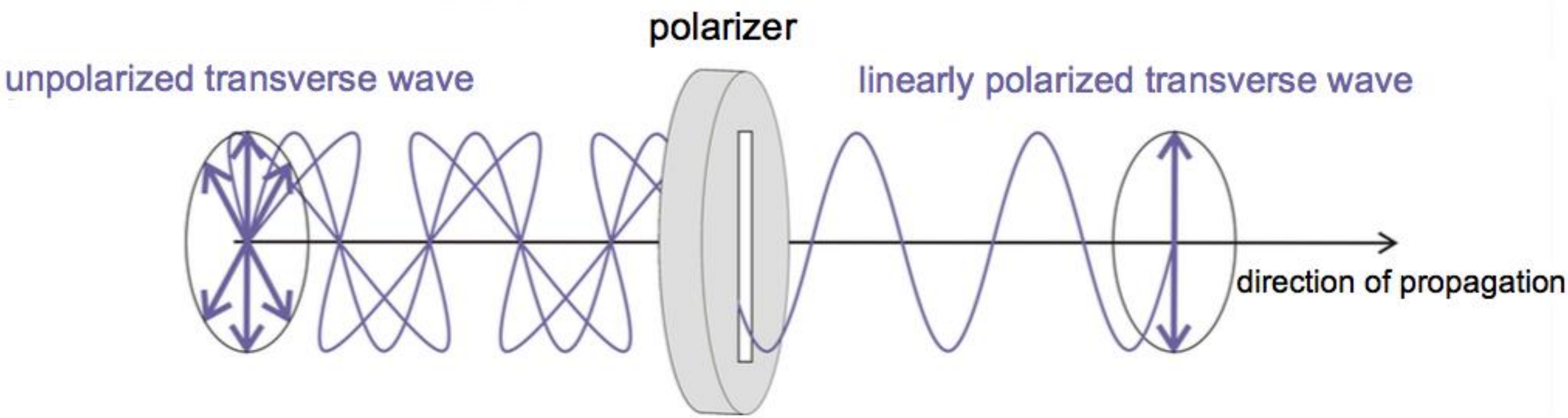
Refraction

$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

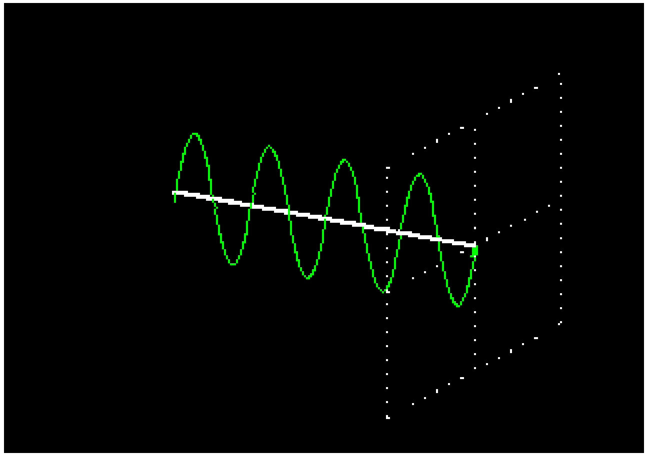
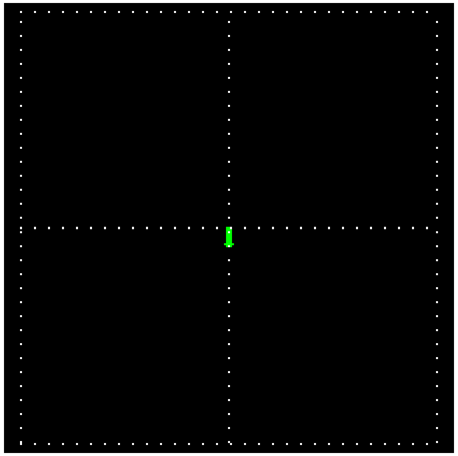
$$\sin \alpha \cdot n_1 = \sin \beta \cdot n_2$$



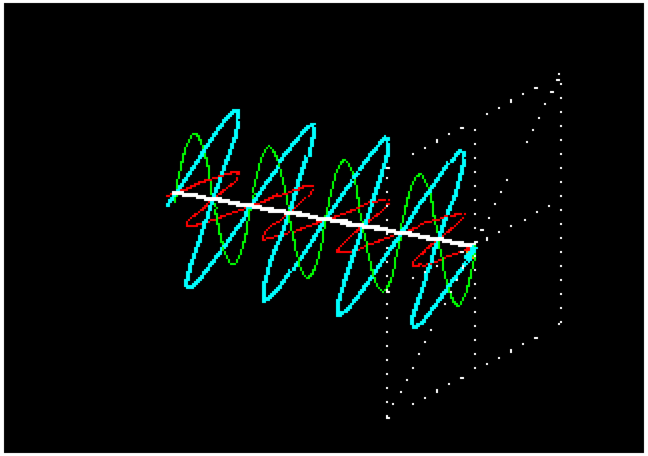
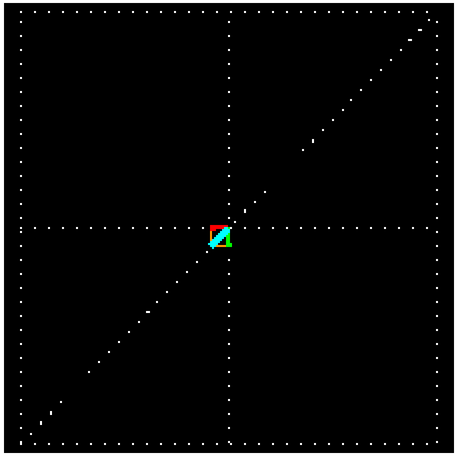
Polarization



usually we draw the E-field only, since the B-field is bound to it.
This makes the graphics simpler.

Summation of two waves with different E field direction

Soundwaves

Range of sounds

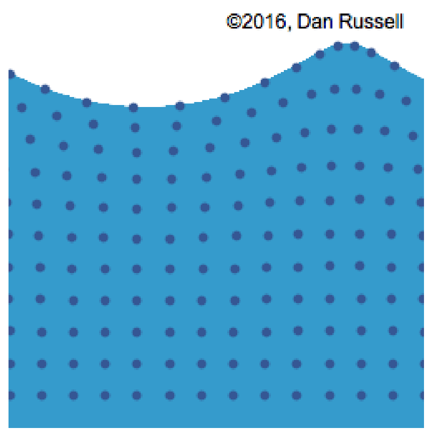
sound range	infrasound	audible sound	ultrasound	hypersound
frequency (Hz)	< 20	20–20 000	20 000–10^9	10^9 <

Speed of sound in various media

medium	c_{sound} (m/s)
air (0ºC, 101 kPa)	330
helium gas (0ºC, 101 kPa)	965
water (20ºC)	1483
fatty tissue	1470
muscle	1568
bone (compact)	3600
iron	5950

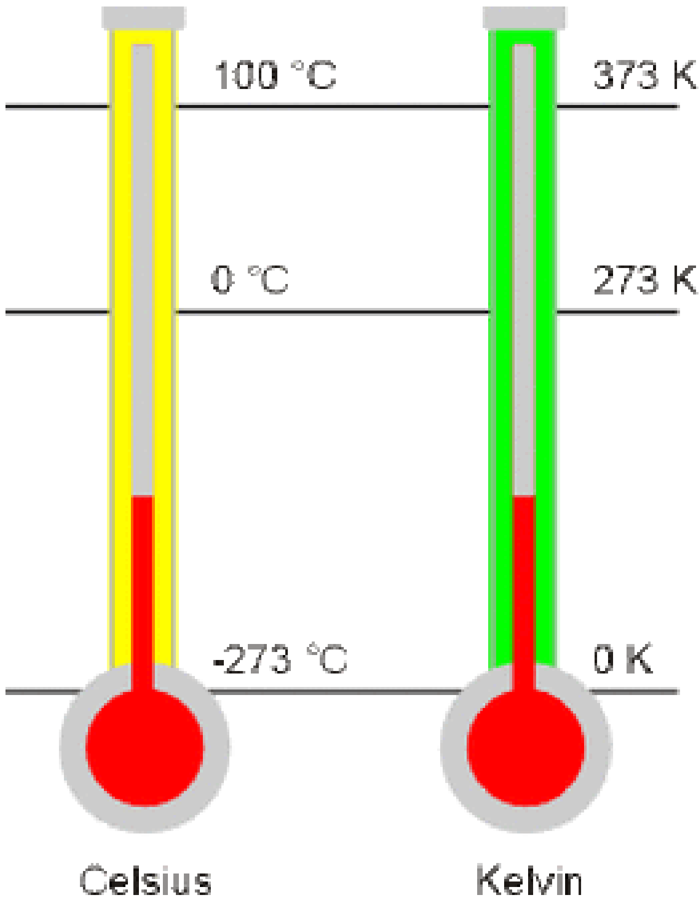
Problems: 8/4 and 8/10

4. Waves are propagating on the surface of water towards the shore with a velocity of 1.5 m/s. The distance between two neighboring crests is six meters. There is a piece of wood somewhere further in the water that turns up and disappears periodically as the water waves when you are looking at it from the shore. Calculate the time interval between two turn-ups.

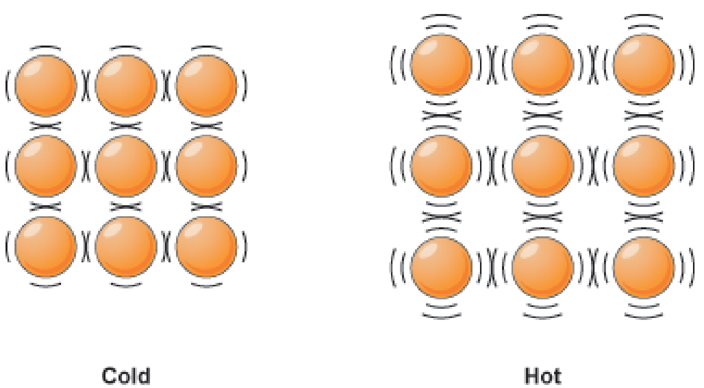


10. A sound wave arrives from air (0ºC) at the water surface (20ºC). Angle of incidence is 10º. Calculate the angle of refraction!

Termodynamics



$$\frac{T}{\mathrm{K}} = \frac{t}{^\circ\mathrm{C}} + 273 \qquad \frac{t}{^\circ\mathrm{C}} = \frac{T}{\mathrm{K}} - 273.$$



Heat capacity (C) and specific heat capacity(c)

$$C = \frac{Q}{\Delta T} = \left[\frac{J}{K}\right]$$

$$c = \frac{C}{m} = \left[\frac{J}{kg \cdot K}\right]$$

$$Q = c \cdot m \cdot \Delta T$$

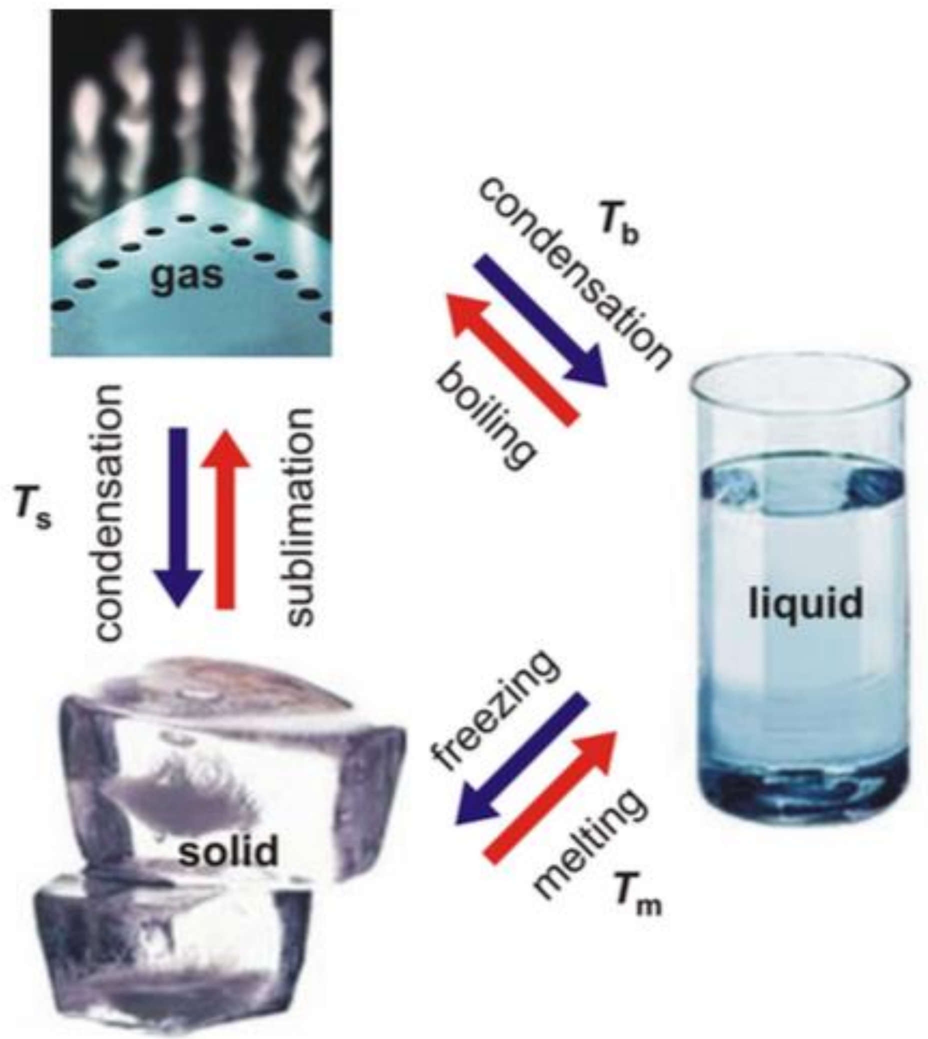
The specific heat capacity of some materials

material	*c* (J/(kg·K))
silver	234
glass	840
water	4180
body tissue (average)	3500

Phase transitions

Specific latent heat

$$L = \frac{Q}{m} = \left[\frac{J}{kg}\right]$$



Gas Laws

Boyle's Law $pV = constant_I$

Charle's Law. $\frac{V}{T} = constant_{II}$

Gay-Lussac's Law $\frac{p}{T} = conatsnt_{III}$

Avogadro's Law $\frac{V}{N} = constant_{IV}$

$$\frac{p}{T} \cdot \frac{V}{N} = k_{III} \cdot k_{IV}$$

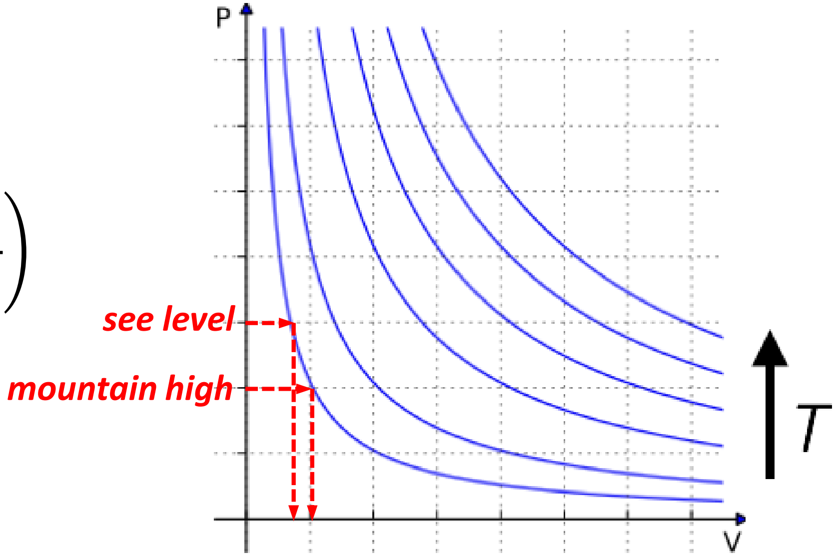
$$k_{III} \cdot k_{IV} = k_B = 1{,}38 \cdot 10^{-23}\, J/K$$

$$pV = Nk_BT$$

$$pV = \frac{N}{N_A} k_B N_A T$$

$$(k_B \cdot N_A = R) \qquad \left(n = \frac{N}{N_A}\right)$$

$$\boxed{pV = nRT}$$



isobaric process – **pressure** stays constant

isothermal process – **temperature** stays constant

isochoric process – **volume** stays constant

Problems: 9/7 and 9/12

7. We throw a 20 g 0 °C ice cube into a glass (2 dl) of warm (30 °C) water. What will be the final temperature after the ice melts? Conditions are same as in problem #6.

The specific heat capacity of some materials

material	*c* (J/(kg·K))
silver	234
glass	840
water	4180
body tissue (average)	3500

Specific latent heat of some materials

material	*L* (kJ/kg)
gold — *heat of fusion*	67
aluminum — *heat of fusion*	396
table salt (NaCl) — *heat of fusion*	517
ice — *heat of fusion*	334.4
water — *heat of vaporization (at 30 °C and 101 kPa)*	2400
water — *heat of vaporization (at 100 °C and 101 kPa)*	2257

12. A metal gas container is left lying under the shining Sun. The initial pressure of the ideal gas inside is 50 bar. Its temperature increases as a result of the sunshine from 12 °C to 72 °C. What will be the final pressure?