Physical Basics of Biophysics

Lecture 8 29. 09. 2023.

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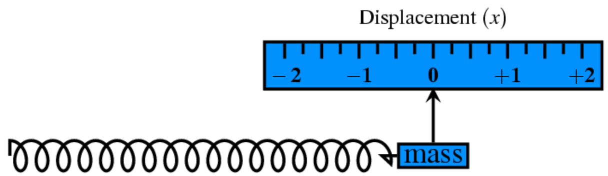
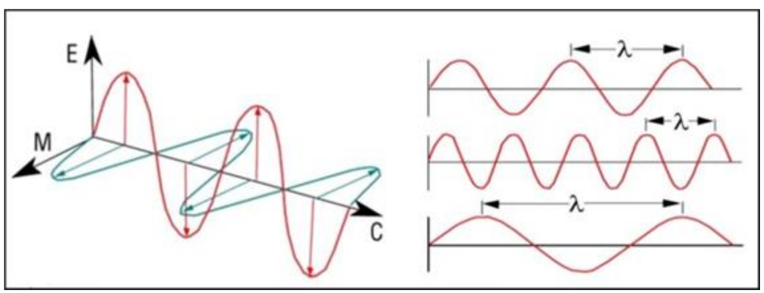
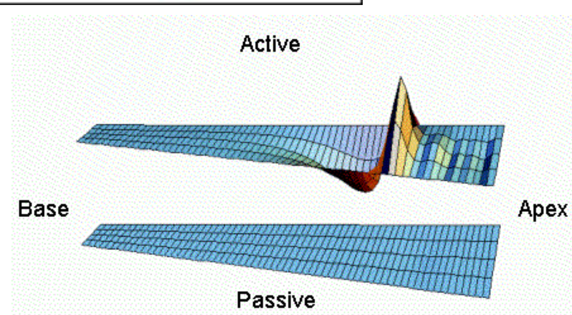
Oscillations and Waves



Basic concepts of oscillations

1. Types of oscillations
2. Free oscillation and eigenfrequency
3. Mass-spring oscillator
4. Driven oscillation
5. Resonance



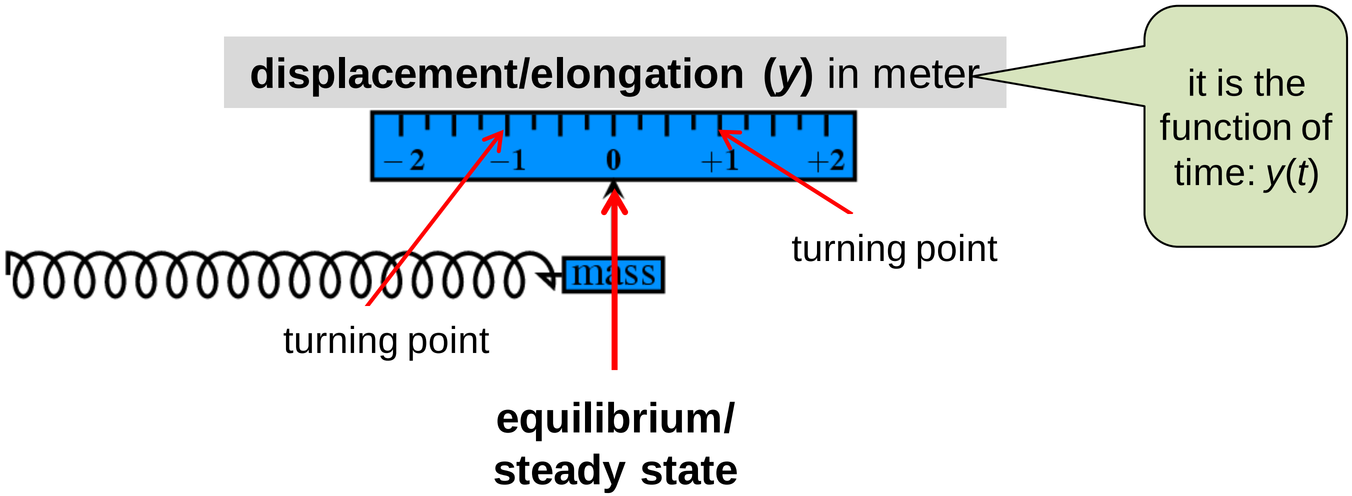


Basic concepts of waves

1. Wavelength
2. Transverse and longitudinal waves
3. Interference
4. Huygens-Fresnel-principle
5. Diffraction
6. Standing waves
7. (linear) polarisation
8. Mechanical waves – sound
9. Electromagnetic waves – light

Basic concepts of oscillations

Oscillator: a physical system capable of oscillation(e.g. mass attached to a spring or pendulum).
Oscillation (mechanical): a periodic (repetitive) motion about a point of equilibrium.



Amplitude (A): maximal displacement

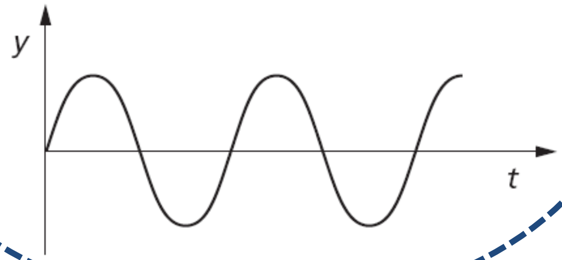
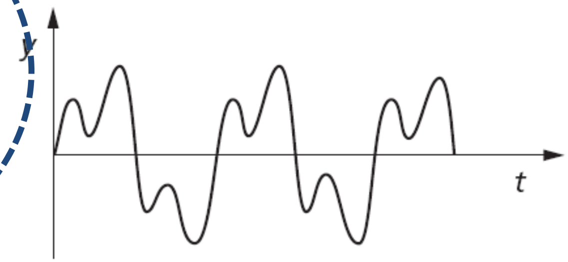
Reminder:

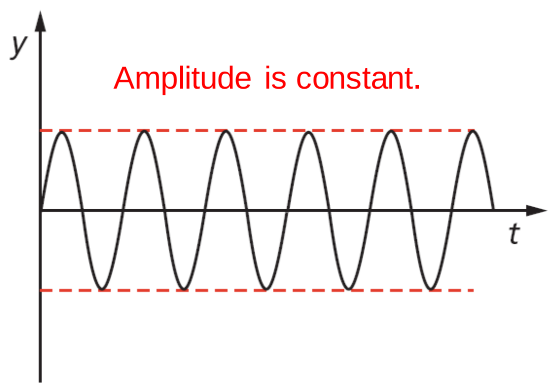
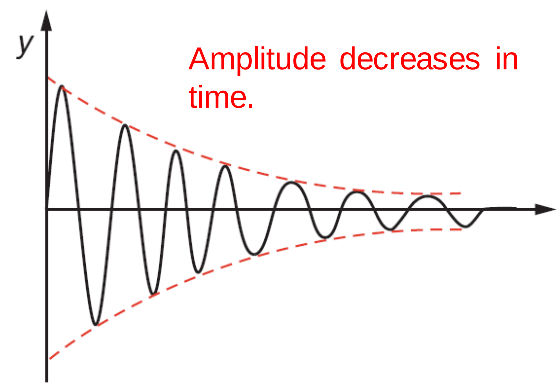
- **Period** (T): duration of one cycle in a repeating event; its base unit is the second (s).
- **Frequency** (f): number of cycles per unit time. The reciprocal of period:

$$f = \frac{1}{T} \quad \left(\frac{1}{\mathrm{s}} = \mathrm{Hz}\right)$$

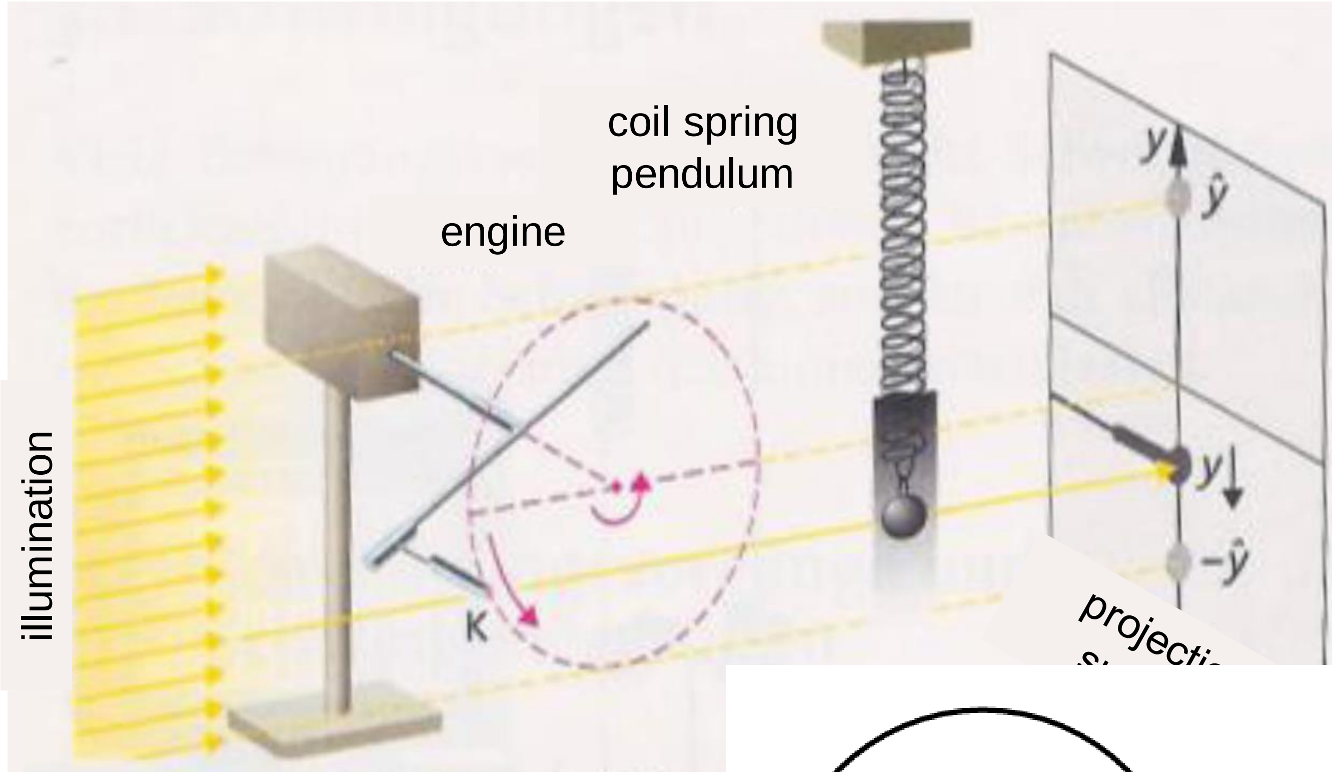
- **Angular frequency** (ω): the number of cycles in 2π. 2π times the frequency : $\omega = 2\pi f$

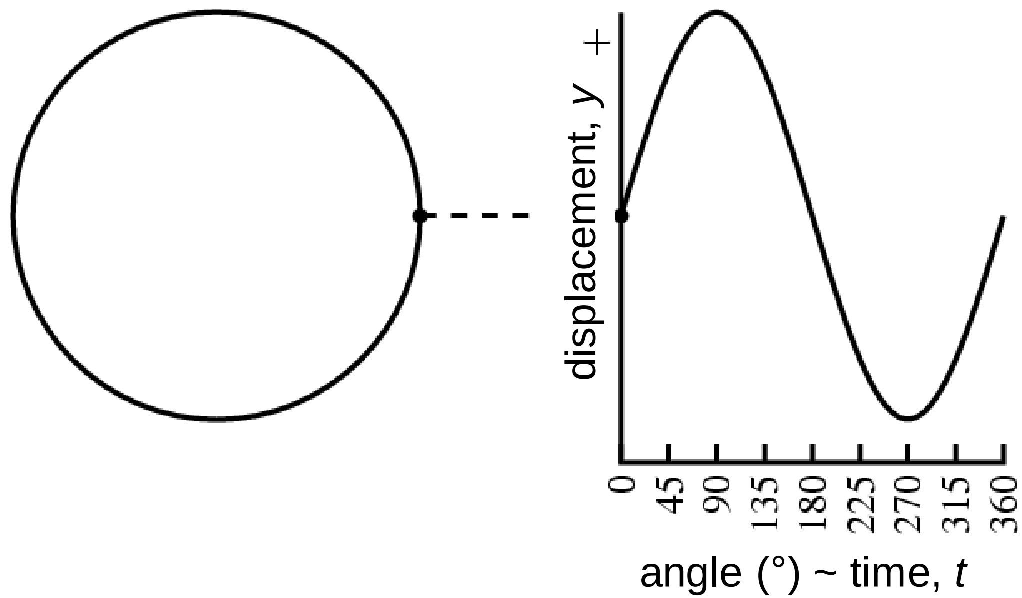
Types of oscillations

Harmonic oscillation (sinusoidal oscillation)	**Non-harmonic oscillation (non-sinusoidal oscillation)**
	
pendulum clock, spring pendulum - spring oscillator	oscillations of the human vocal cord shock absorbers in cars

Free oscillation	**Dampeded oscillation**
	
speaker diaphragm during a sound at a given volume	Pendulum which is left to itself, vibration damper

Uniform circular motion– harmonic oscillation

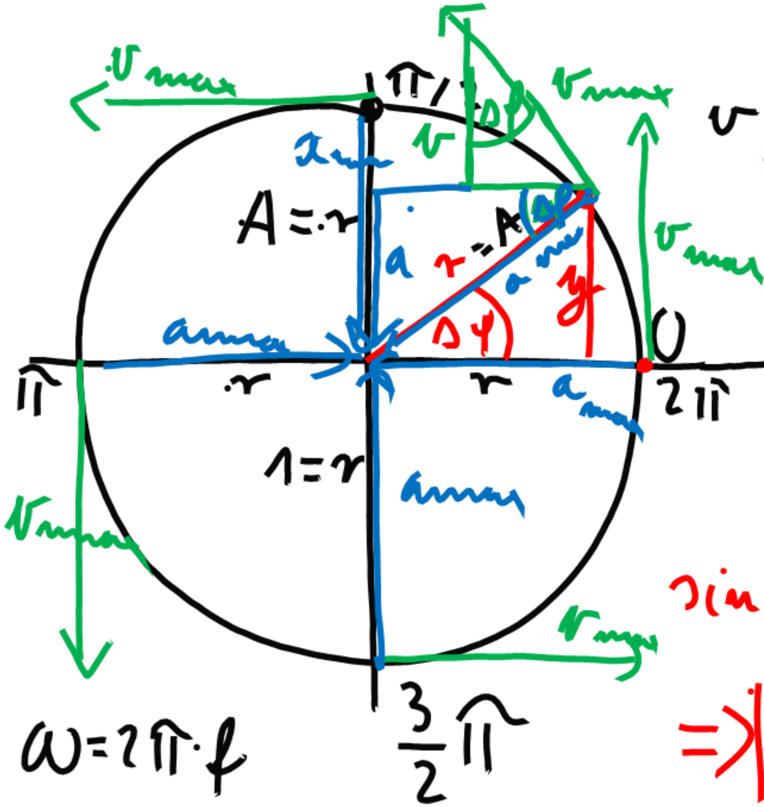




The general form of the displacement-time function:

$$y = A \cdot \sin(\omega t + \varphi_0)$$

Displacement, velocity, acceleration, force

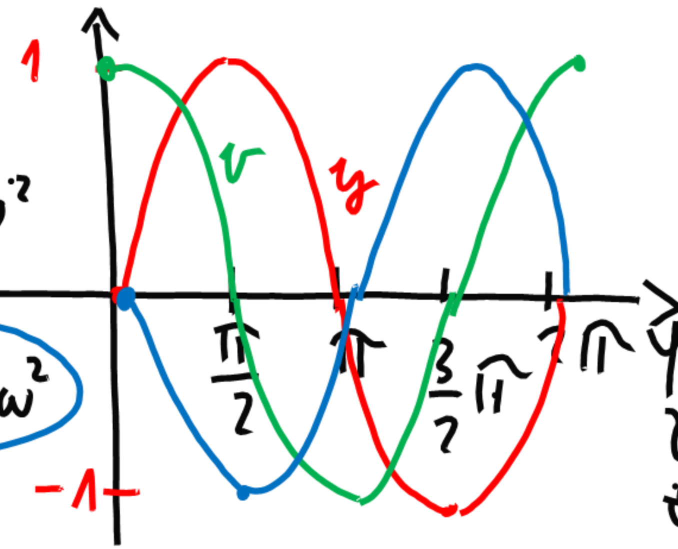


$v = r \cdot \omega$

$A = r$

$a_{cp} = \frac{v^2}{r} = \frac{r^2 \cdot \omega^2}{r}$

$$a_{cp} = r \cdot \omega^2 \quad (r \hat{=} A)$$



$\sin\varphi = \frac{y}{A} \Rightarrow y = A \cdot \sin\varphi$

$\omega = 2\pi \cdot f$

$\cos\varphi = \frac{v}{v_{max}} \Rightarrow v = v_{max} \cdot \cos\varphi$

$$v = A \cdot \omega \cdot \cos\varphi$$

$\sin\varphi = \frac{a}{a_{max}} \Rightarrow a = a_{max} \cdot \sin\varphi$

$$a = -A \cdot \omega^2 \cdot \sin\varphi$$

$F = m \cdot a$

$F = -m \cdot A \cdot \omega^2 \cdot \sin\varphi$

$$F = -m \cdot \omega^2 \cdot y$$

$F = -k \cdot y$

$-k \cdot y = -m \cdot \omega^2 \cdot y$

$k = m \cdot \omega^2$

$2\pi \cdot f = \omega = \sqrt{\frac{k}{m}}$

$$f = \frac{1}{2\pi} \cdot \sqrt{\frac{k}{m}}$$

Free oscillation

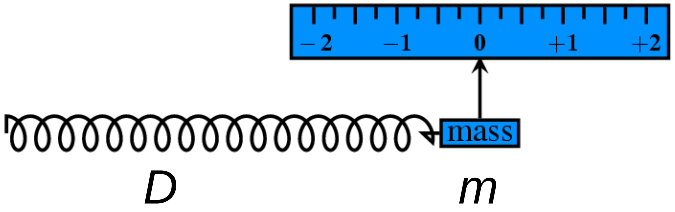
Prerequisite:
We displace the system and and release it, creating an oscillation.

Free oscillation: oscillation that proceed on its own without external influence.

Eigenfrequency (natural frequency): frequency of a free oscillation.

It is determined by the properties of the oscillator (mass, geometric quantities, material properties, etc.).

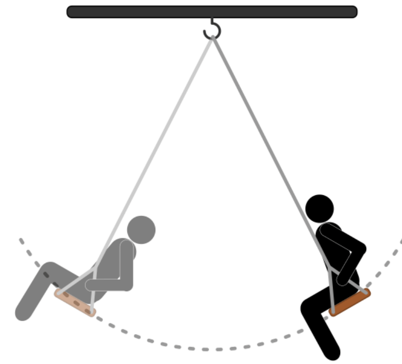
Mass-on-spring oscillator



$$f_{\text{natural}} = \frac{1}{2\pi}\sqrt{\frac{D}{m}}$$

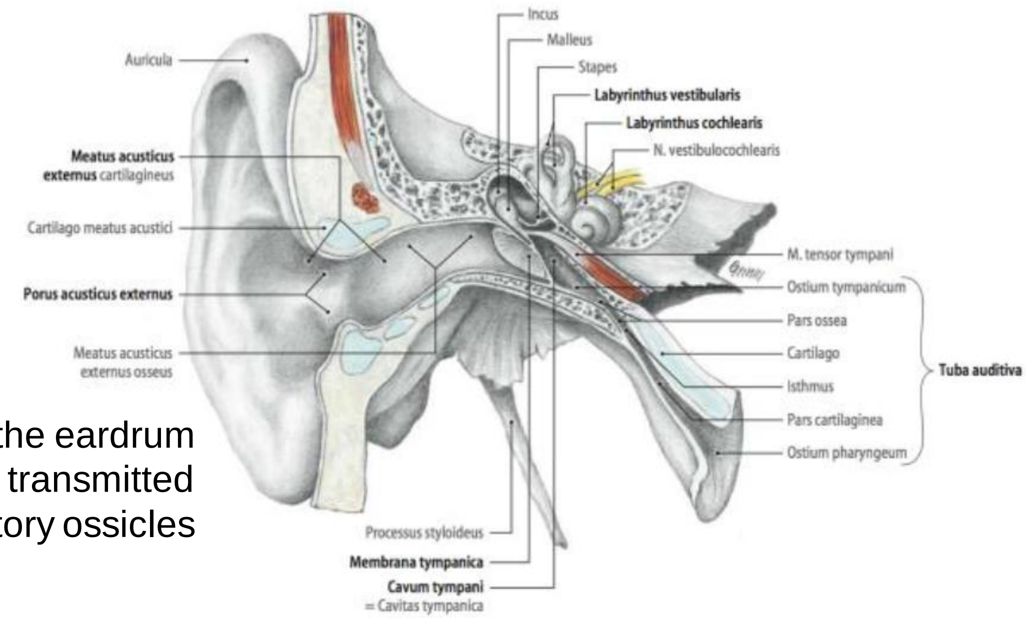
Comment:
The equation is only valid in the ideal case, i.e. the oscillation is harmonic (not damped). In fact, there is always a loss of energy (friction, air resistance,…), so the oscillation is damped.

Driven oscillation



Oscillation caused by a periodic **external force**.

If one of the tuning forks is hit, the resulting air pressure fluctuations will also cause the other tuning fork to vibrate (provided that both forks are tuned to the same pitch).

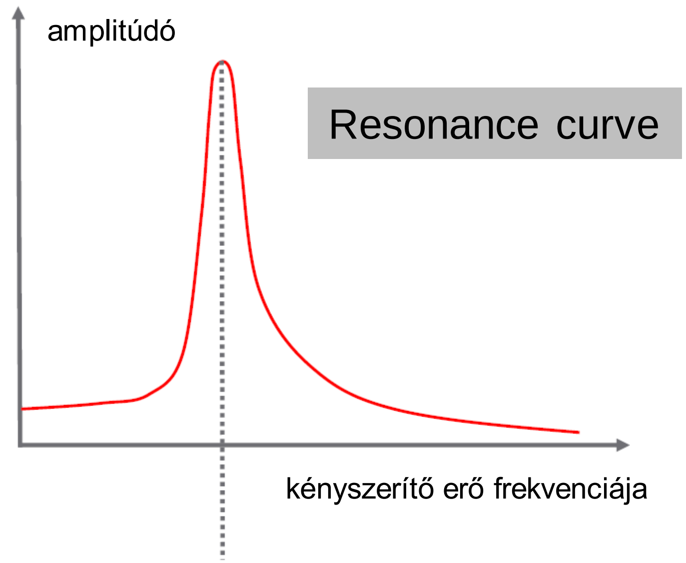


Air pressure fluctuations vibrate the eardrum and then these vibrations are transmitted through the auditory ossicles

- During a driven oscillation, we can maintain the harmonic oscillation with a constant amplitude with the driving force, despite the energy losses.
- The oscillationg system **takes up the frequency of the driving oscillation**.

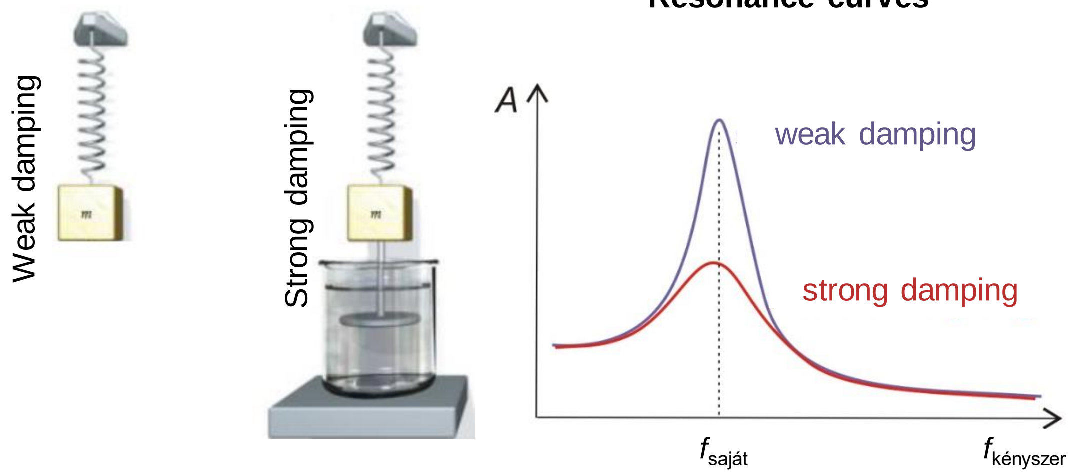
Resonance

- If energy is periodically transmitted to a oscillating system by means of an **external driving system**, **driven oscillation** will develop after a certain settling time.
- Depending on the frequency of the driven oscillation, oscillations of different amplitudes are generated.
- If the **frequency of the driven oscillation coincides** with the **natural frequency of the system**, a particularly strong, high-amplitude driven oscillation is generated.
- A particularly high-amplitude oscillation at a given frequency is called a **resonance**, and a frequency characteristic of a phenomenon is called a **resonance frequency**.

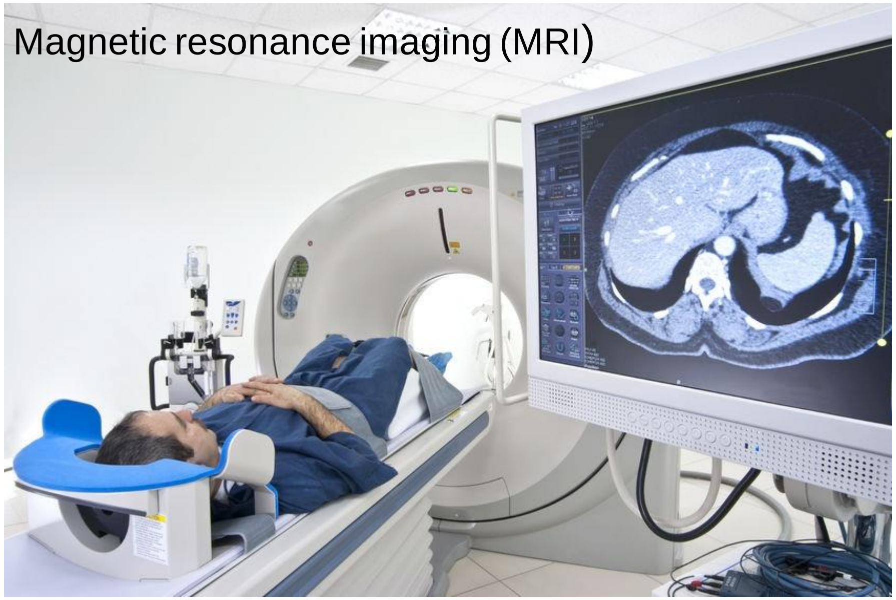


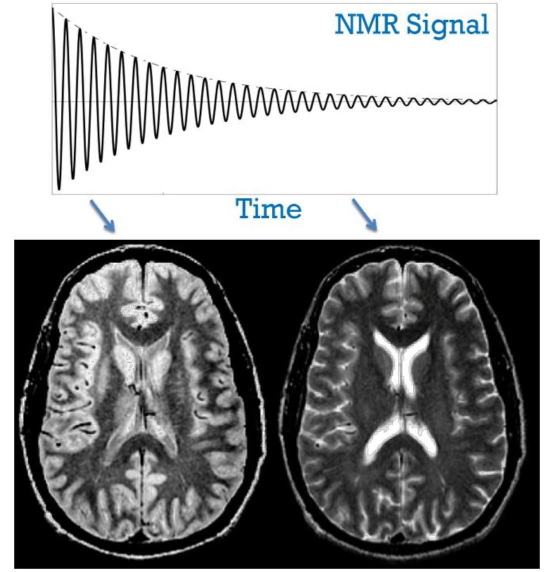
resonance frequency = Natural frequency of the oscillator

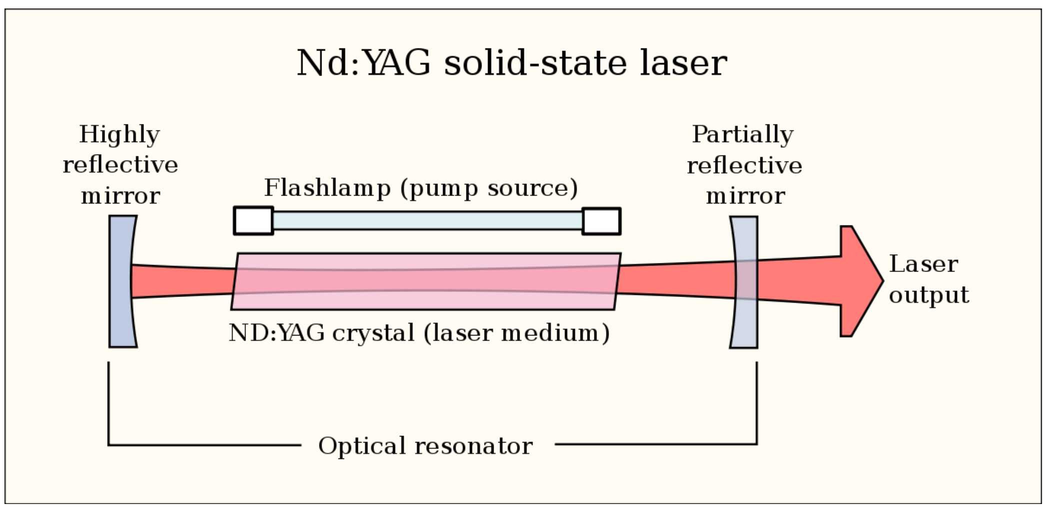
Resonance curves



Comment:
The phenomenon of resonance is used in several technical devices (eg nuclear magnetic resonance spectroscopy and imaging, laser, ...)



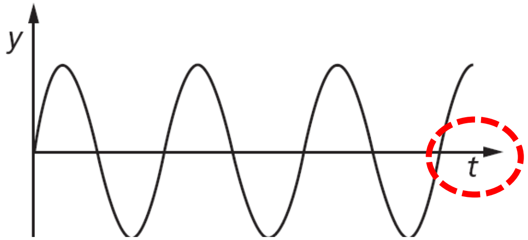


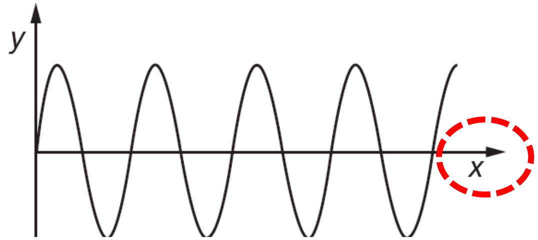


Homework: Chapter 6 and 7

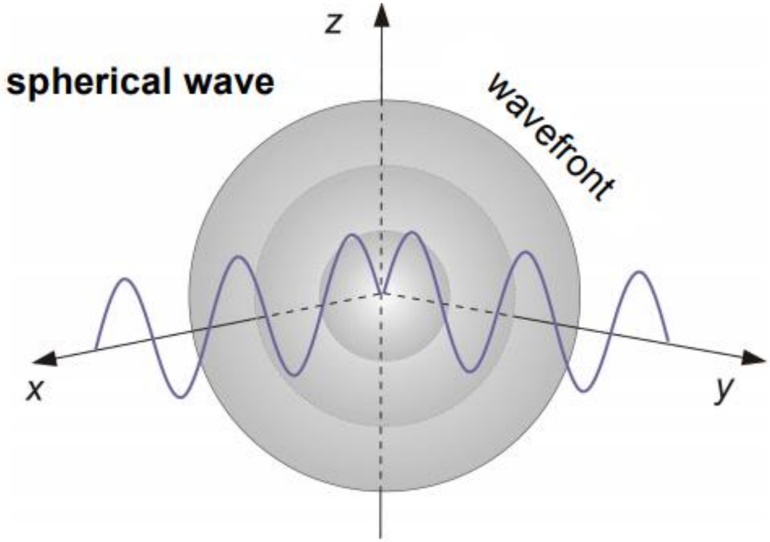
Waves – basic concepts

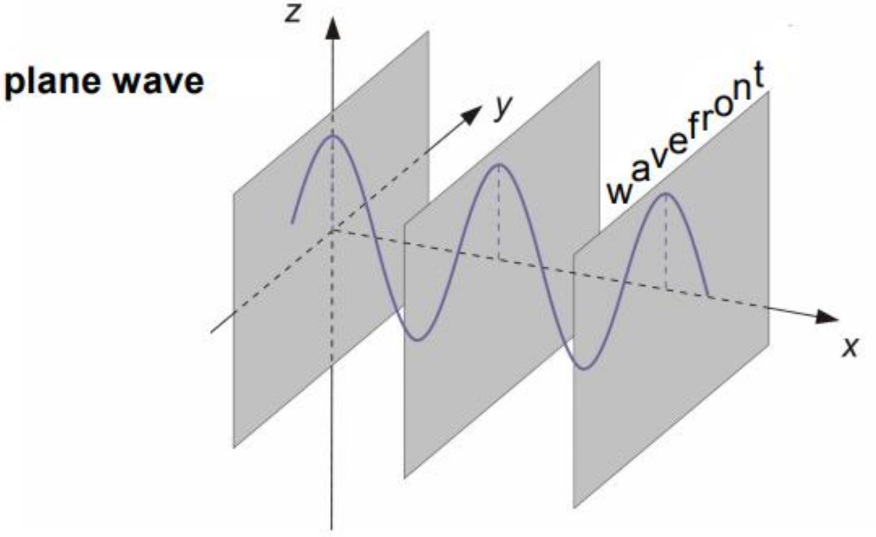
- Propagation of an oscillation in a medium capable of oscillation.
- Periodic (repeated) changes in a physical quantity over **time** and **space**.



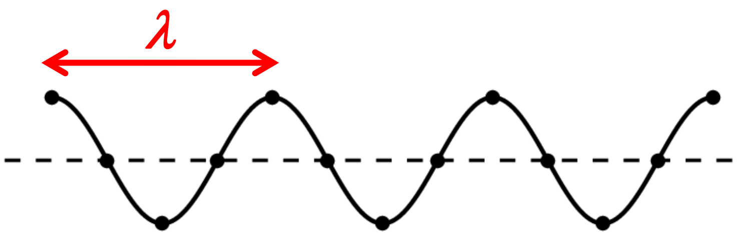


Wavefront: The surface containing points of waves in identical phase - in the same state of oscillation.



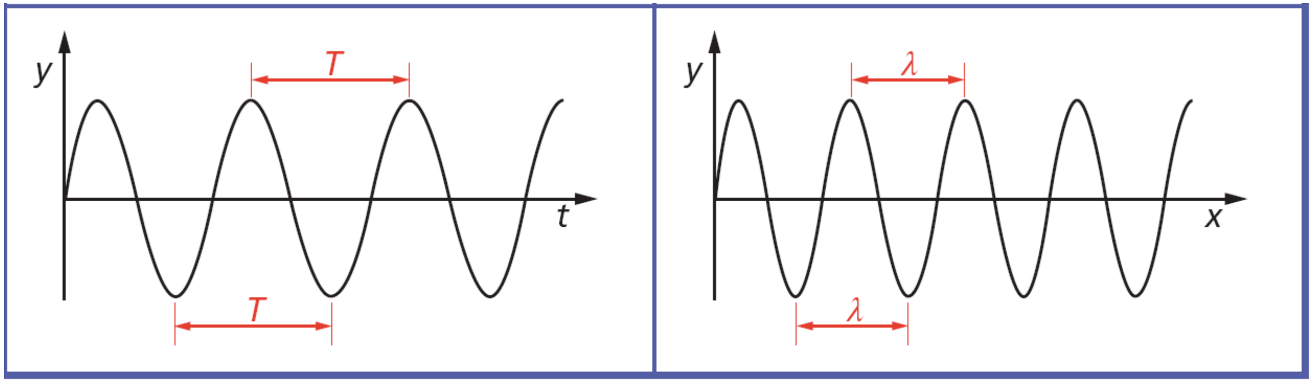


Wavelength



Wavelength (λ): The length of a wave, the distance between two consecutive points of the same phase.

- The **wavelength** is an amount analogous to the **period**. **Wavelength** characterizes **spatial** periodicity, while **period** is **temporal** periodicity.



Correlation between wavelength and period time (or frequency):

$$c = \frac{\lambda}{T} = \lambda \cdot f$$

Velocity of propagation

***c**eleritas (lat.)* - swiftness

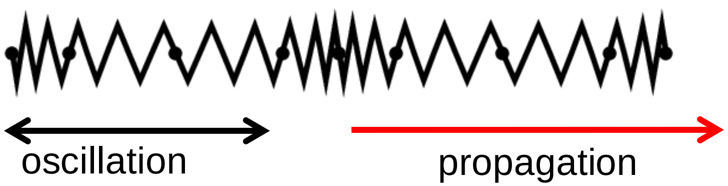
Comment:
The relation is generally valid for all types of waves (be they mechanical, electromagnetic or even material waves).

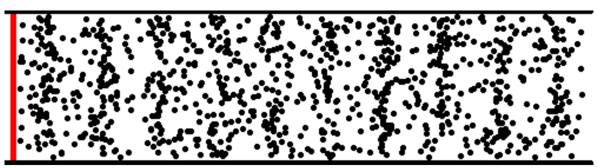
Longitudinal and transverse waves

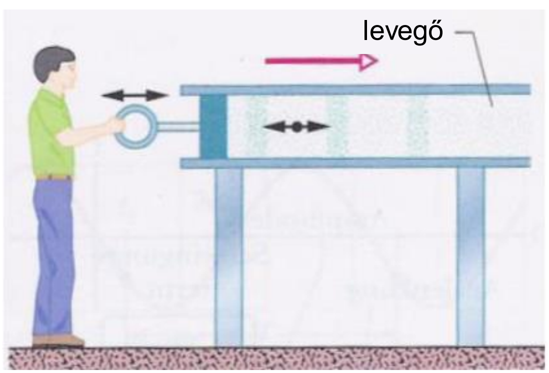
Depending on the relationship between the **direction of displacement (oscillation)** and the **direction of propagation**, we can distinguish between longitudinal and transverse waves.:

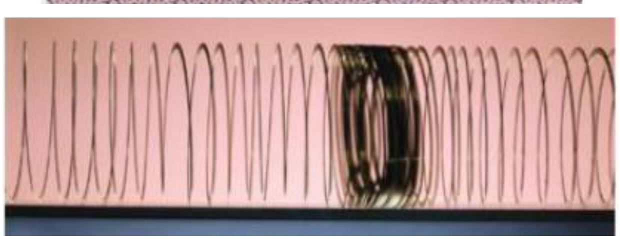
- **Longitudinal waves:** The direction of oscillation is **parallel** to the direction of propagation.
- **Transverse waves:** The direction of oscillation is perpendicular to the direction of propagation.

Longitudinal waves

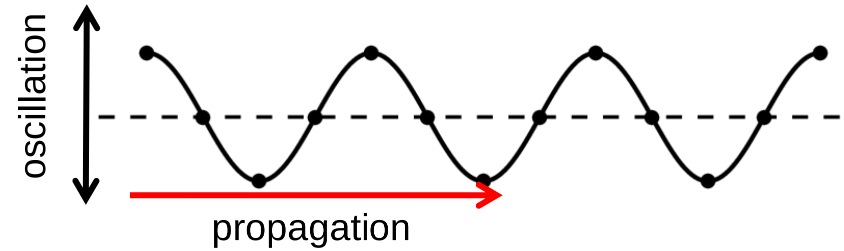


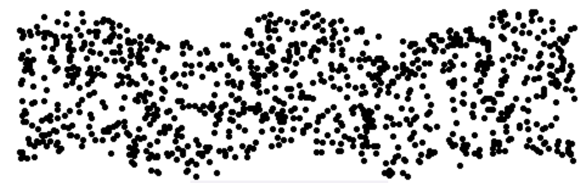


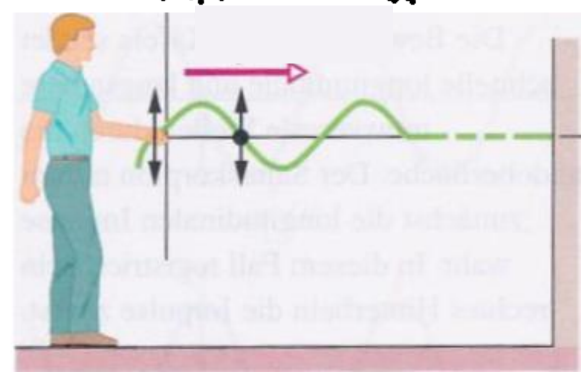




Transverse waves





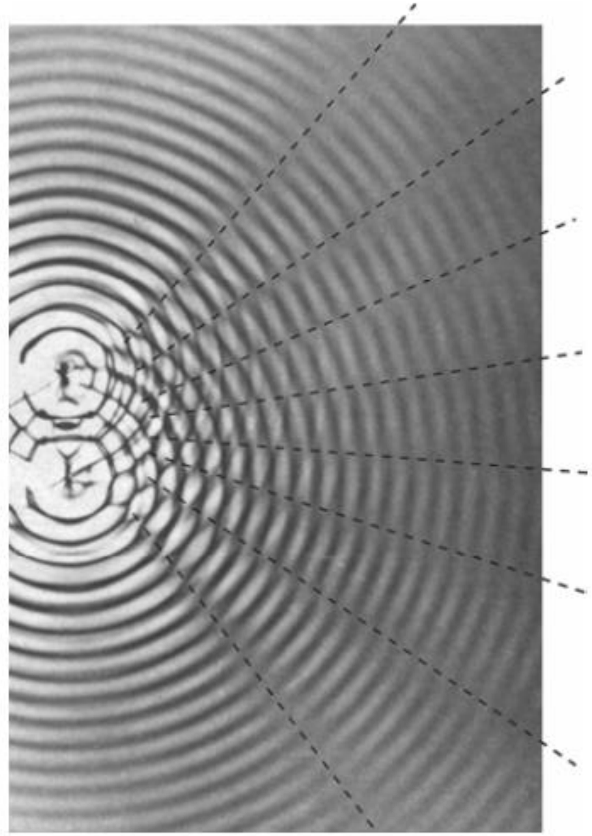


Wave phenomena

The „water wave" is directly observable. Because it varies slowly enough (small f) and is large enough (long λ).

But e.g. the „light wave" is not like that.

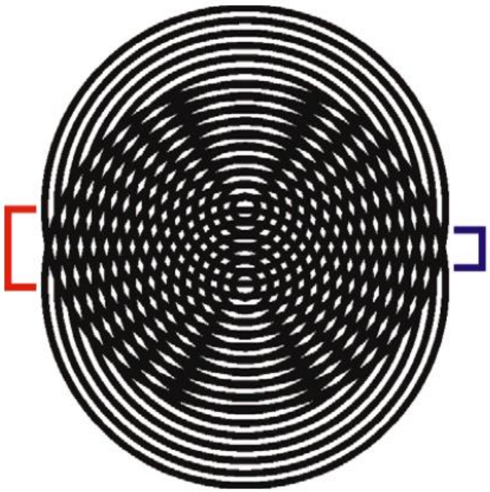
However, under certain conditions, **patterns** can appear that **do not change** over time (or very little), and their size can be significantly larger λ.



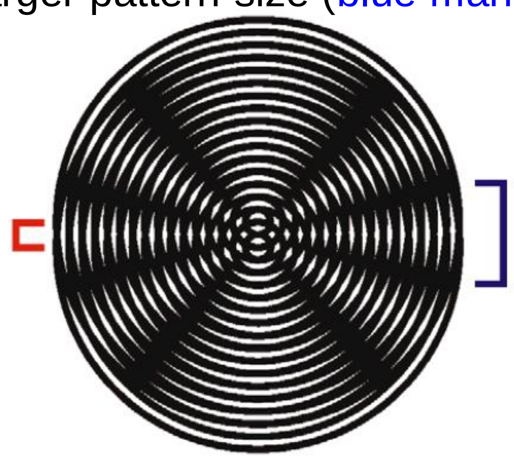
Interference

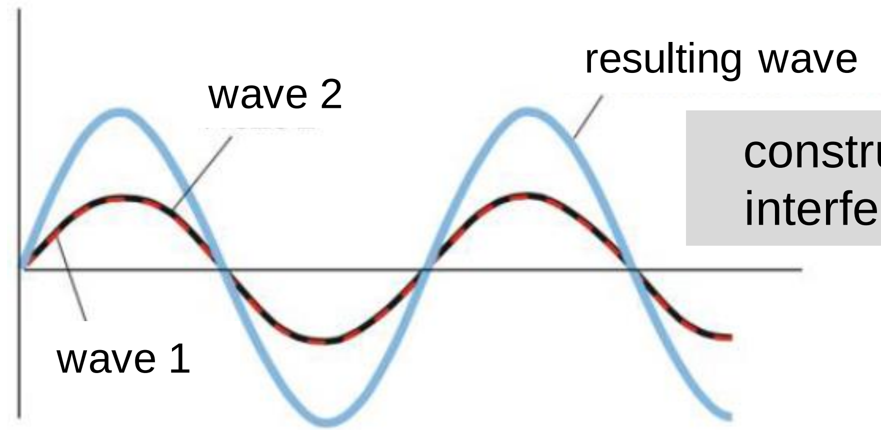
The **most important phenomenon** related to waves is **interference**. It occurs when two or more waves meet. Conditions for the visibility of patterns:

- Coherent waves (e.g. constant phase difference $\Delta\phi$ = const.)
- The distance between sources is comparable to the wavelength

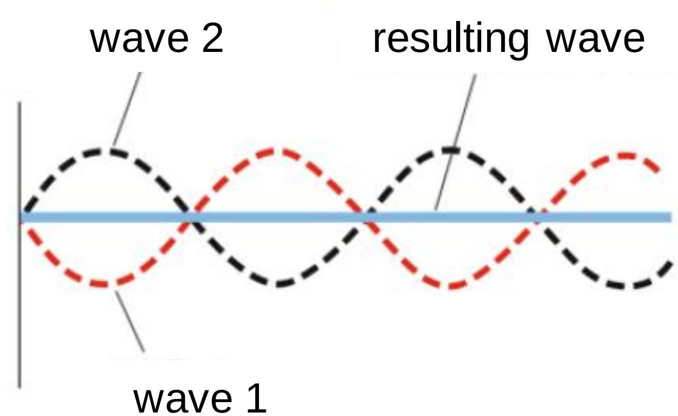


Smaller source spacing (red mark), larger pattern size (blue mark).



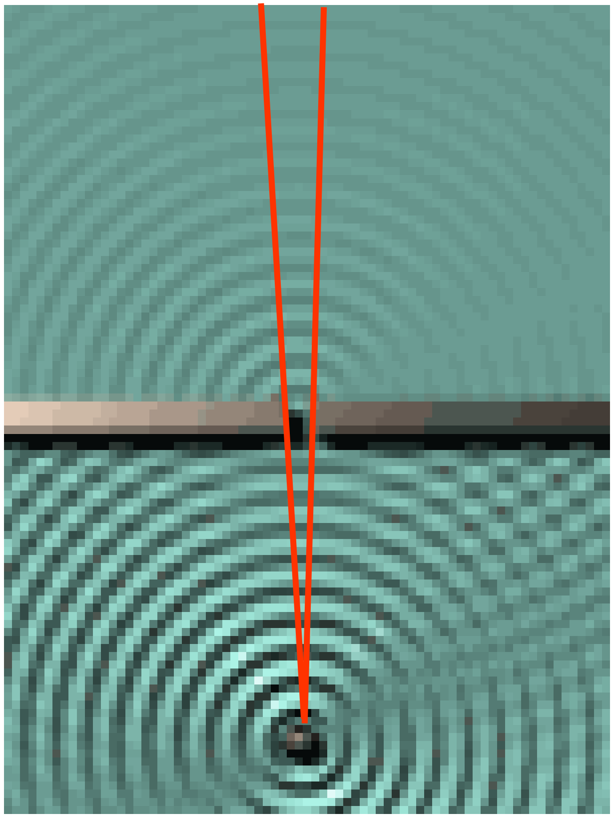


constructive interference

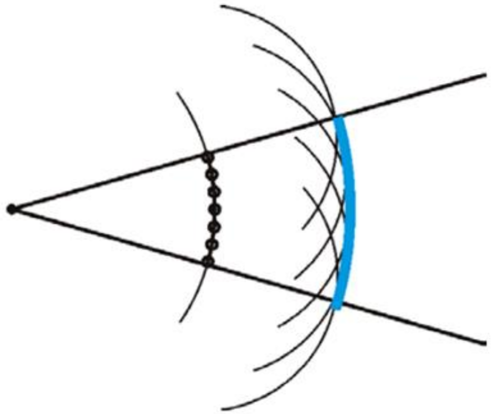


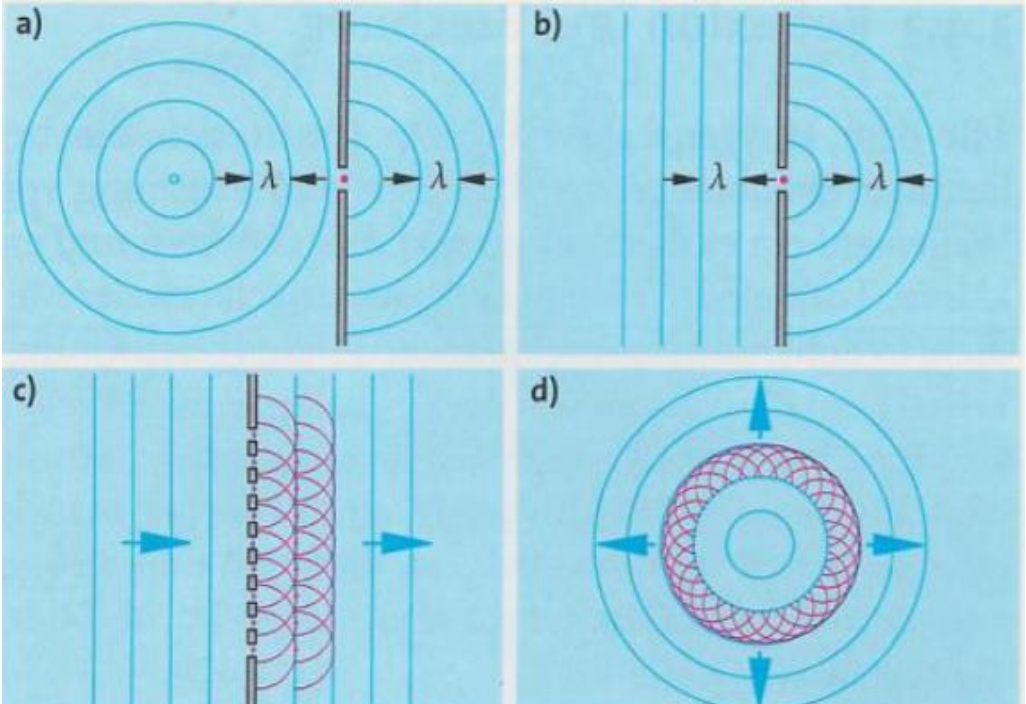
destructive interference

Huygens–Fresnel principle



- A **model** to describe wave propagation
- **Huygens**: Each point on the wavefront can be considered as the starting point for a new spherical wave, called the **elementary wave**
- The elementary wave propagates at the same speed and frequency as the original wave
- The **common envelope** of the elementary waves results in a **new wavefront**
- **Fresnel**: When the new envelope appears, the **principle of superposition** applies: waves pass through each other - **they interfere**

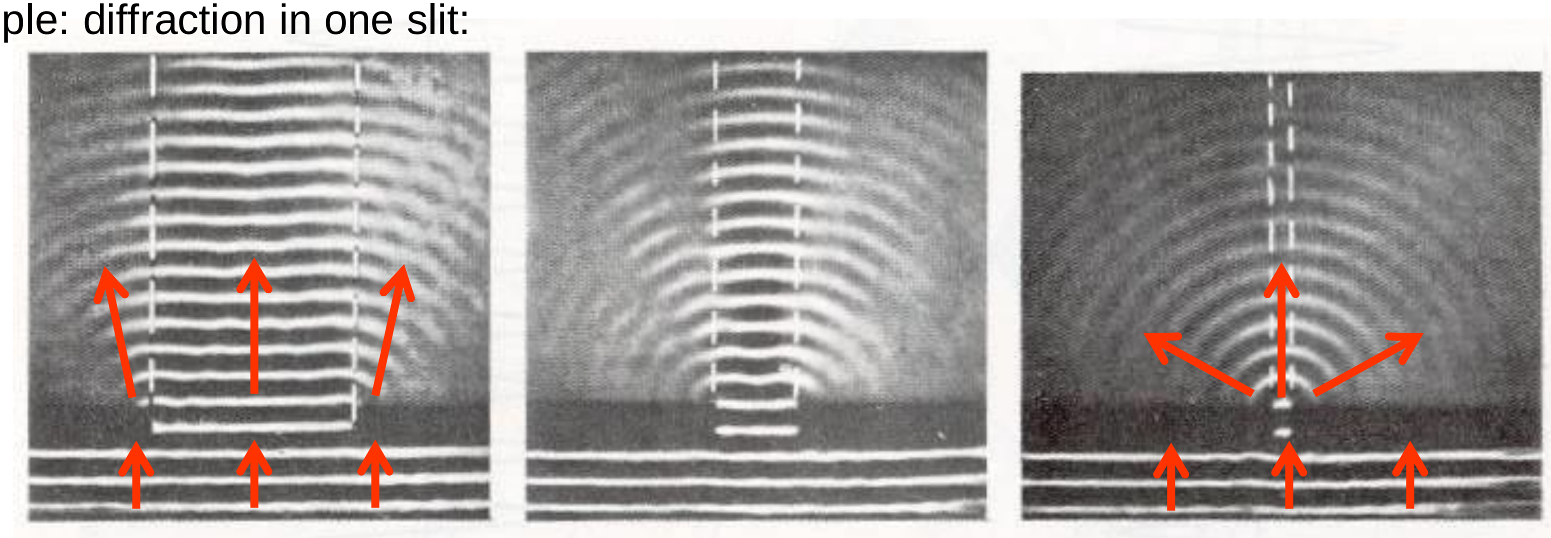




Diffraction

Change of the direction of wave propagation on obstacles and **openings** in the path of the wave (not at the interface of two media!).

Example: diffraction in one slit:

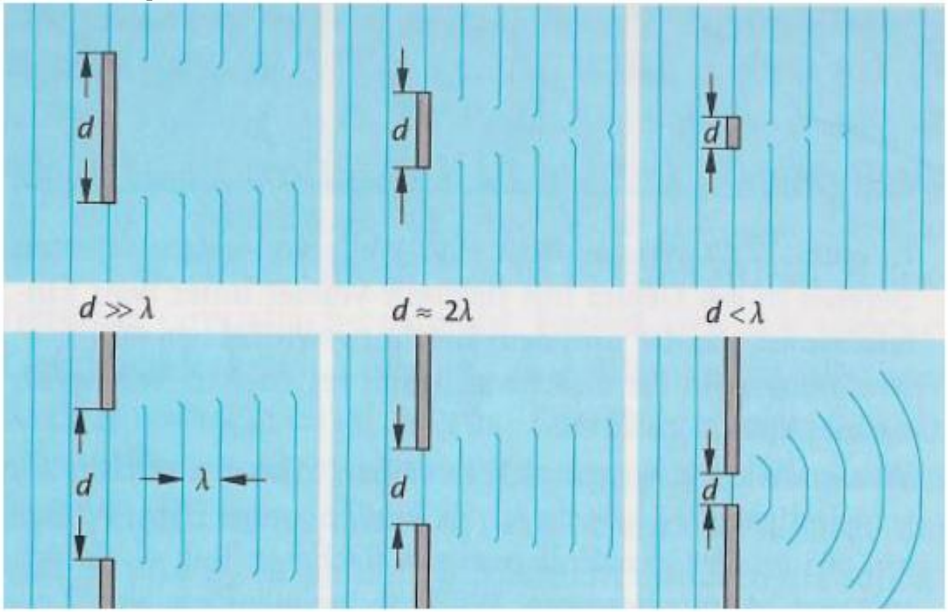


$\lambda \ll d$ $\lambda \approx d$

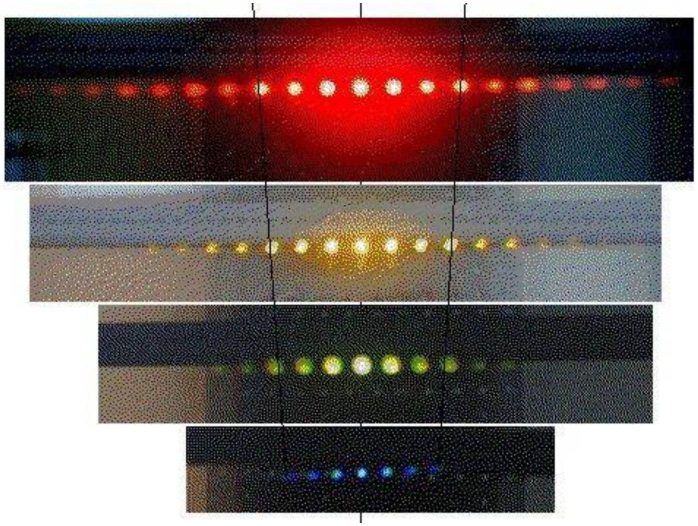
wavelength slit width

The waves **also penetrate behind an opening** into a range that could be termed as “**shadow space**” based on a simple **geometric** expectation.

- Depending on the size of the **obstacle** or **opening** and the **ratio of the wavelength**, the **diffraction is more or less pronounced**
- The phenomenon of **diffraction is all the more pronounced**:
    - the **smaller the obstacle or slit** (for a given wavelength)



    - The **larger the wavelength** (for a given slit width)

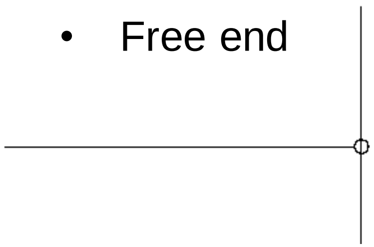


Application:
Diffraction methods
Refraction is the reason for the finite resolution of all optical devices, e.g. microscope, eye

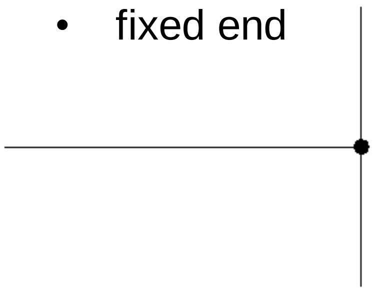
Standing waves

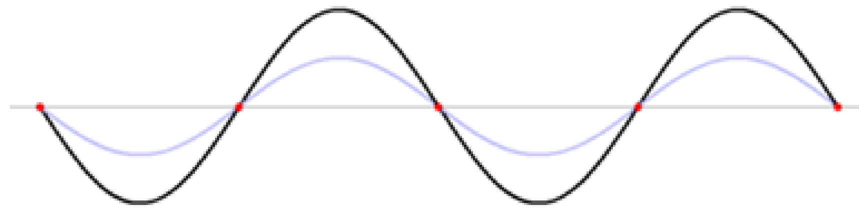
The reflection of a wave

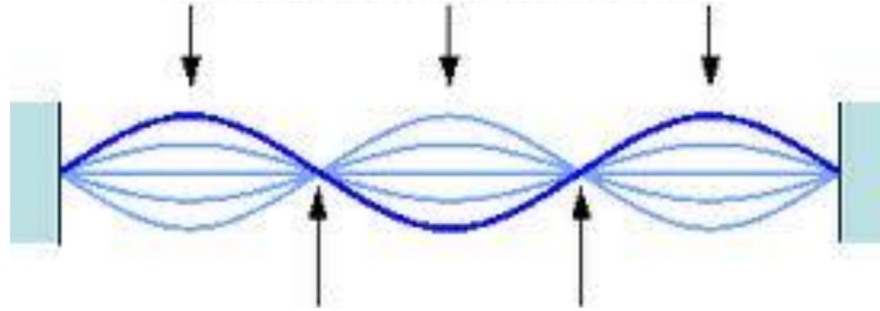
- Free end



- fixed end



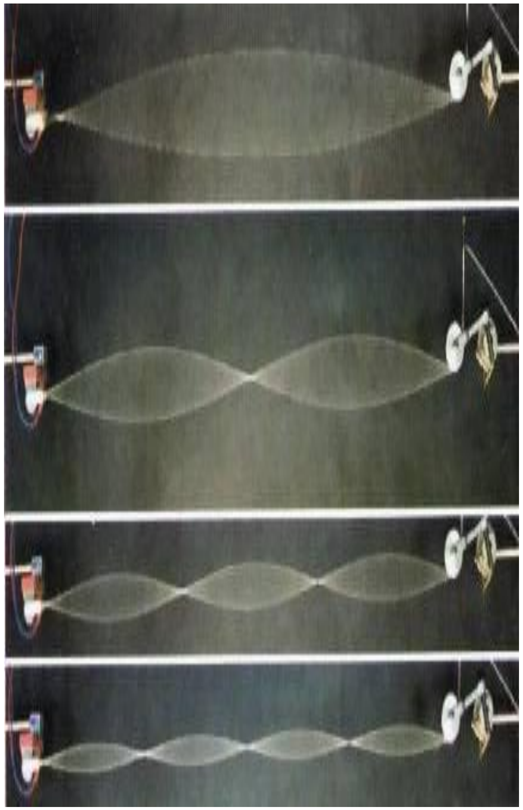


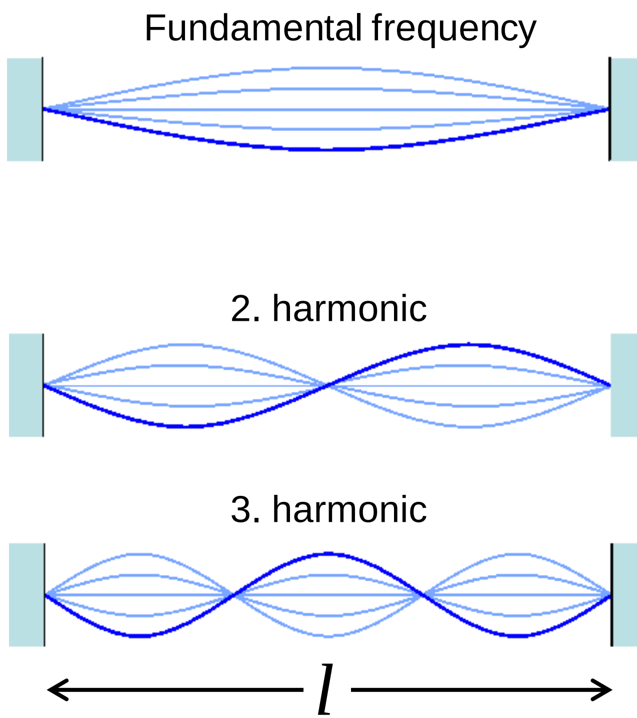


- It is the result of the **interference of two plane waves** of the same frequency and amplitude but in **opposite propagation directions** (e.g. superimposition of reflected and incident waves)
- Each point within a node oscillates with the same phase but different amplitude
- At the fixed end there is a node
- At the free end an anti-node

Standing waves

Example: A system with two fixed ends (both nodes)





$$\lambda_0 = 2 \cdot l \Rightarrow f_0 = \frac{c}{2l}$$

f_0 – fundamental frequency

$$\lambda_1 = l \Rightarrow f_1 = \frac{c}{l} = 2 \cdot f_0$$

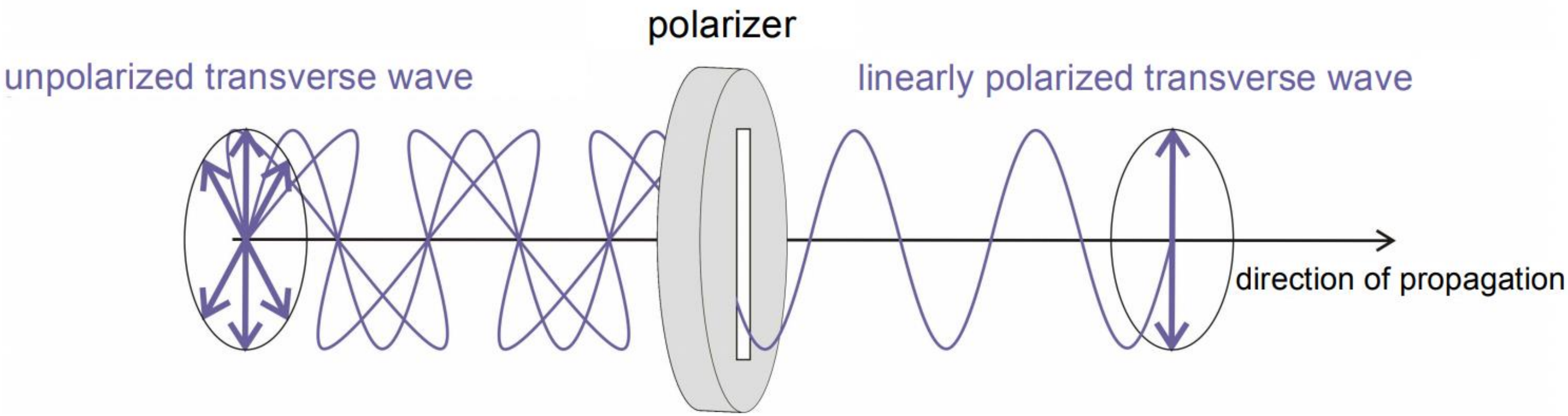
$$\lambda_2 = \frac{2}{3} \cdot l \Rightarrow f_2 = \frac{3c}{2l} = 3 \cdot f_0$$

Comment:
The basic frequency of a string of a violin can be set e.g.:

- by changing the length of the string
- By changing the vibration state of the string ($\rightarrow$ speed of propagation)

(Linear) polarisation

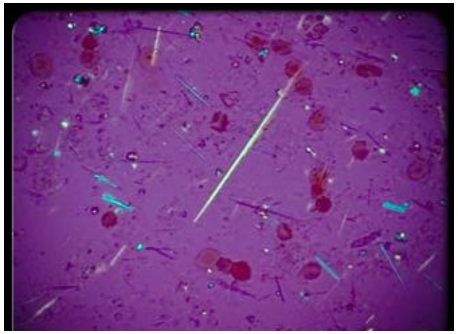
- In the case of **transverse waves**, the direction of oscillation and the direction of propagation are perpendicular to each other
- The **direction of the oscillation is not yet precisely determined**, although it is always perpendicular to the direction of propagation (= unpolarized wave)
- The **"selection" of one direction of oscillation (plane of oscillation)** by means of a polarizer is called linear polarization. **Only transverse waves can be polarized!**



Using polarized light:

Polarised light in microscopy

gout

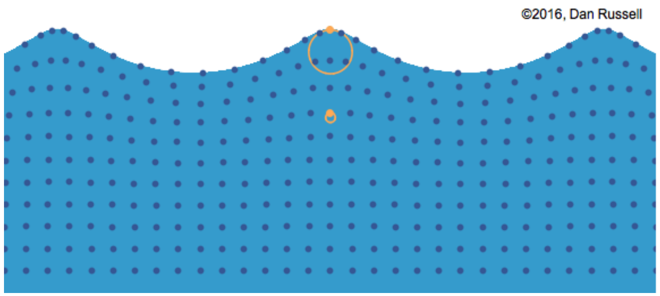


uric acid crystals in a microscope with polarizer

Mechanical waves

- They are bound to material movements (they need a medium to propagate)
- The movement of the components of the medium e.g.:
  - waves of water (water)
  - sound waves (air)

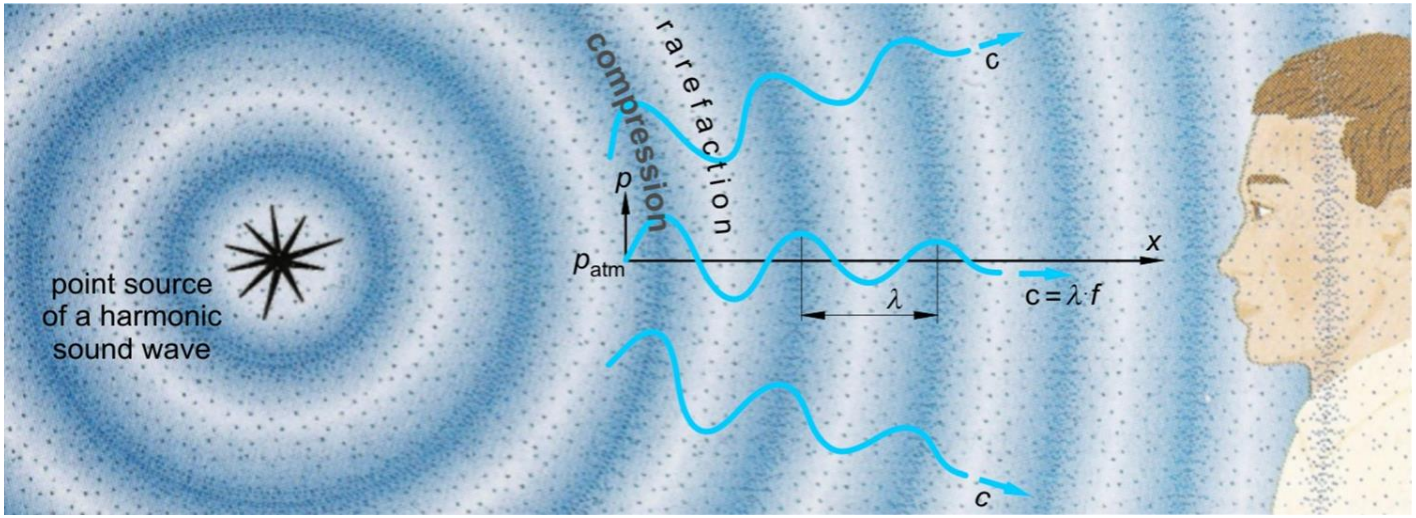
- They can also be longitudinal and transverse waves.



- **Mechanical longitudinal** waves are able to propagate in **all media**, **mechanical transverse** waves **only in solids**. (However, the surface waves of water are partially transverse).

- The propagation of waves is accompanied by **energy transport**, but **not** by material transport.

Sound



- Sound waves are mechanical waves, which can be divided into four ranges based on human hearing:

Range of sounds

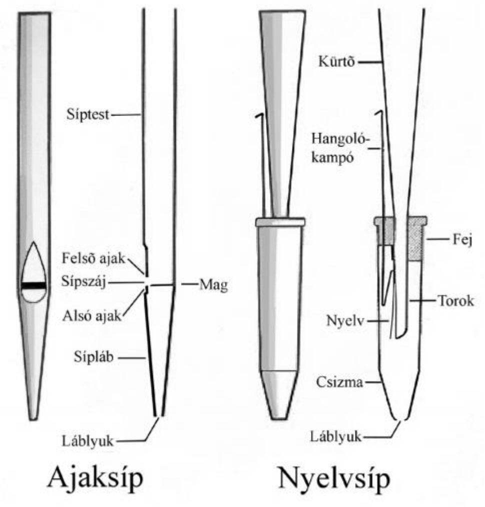
sound range	infrasound	audible sound	ultrasound	hypersound
frequency (Hz)	< 20	20–20 000	20 000–10^9	10^9 <

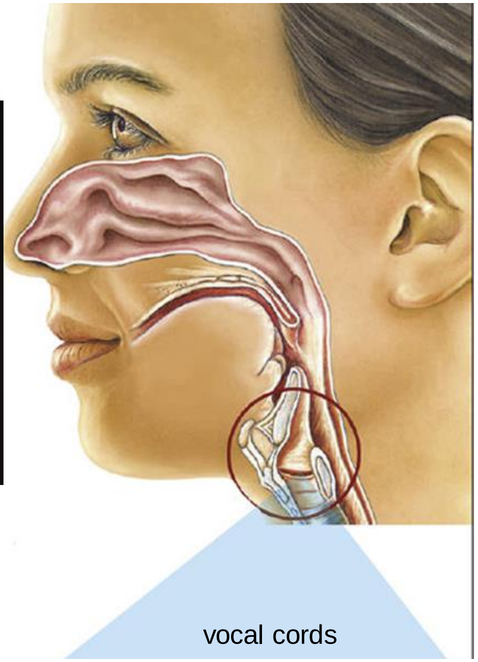
- The speed of sound is generally lower in gases than liquids and lower in liquids than in solids.

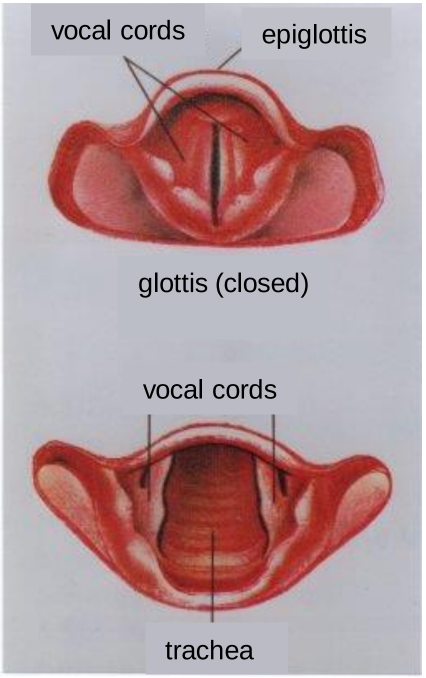
Speed of sound in various media

medium	c_{sound} (m/s)
air (0 ºC, 101 kPa)	330
helium gas (0 ºC, 101 kPa)	965
water (20 ºC)	1 483
fatty tissue	1 470
muscle	1 568
bone (compact)	3 600
iron	5 950

Generating sound waves







Problem

The dolphin emits a sound with a wavelength of 7 mm.

a) Calculate the frequency in water!

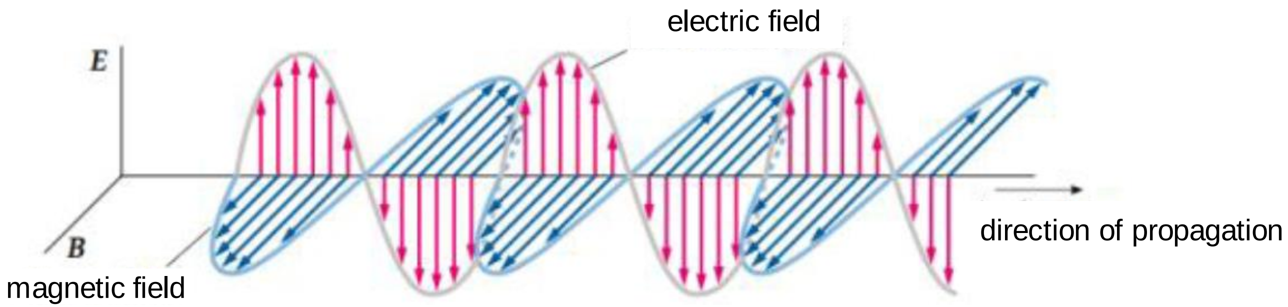
b) Which frequency range does this sound belong to?

c) Sound waves travel from water to air. Calculate the frequency in the air!

d) Calculate the wavelength in air!

Electromagnetic waves

- Waves of electric and magnetic fields



- Electromagnetic field is the oscillating medium, so these waves **can propagate in a vacuum.**

- Transverse waves (so these can be polarized)

- All electromagnetic waves propagate in a vacuum at the same speed, the speed of light:

$$c = 299\,792\,458 \text{ m/s} \approx 3 \cdot 10^8 \text{ m/s}$$

Light - the best known electromagnetic wave

- The spectrum of visible light is approx. 380 nm to 780 nm (VIS-range)
  → **400 nm–800 nm**