Physical bases of biophysics

Lecture 4   17. 09. 2021.

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Mechanics – Work and Energy

1. **Work and energy**
   - Work-energy theorem
   - Lifting up an object and potential energy
   - Accelerating an object and kinetic energy
   - Streching an object and elastic potential energy
   - Types of energy
2. **Power**
3. **Conservation of energy**
4. **Mass-energy equivalence**

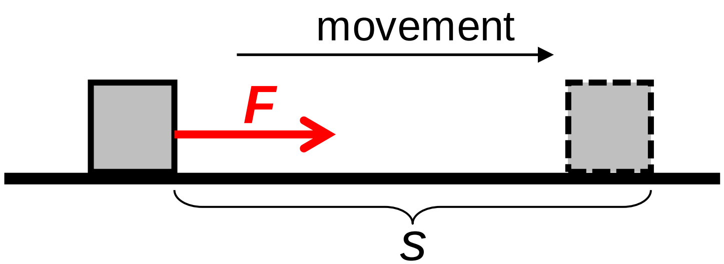
Force and energy

How can we exactly describe the following interaction?

- The challange for the the competitor is to interact with the fire engine and change its state of motion.
- With the **force *F*** we can specify how **strongly** the man must pull the fire engine.
- The **magnitude of this force** remains **unchanged** however, regardless of whether the fire engine has to be towed over 2 meters or 20 meters.
- The man, on the other hand, will be less tired if he only has to push for 2 meters.
- So the interaction between the man and the fire engine <u>cannot be described solely in terms of force</u>. We need a **new physical quantity**, which **also takes into account how long the interaction "works" along the way** $\rightarrow$ „**Work**".

Work

If the direction of movement and the force are the same:

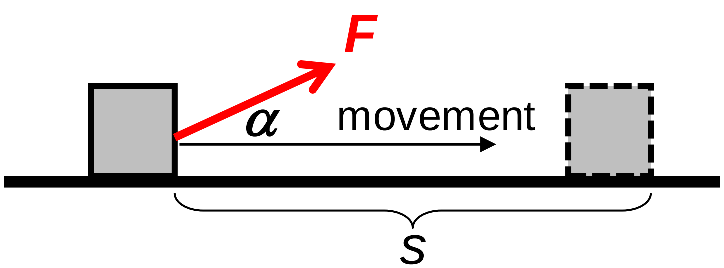


work (*W*): $W = F \cdot s \quad (\mathrm{N} \cdot \mathrm{m} = \mathrm{J})$

scalar

joule
(also unit of energy)

If the direction of movement and the force are at an α angle to each other:



work (*W*): $W = F \cdot s \cdot \cos\alpha$

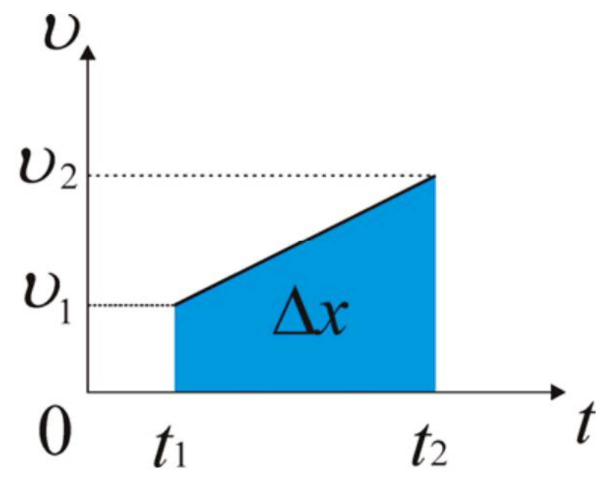
scalar

Calculate the work done by the man when pulling the fire engine horizontally for 30 m with a force of 1400 N.

Work-energy theorem

If the force is constant (and $\alpha = 0$):

$$F = ma = m\frac{\Delta \upsilon}{\Delta t} = m\frac{\upsilon_2 - \upsilon_1}{t_2 - t_1}$$



The displacement would be $\Delta x = v\Delta t$,
but v also changes:

$$\Delta x = \frac{(\upsilon_1 + \upsilon_2)(t_2 - t_1)}{2}$$

$$W = F\Delta x = m\frac{\upsilon_2 - \upsilon_1}{t_2 - t_1}\frac{(\upsilon_1 + \upsilon_2)(t_2 - t_1)}{2} = m\frac{(\upsilon_2 - \upsilon_1)(\upsilon_1 + \upsilon_2)}{2}$$

$$W = m\frac{(\upsilon_2{}^2 - \upsilon_1{}^2)}{2} = \frac{1}{2}m\upsilon_2{}^2 - \frac{1}{2}m\upsilon_1{}^2 = E_{\mathrm{kin}\,2} - E_{\mathrm{kin}\,1} = \Delta E_{\mathrm{kin}}$$

Kinetic energy (E_{kin})

$$\boxed{\sum_{i=1}^{n} W = \Delta E_{\mathrm{kin}}}$$

Result of work: greater E_{kin}

Work and energy

- He <u>performs work</u> on the fire engine.
- Meanwhile he <u>**tranfers**</u> <u>energy</u> to it.
- So the man <u>loses</u> energy, while the fire engine <u>gains</u> energy.

- He <u>performs work</u> on the weight while lifting it up.
- Meanwhile he <u>**tranfers**</u> <u>energy</u> to it.
- So the man <u>loses</u> energy, while the weight <u>gains</u> energy.

- She <u>performs work</u> on the bow while tightening the nerve.
- Meanwhile she <u>**tranfers**</u> <u>energy</u> to the bow.
- So she <u>loses</u> energy, while the bow <u>gains</u> energy.
- This energy is stored in the bow. During shooting, the bow does work on the arrow, which gains energy.

work ≡ „transfer of energy” – „energy change”

energy ≡ „stored work” – „ability to perform work”

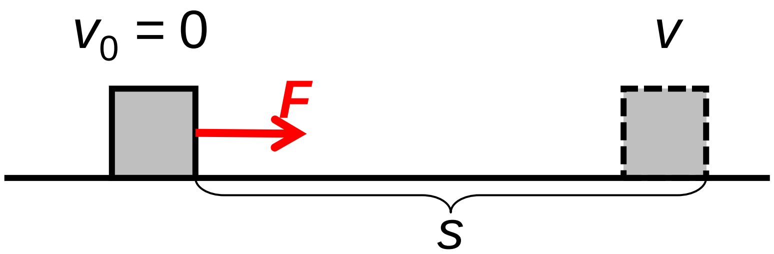
Work describes a **process**, **energy** a **state**.

Energy cannot be created or destroyed.
Possibilities: one system can **transfer** it to another, or one form of energy can be **transformed** into another form of energy.

Kinetic energy

(work done while accelerating an object)

Work done while accelerating an object from 0 to *v:*



$$W = F \cdot s = \frac{1}{2} m v^2$$

The work done appears as an energy connected to motion – ***kinetic energy***:

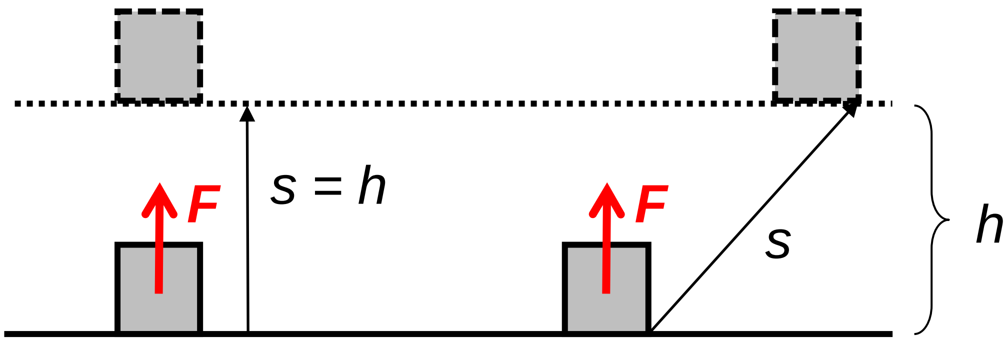
$$E_{\mathrm{kin}} = \frac{1}{2} m v^2$$

The stored kinetic energy can be transformed in various ways:

Gravitational potential energy

(work done while lifting up an object)

Work done when *lifting a body* to a height *h:*



$$\boxed{W = F \cdot s = mgh}$$

The work done is stored as an energy connected to the position – ***potential energy*** :

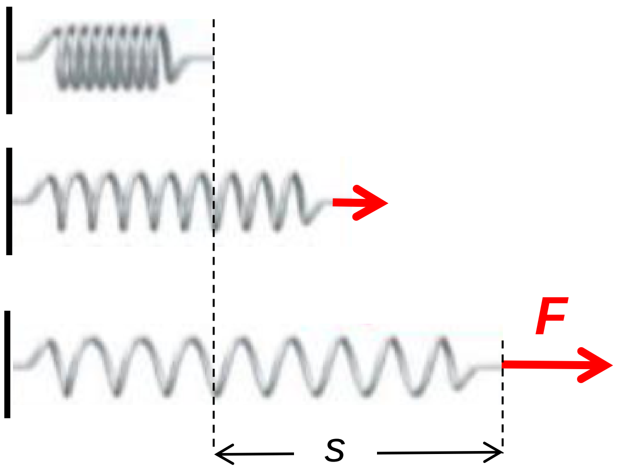
In gravitational field: $\boxed{\Delta E_{\mathrm{pot}} = mg\Delta h}$

- Defining refence level h=0 is arbitrary.

The stored potential energy can be used for various purposes:

Elastic potential energy

Work done by stretching a spring (or stretching a bow):



$$\boxed{W = F \cdot s = \frac{1}{2} k s^2}$$

The stored flexible energy can be used for various purposes:

The work done is stored as configuration dependent **elastic potential energy**:

$$W = \Delta E_{\mathrm{pot}}$$

$$\boxed{E_{\mathrm{el}} = \frac{1}{2} k s^2}$$

How much energy does the Achilles tendon store at 2 mm elongation if its spring constant is $1{,}2 \cdot 10^5$ N/m:

Types of energy

<u>We can come across many types of energy:</u>

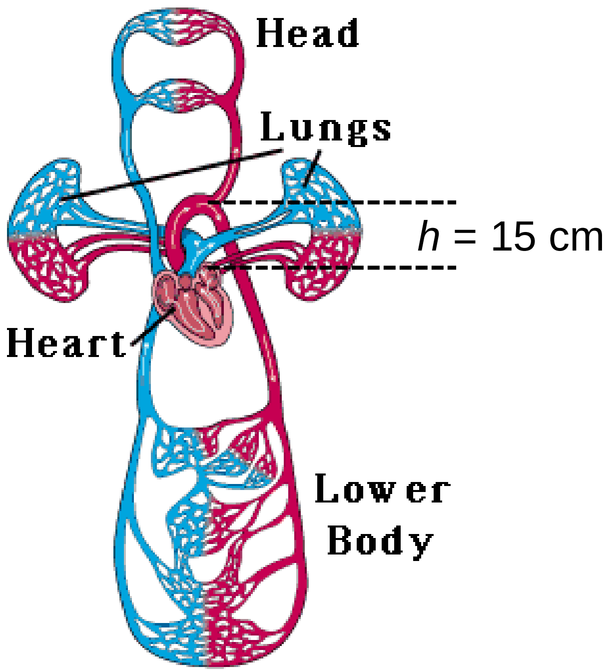
- Mechanical energy
- Thermal energy (heat)
- Electrical energy
- Magnetic energy
- Chemical energy
- Nuclear energy
- Internal energy

Potential energy

Kinetic energy

Elastic potential energy

Problem



The contraction of the left ventricle of the human heart accelerates blood to a flow velocity of 40 cm / s and raises it 15 cm higher to the aortic arch. During one heart contraction, 60 g of blood is pumped out. Calculate

a) the work needed to accelerate the blood

b) the work needed to lift the blood

c) the total work of the heart during one contraction

Power

- How **strongly** the man must pull the fire engibe can be given by the **force *F***. How much work do you do if you pull on a **distance *s*** can be given with the **work *W***.

- The force and work remain the same if you cover the distance in 2 minutes or 20 minutes.

- We need a **new quantity** that also takes the **time into account** → „**Power**".

Power

power (*P*): $P = \frac{W}{t}\ \left(\frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W}\right)$

Watt

“speed” of the work done

?

Continuation of the previous problem: Calculate the power of the left ventricular musculature if the contraction time is 0.2 s!

?

Continuation of the previous problem: Calculate the power of the man, if he pulls the fire engine to a distace of 30 m in 41 s!

Conservation of energy

Conservation of energy (in general):

Energy cannot be created nor be destroyed.
Possibilities: one system can transfer it to another, or one form of energy can be converted to another form of energy.

If we can neglect friction (and do not take into account other electrical and magnetic phenomena), then the **law of energy conservation** *in a closed system* applies **to mechanical energy types** as follows:

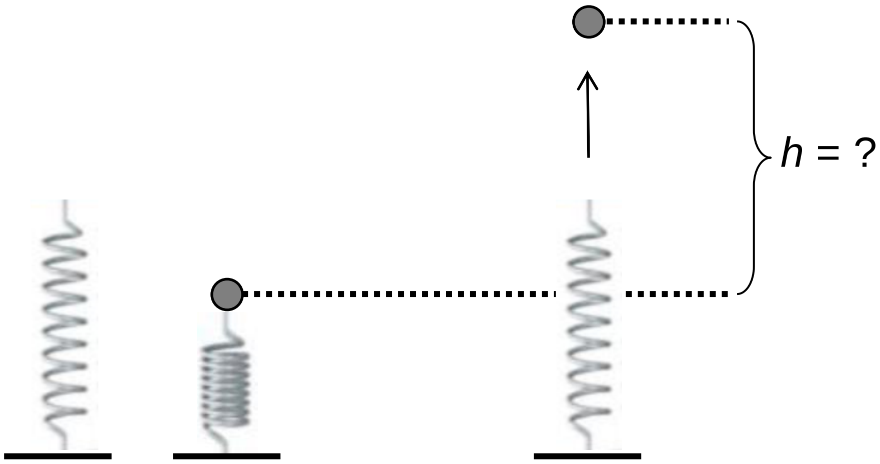
$$\sum E_i = E_{\mathrm{pot}} + E_{\mathrm{kin}} + E_{\mathrm{el}} = const.$$

So:

Time 1.:	$E_{\mathrm{pot},1}$	Time 2.:	$E_{\mathrm{pot},2}$
	$E_{\mathrm{kin},1}$		$E_{\mathrm{kin},2}$
	$E_{\mathrm{el},1}$		$E_{\mathrm{el},2}$

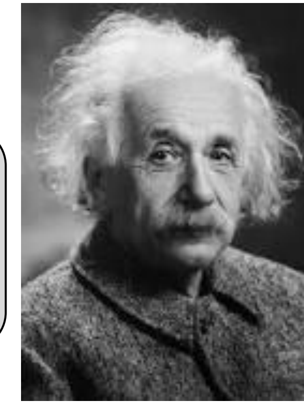
$$\boxed{E_{\mathrm{pot},1} + E_{\mathrm{kin},1} + E_{\mathrm{el},1} = E_{\mathrm{pot},2} + E_{\mathrm{kin},2} + E_{\mathrm{el},2}}$$

Problem



How strongly do we have to compress the coil spring with a spring constant of 2000 N / m for the ball with a mass of 30 g to fly to a height of 10 m?

Mass-energy equivalence



According to the theory of relativity, mass and energy are equivalent and are related to eachother as follows:

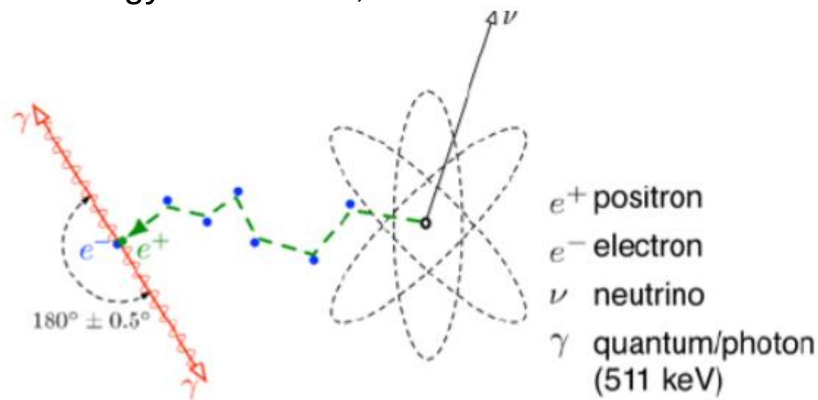
$$E = m \cdot c^2$$

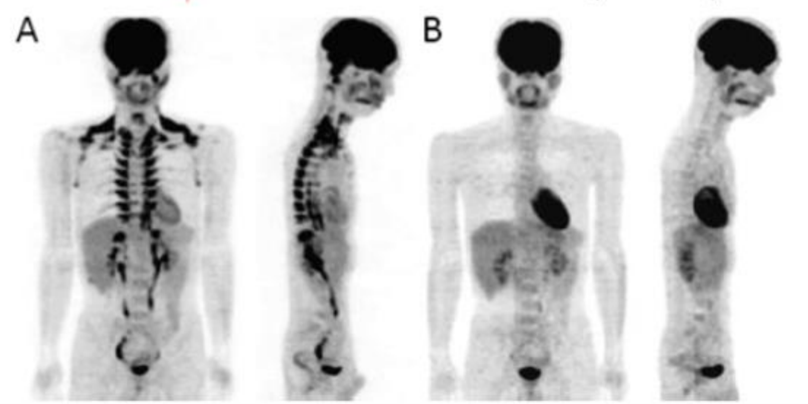
rest energy equivalent to the mass

mass of one particle

speed of light (in vacuum): $3 \cdot 10^8$ m/s

The relationship can be used for phenomena in which particles disappear while their mass is converted into energy, or when new particles are formed from energy.
Examples: pair production – interaction of γ-photons of higher energy with matter, annihilation - PET

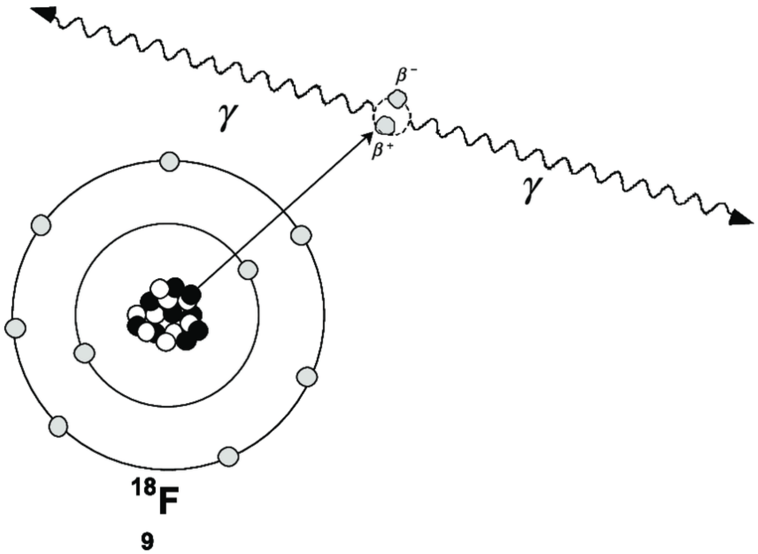




- In the case of **positron emission tomography (PET)**, an isotope emitting **β-positive** particles (positron) is administered to the body.
- During the decay of isotopes a **positron** is formed, which is an **anti-particle**.
- The **positron merges with an electron** after a short lifetime and both are converted into energy (annihilation - destruction). Two **γ-photons** appear, which can then be detected.
- Both γ-photons have an **energy of 511 keV**.

Problem

Why is the energy of γ-photons generated during annihilation exactly 511 keV ?



Homework: Chapter 5