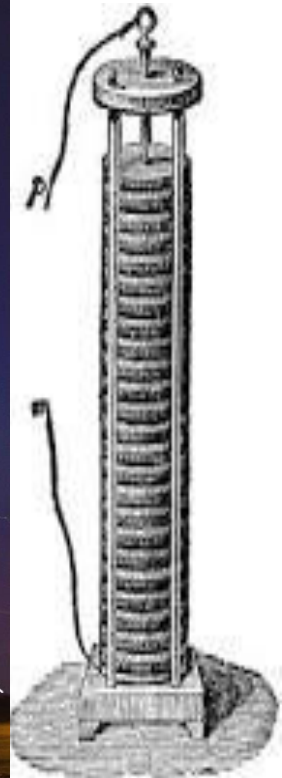
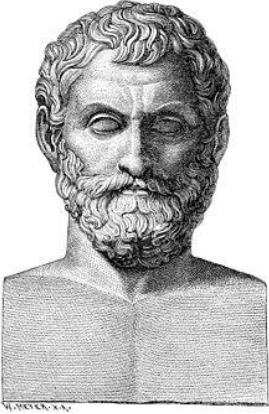


# Electricity theory



G.Schay

## Electrostatics: charges in resting state (not moving)



Discovery:

Thales of Mile (624BC-546BC)

Rubbed amber can attract  
fur/feather

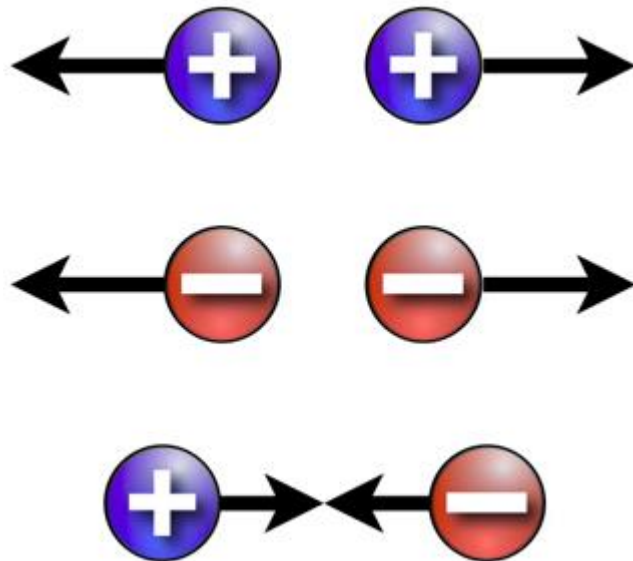


“electron” was the name of amber in greek 😊

## Experiments:

When we rub materials then they accumulate “charge” and attract or repel each other.

(triboelectric effect)



### Triboelectric series:

**Most positively charged**

+

Polyurethane foam

Hair, oily skin

Nylon, dry skin

Glass

Acrylic, Lucite

Leather

Rabbit's fur

Quartz

Mica

Lead

**Cat's fur**

Silk

Aluminium

Paper (Small positive charge)

Cotton

Wool (No charge)

**0**

Steel (No charge)

Wood (Small negative charge)

**Amber**

Sealing wax

Polystyrene

Rubber balloon

Resins

Hard rubber

Nickel, Copper

Sulfur

Brass, Silver

Gold, Platinum

Acetate, Rayon

Synthetic rubber

Polyester

Styrene and polystyrene

Orlon

Plastic wrap

Polyethylene (like Scotch tape)

Polypropylene

Vinyl (PVC)

Silicon

Teflon (PTFE)

Silicone rubber

Ebonite

-

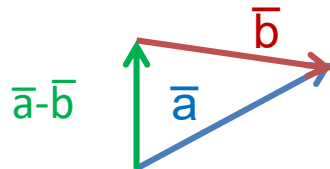
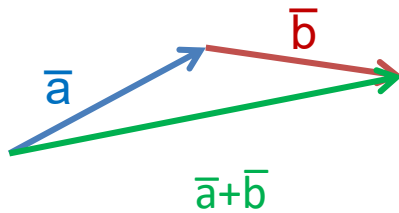
**Most negatively charged**

Here we need **vectors**, since everything has **direction + size**

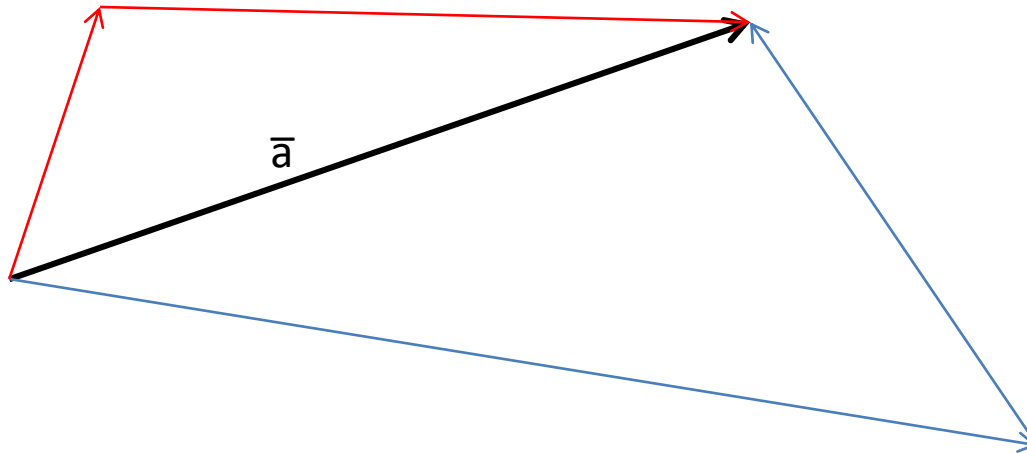
Force: something which describes an interaction.

VECTOR: has size AND direction.

Vector algebra: calculation rules with vectors:



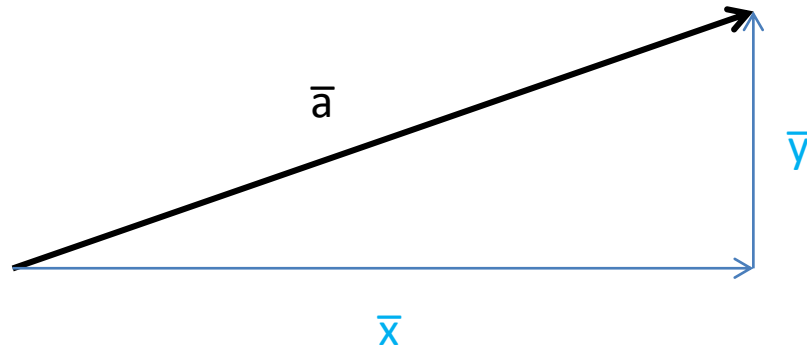
The algebra makes it possible to **break up** vectors, just like numbers.  
Az algebra lehetőséget ad a vektorok FELBONTÁSÁRA is!  
( $10=3+7$  or  $10=4+6$  or  $10=5.01 + 4.99$ , etc)



We have endless possibilities, with size and direction being free.

**We always choose the breakup which makes our life easy ☺**

Usually we use two INDEPENDENT directions, so we can focus on the directions one by one.



Everything we calculate using vectors enables a breakup.

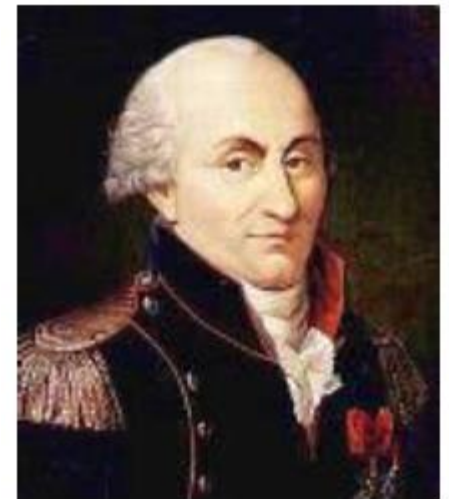
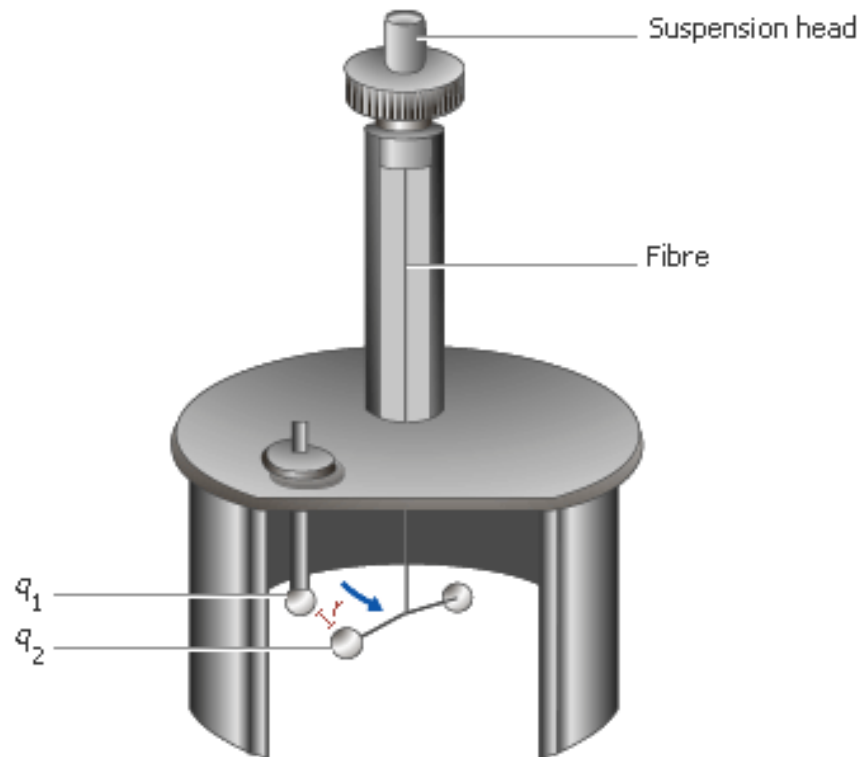


## The electroscope

As the two rods (the base and the rotating) are charged equally, they repel each other. This enables the rotatable bar to increase the distance to the base rod.

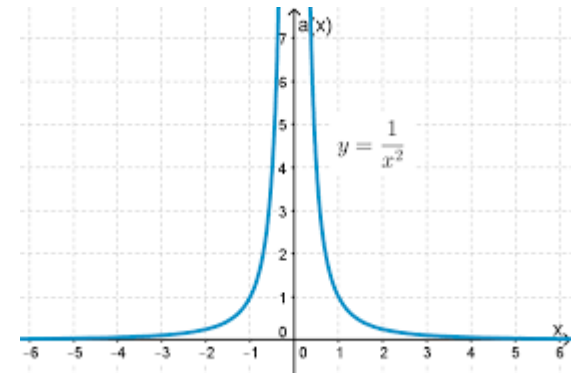
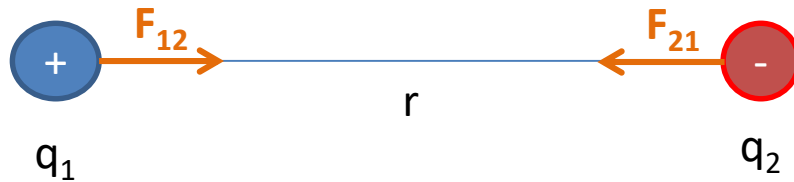
First quantitative measurements:

Coulomb's Force



*Charles A. de Coulomb*  
1736-1806





In size  $F_{12}=F_{21}$

$$F = k \frac{q_1 q_2}{r^2}$$

Very similar to gravity,  
**inverse square law**

There exists a minimum charge

$$e = 1,6 \cdot 10^{-19} \text{C}$$

There is no charge on it's own, it is always carried by some material

The modern way of defining the constant:

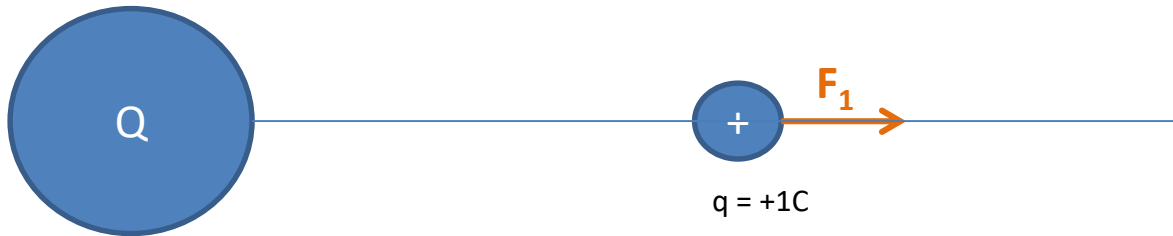
$$\begin{aligned} k_e &= \frac{1}{4\pi\epsilon_0} = \frac{c_0^2 \mu_0}{4\pi} = c_0^2 \times 10^{-7} \text{ H} \cdot \text{m}^{-1} \\ &= 8.987\,551\,787\,368\,176\,4 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \end{aligned}$$

What transmits the force?

Electrical **field**

Force field: a type of vector, which is present in all points in space, but may depend on the position. The vector is related to the force acting on an object at the same position.

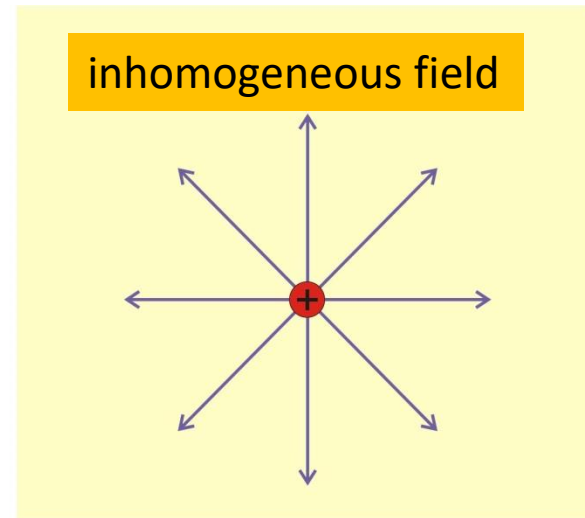
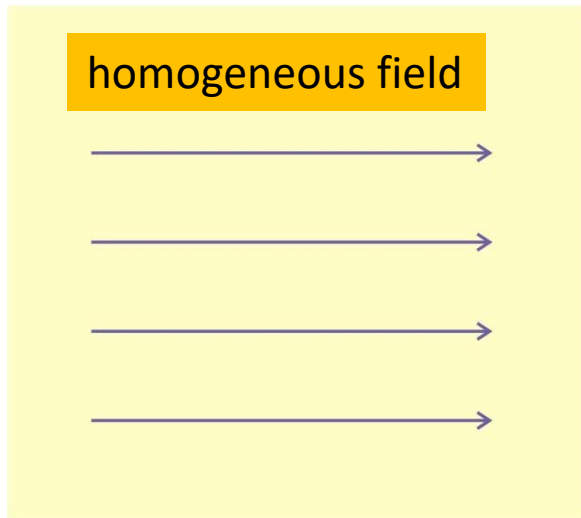
Let's define the **electric field** as the force acting on +1C charged positive testing object  
**E**



With this  $F = q \cdot E$   
And  $E = k \cdot Q / r^2$

Unit:  $[F] / [q] = \text{N/C}$

In a field, we can draw the directions of the force with lines: **field lines**



Rules:

- 1: the direction of the force is parallel to the tangential of the field lines.
- 2: the size of the force is proportional to the density of lines.

Work in an electric field:

$W = F \cdot s$ , so here we also have a work if we move parallel to the field lines.

$$F = q \cdot E$$

So

$$W = q \cdot E \cdot s$$

Here it is convenient to have  $W = q \cdot \Delta\phi$ , and define the electric potential  $\phi$ .  
(just like  $E_{\text{pot}} = mgh$ )

So  $\phi = E \cdot s$ , BUT we need a 0=point.

**Let's define  $\phi = 0$  IF we are infinitely far away**

Now we can say that **the electric potential  $\phi$  is equal to the work needed to move +1C charge from infinity to the given position.**

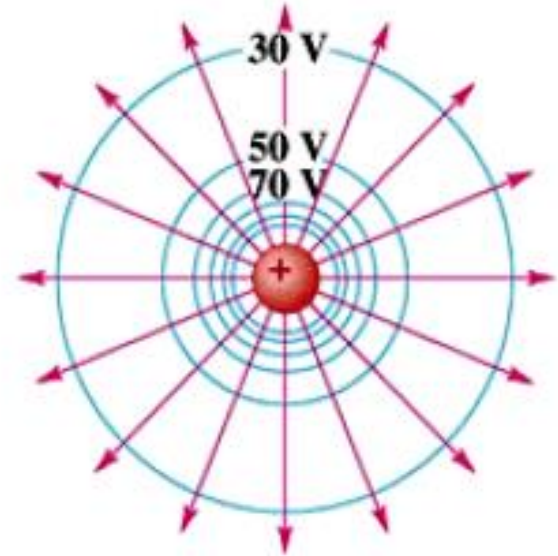
Since  $E$  is a conservative force field (just like gravity) the way is not important, just the start and end positions.

**$U = \Delta\phi$ , electric voltage. Unit:  $[W]/[q] = J/C = \text{Volt } [V]$ .**

So we have  **$W = q \cdot U$**

We have now **field lines** and **equipotential lines**

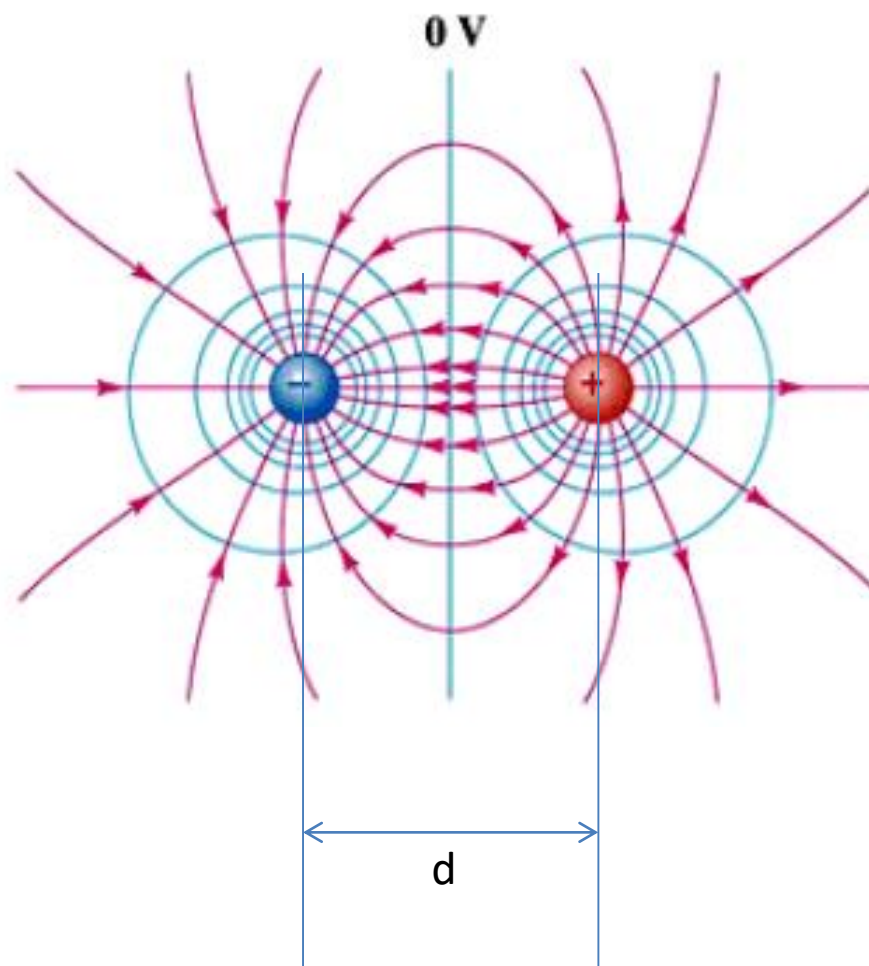
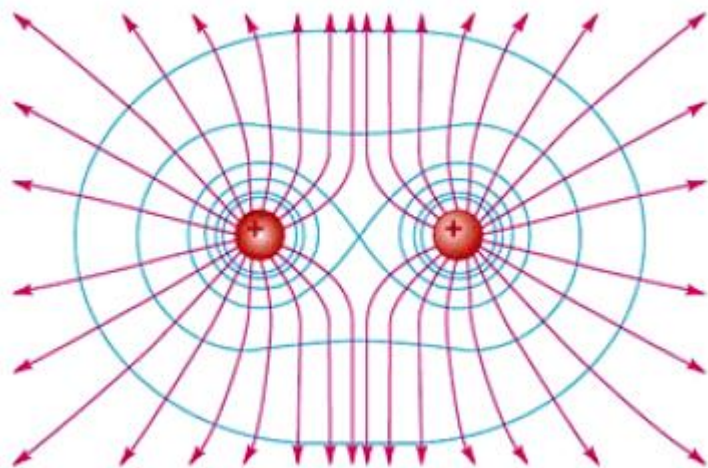
Notice that they are always perpendicular!  
( $\Delta\phi = 0$  if  $F \cdot \Delta s = 0 \rightarrow \cos\alpha = 0$ , so  $\alpha = 90^\circ$ .)



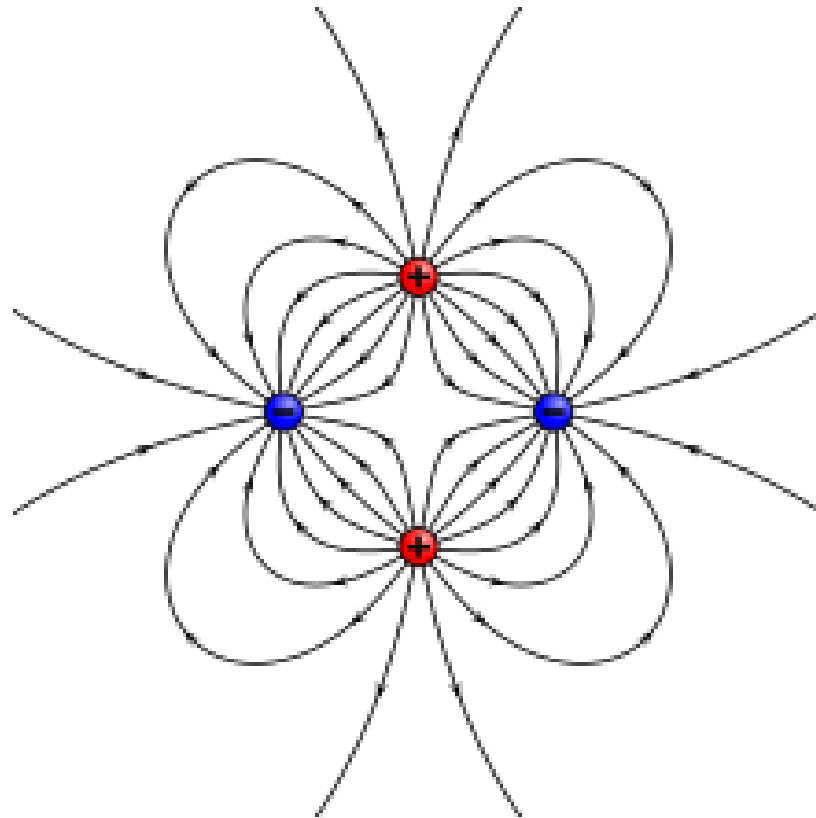
We can make geometrical constructs of charges...

Dipole

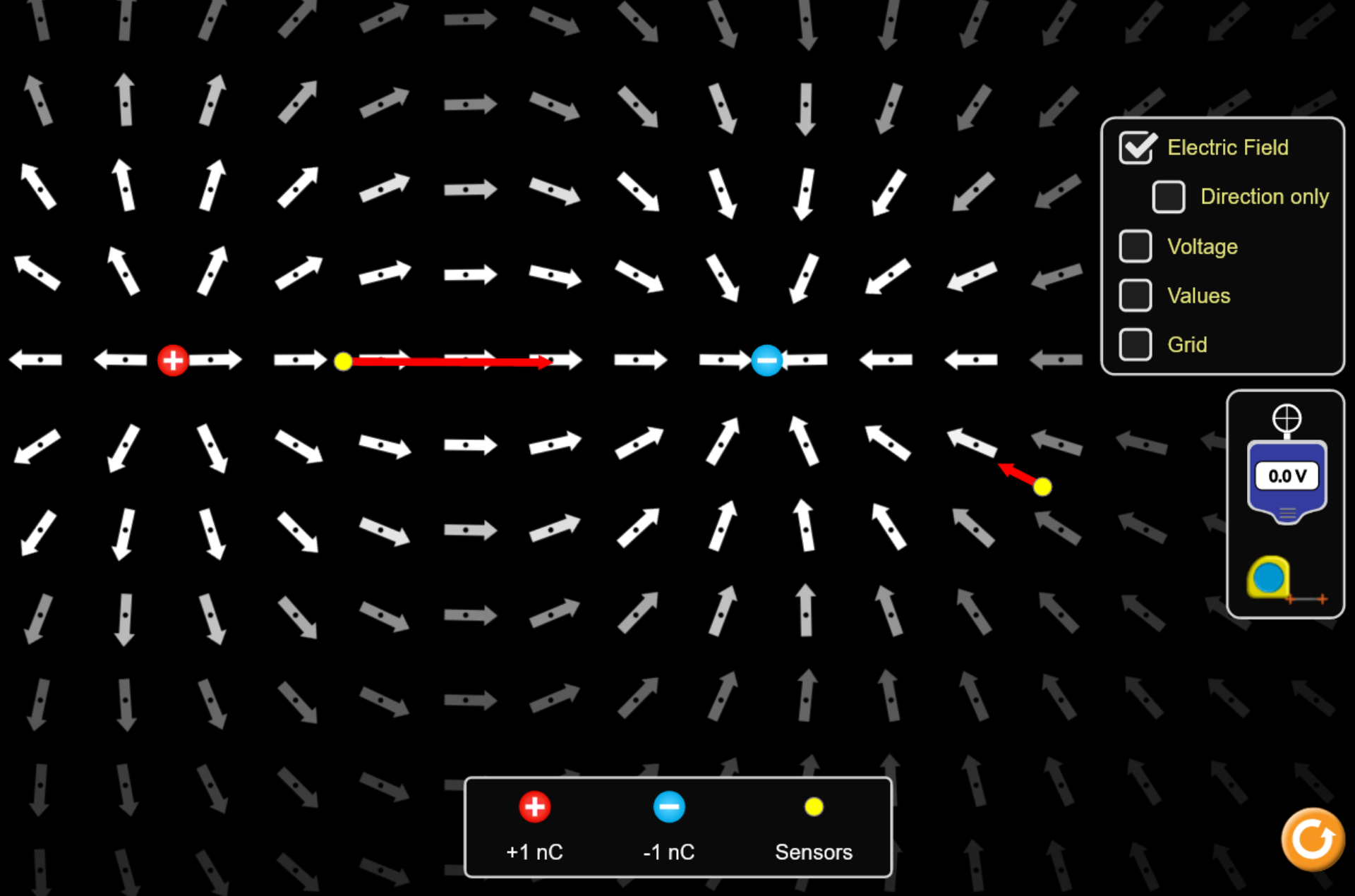
Dipole moment:  $p = q \cdot d$



Quadrupole...



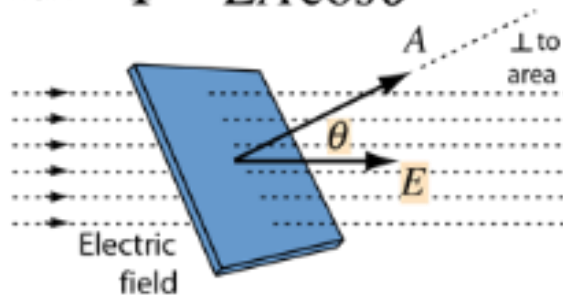
All kinds of charge distributions  
can be modelled by a series of  
geometrical objects together.  
(multipole expansion)





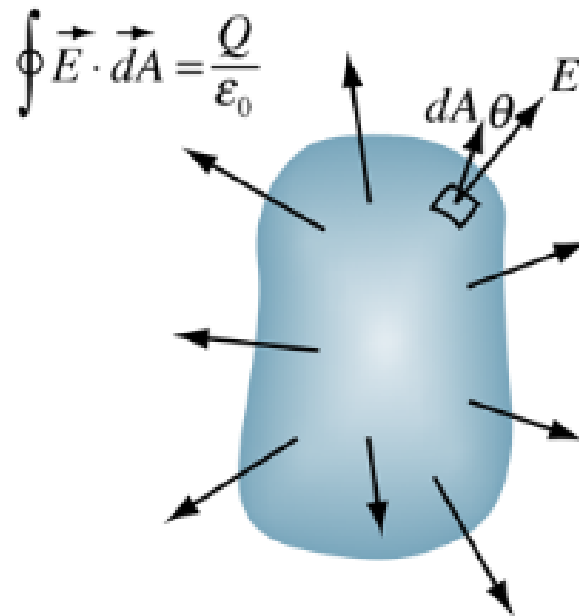
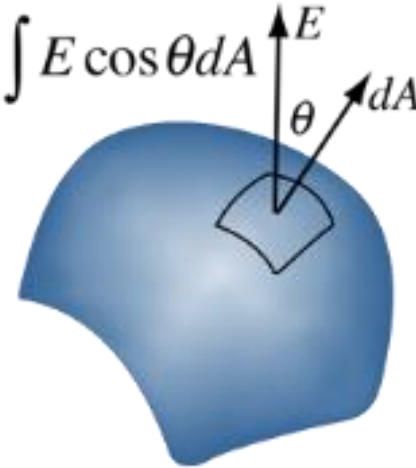
## Flux and Gauss's law

$$\text{flux} = \Phi = EA \cos \theta$$

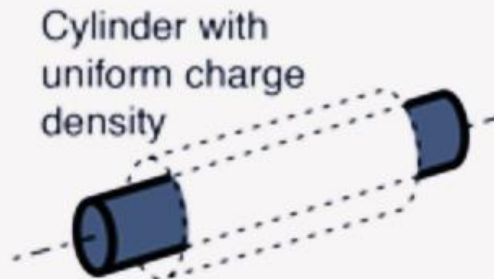
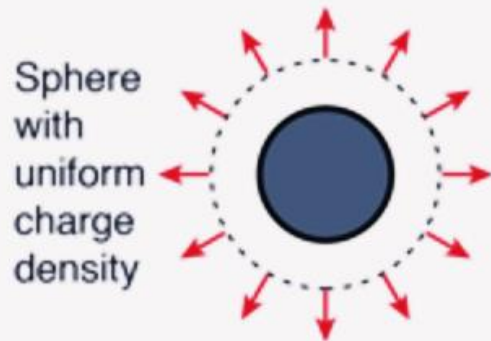
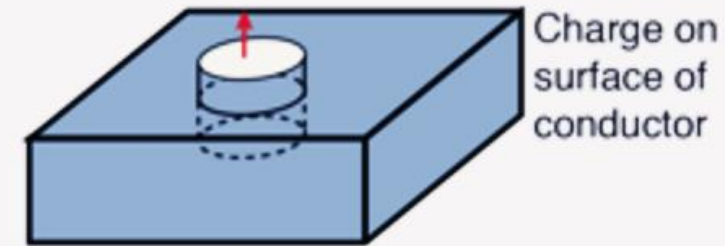
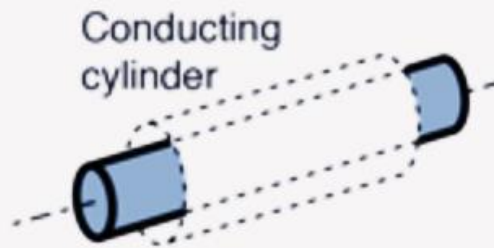
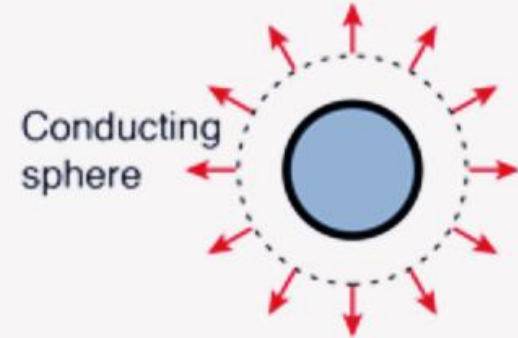
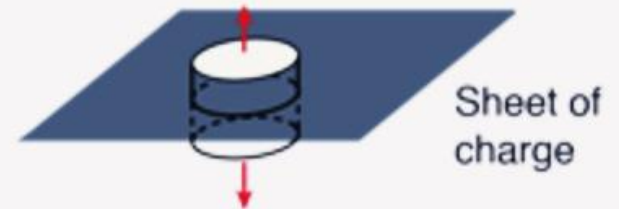
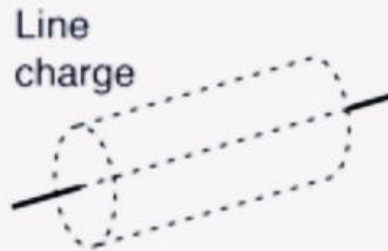
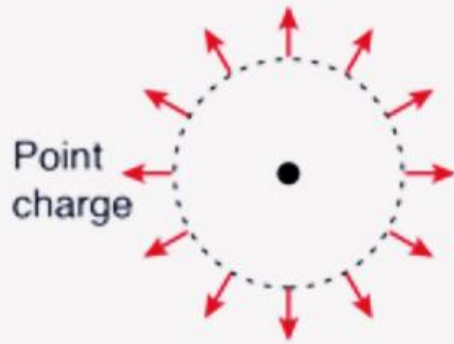


Electric flux:

$$\Phi = \int E \cos \theta dA$$



# applications of the Gauss's law



## Storing charges: the capacitor

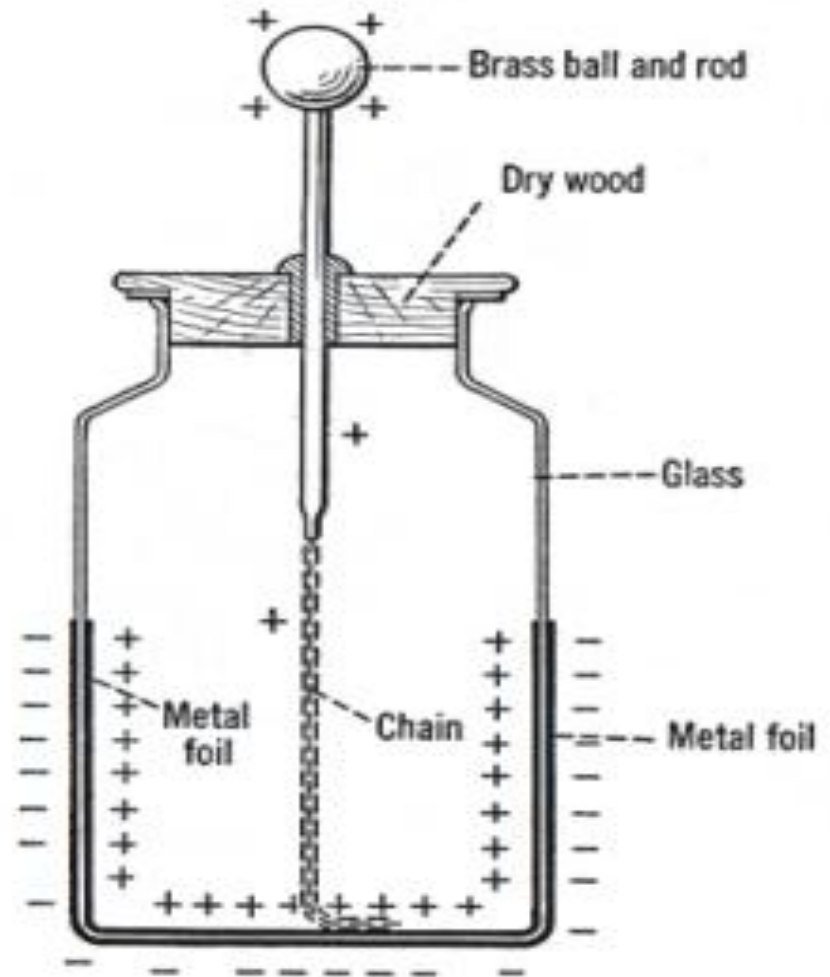
We need work to move the charge into the capacitor.

(like charges repel each other...)

The CAPACITY (C) or CAPACITANCE is a measure of the storing capability of the device:  $Q = C \cdot U$ , or

$$C = Q/U$$

Unit:  $C/V = F$  (Farad)



Leyden jar

Simplest case for calculation:

Flat panel capacitor

Here the  $E$  is constant, homogeneous field.

So  $U = E \cdot d$

But

$E = \sigma / \epsilon_0$ , where  $\sigma$  is the charge density ( $\sigma = Q/A$ )

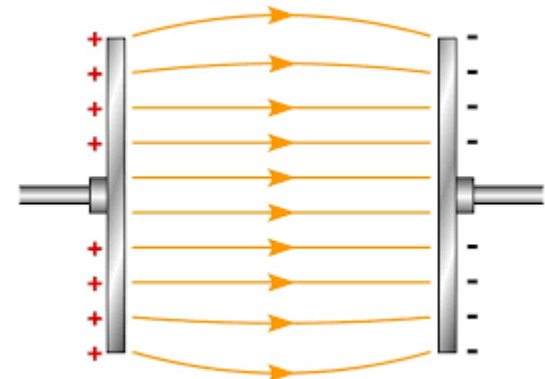
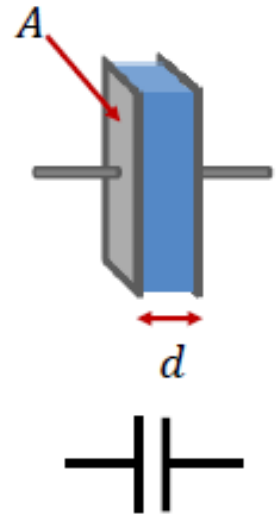
And  $\epsilon_0$  is the vacuum permittivity (see later)

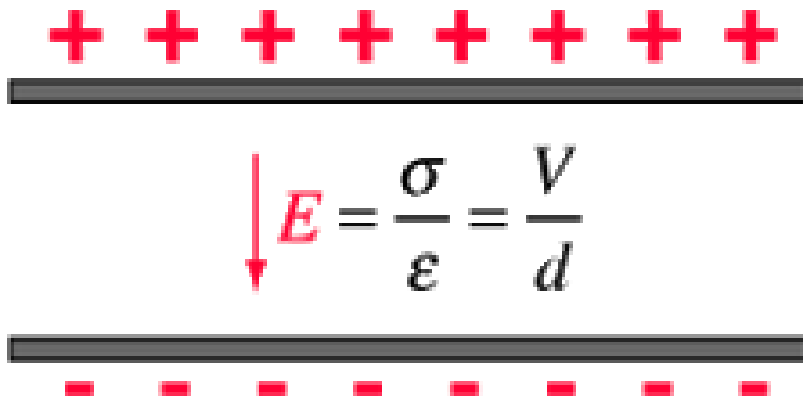
With this  $C = Q/U = Q / (Q/A \cdot 1/\epsilon_0 \cdot d) = \epsilon_0 \cdot A/d$

If we have some material in the gap then:

$C = \epsilon_0 \epsilon_r \cdot A/d$

Where  $\epsilon_r$  is the permittivity of the material.



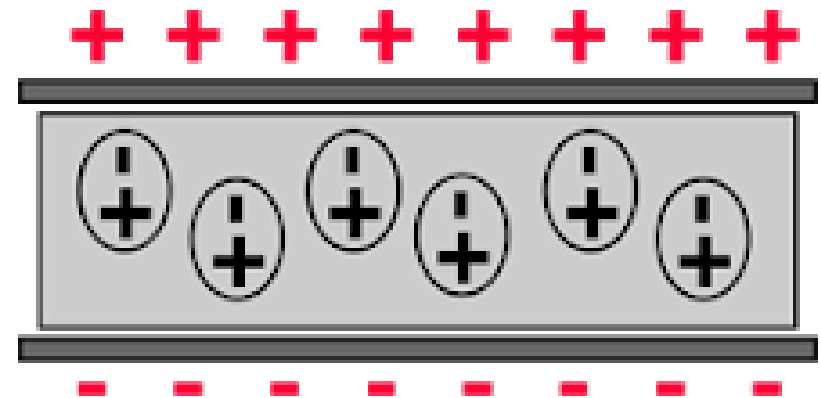


$$E = \frac{\sigma}{\epsilon} = \frac{V}{d}$$

For air,  $\epsilon \approx \epsilon_0$

$$C = \frac{\epsilon_0 A}{d}$$

The capacitance is increased by the factor  $k$ .



$$E_{\text{effective}} = E - E_{\text{polarization}} = \frac{\sigma}{k\epsilon_0}$$

$$C = \frac{k\epsilon_0 A}{d}$$

The electric field strength is reduced by the polarization of the material

Electric field in materials:

The electric field creates (induces) dipoles in a material, which has its own field. This adds up to the original, so we get a modified field.

$$\mathbf{D} = \mathbf{P} + \epsilon_0 * \mathbf{E}$$

Where  $\mathbf{D}$  is the electric displacement vector, and  $\mathbf{P}$  is the polarization.  
( $\mathbf{P}$  is the dipole moment of a volume element in the material)

Usually  $\mathbf{P} = \chi * \epsilon_0 * \mathbf{E}$ , so

$$\mathbf{D} = \epsilon_r * \epsilon_0 * \mathbf{E}$$

If we have no material, just classical vacuum then  $\mathbf{D} = \epsilon_0 * \mathbf{E}$ .

Permittivity: how much charge is needed to generate a given flux.  
(flux measures the electric field going through a surface)

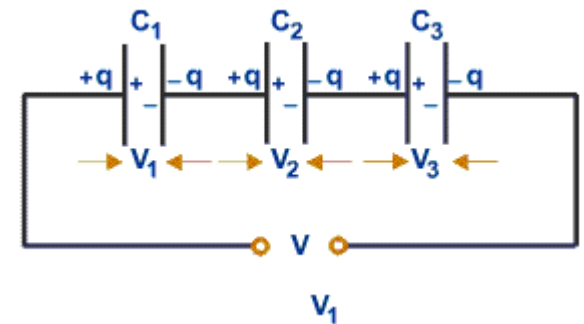
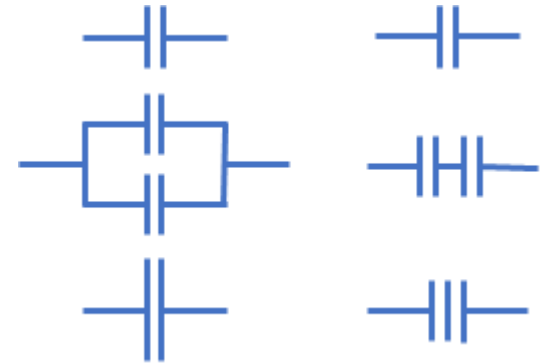
Circuits of capacitors:

Parallel: more surface, more capacity

$$C_{\text{tot}} = C_1 + C_2$$

Series: more distance, less capacity

$$1/C_{\text{tot}} = 1/C_1 + 1/C_2$$



We can calculate the work needed to charge up a capacitor

$$\Delta W = U * \Delta q$$

But

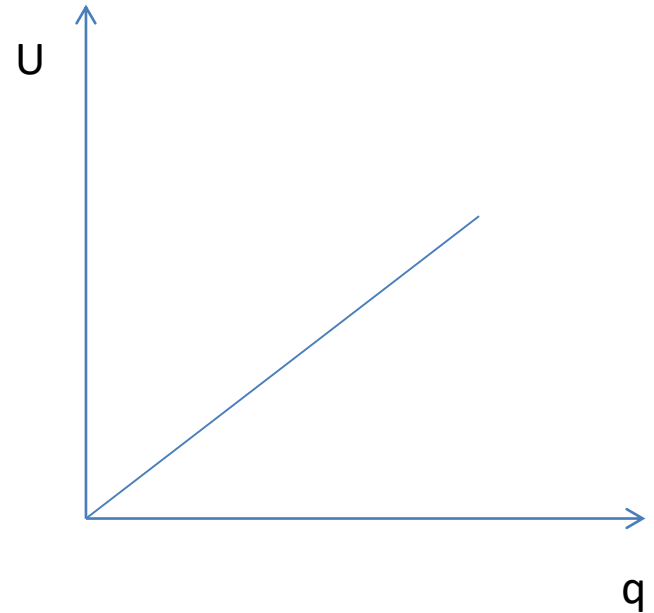
$C = Q/U$ , so the slope is  $1/C$  here.

The total work is

$U_{\text{average}} * Q$ , but  $U = Q/C$ , and  $U_{\text{average}} = \frac{1}{2} U_{\text{max}}$

Which is

$$\frac{1}{2} Q^2/C = \frac{1}{2} C U^2$$





### Some calculation problems:

A  $\text{Cl}^-$ -ion and a  $\text{Na}^+$ -ion are exactly on the opposite sides of a cell membrane. By what force do they attract one other if the thickness of the membrane is 6 nm?

The membrane potential measured between the two sides of an excitable cell is  $-90$  mV. Suppose that the electric field in the 10 nm thick membrane is homogeneous. Find the field strength.

A capacitor of 50 nF capacitance has a charge of 30  $\mu\text{C}$ . Determine

- a) the voltage of the capacitor and
- b) the energy stored in the capacitor.

$$\begin{aligned} k_e &= \frac{1}{4\pi\epsilon_0} = \frac{c_0^2\mu_0}{4\pi} = c_0^2 \times 10^{-7} \text{ H} \cdot \text{m}^{-1} \\ &= 8.987\,551\,787\,368\,176\,4 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \end{aligned}$$

## Moving charges

We measure how much charge flows in a given time

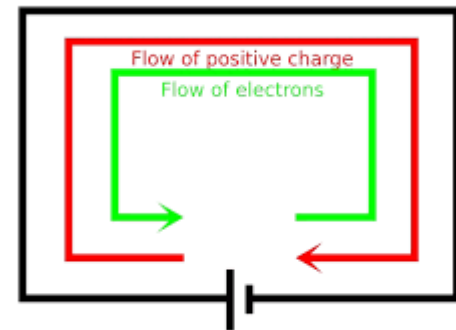
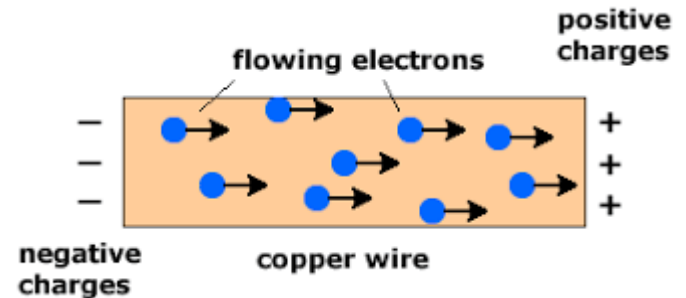
$$I = \frac{\Delta q}{\Delta t} \quad [\text{A}] \text{ Amper}$$

$$1\text{A} = 1\text{C}/1\text{s}$$

The traditional direction is for the POSITIVE charge,  
Which is the lack of electrons.

So the real flow is always opposite to the  
“technical” direction!

What can move is the electrons.  
(if there are free, movable electrons in a **conductor**  
If not then it is an **insulator**.)



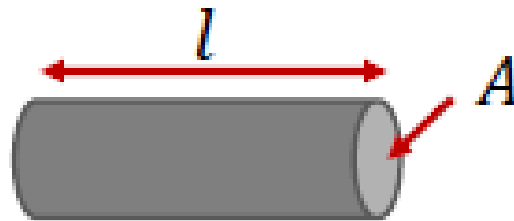
## Resistors

When electrons move in materials they hit the atoms and loose speed.  
Just like friction!

Ohm's law: the flow is determined by the voltage  
and the material:

$$I = U/R$$

$$R = \frac{U}{I} \quad [\Omega] \text{ Ohm}$$



$$R = \rho \frac{l}{A}$$

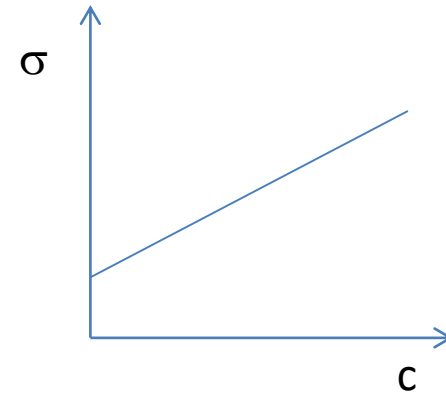
Specific resistance or resistivity

Conductance / conductivity: the inverse of the ohmic resistivity

$G = 1/R$  : conductance (1/Ohm = Siemens)

$\sigma = 1/\rho$  : conductivity

We usually use this for electrolytes, etc.  
( $\sigma$  is proportional to the concentration)



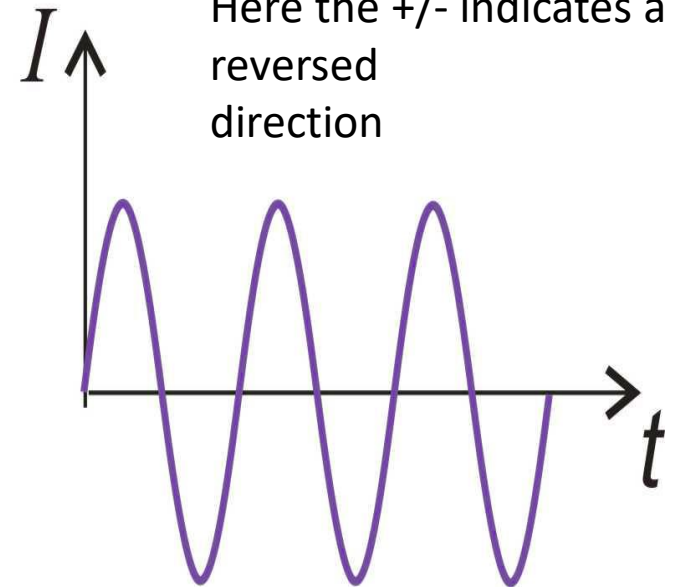
The movement can be unidirectional or changing:

DC: direct current



AC: alternating current

Here the +/- indicates a reversed direction



$$I = I_{max} \cdot \sin \omega t$$

$$U = U_{max} \cdot \sin(\omega t + \varphi)$$

$$I_{eff} = \frac{I_{max}}{\sqrt{2}} \quad U_{eff} = \frac{U_{max}}{\sqrt{2}}$$

$I_{rms}$

$U_{rms}$

Circuits of resistors:

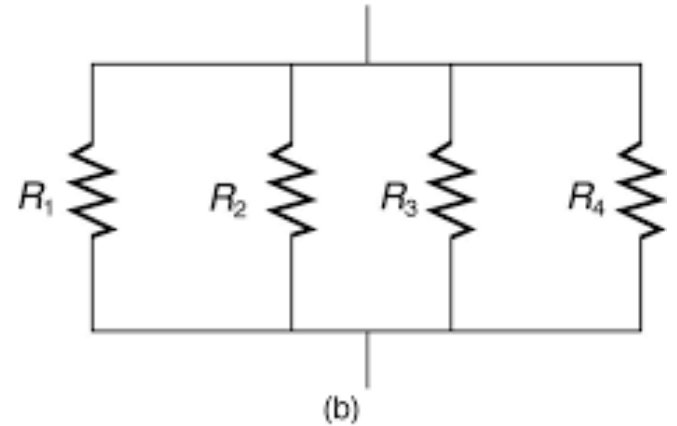
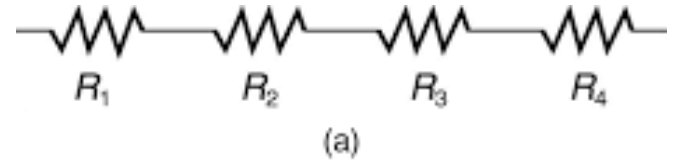
Series: more length => more R

$$R = R_1 + R_2$$

Parallel: more space to go => less R

$$1/R = 1/R_1 + 1/R_2$$

More length to go



More ways to go,  
more area for the flow

Heat of the friction: Joule's work / Joule's heat

$$W = U \cdot I \cdot t \text{ . Unit: joule (J)}$$

It is general: the work done by the electric field to move the charges is transformed into heat if the current flows through a resistor.

BUT

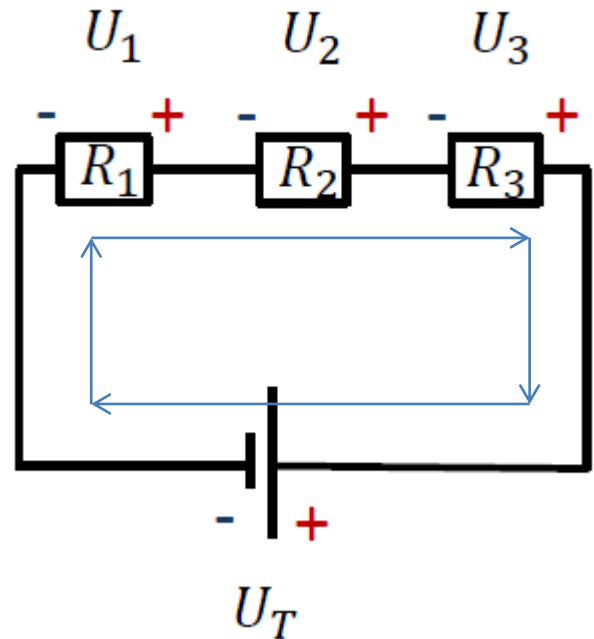
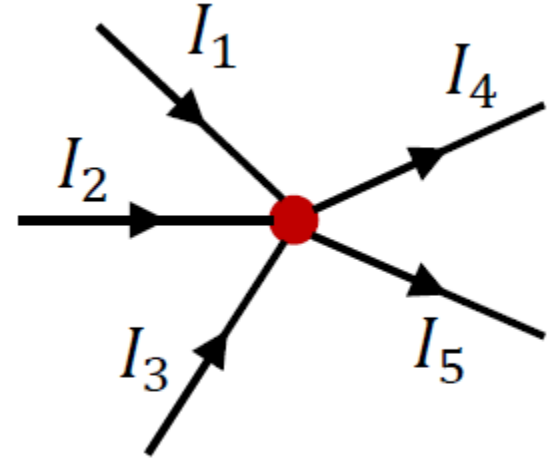
The electrical field can do other types of work too (electric motors, LED, etc).

Do not forget the energy conservation law!

$$\text{Power: } P = U \cdot I, \text{ unit: Watt (W)}$$

## Kirchoff's laws

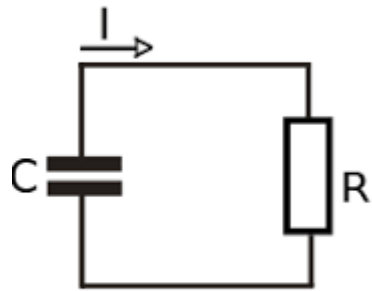
- I. Junction law: the sum of currents must match the storage in the junction. If there is no storage, then the sum is 0 (taking direction into account by +/-)
- II. Loop law: in a closed loop the sum of electrical potential changes is 0. Here we have to take the +/- again into account, and use a single direction for the loop.



Gustav Robert Kirchhoff  
1824-1887



## RC circuit



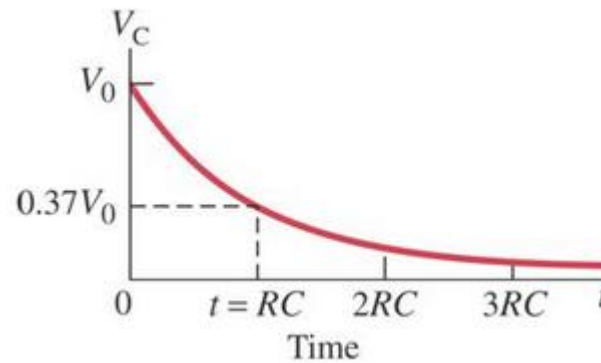
Discharge a capacitor through a resistor

$C = Q/U_c$ , so  $U_c = Q/C$ , but  $Q = I \cdot t$ , and  $I = U_c/R$ .

What we get for the loop eq. is:  
 $I \cdot R + U_c = 0$

$\Delta Q / \Delta t = -Q/RC$   
 $\tau = RC$

$$U_c = U_0 \cdot e^{-\frac{t}{R \cdot C}}$$



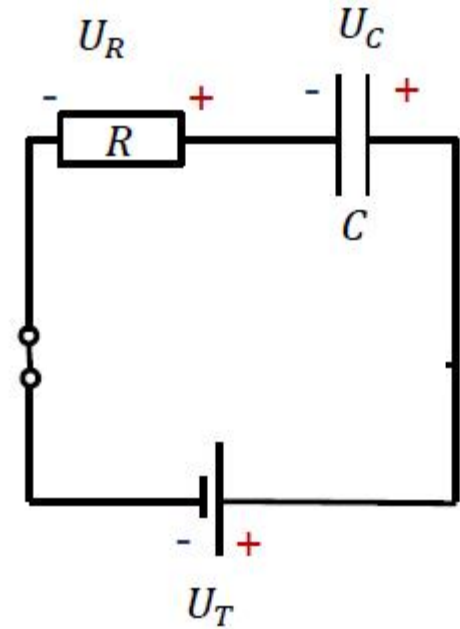
Charging up:

Here the loop equation includes the battery:

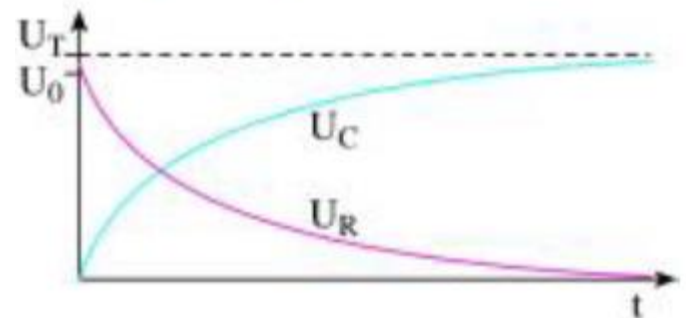
$$I \cdot R + U_C = U_T$$

So

$$\Delta Q / \Delta t = -Q / RC + U_T / R$$



$$U_C = U_T \cdot \left(1 - e^{-\frac{t}{RC}}\right)$$



## Capacitor in a circuit

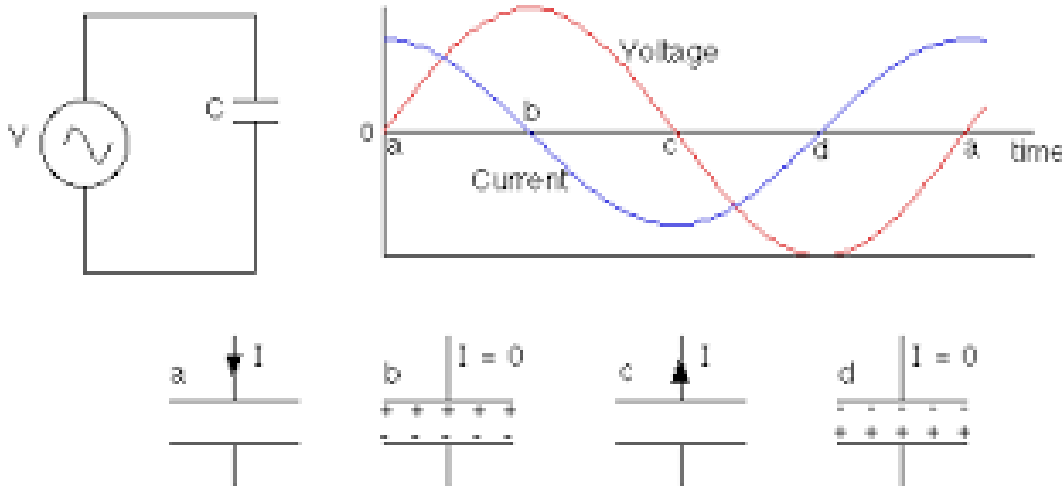
After charging/discharging in a DC circuit it is like a break.

In an AC it is periodically charged/discharged  
So it acts as a resistor-like object,  
BUT it is in a different phase than pure resistors, here **there is a phase shift!**

Capacitive reactance

$$X_C = \frac{1}{\omega C}$$

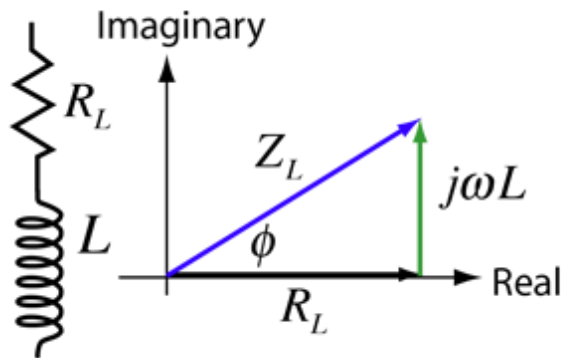
More capacity, less reactance  
higher freq, less reactance



2022 QR



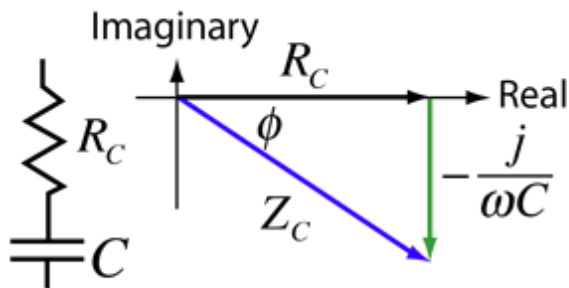
**Extension material:** since the phase shift causes a shift between  $I$  and  $U$ , the AC resistance or IMPEDANCE can not be calculated so easily, but if we let the reactance behave like a vector then it is straightforward.



Cartesian form:  $Z_L = R_L + j\omega L$

Polar form:  $Z_L = |Z_L| e^{j\phi}$

where  $|Z_L| = \sqrt{R_L^2 + \omega^2 L^2}$   
 $\phi = \tan^{-1} \frac{\omega L}{R_L}$



Cartesian form:  $Z_C = R_C - \frac{j}{\omega C}$

Polar form:  $Z_C = |Z_C| e^{j\phi}$

where  $|Z_C| = \sqrt{R_C^2 + \left[\frac{-1}{\omega C}\right]^2}$   
 $\phi = \tan^{-1} \frac{-1}{\omega C R_C}$

