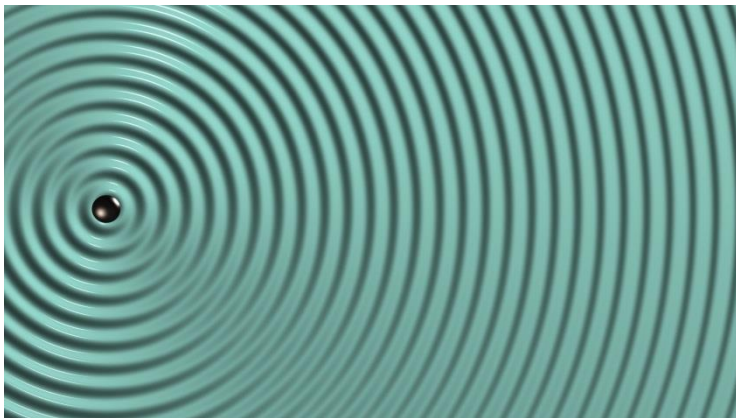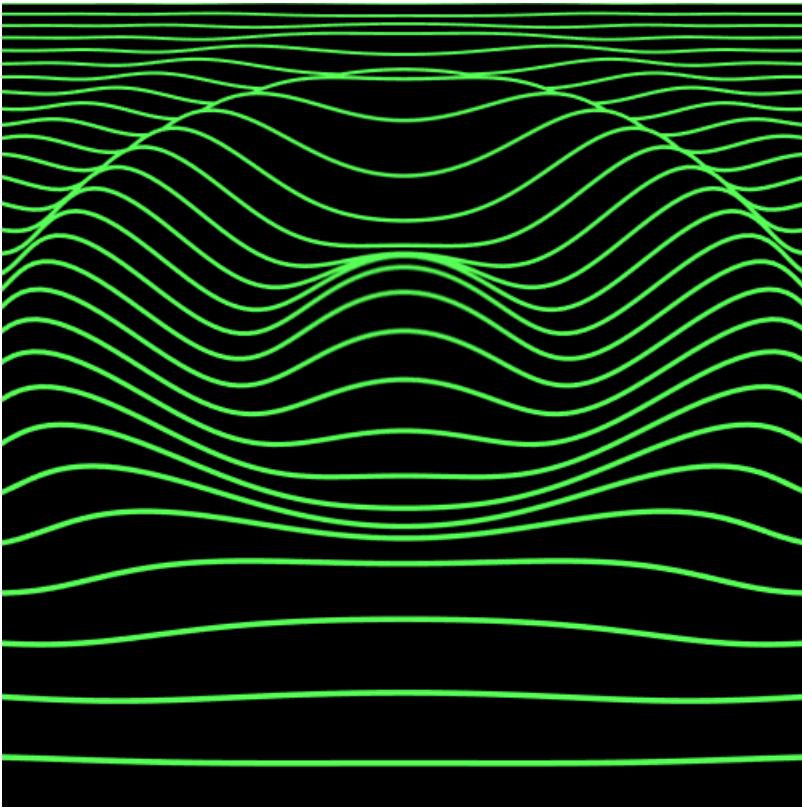# Wave optics

Waves are everywhere, not just in optics!
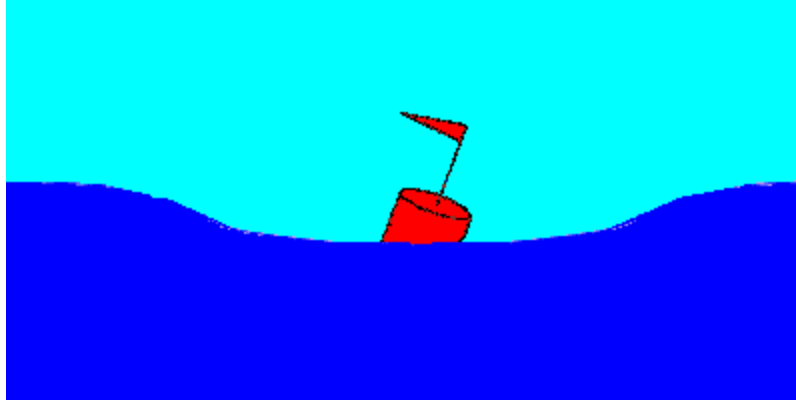
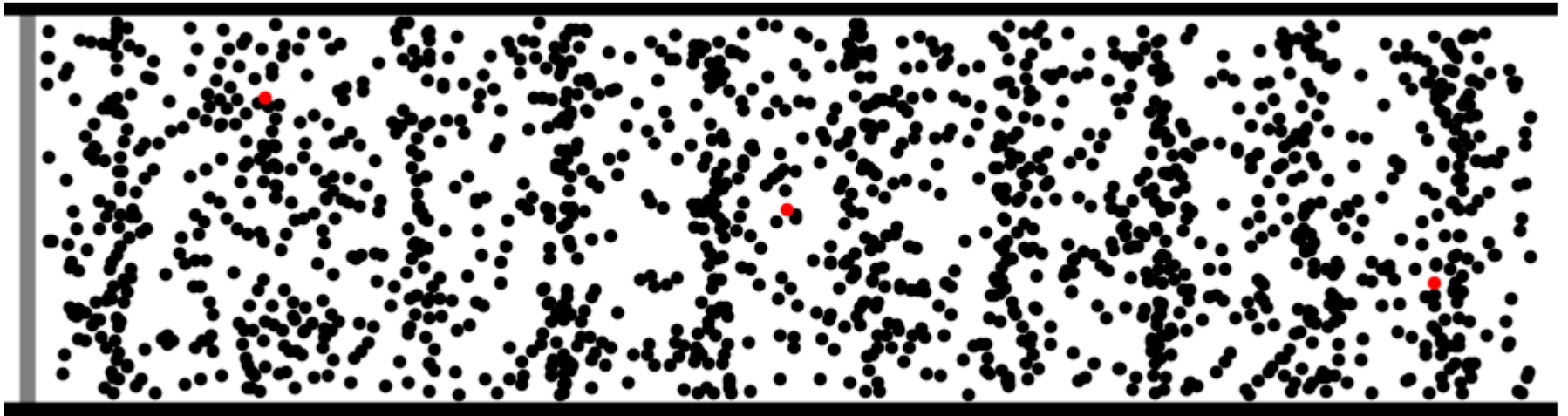G.Schay

Waves are seen most often in water:

Waves can be described by the wave equation, which relates
the motion of individual parts of the medium to the observed wave.

It is important to note, that as the waves propagate, the parts
of the medium (here the water molecules) stay "in place", which means
there is no net transport of material.

Longitudinal waves:
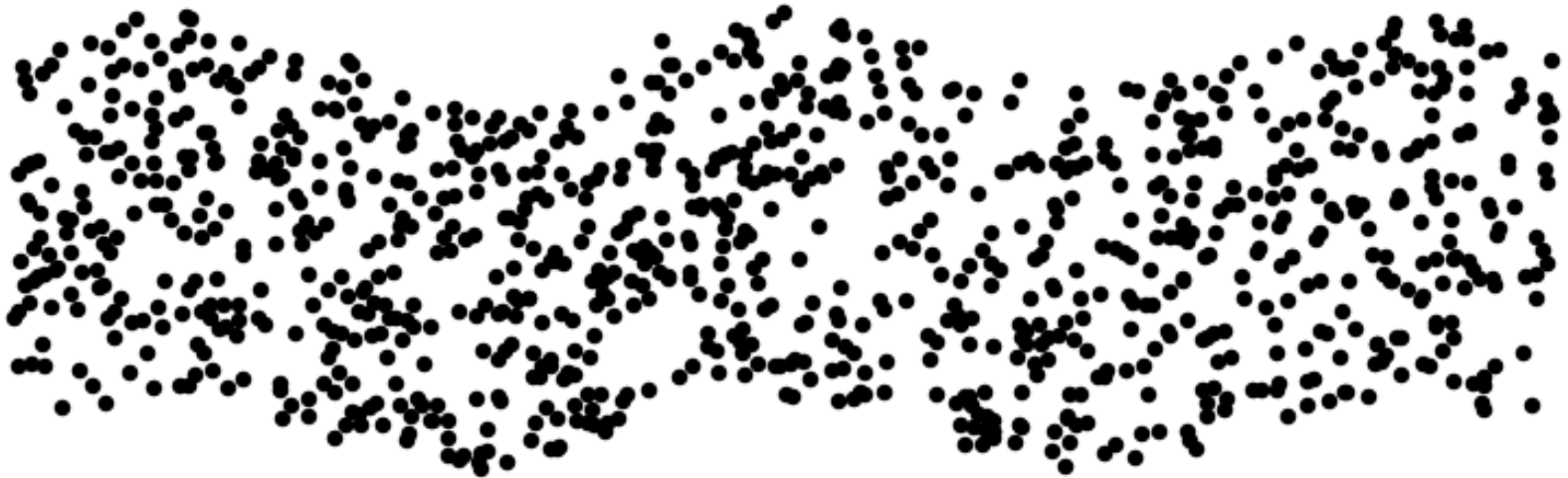propagation direction is parallel to the "motion"

©2011. Dan Russell

Moving surface (wave "source")

Transversal wave – such as light, or sound in some cases in solids

Transversal: wave propagation is perpendicular to the "motion"

As an observation we can say that as waves travel or **propagate** the change in the state of the parts of the medium is moving.
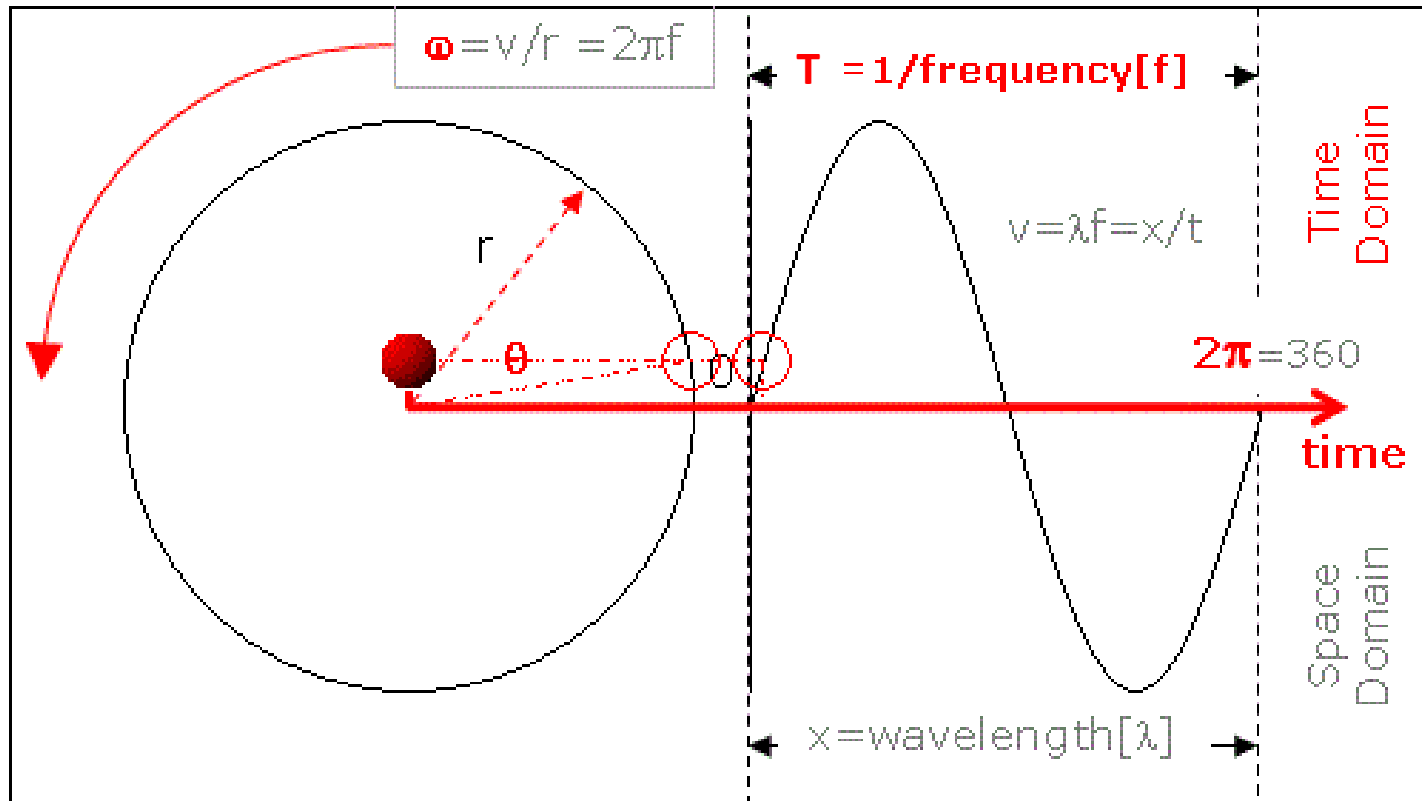
Here the "state of the parts" simply means the deflection of the string at a given point.

The wave is moving forward (propagating) with a speed of "v", any point's position is dependent on both space and time.

Most important motion type is the harmonic motion. (produced by a harmonic oscillator)

The wave equation is a bit complicated:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

We take the change of any property (here "u") in time (du/dt) and also in space (du/dx), but we need to take the change of the change (d²u/du²), and these are linked by the propagation speed (or other named phase velocity) (here as "c").

A simple solution for u(x,t) is:

u(x,t) = A * sin(k*x + ω*t + ϕ)
where
A is the amplitude of the wave, k is the wavenumber, and ω is the angular frequency

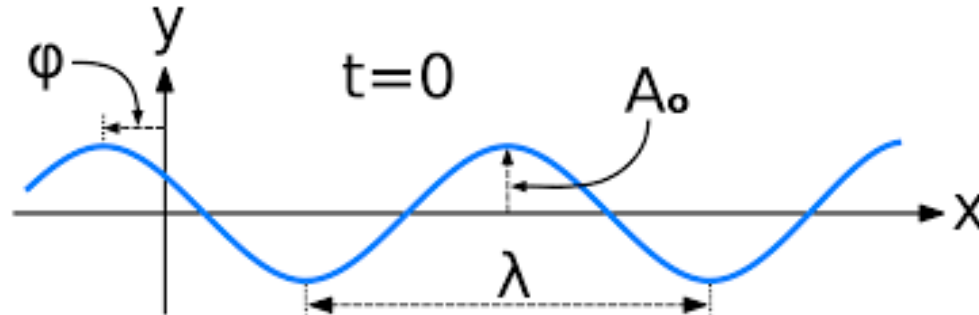ω = 2πf , where f = 1/T [Hz], while T is the period time.
ω = c*k defines the wavenumber, which can be written as k = 2π/λ.
here λ is the wavelength.
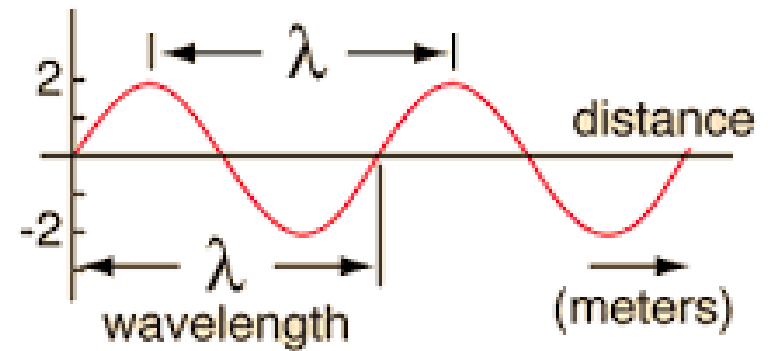
Graphical representation of the solution
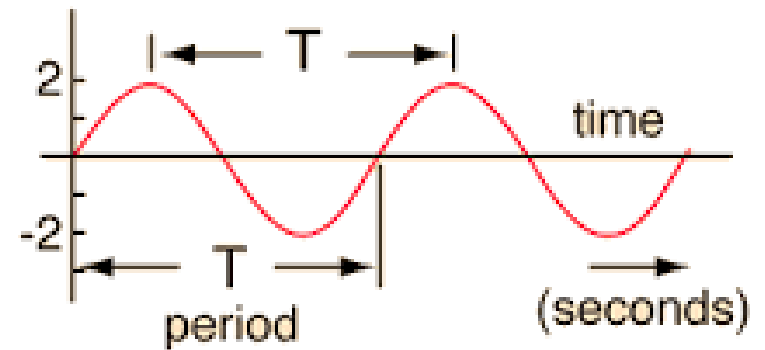

φ

y

t=0

A_o

λ

X

Snapshot at a given time

2

-2

λ

λ

wavelength

distance

(meters)

Time evolution at a given point

2

-2

T

T

period

time

(seconds)

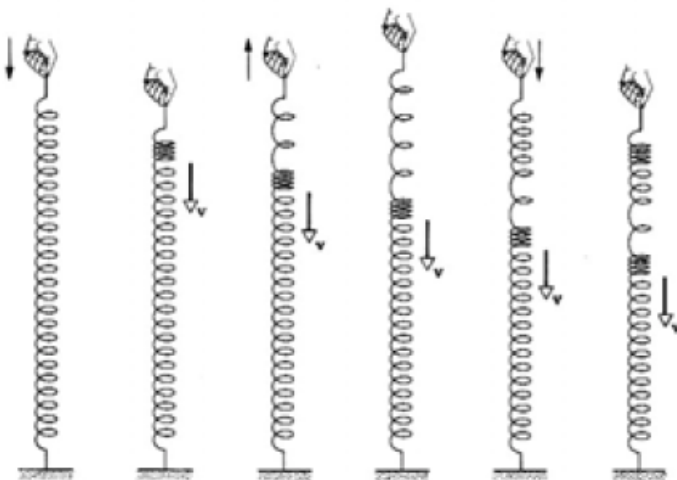# Different types of waves

- **According to source:**
  1. Mechanical: elastic deformation propagating through elastic medium
  2. Electromagnetic: electric disturbance propagating through space (vacuum)
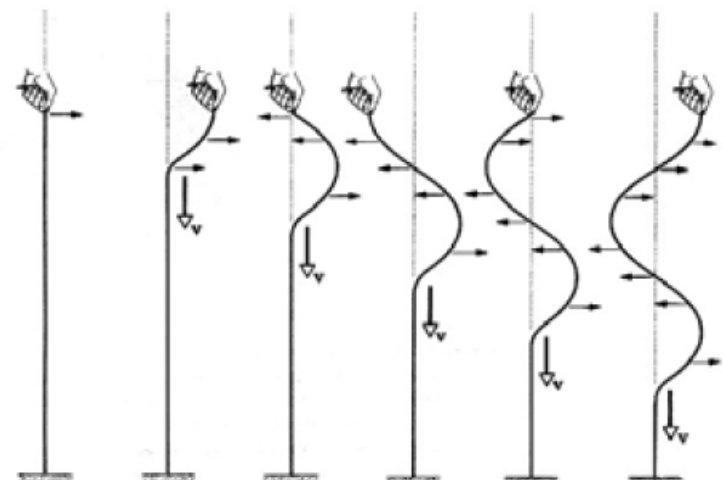
- **According to propagation dimension:**
  1. One-dimensional (rope)
  2. Surface waves (pond)
  3. Spatial waves (sound)

- **According to relative direction of oscillation and propagation:**
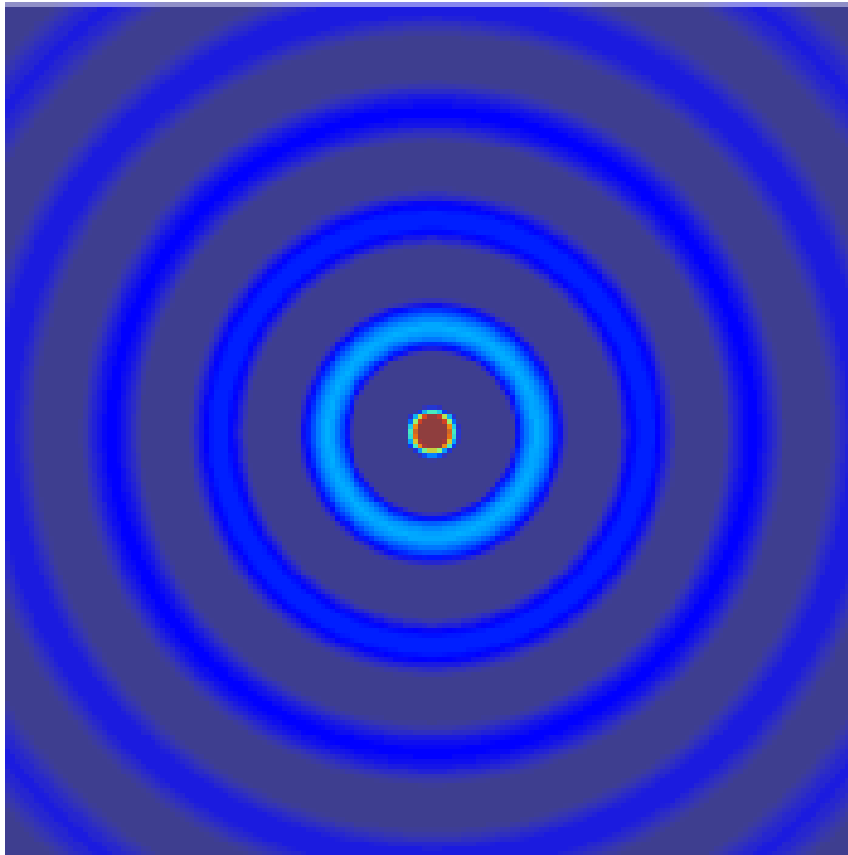
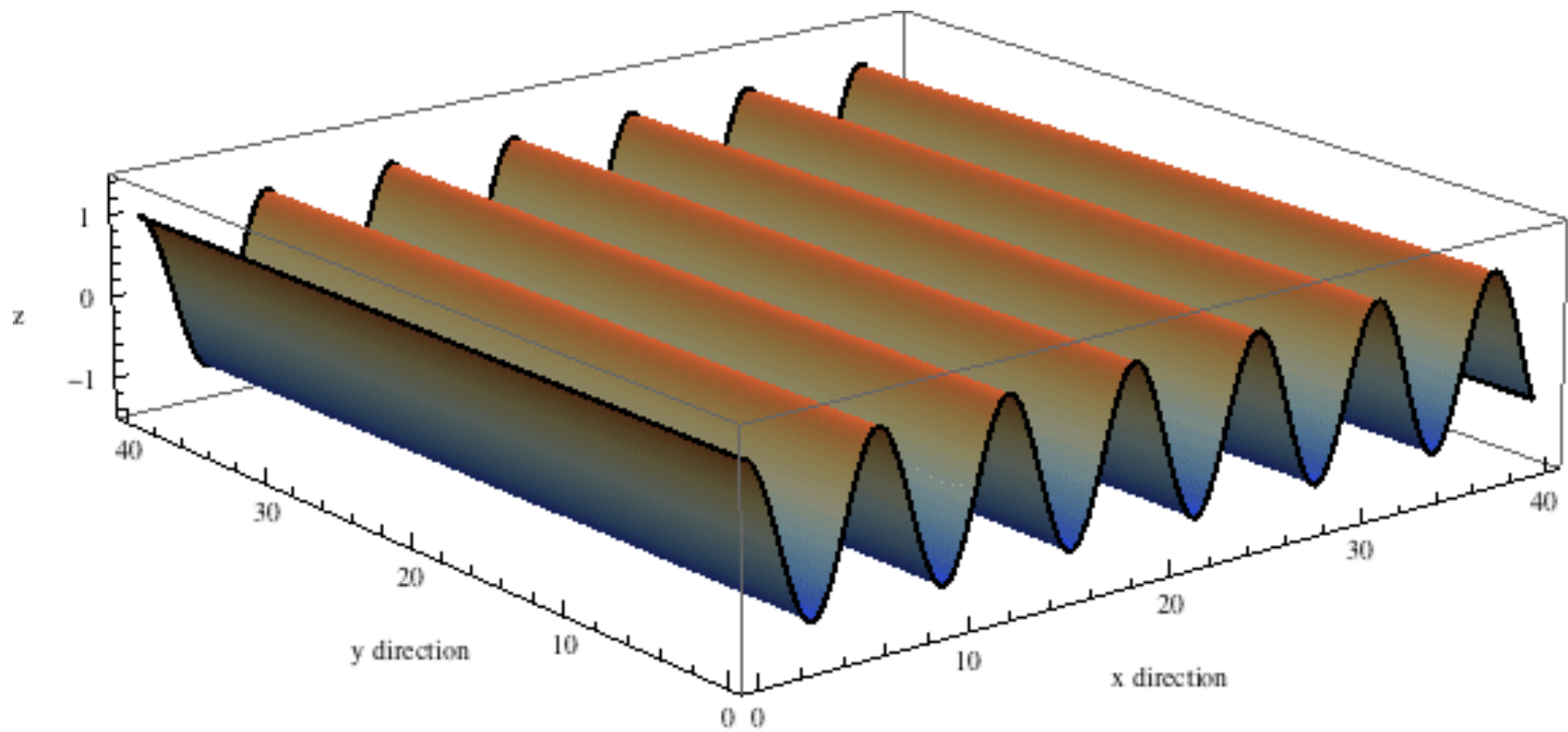 1. Longitudinal

  2. Transverse

Point source: radiating in all directions along a sphere.

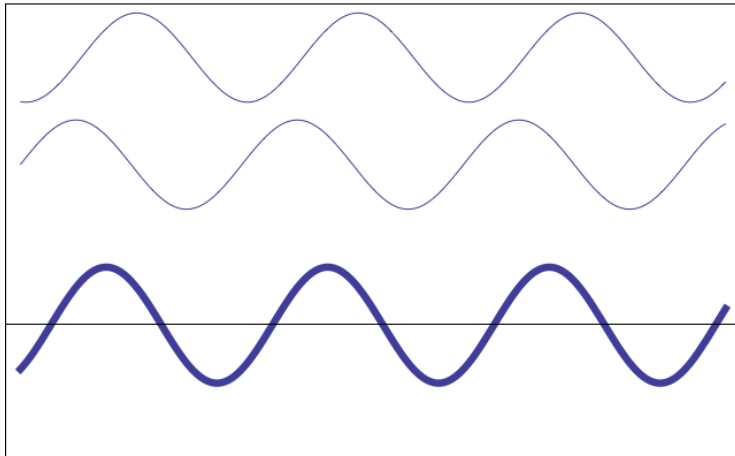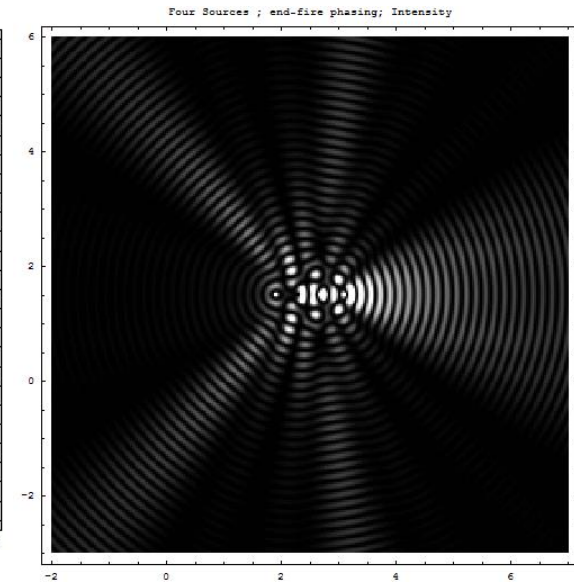This is a transverse wave example, longitudinal is also possible.

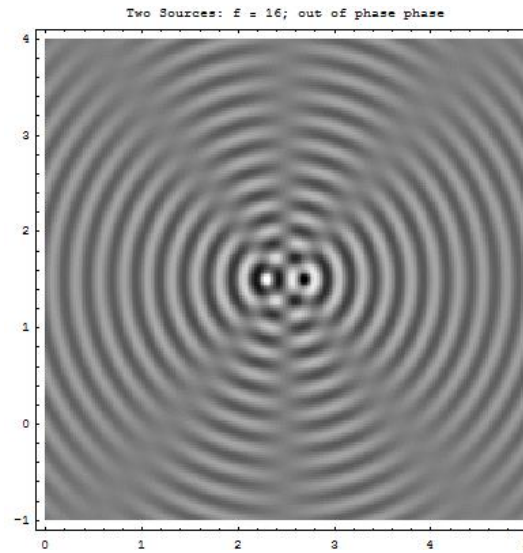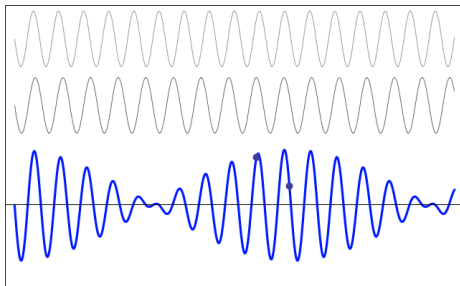Plane wave (again transversal)

Superposition of waves:
the "displacements" caused by each individual waves add up at every point

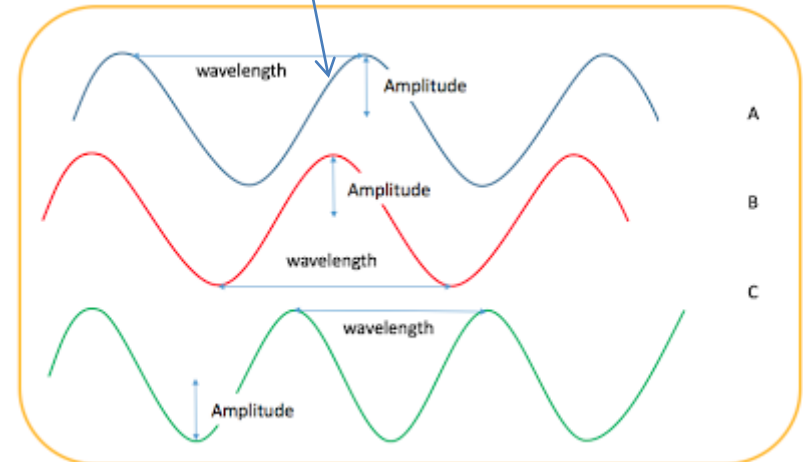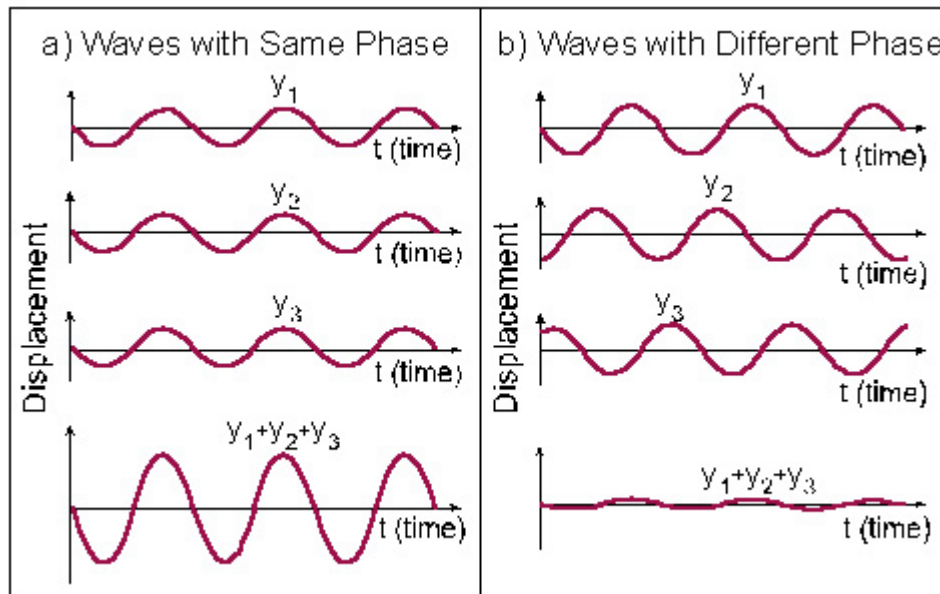$$u(x,t) = A_1 * \sin(k_1 * x + \omega_1 * t + \phi_1) + A_2 * \sin(k_2 * x + \omega_2 * t + \phi_2) + \ldots$$

Two Sources: f = 16; out of phase phase

Four Sources ; end-fire phasing; Intensity

https://www.acs.psu.edu/drussell/demos/superposition/superposition.html

Waves can be coherent or incoherent.
Coherent waves have the same frequency and a constant phase difference

Only coherent waves can have a stable interference pattern,
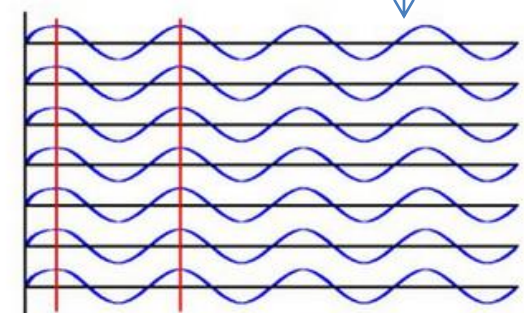incoherent waves on average sum up close to 0.

a) Waves with Same Phase

y_1

t (time)

y_2

t (time)

y_3

t (time)

$y_1+y_2+y_3$

t (time)

b) Waves with Different Phase

y_1

t (time)

y_2

t (time)

y_3

t (time)

$y_1+y_2+y_3$

t (time)

Displacement

wavelength   Amplitude   A

Amplitude   B

wavelength

wavelength

Amplitude   C

If the phase difference is 0 (or 1,2,3.. *2π),
we have constructive interference.

Constructive interference:
the resulting amplitude (A_{sum})
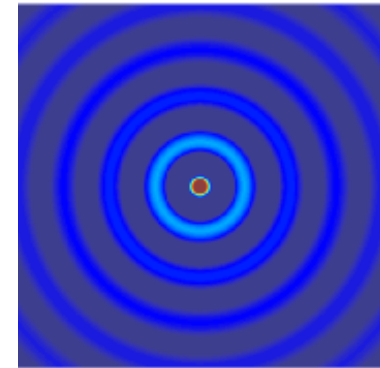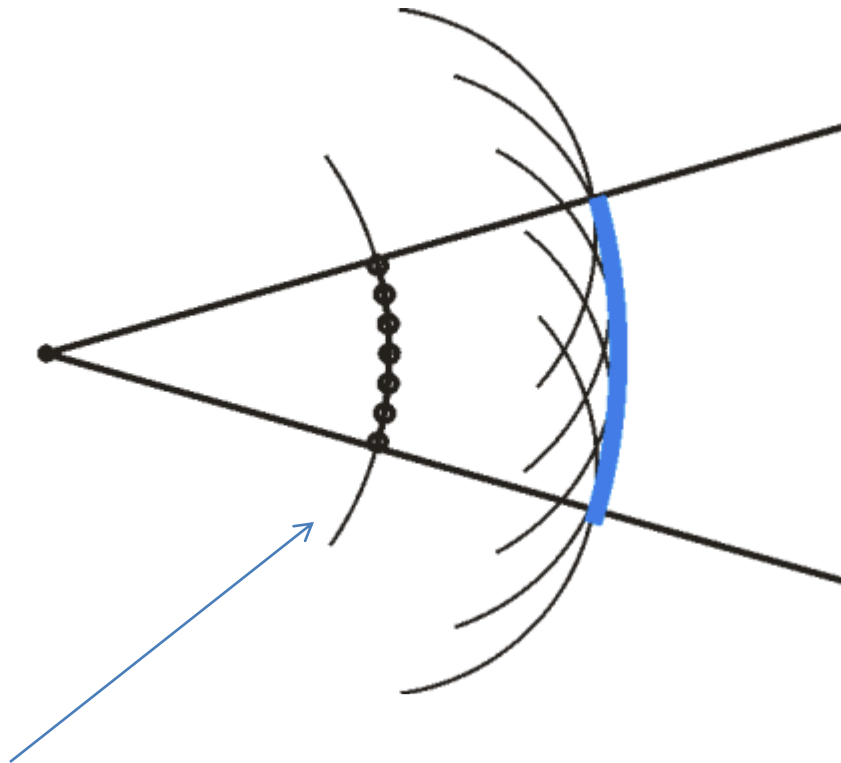after summation is maximal.
Destructive: A_{sum} = 0

Incoherent light waves

Coherent light waves

The Huygens-Fresnel principle describes the propagation
of waves, it can be used to explain most of the experimental results
(but not the quantum-mechanical ones!)

In short: every point at the wavefront acts as a new sperical point source,
and the resulting new wavefront can be computed as a superposition of all of the
waves generated in this way.

Christiaan **Huygens**
(1629-1695)

Augustin-Jean **Fresnel**
(1788–1827)

Wavefront: a surface containing points affected in the same way by a wave at a given time.

Some experiments, observations, which can only be understood
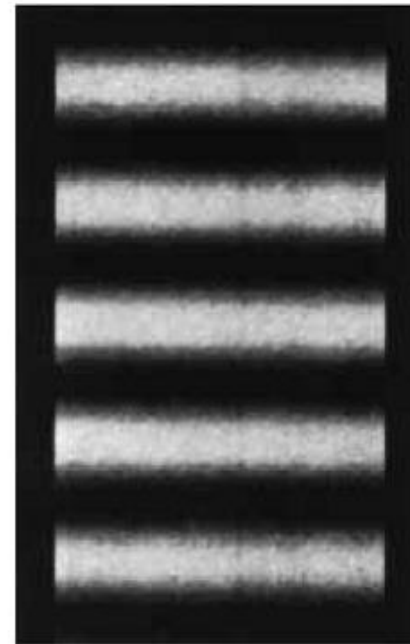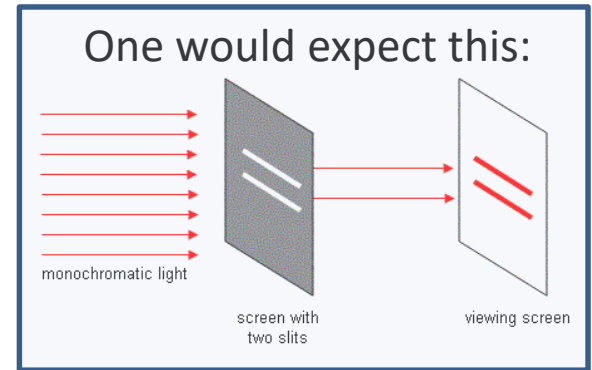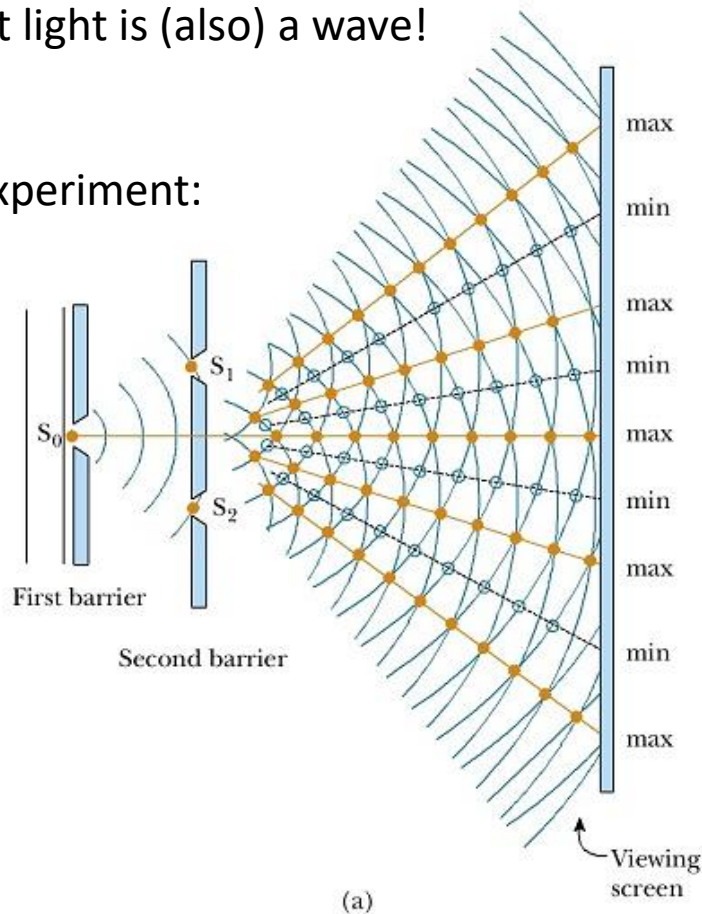with the help of wave theory.

Young's two-slit experiment
Diffraction on grating
Interference patterns

this proves that light is (also) a wave!

The two-slit experiment:

One would expect this:

monochromatic light

screen with
two slits

viewing screen

max
min
max
min
max
min
max
min
max

S₀

S₁

S₂

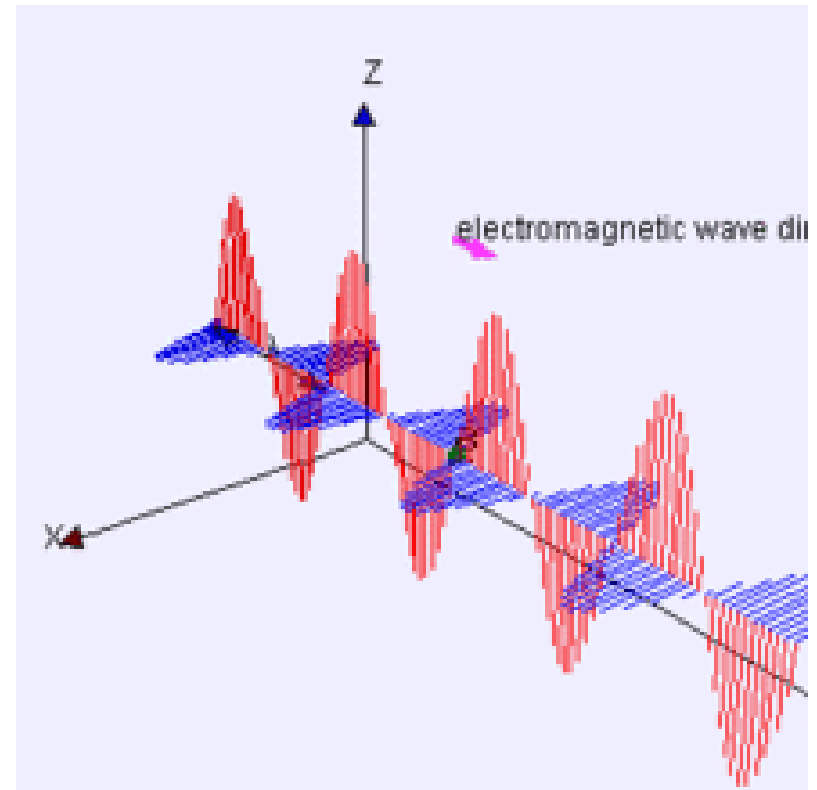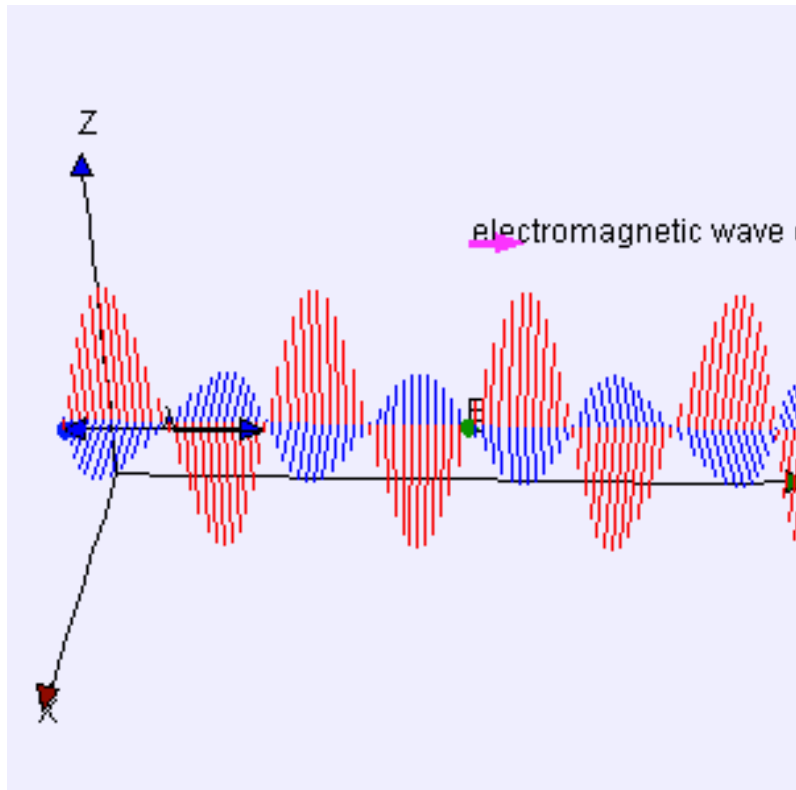First barrier

Second barrier

Viewing
screen

(a)

(b)

Thomas Young

Since we have interference, we must assume light is a wave.
If so, then we have a wave equation for it. Since it is electro-magnetic, we have two oscillating quantities: electric field strength (**E**) and magnetic field strength (**B**).

For EM waves we have two equations, and the wave can travel to x,y,z directions, so the equations are a bit even more complicated:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

Here the ∇ means the $d^2/d...^2$ in all directions
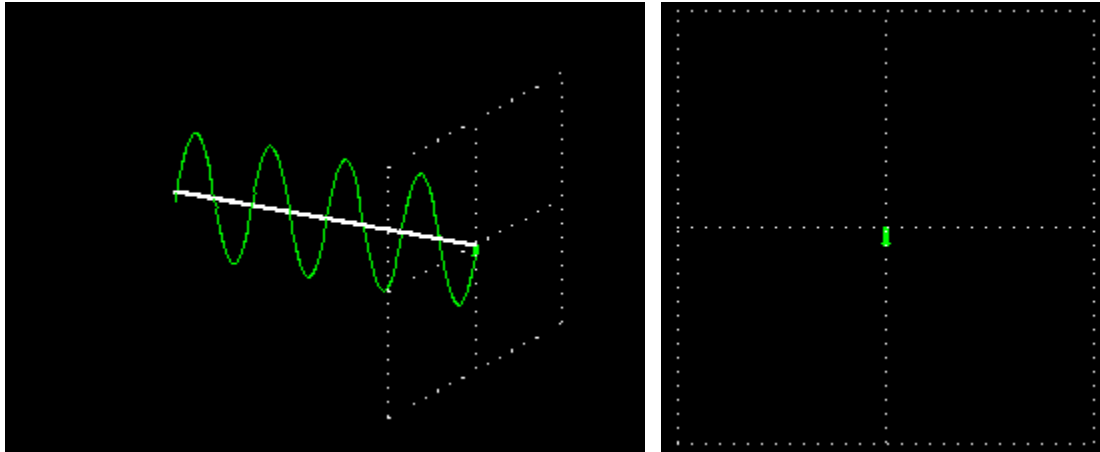
The solution is again a sine or cosine wave:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$
$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$
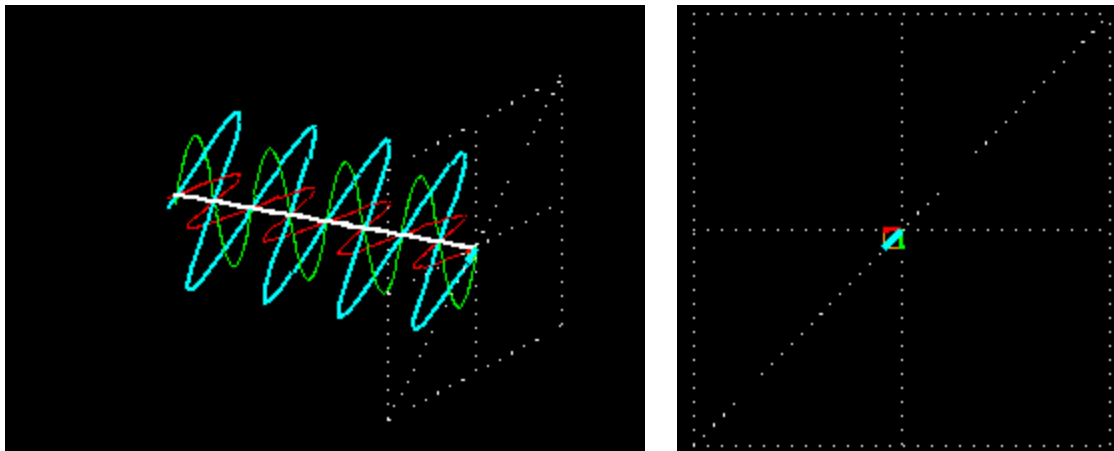
At any point of the observation we have to add all of the incoming sine waves, and that gives the net value of **E** and **B**.

Remember: incoherent waves add up to practically 0, while coherent ones can add up from 0 to a maximum, depending on the phase difference.
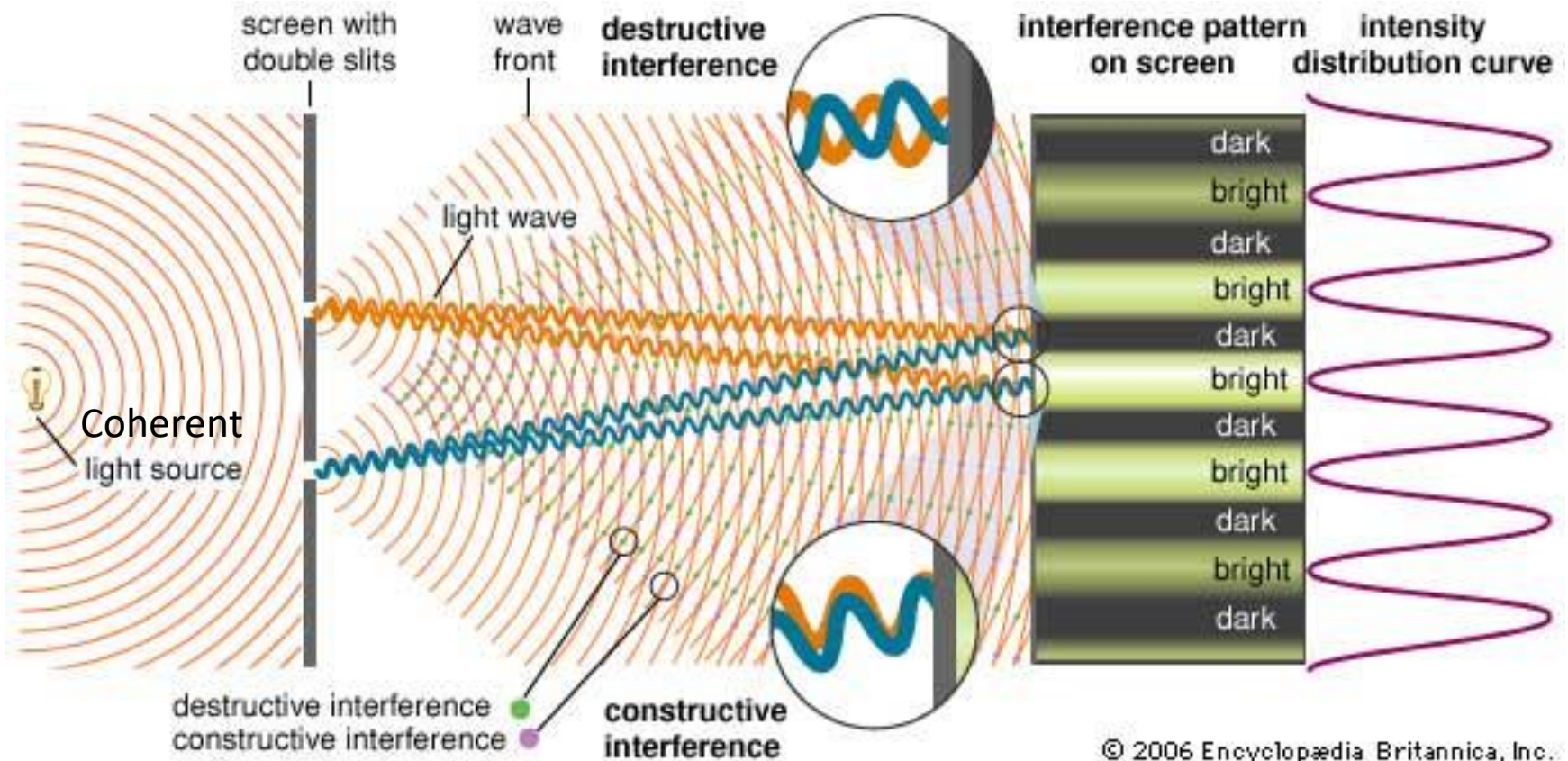
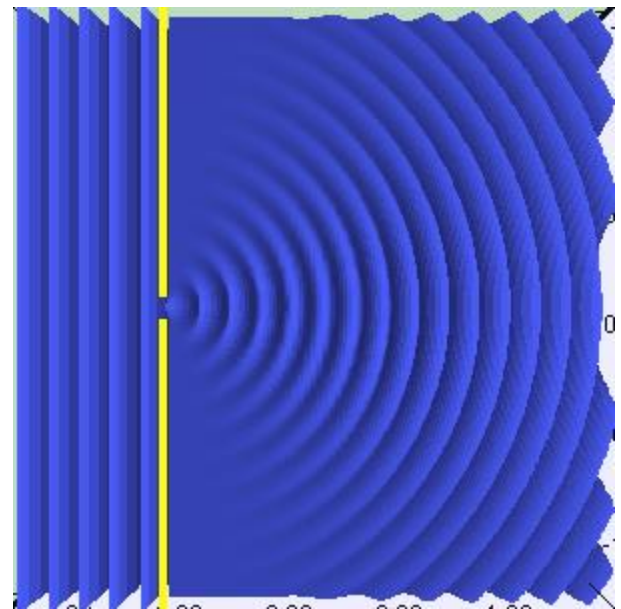usually we draw the E-field only, since the B-field is bound to it. This makes the graphics simpler.
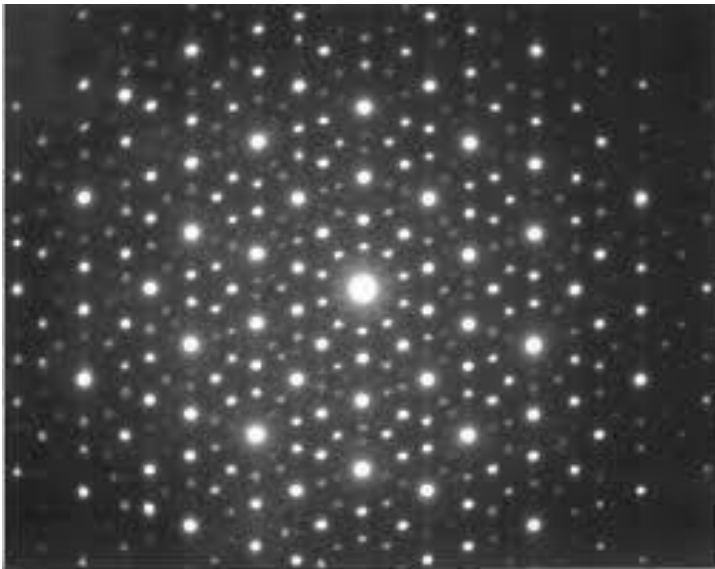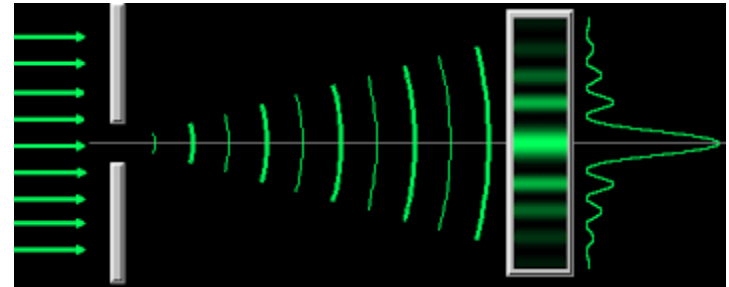

Summation of two waves with different E field direction

The explanation of the periodic profile : interference of waves, Huygens principle.

screen with double slits
wave front
destructive interference
interference pattern on screen
intensity distribution curve

light wave

Coherent light source

dark
bright
dark
bright
dark
bright
dark
bright
dark
bright
dark

destructive interference
constructive interference
constructive interference

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Diffraction and interference patterns with coherent light

propagation direction

Diffraction of waves on water surface

THANKASSERY HARBOUR
The Largest in Size in Kerala

To understand the patterns we need to calculate the phase for each wave at the screen.

paths out of phase
destructive interference

paths in phase
constructive interference

Dark
(destructive
interference)

Bright
(constructive
interference)

θ

λ

(a)

(b)
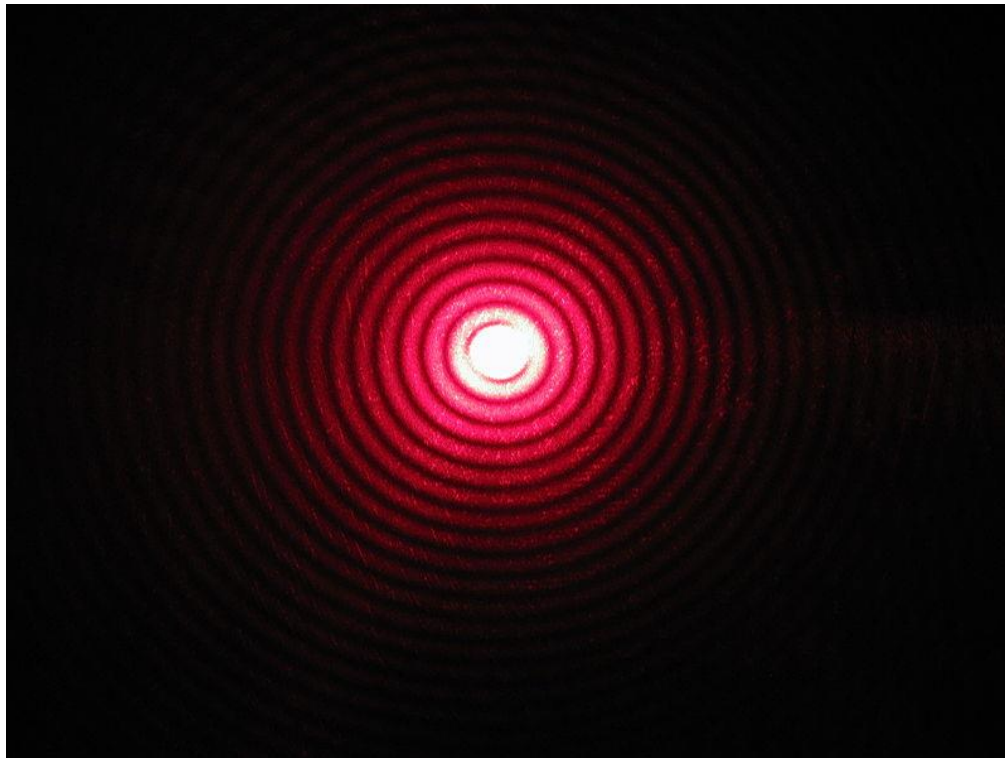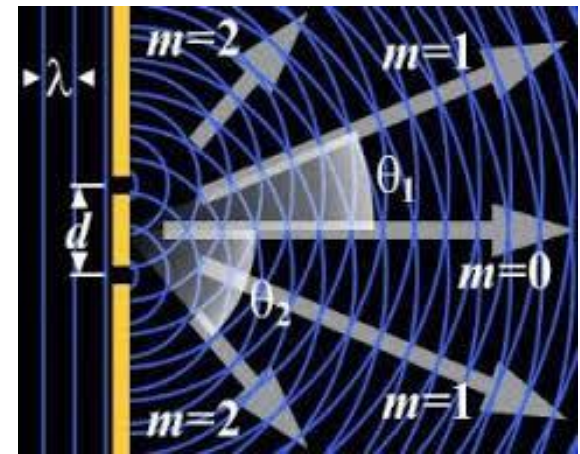
$m=2$ $m=1$

λ

θ_1

d

$m=0$

θ_2

$m=2$ $m=1$

Diffraction on optical grating:

Incoming plane wave of light

P — First-order maximum ($m = 1$)

Central or zeroth-order maximum ($m = 0$)

First-order maximum ($m = 1$)
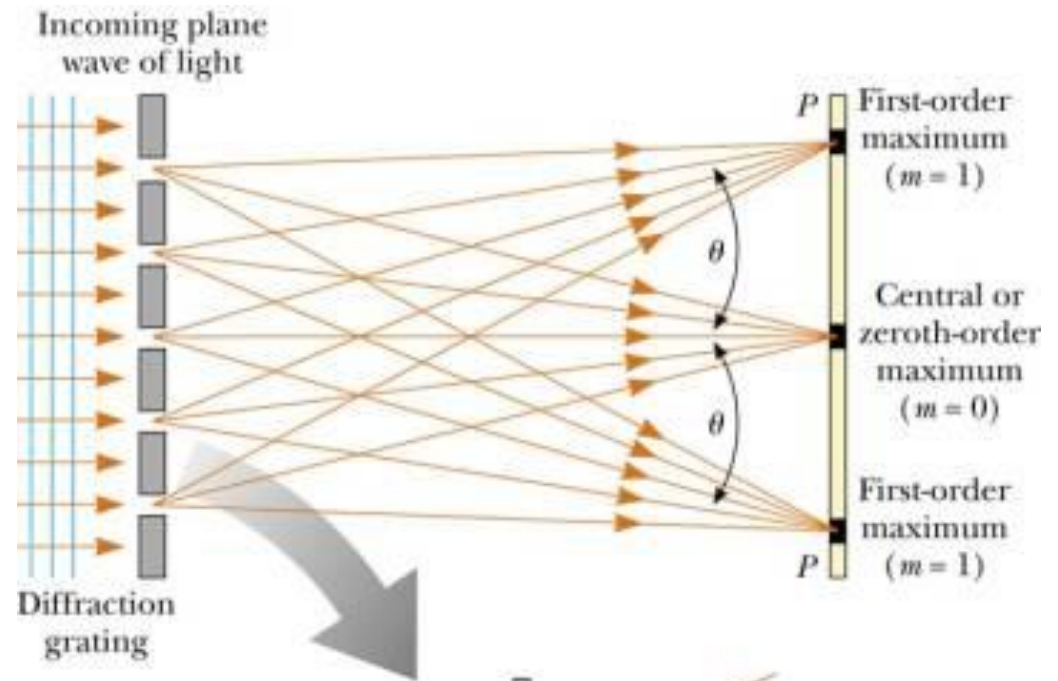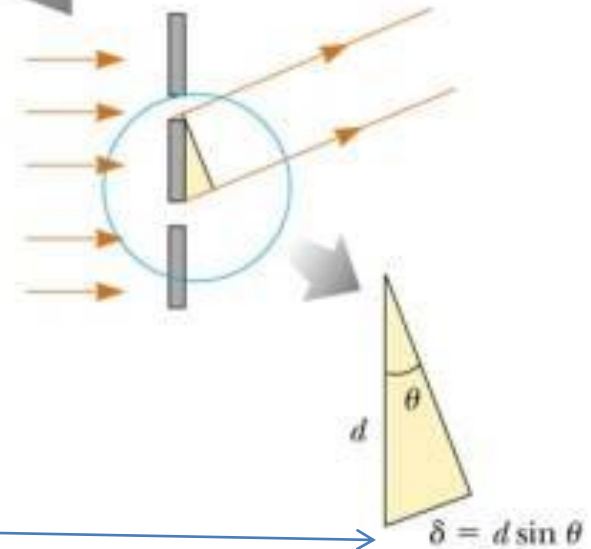
Diffraction grating

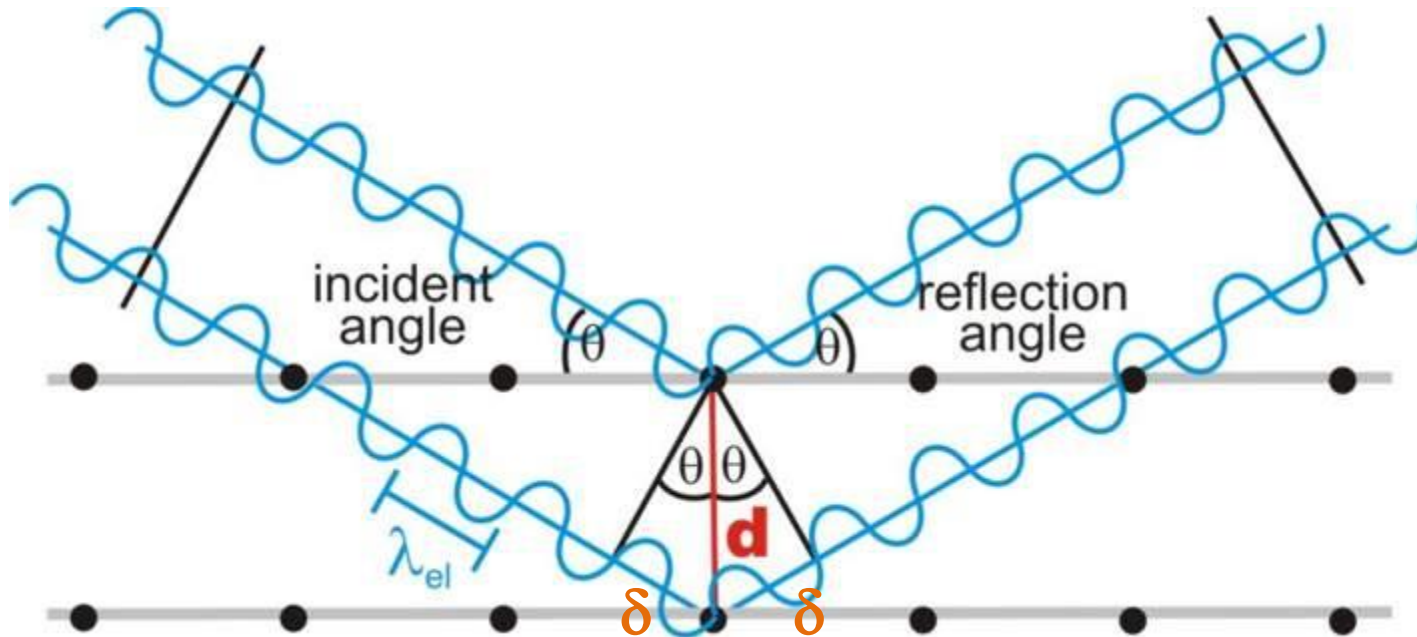Due to the path difference of δ a phase difference of $2\pi*\delta/\lambda$ arises between waves.

(n.B.: during a length of λ, the time it takes for the light to travel is T, under which the change in the phase is exactly 2π)

Constructive interference is only possible if the phase difference is 0,1,2,3,… * 2π. This means that δ has to be 0,1,2,3,… * λ

$\delta = d \sin \theta$

On a reflection grating the concept is the same, but the phase difference (due to **path difference 2*δ**) is twice as much as on a transmission grating.
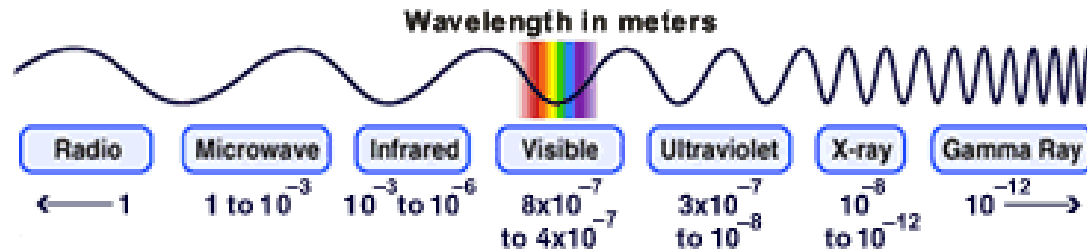
Applications:

Optical grating can separate the different frequencies -> recording of a spectrum
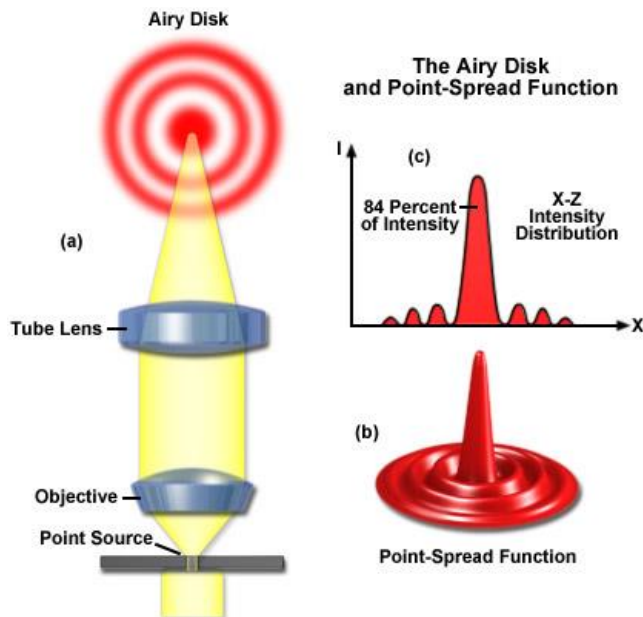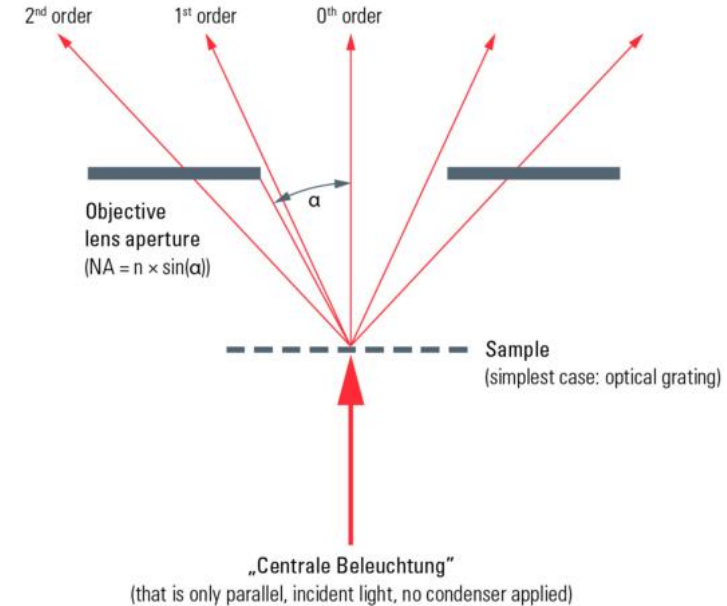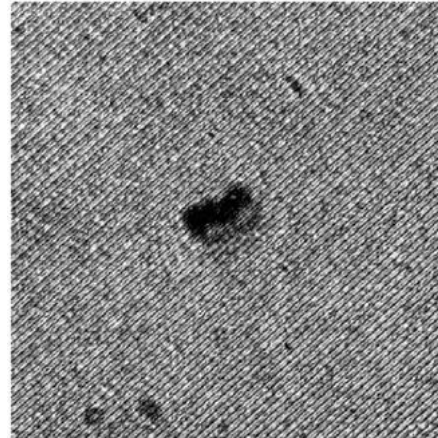
Resolving power of a microscope is limited by the wave's diffraction.

Abbe's principle:
There is **only** an image formation in the microscope **if** at least the first order diffracted waves enter the objective lens.

2nd order   1st order   0th order

Objective lens aperture
$(NA = n \times \sin(\alpha))$

Sample
(simplest case: optical grating)

„Centrale Beleuchtung"
(that is only parallel, incident light, no condenser applied)

Airy Disk

The Airy Disk and Point-Spread Function

(c)

84 Percent of Intensity

X-Z Intensity Distribution

(a)

Tube Lens

(b)

Objective

Point Source

Point-Spread Function

$$d = \frac{\lambda}{2 n \sin \alpha}$$

Ernst Abbe
(1840-1905)

modern version: $\delta = 0.61 \dfrac{\lambda}{n \sin \omega}$

Demonstration of the Abbe's principle with a slit+stage micrometer under the microscope

a)

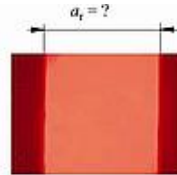detailed image of the optical grating (many higher order maxima)

objective

directions of the interference maxima

slit (opened)

specimen (stage micrometer as an optical grating)

$d = 10\ \mu m$

monochromatic light source $\lambda_r = 618\ nm$ (red)

b)

$a_r = ?$

image of the "red" slit

blurred image of the optical grating (limiting case, 10 μm lines are just not visible)

$(n \sin \omega \approx a / 2b)$

modified Abbe's-formula

$$d' = 0.61\ \frac{\lambda_r}{a_r / 2b}$$

$d' \sim 10\mu m$?

$a_r / 2 = ?$ "red" slit (a_r) in the limiting case

$\omega_r\ \omega_r$

$b = 3.15\ mm$

monochromatic light source $\lambda_r = 618\ nm$ (red)
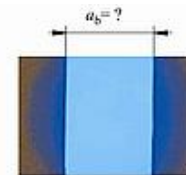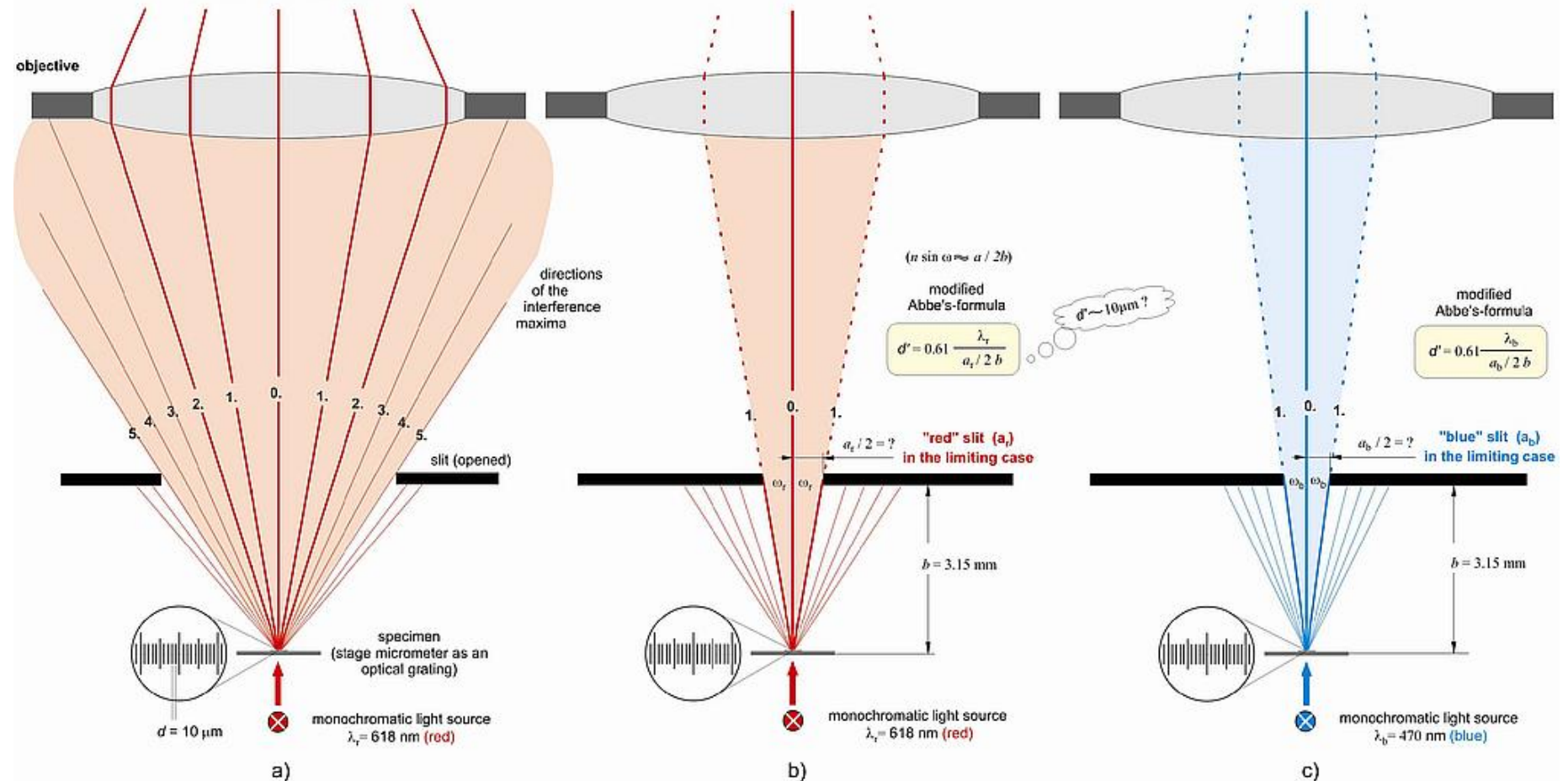
c)

$a_b = ?$

image of the "blue" slit

blurred image of the optical grating (limiting case, 10 μm lines are just not visible)

modified Abbe's-formula

$$d' = 0.61\ \frac{\lambda_b}{a_b / 2b}$$

$a_b / 2 = ?$ "blue" slit (a_b) in the limiting case

$\omega_b\ \omega_b$

$b = 3.15\ mm$
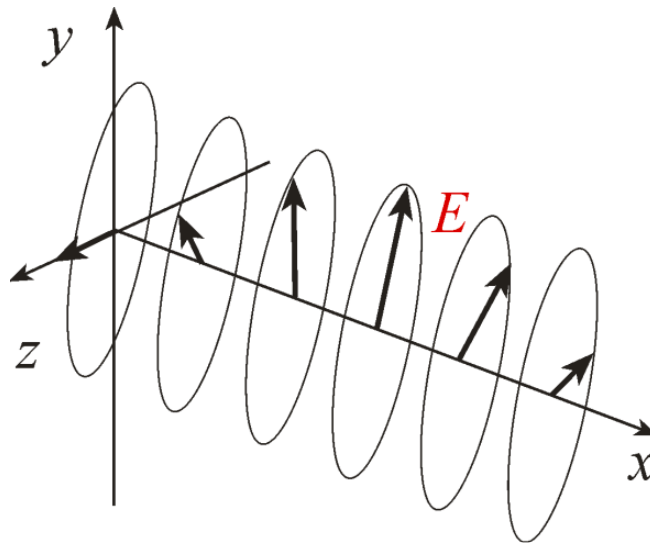
monochromatic light source $\lambda_b = 470\ nm$ (blue)
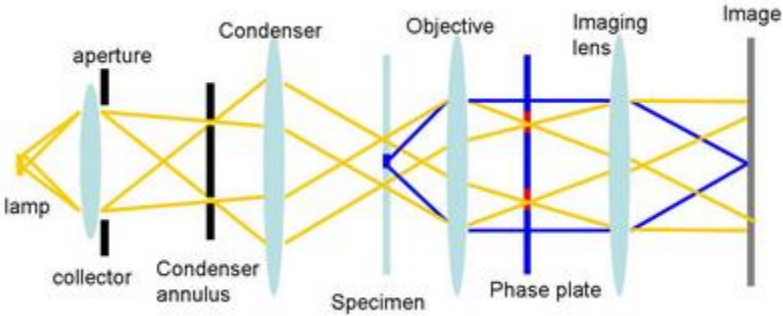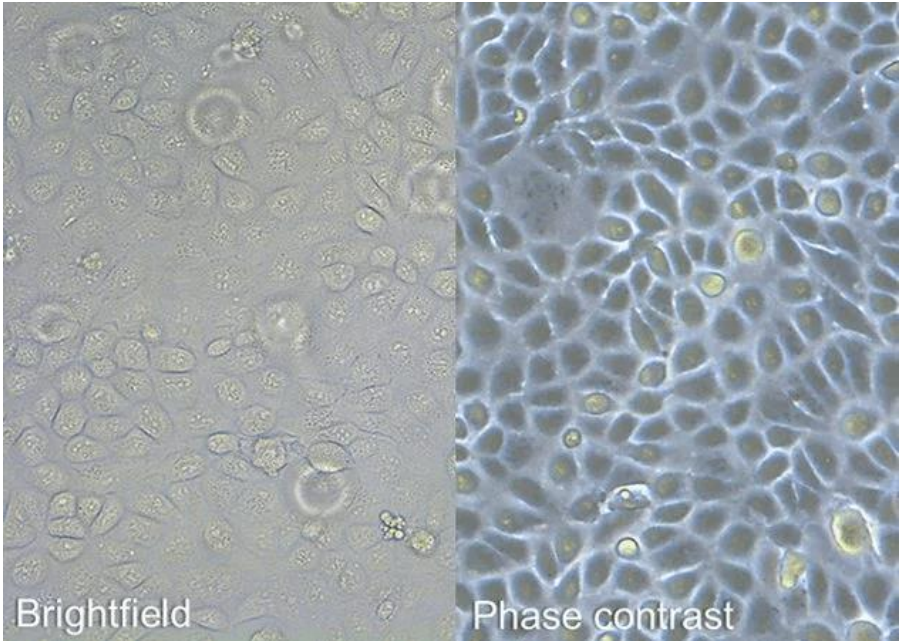
Polarization
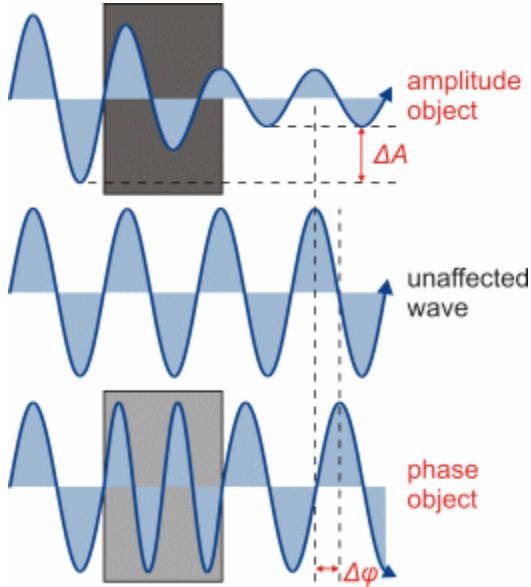
Linearly polarized light
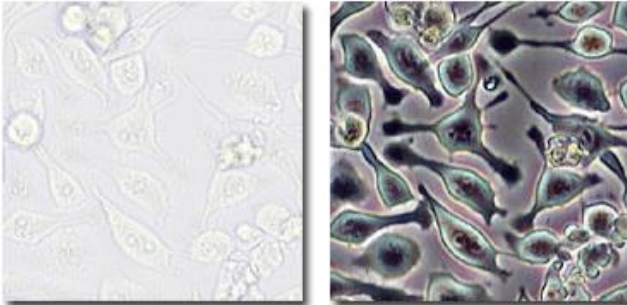
Circular or elliptical polarization

Phase contrast microscopy

The phase object is transparent,
but the refractive index is different,
so the speed of light is different,
which builds up a phase difference during
the way through the object.
This is not observable to the human eye, but
can be made visible with the phase-contrast
microscope, which creates intensity changes
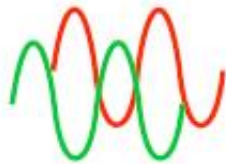from the phase changes.

Brightfield

Phase contrast

It creates the contrast (amplitude change) by coherently re-interfering a reference, or surround beam (S),
with a diffracted beam (D) from the specimen. The S beam is shown in yellow, and the D beam is shown in blue.

Cancellation
$\Delta\varphi=-\lambda/4-\lambda/4=-\lambda/2$
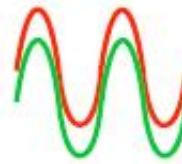
Amplification
$\Delta\varphi=+\lambda/4-\lambda/4=0$

image plane

bright   dark   bright   dark

light-absorbing layer

diffraction maxima

phase plate

In the thick region (at diffraction maxima): $\lambda/4$ phase retardation

principal maximum

Frits Zernike (1888-1966) Nobel-prize

Right ray: $\Delta\varphi=+\lambda/4$
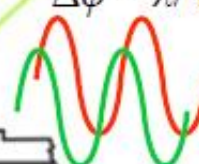Phase of diffracted ray accelerated relative to that of principal ray
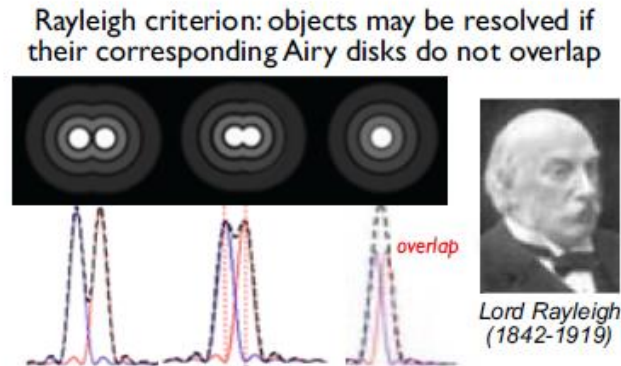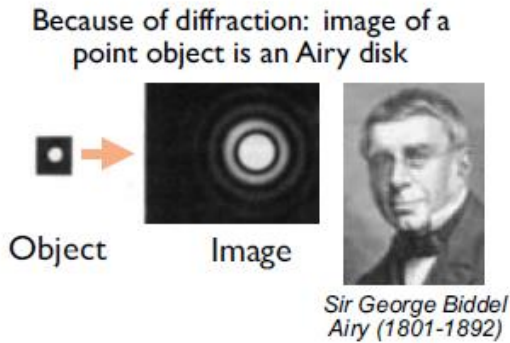
$\Delta\varphi=-\lambda/4$ Left ray: Phase of diffracted ray retarded relative to that of principal ray

phase object

Diffraction is also present in the human eye:

Because of diffraction: image of a point object is an Airy disk

Object → Image

Sir George Biddel Airy (1801-1892)

Rayleigh criterion: objects may be resolved if their corresponding Airy disks do not overlap
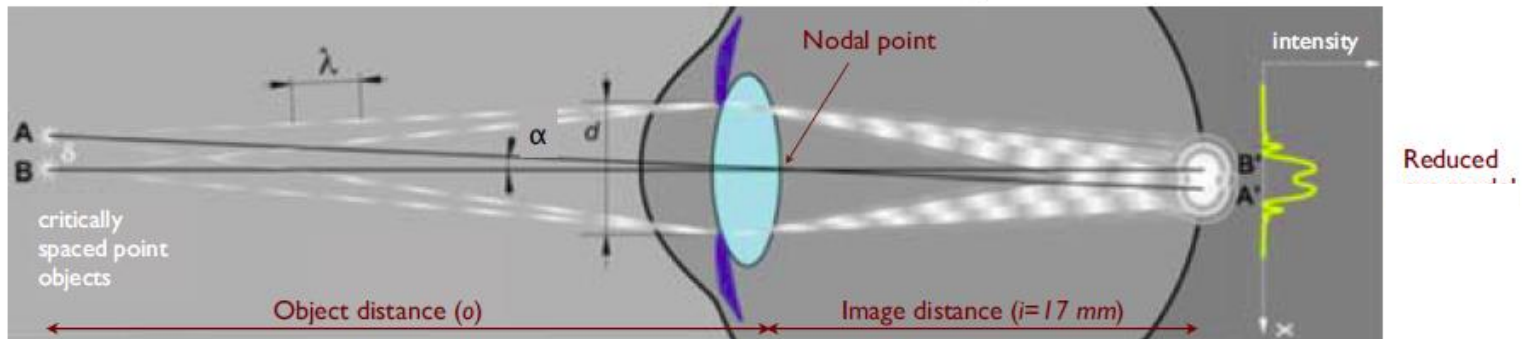
overlap

Lord Rayleigh (1842-1919)

Smallest resolved distance has a limit (Abbe equation):

$$d = \frac{0.61\lambda}{n \sin\alpha}$$

λ = wavelength
n = refractive index of medium
α = angle between axis and outermost ray

Ernst Abbe (1840-1905)

Diffraction limit of the human eye

Nodal point

intensity

λ

A
B

α d

B'
A'

Reduced

critically spaced point objects

Object distance (o)

Image distance (i=17 mm)

For a detailed explanation, and more figures: see the "optics of the eye" lab manual!

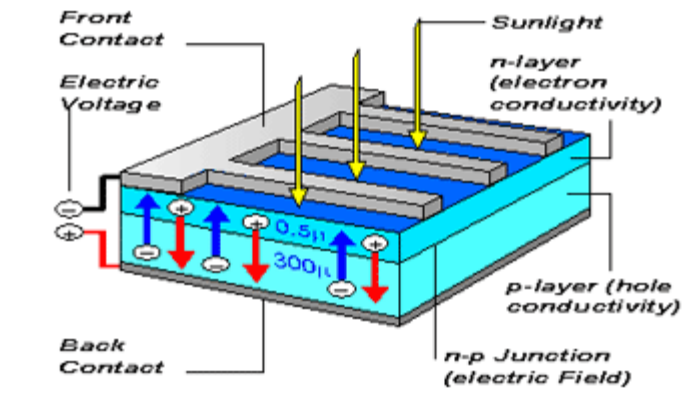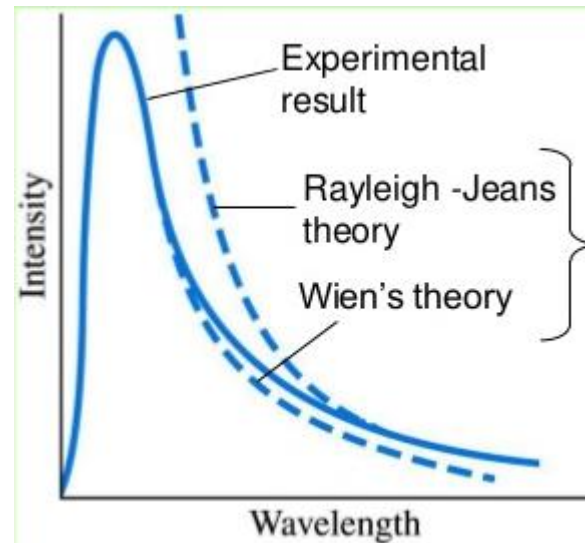There are some experiments, observations which still can not be explained alone by the wave theory
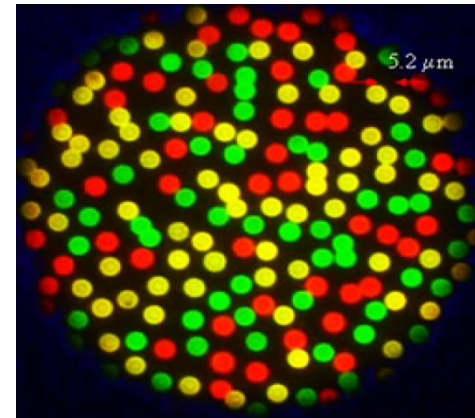
Photoelectric effect
Quantum dots
Fluorescence
Lasers
Black body radiation

-> **light is a wave AND a particle (this leads us to quantum physics)**

Feedback