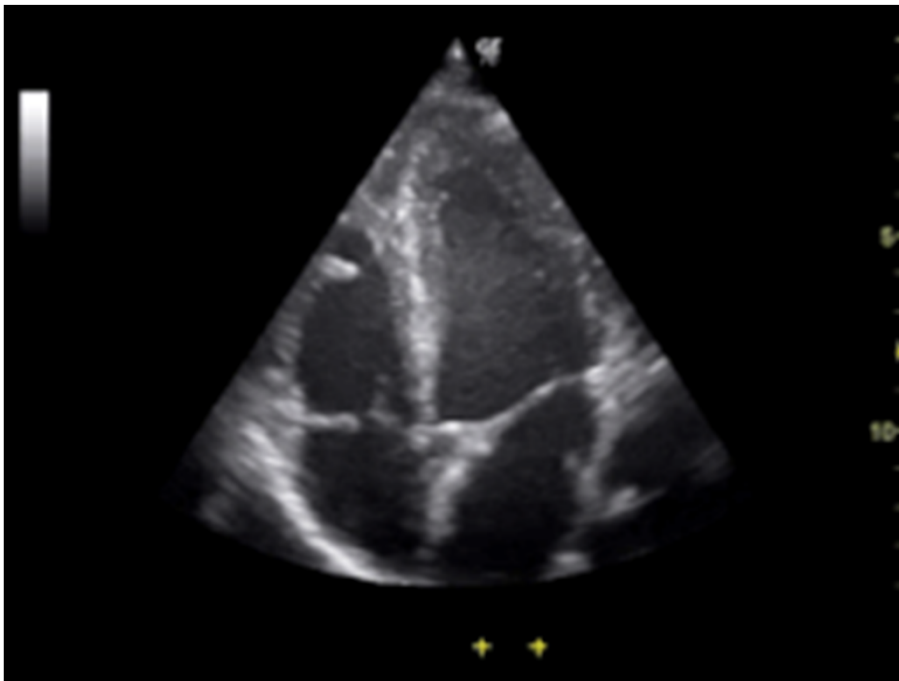
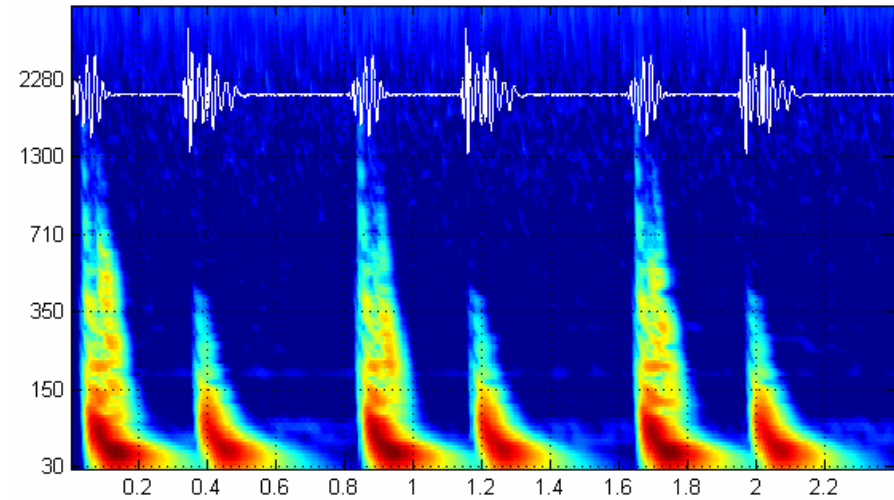
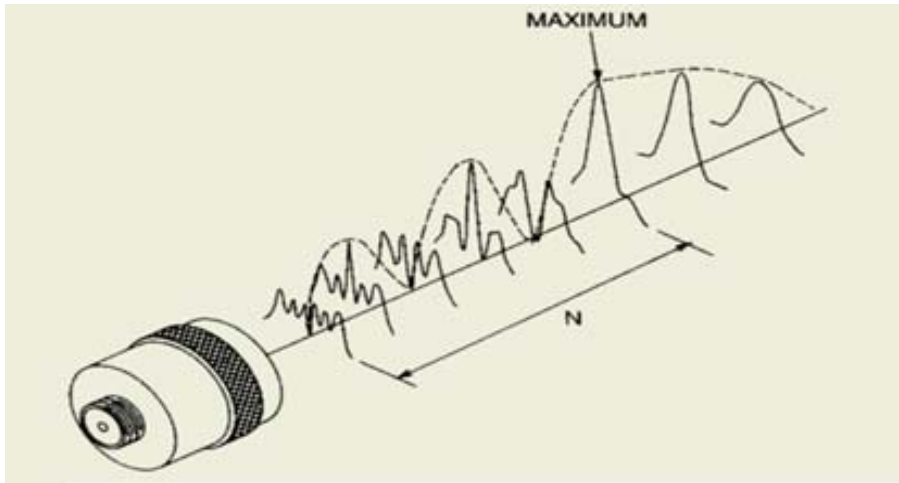
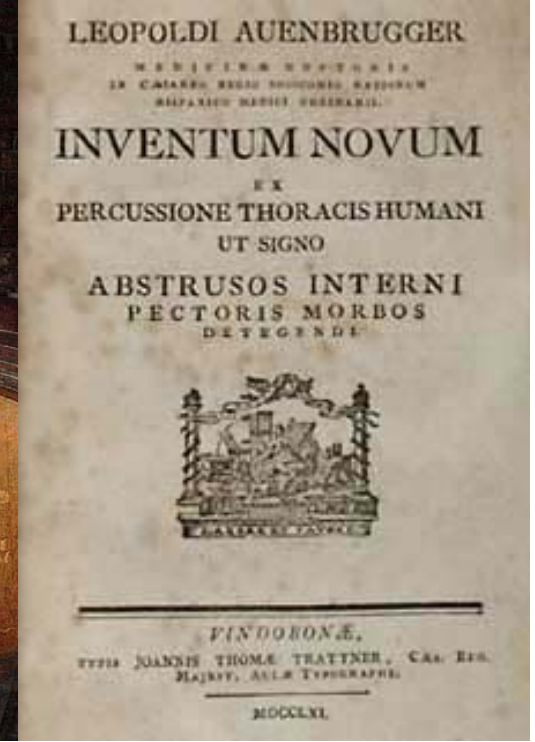


Physics of ultrasonography



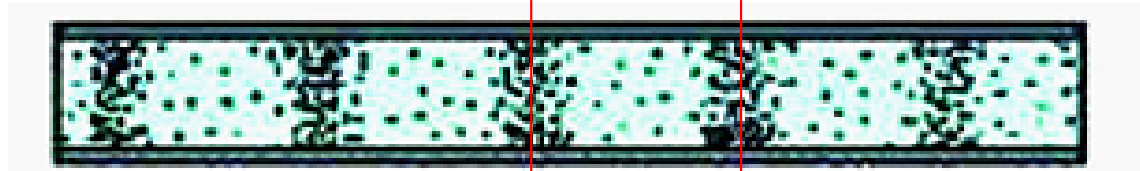


question in the cellar of a pub: how much wine is in the barrel?
medical question: how much air is in the lungs?

Auenbrugger (son of an innkeeper in Graz, 1761): **percussion**
for testing the air content of hollow organs

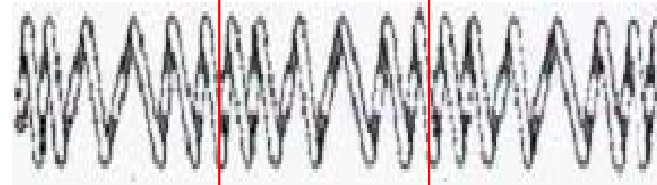
Sound: mechanical wave (model)

whistle



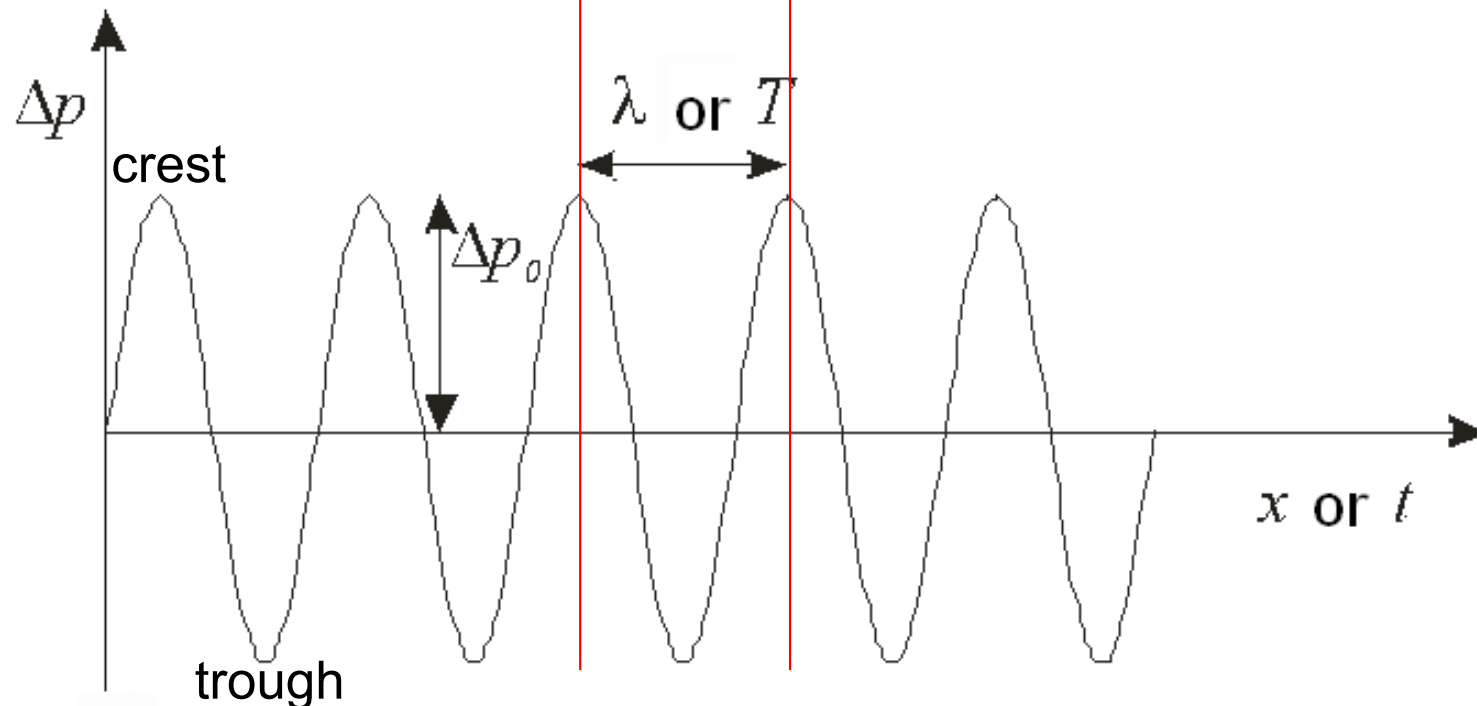
spring

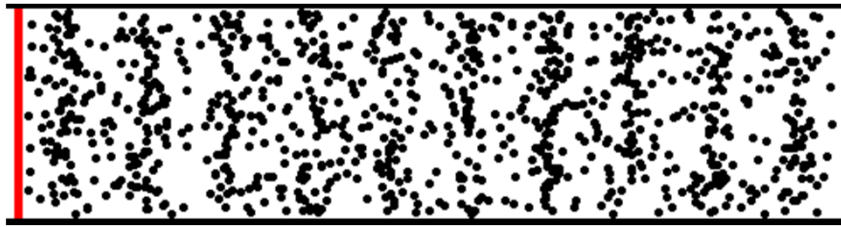
compression
rarefaction



spatial and
temporal
periodicity

function





longitudinal wave
(in the interior of liquids and
gases only this type)



transverse wave

hydrostatic
pressure

pressure change,
sound pressure

$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

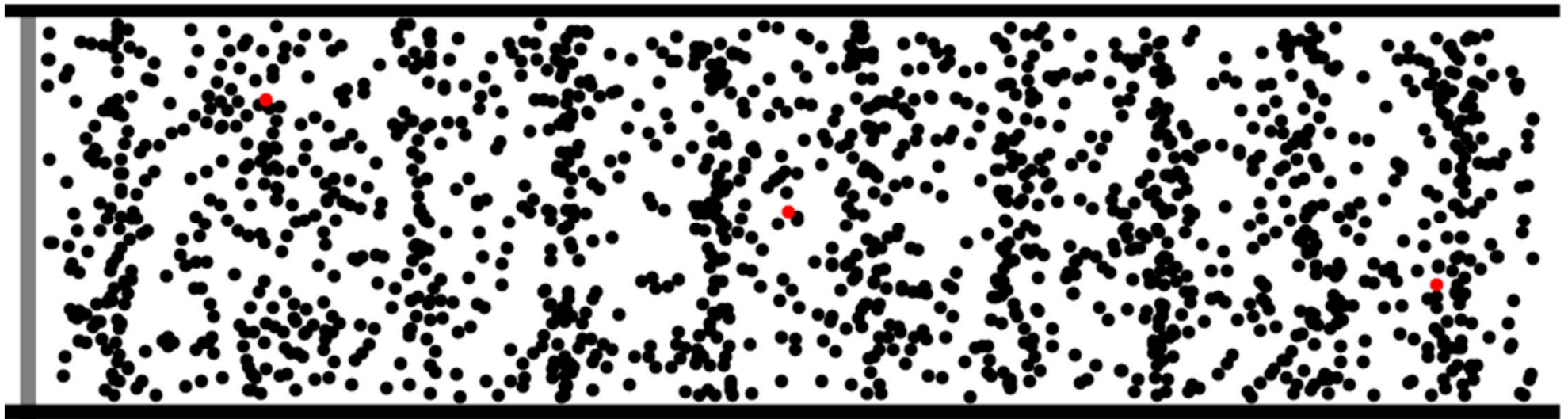
pressure DC + AC amplitude phase

$$\Delta p(t, x) = \Delta p_{\text{max}} \sin \left[2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \right]$$

$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$



longitudinal wave

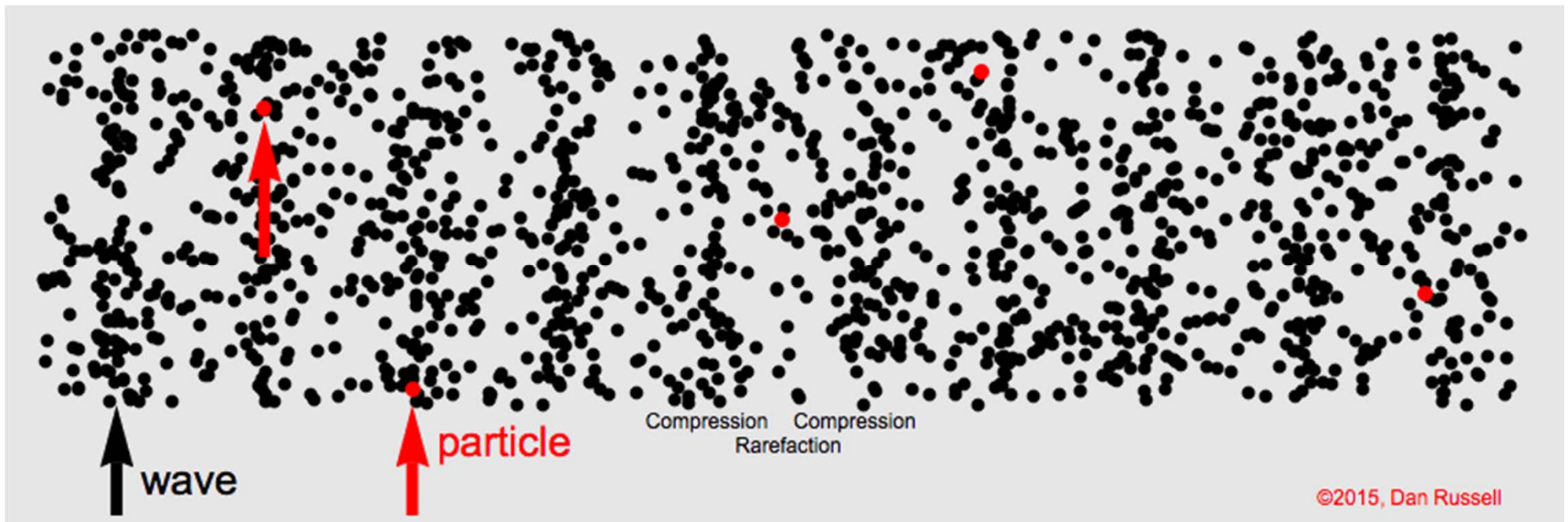


©2011. Dan Russell

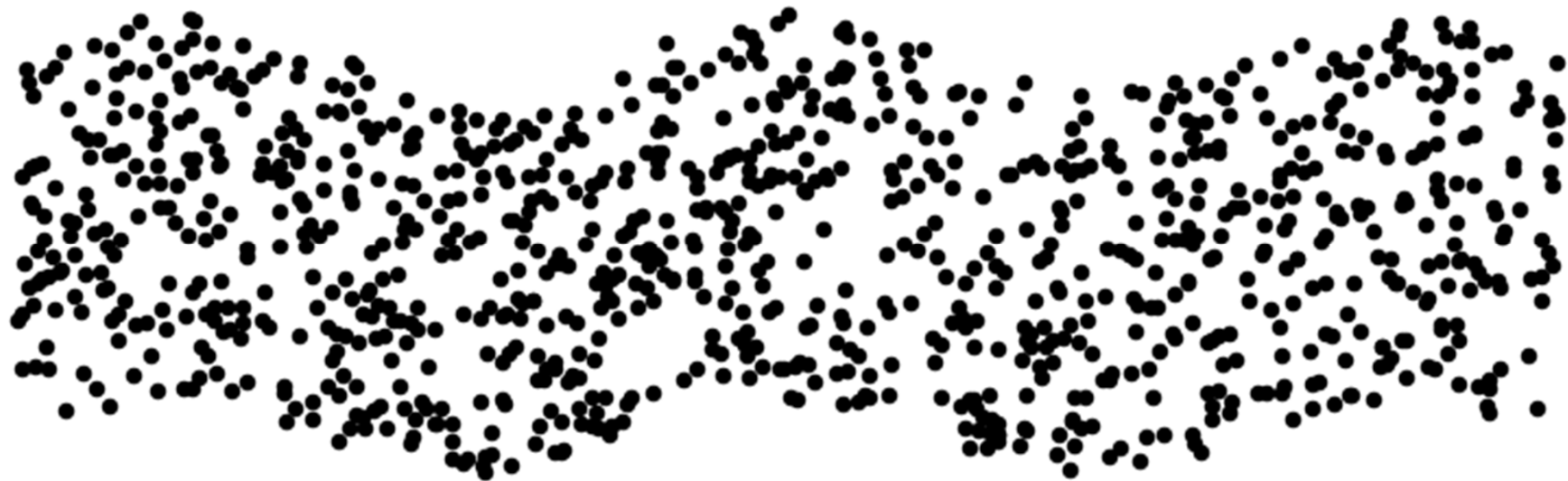


moving surface (source of wave)

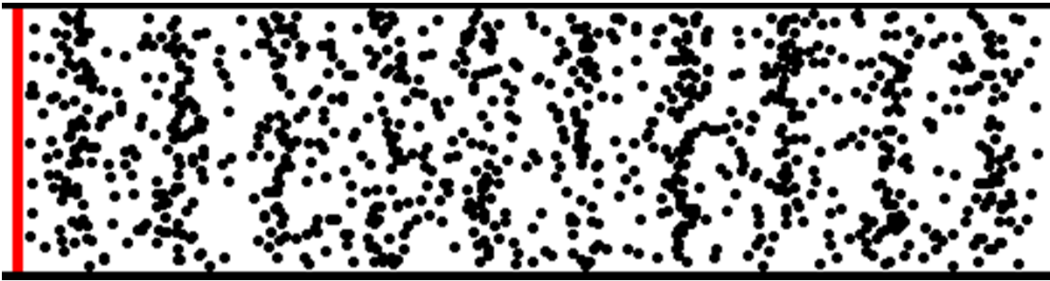
longitudinal wave



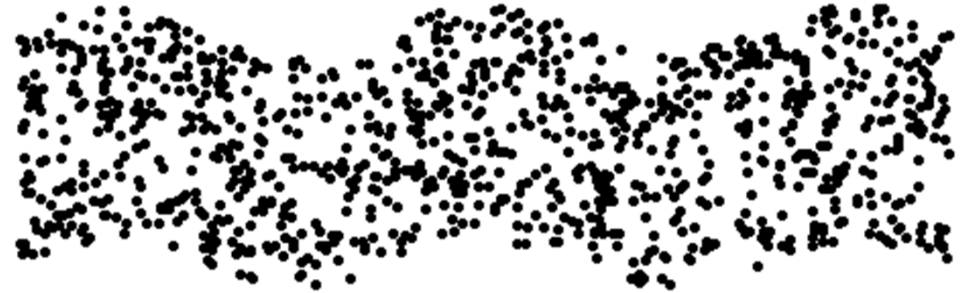
transverse wave



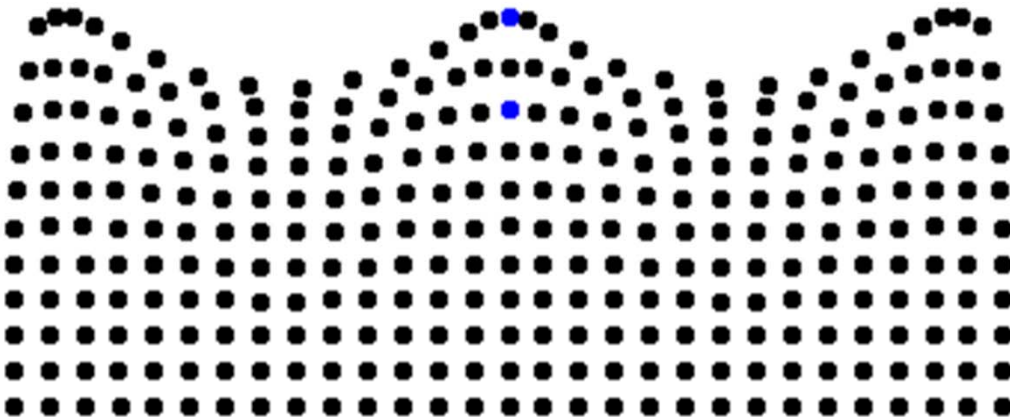
longitudinal wave



transverse wave

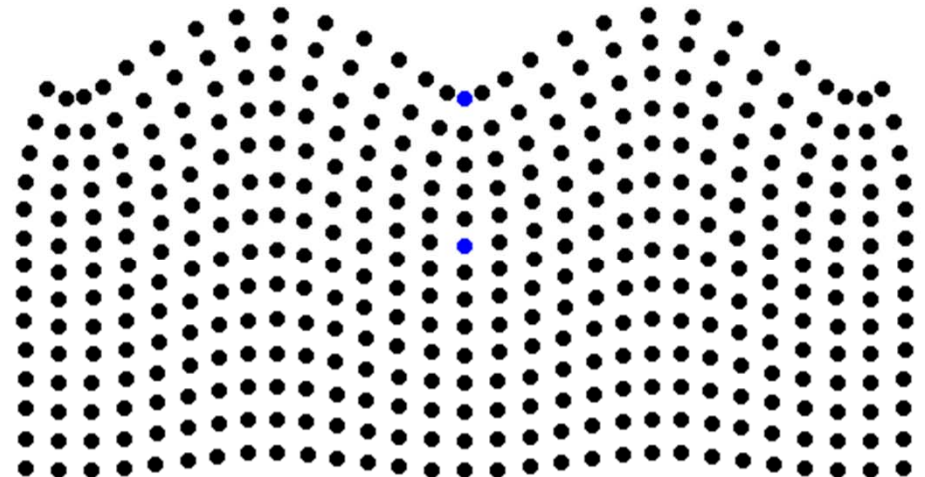


surface wave

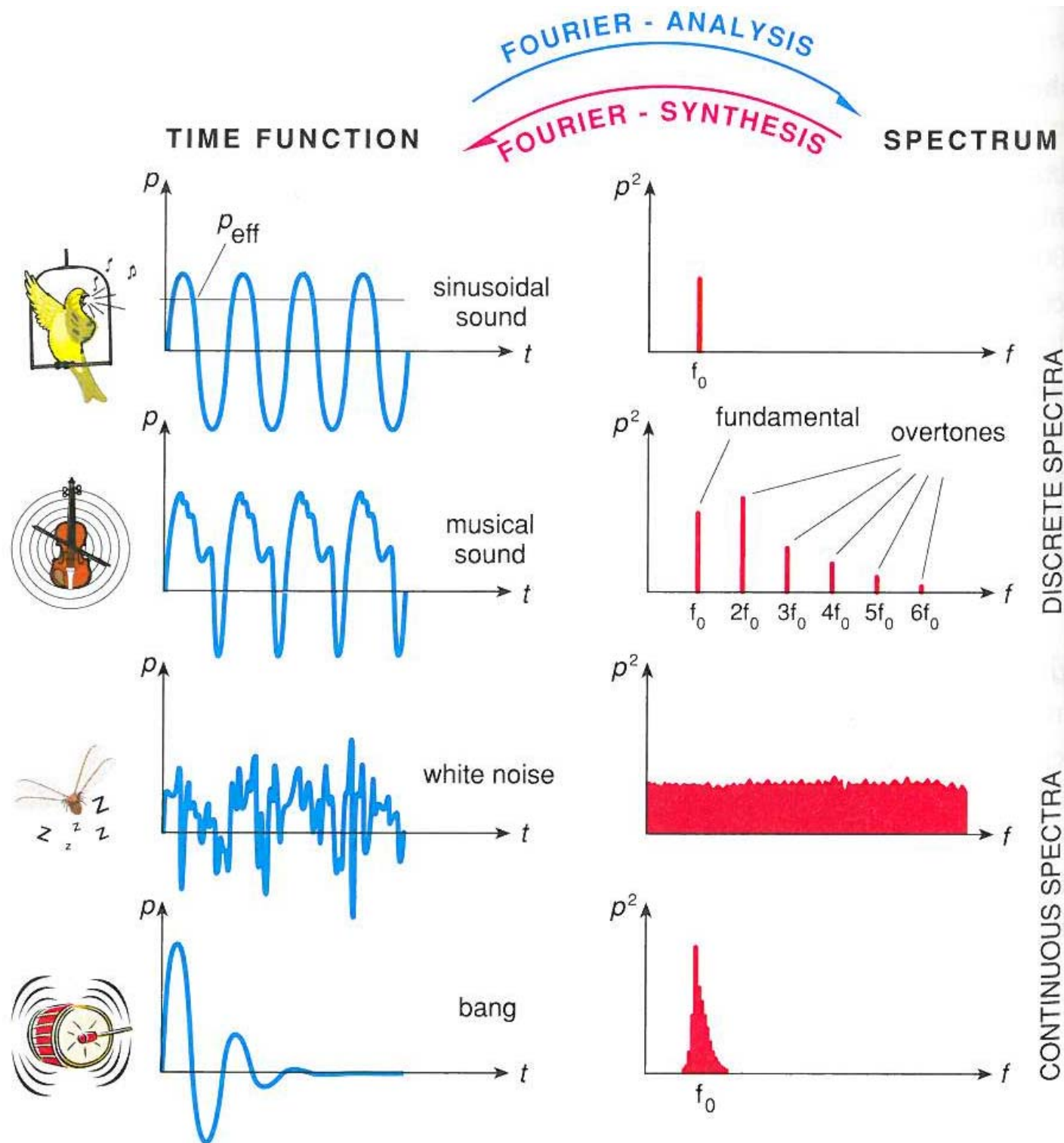


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Rayleigh wave



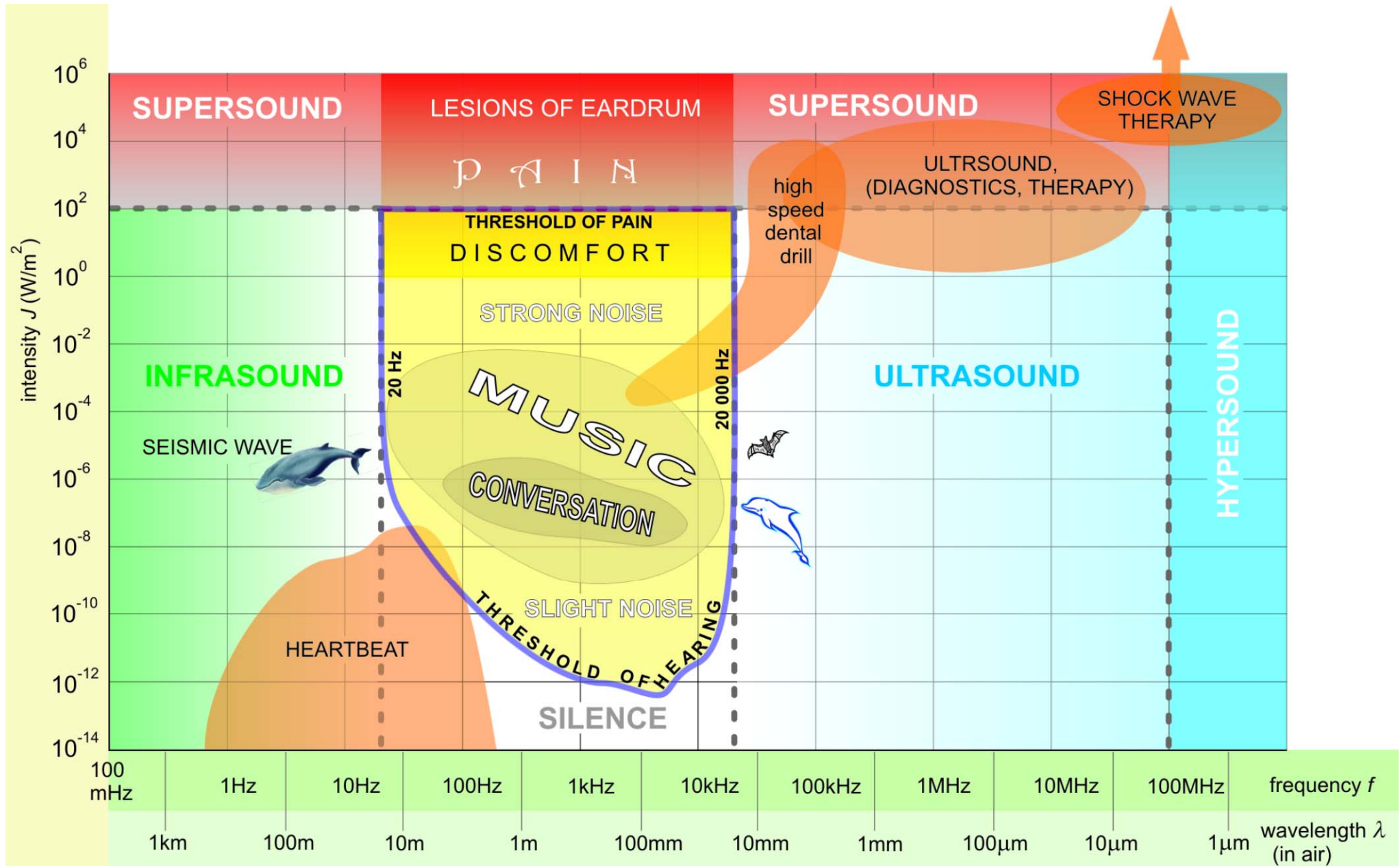
©1999, Daniel A. Russell



pitch:
frequency of the
fundamental

timbre (tone colour):
relative strengths of
overtones/harmonics
(spectrum)

Frequency and intensity regions of sounds



The role of elastic medium

$$\kappa = -\frac{\frac{\Delta V}{V}}{\Delta p}$$

compressibility

relative volume decrease
over pressure

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

speed of sound

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

acoustic **impedance**
(definition)

$$Z_{\text{el}} = \frac{U}{I}$$

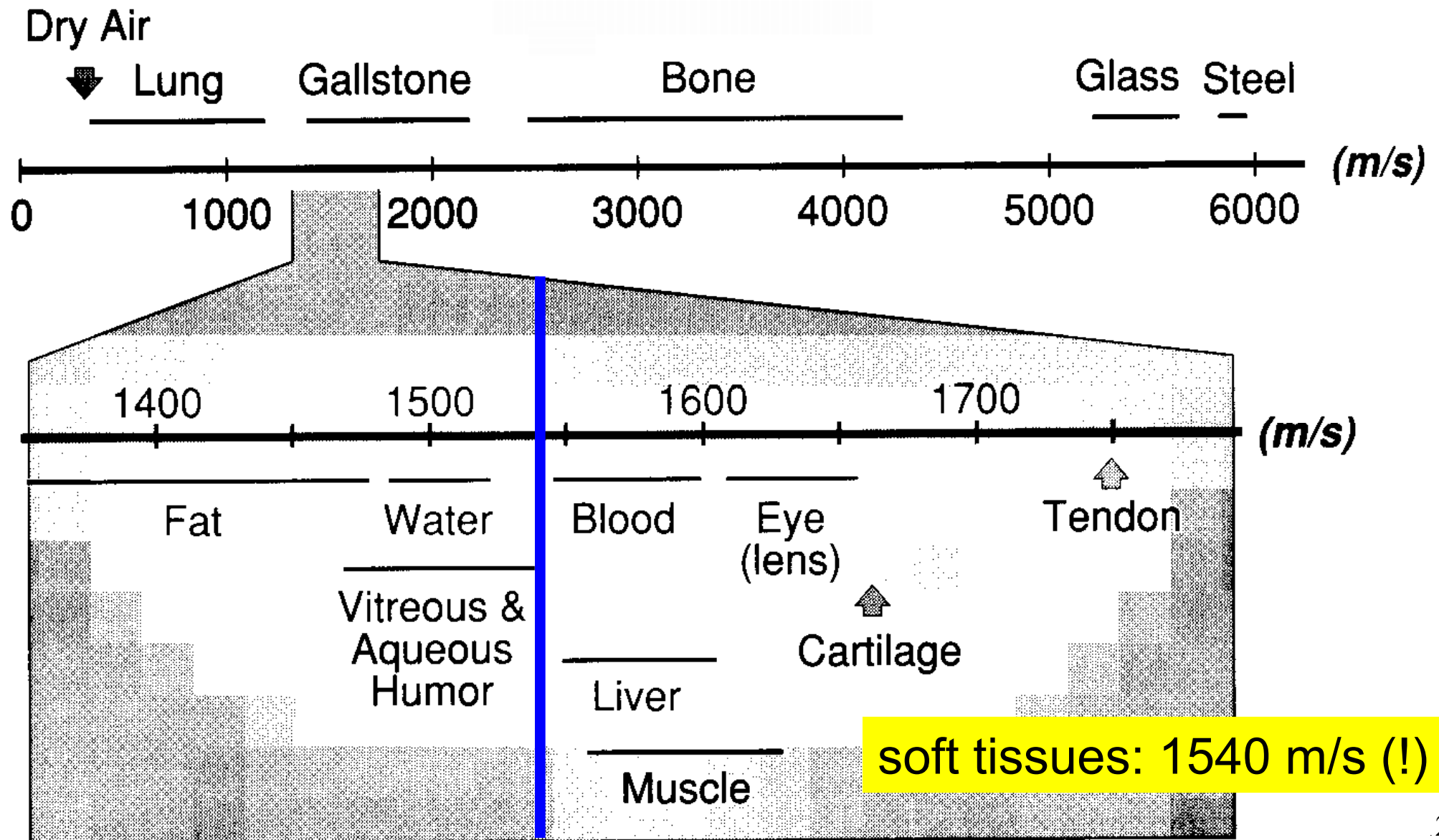
$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance**
(useful form)





Speed of sound/US in different media



Intensity of US

$$J = \frac{1}{Z} \Delta p_{eff}^2$$

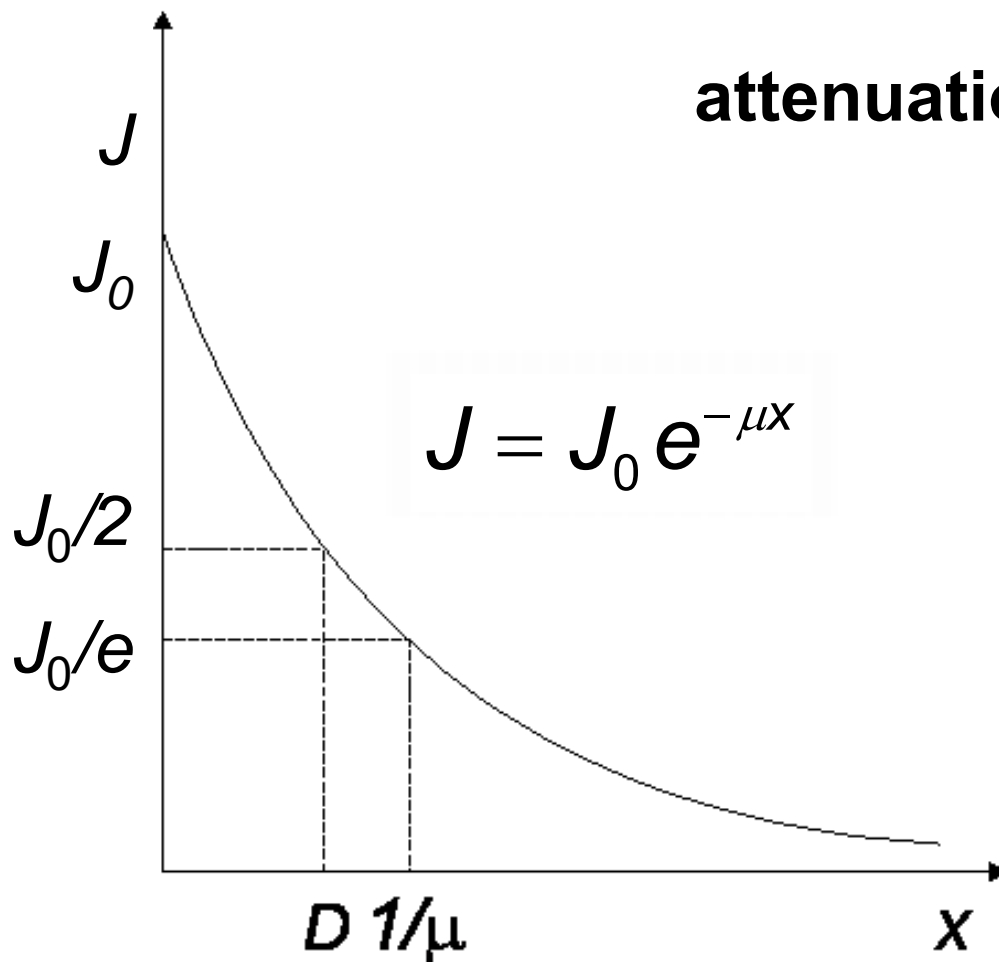
$$P_{el} = \frac{1}{Z_{el}} U_{eff}^2$$

intensity =

energy-current density

electric analogy

Loss of energy during propagation (absorption)



attenuation: $\alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

μ is proportional to
frequency in the
diagnostic range

**specific
attenuation:**

$$\frac{\alpha}{f \cdot x}$$

μ is proportional
to frequency in
the diagnostic
range

$$\mu \sim f^k, \quad k \sim 1(?)$$

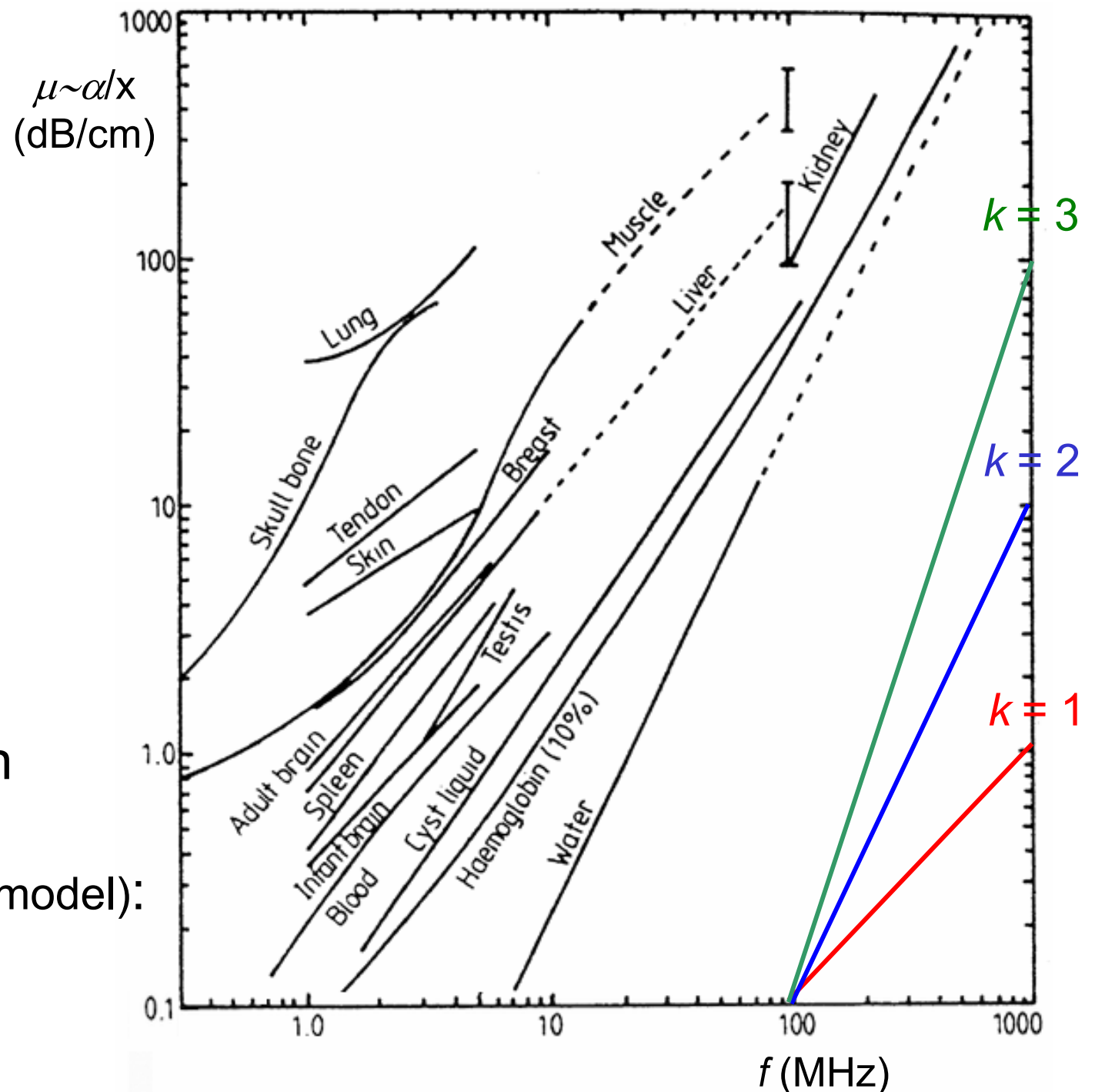
$$\log \mu \sim k \log f$$

if the graph is a linear,
the power function
approximation is valid

specific attenuation
for soft tissues

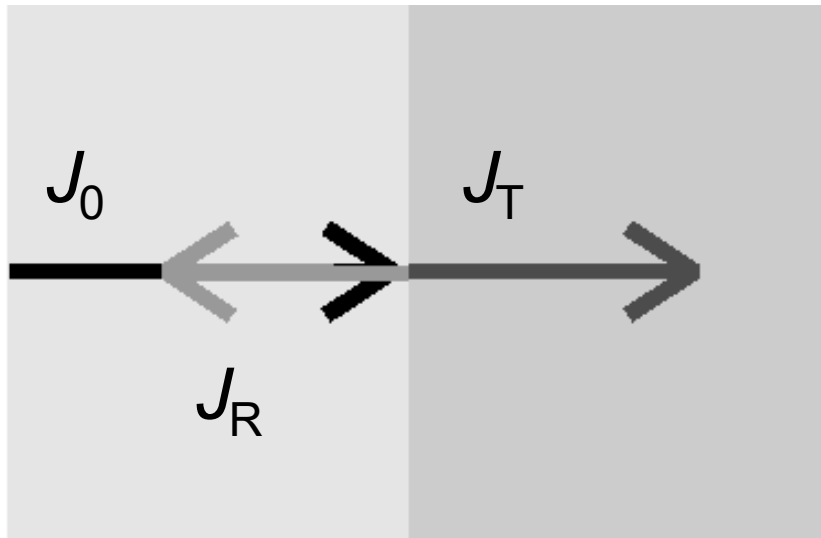
(homogeneous tissue model):

$$\frac{\alpha}{f x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$



Phenomena at the boundary of different media

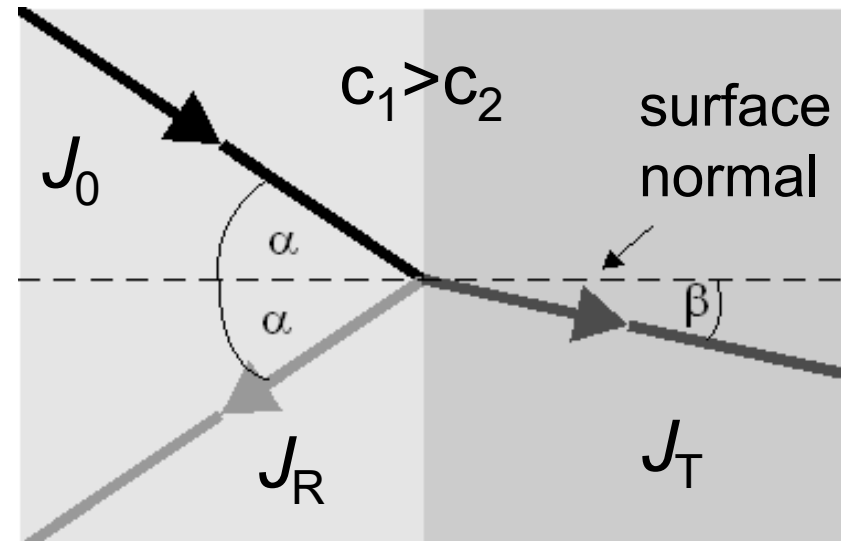
normal/perpendicular
incidence



$$J_0 = J_R + J_T$$

reflection and transmission
(penetration)

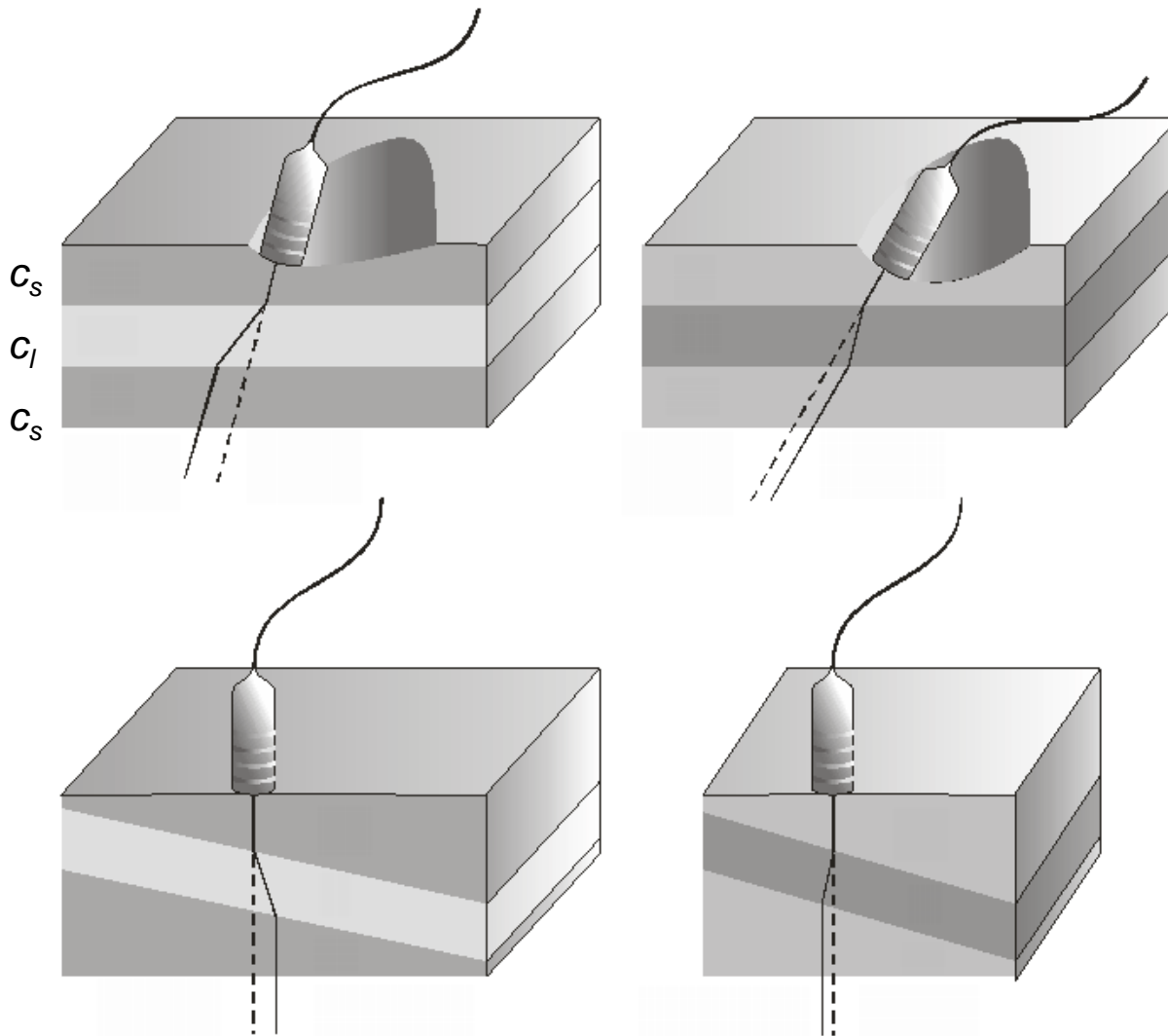
skew incidence



$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

Snell's law

Phenomenon of skew or normal incidence and skew boundaries



position in the image and the real position are different



Reflection (normal incidence)

reflectivity:

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

<i>boundary surface</i>	<i>R</i>
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

“full” reflection:

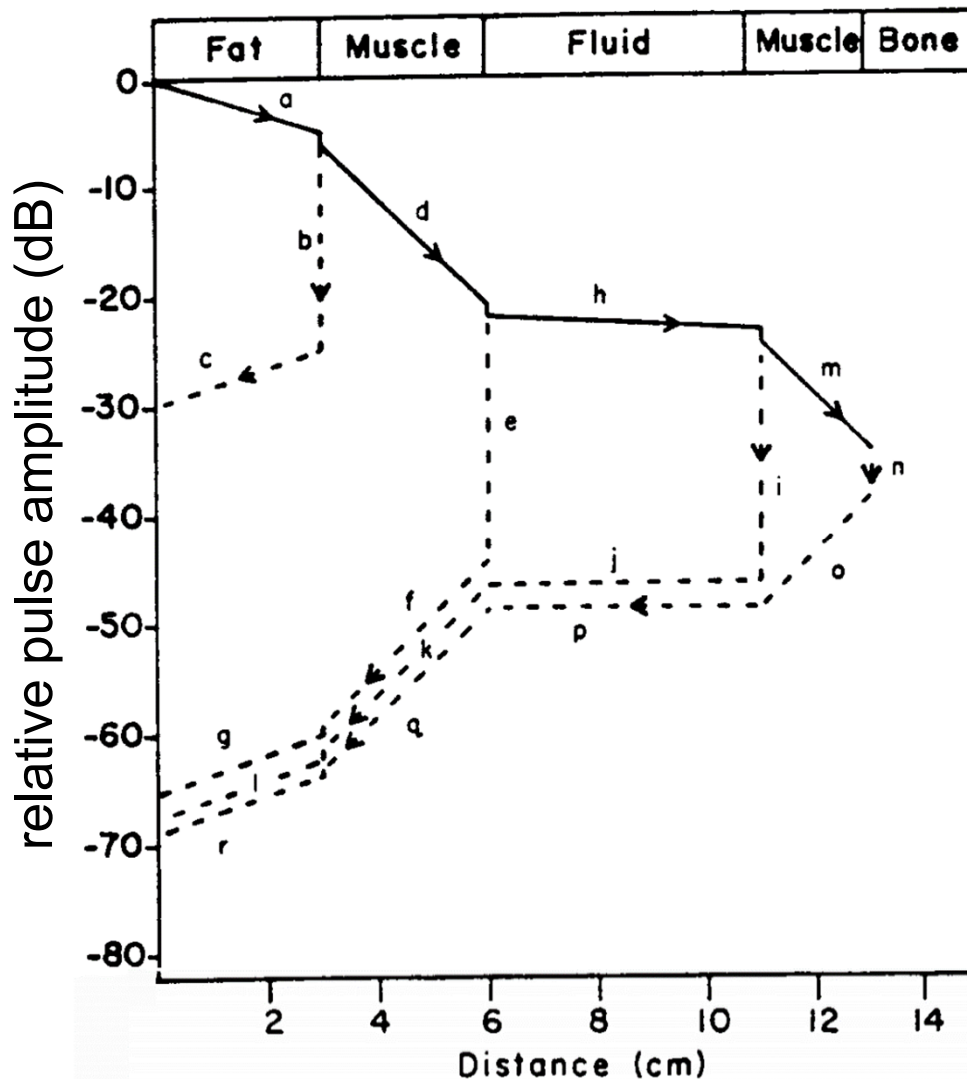
$$Z_1 \ll Z_2, \quad R \approx 1$$

optimal coupling:

$$Z_{\text{connecting}} \approx \sqrt{Z_{\text{source}} Z_{\text{skin}}}$$



Absorption and reflection



the later comes back the reflection, the deeper lays the reflecting surface and the weaker is the intensity

run time dependent amplification

TGC: time gain compensation

DGC: depth gain control

boundary surface	R	$10\lg R$ (dB)	T	$10\lg T$ (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

Generation of US. Piezoelectric effect

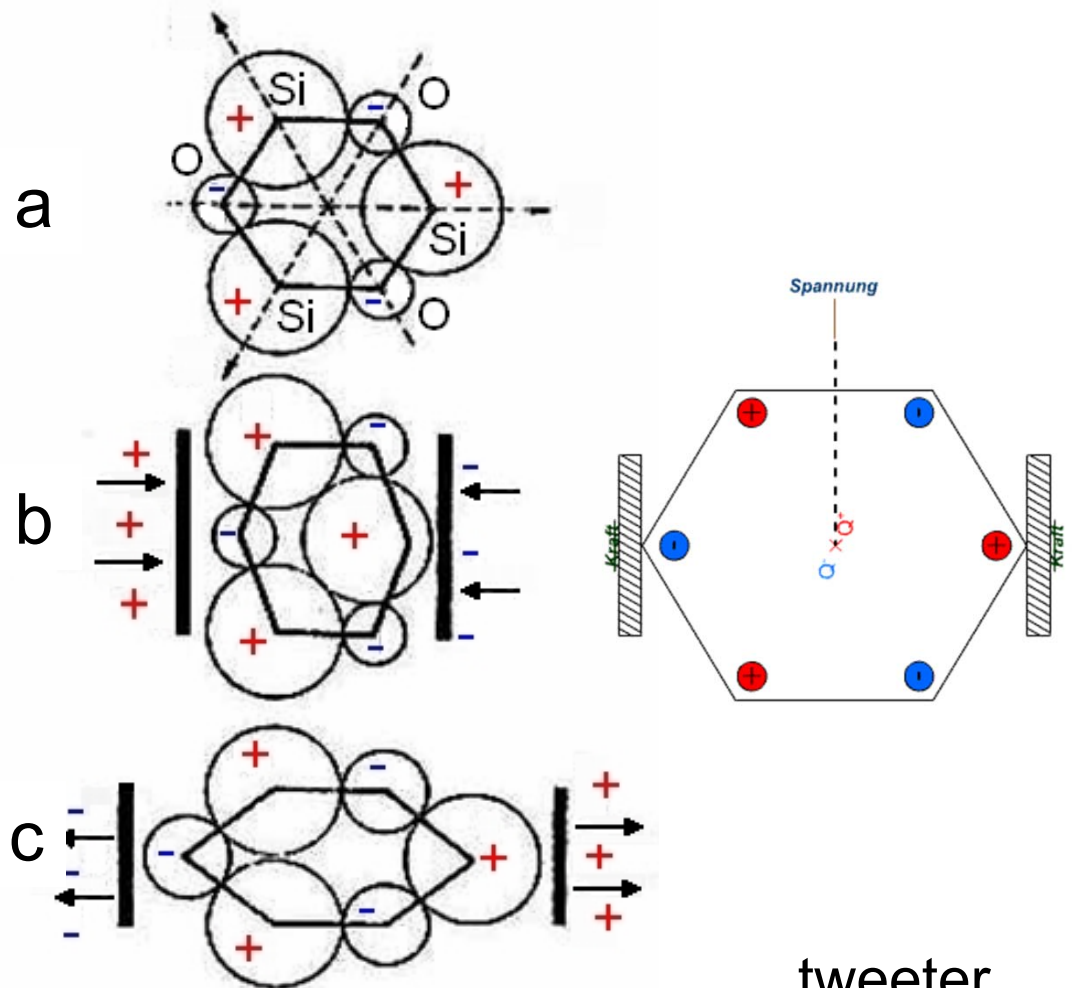
production: inverse ~
detection: direct ~

source of electric signal
(sine wave oscillator)+
transducer (piezo-crystal)

(a) Center of charge of
positive and negative
charges coincides.

(b) and (c) As a result of
pressure , the charge
centers are separated, i.e.
a potential difference
arises (direct ~).

The crystal is deformed
when voltage is applied
(inverse ~).



at home:
gas igniter



tweeter

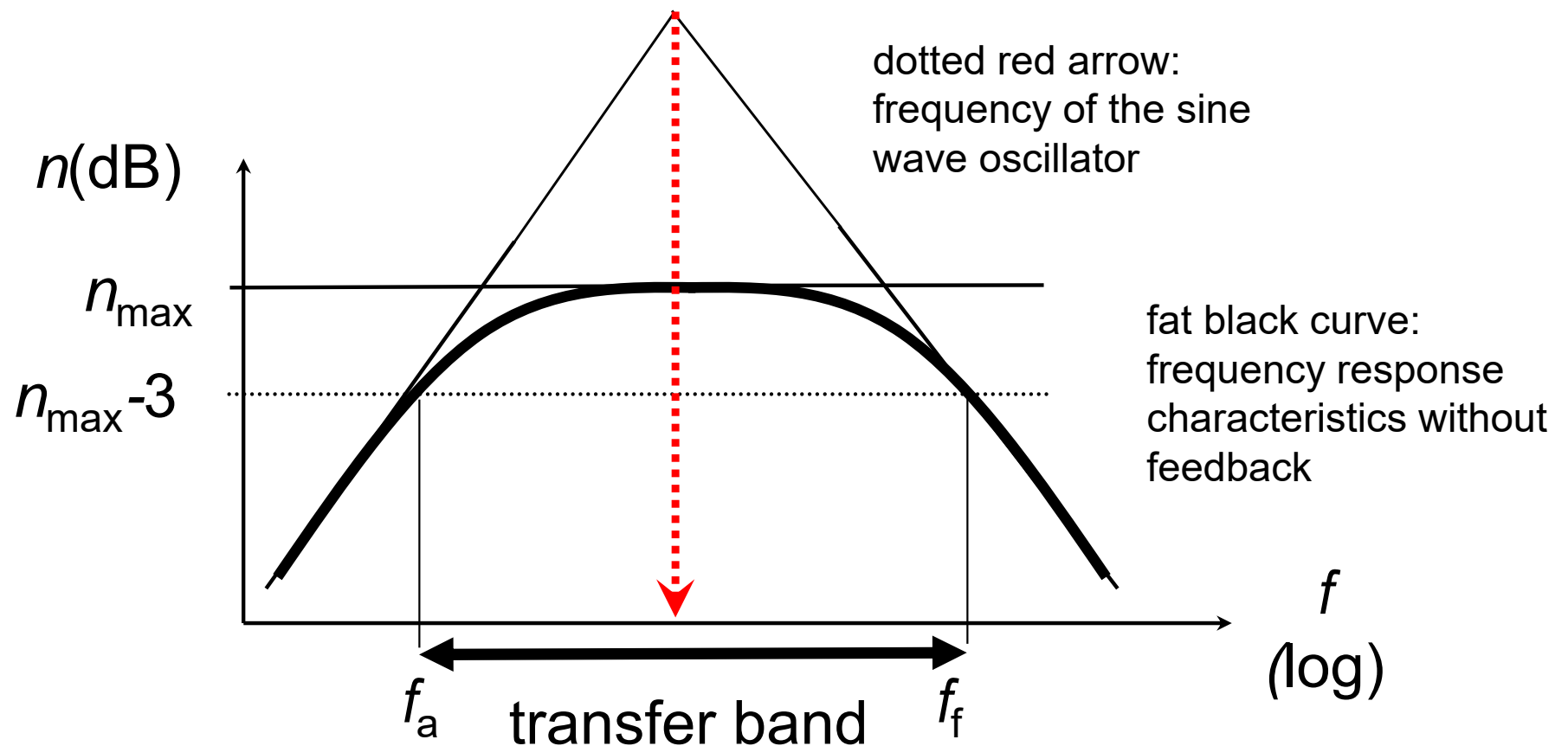


Source of electric signal : sine wave oscillator

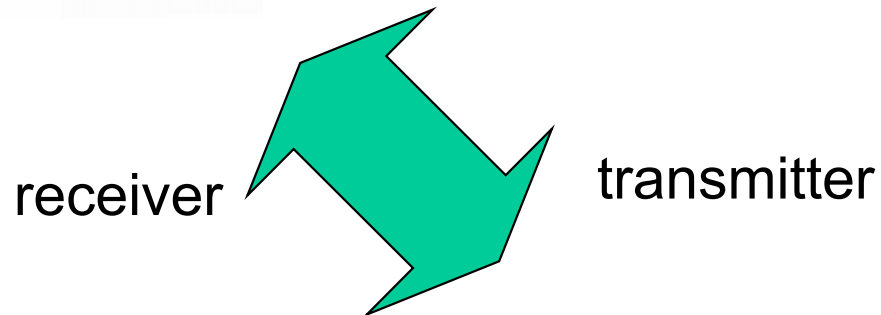
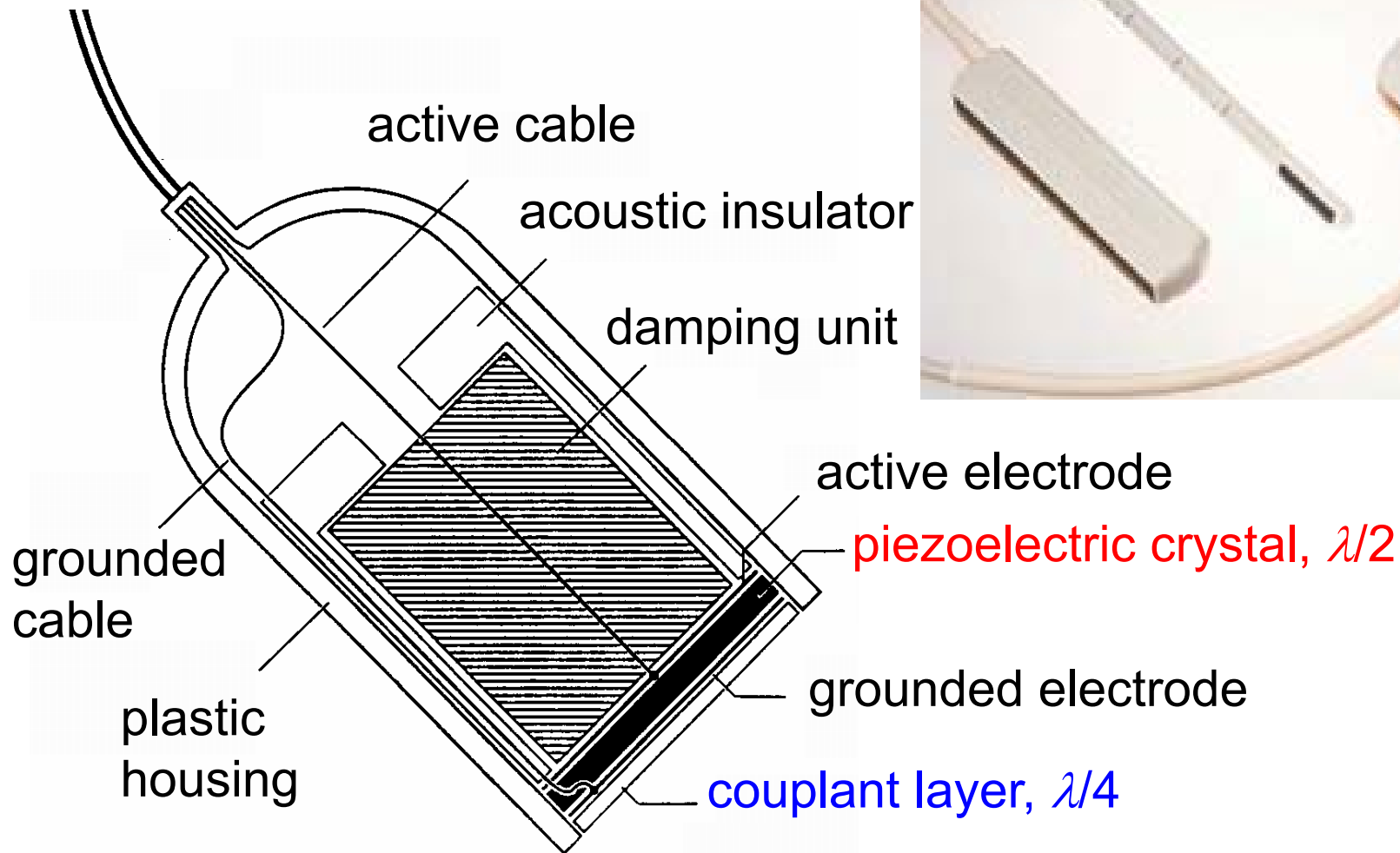
amplifier with positive feedback

$$A_{U, \text{feedback}} = \frac{A_U}{1 - \beta A_U}$$

$\beta A_U = 1$, amplification = „infinity“ → sine wave oscillator
no input signal, output signal: sine voltage



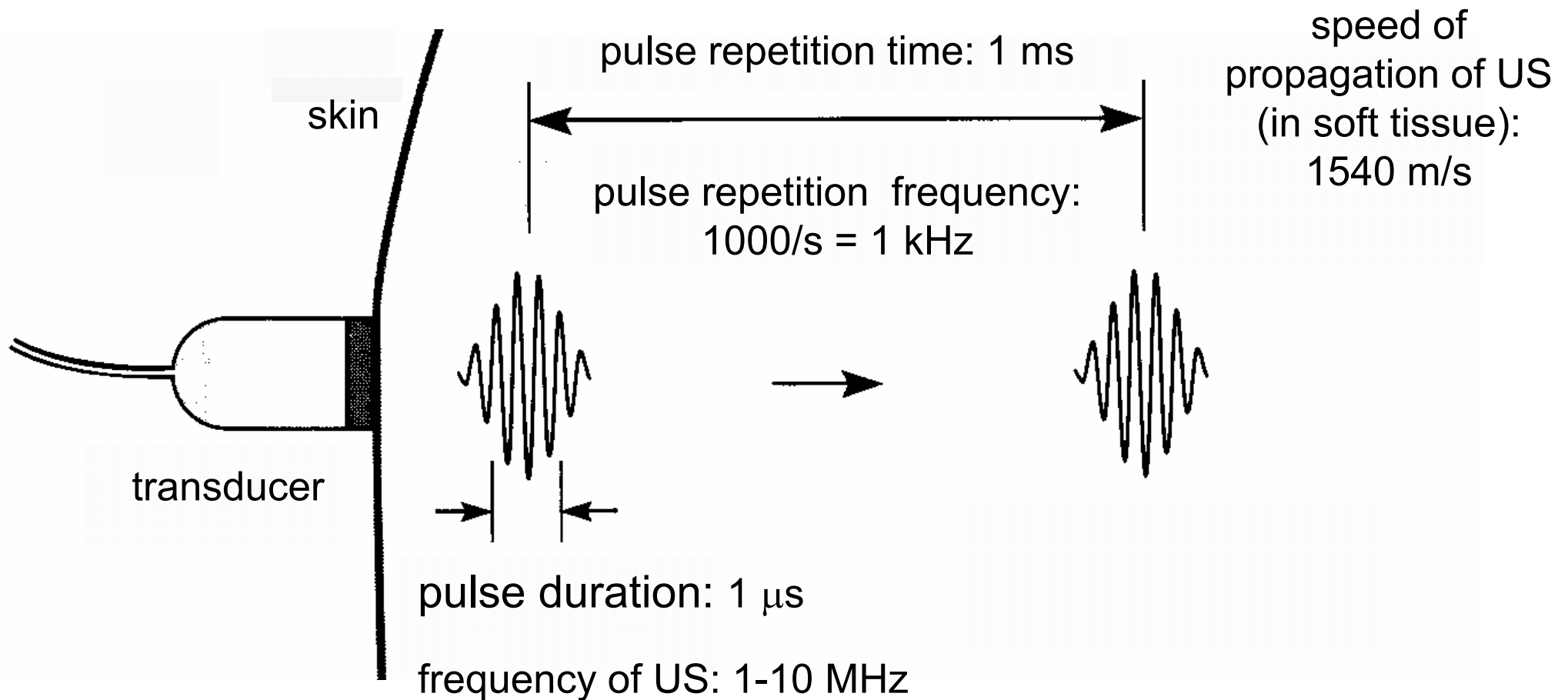
Ultrasound transducer



Characteristic of US pulses

transducer: transmitter and receiver is the same unit

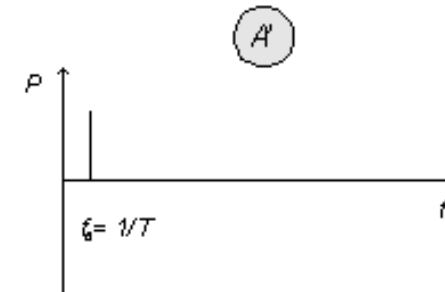
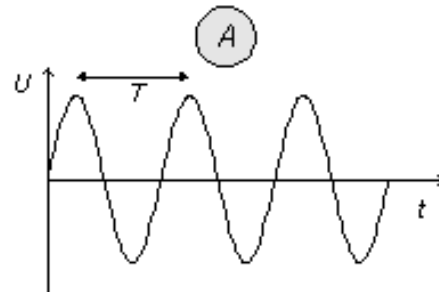
time sharing mode: pulses instead of continuous wave US



Time function

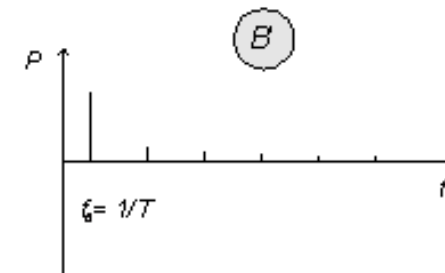
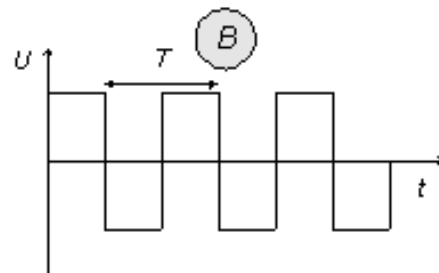
Spectrum

sine function



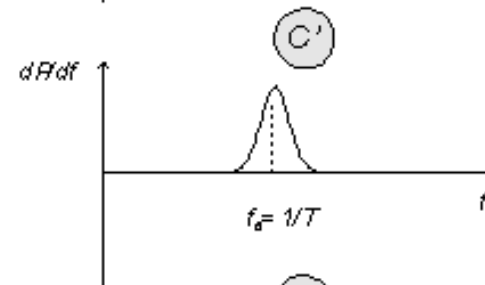
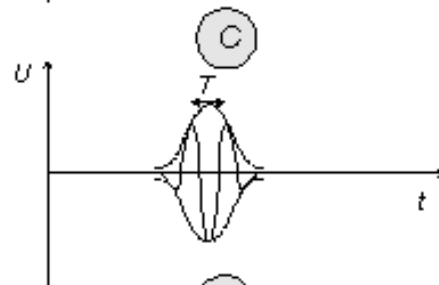
line spectrum (1 line)

square function



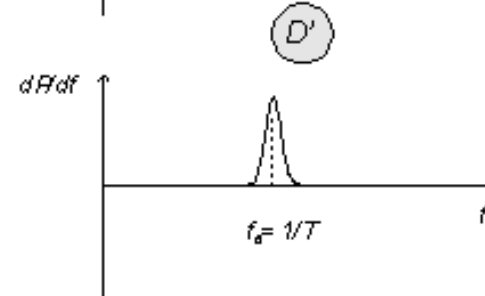
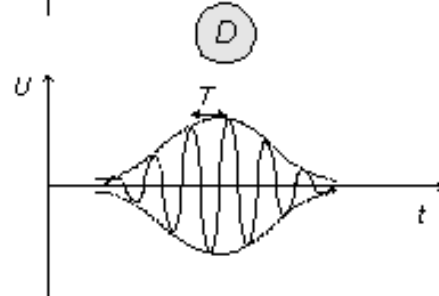
line spectrum

sine wave
pocket (some
„periods”)



band spectrum

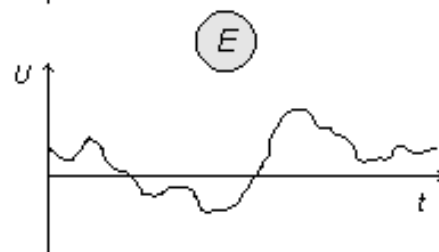
sine wave
pocket (several
„periods”)



band spectrum

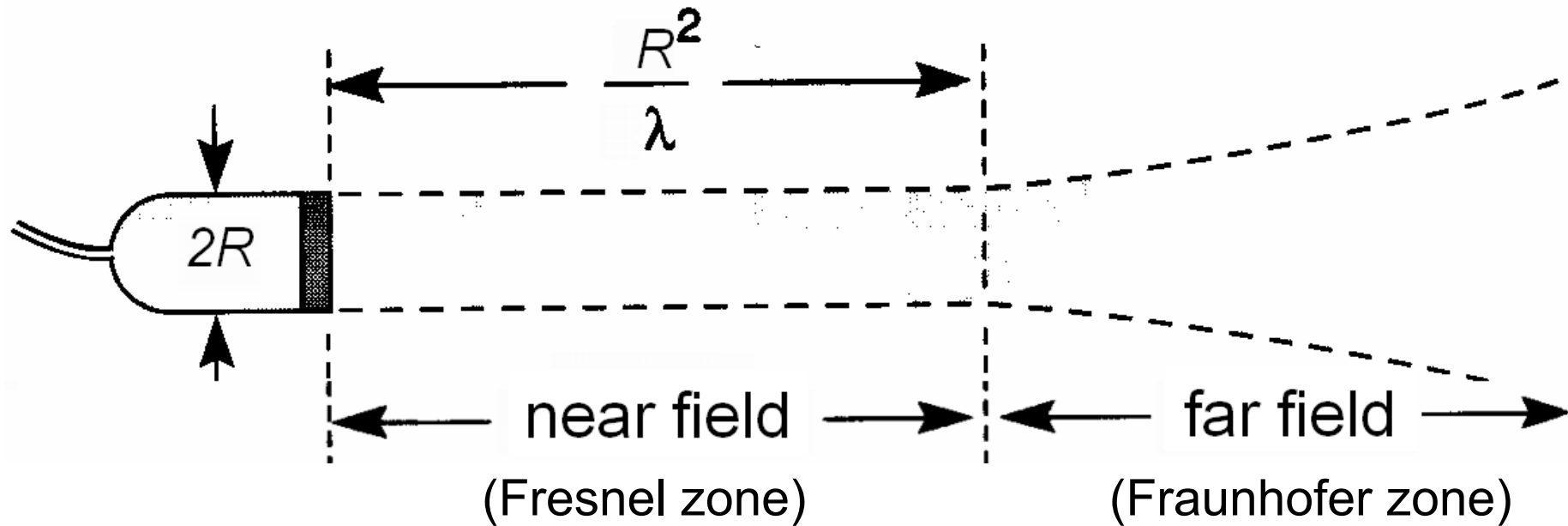
US pulse

aperiodic
function

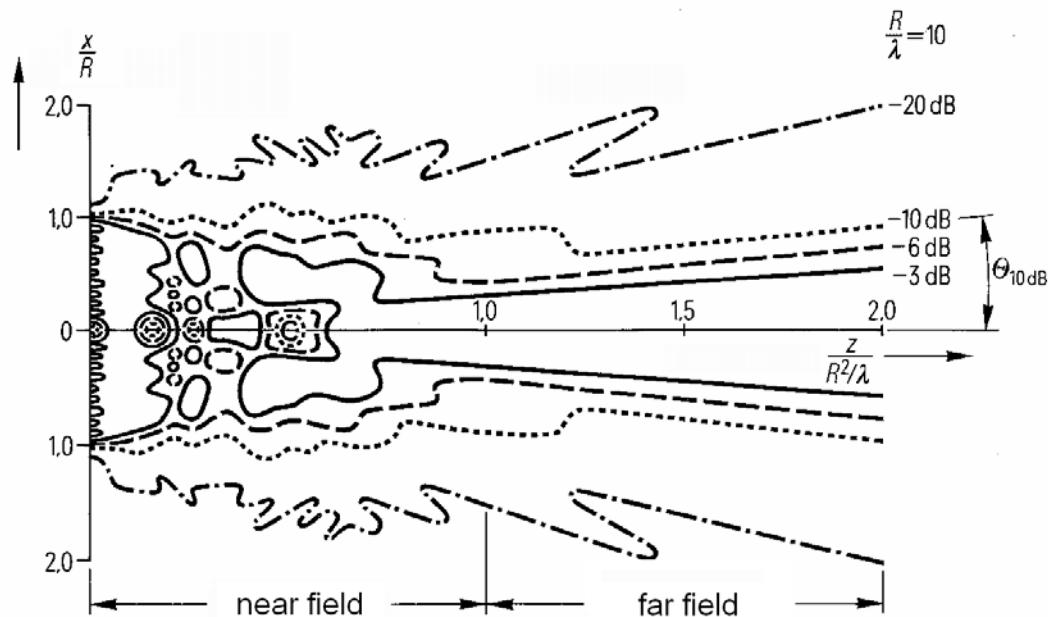


continuous spectrum

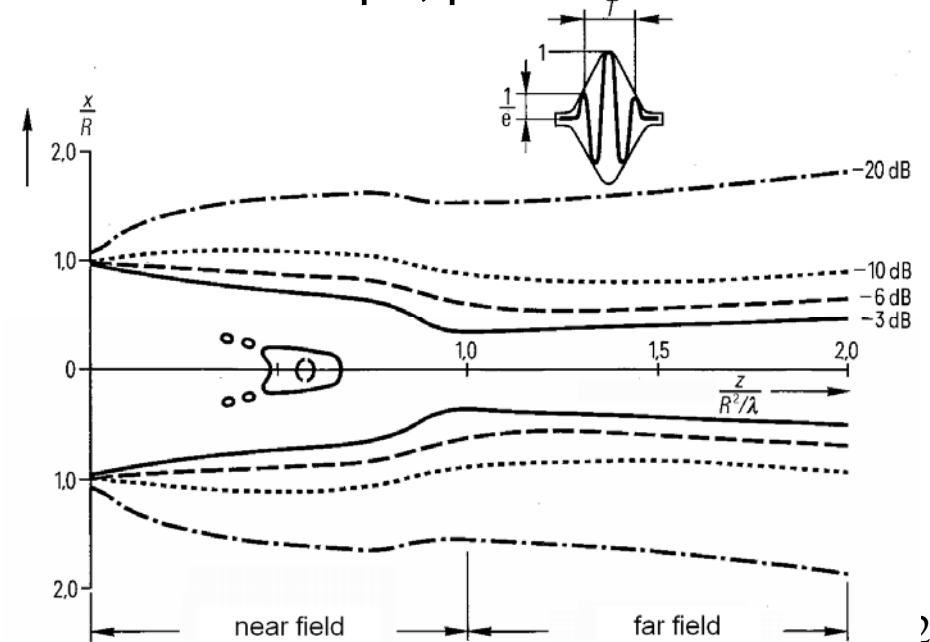
US beam shape (simplified version)



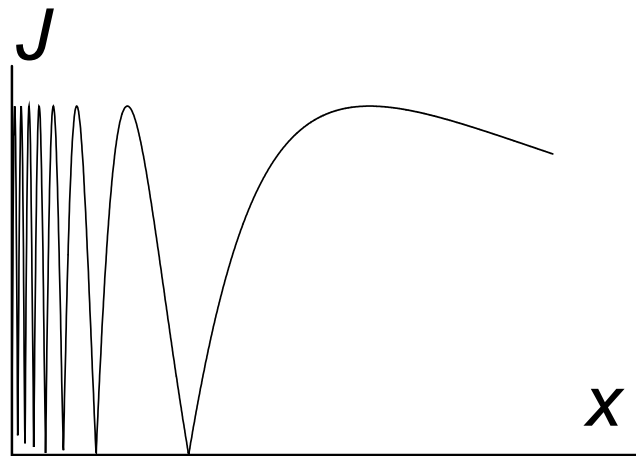
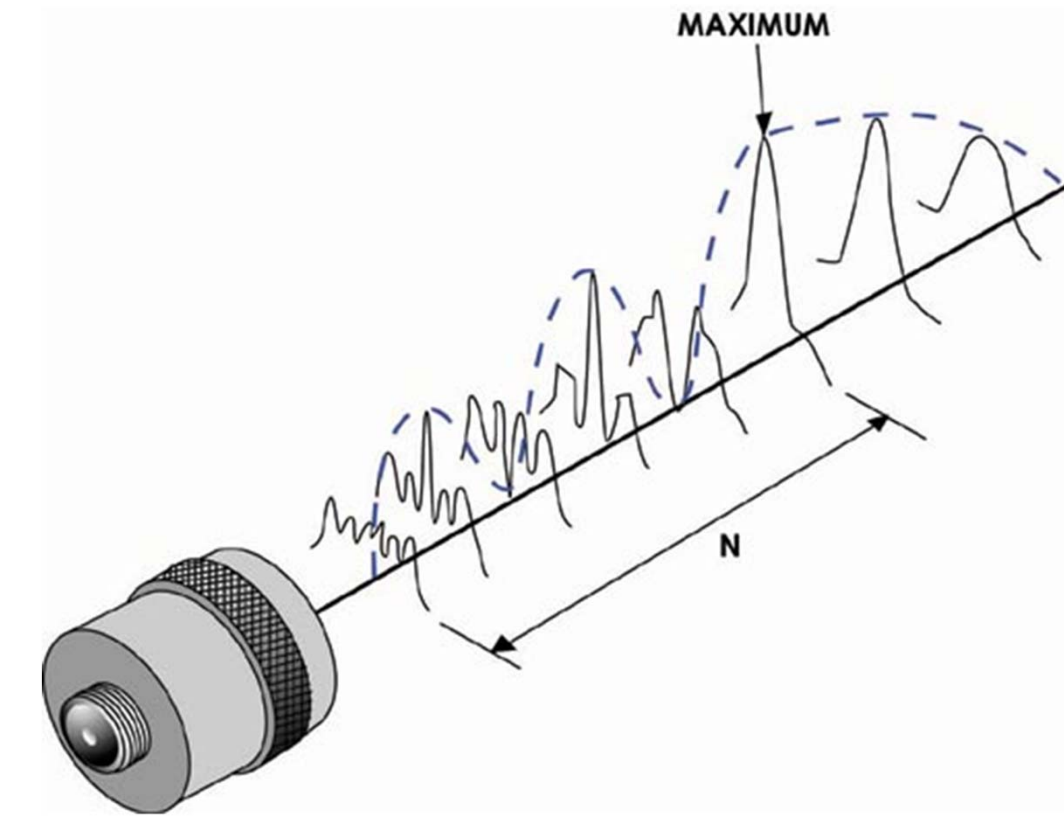
Beam shape, continuous wave US



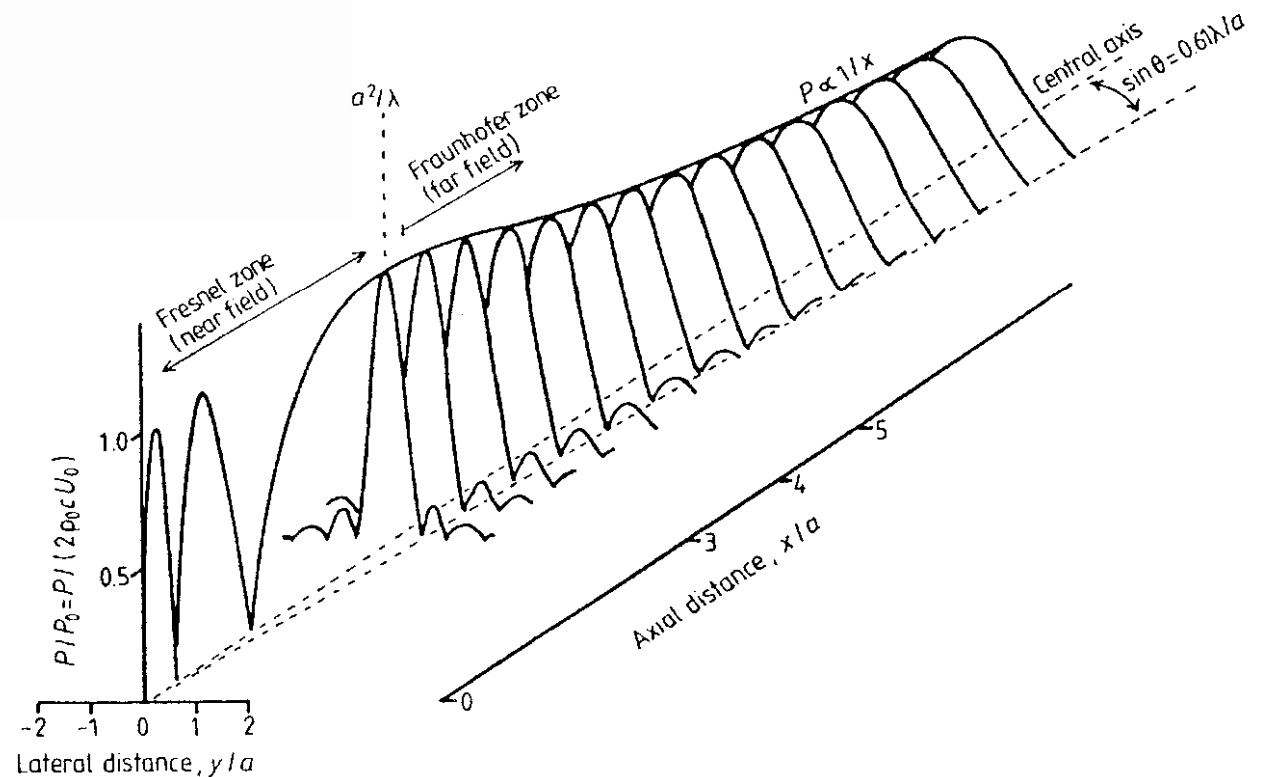
Beam shape, pulsed wave US



Perspective view of the US beam



change of the intensity
in the axial direction



Resolving limit, resolution

Resolving limit is the distance between two object details which can be just resolved as distinct objects (the smaller the better).

Resolution (resolving power): the reciprocal of the resolving limit (the greater the better)

Axial resolving limit depends on the pulse length. Pulse length is inversely proportional to the frequency.

Lateral resolving limit is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

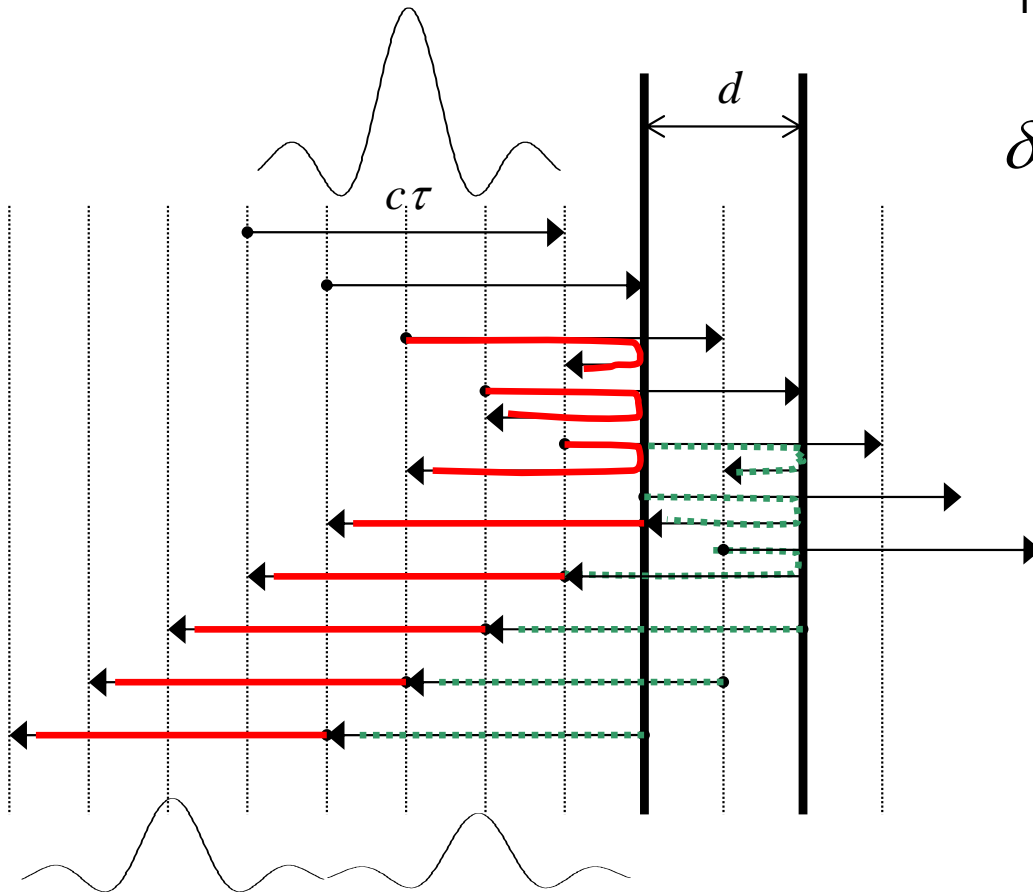
Typical values	frequency (MHz):	2	15
	wavelength (in muscle) (mm):	0.78	0.1
	penetration depth (cm):	12	1.6
	lateral resolving limit (mm):	3.0	0.4
	axial resolving limit (mm):	0.8	0.15

Axial resolving limit

τ : pulse duration

$c_1\tau \cong c_2\tau = c\tau$ pulse length

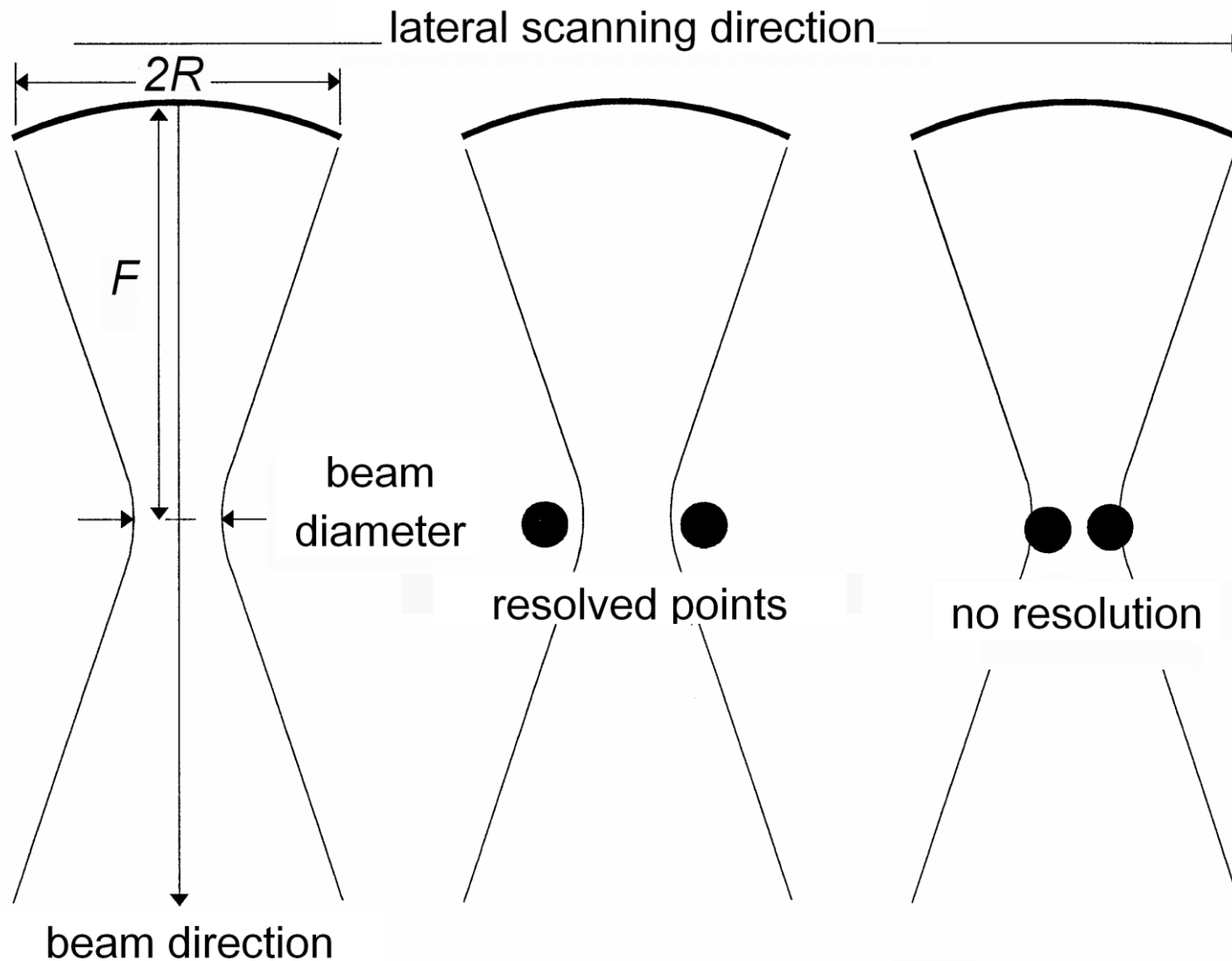
$\delta_{ax} = d = \frac{c\tau}{2}$ resolving limit



The axial resolving limit is the half of the pulse length. The echos from the adjacent surfaces in this case just hit another.

$$\tau \sim T = \frac{1}{f}$$

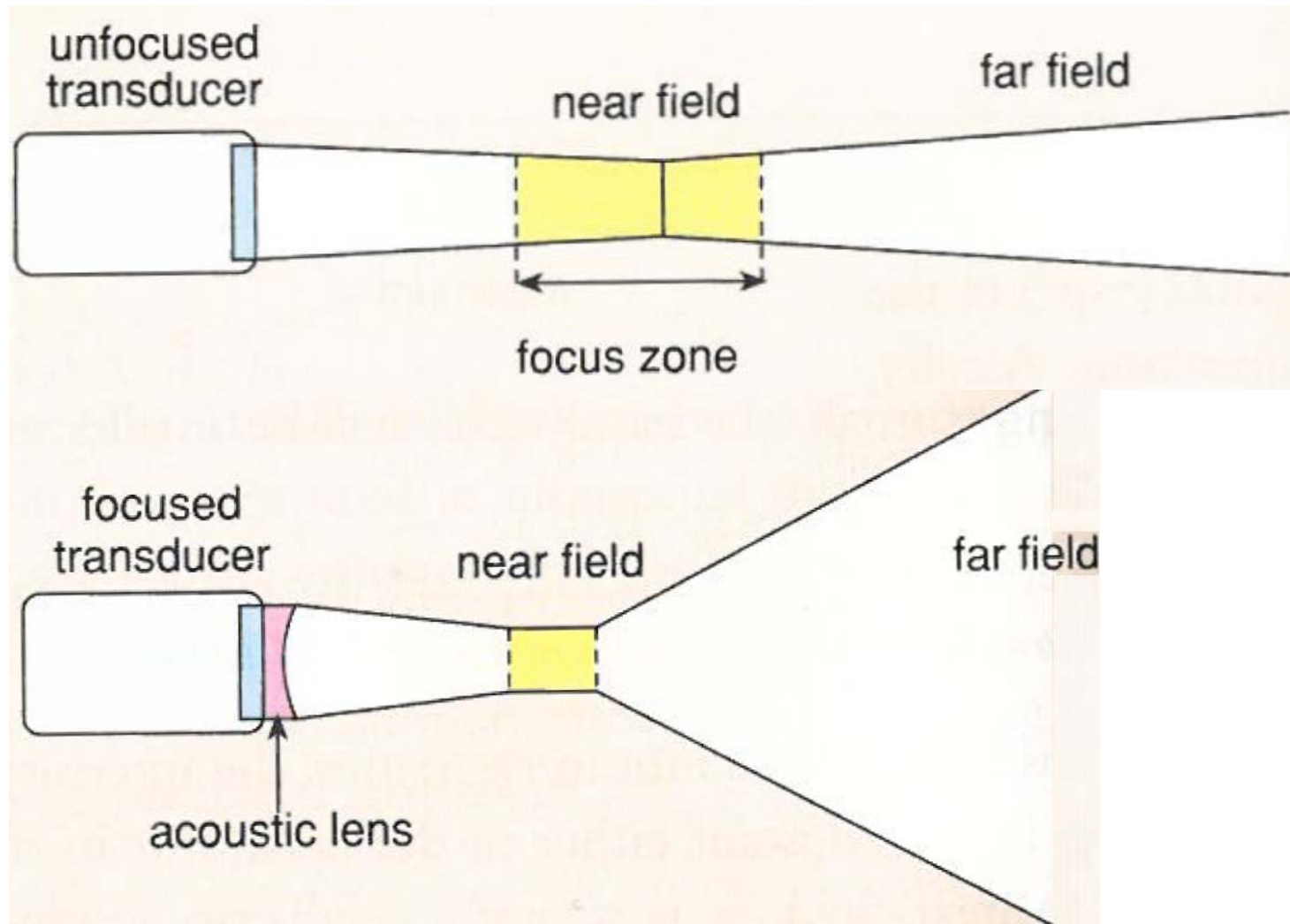
Lateral resolving limit



$$\left(\delta_{\text{lat}} \sim \frac{F}{2R} \cdot \lambda \right)$$

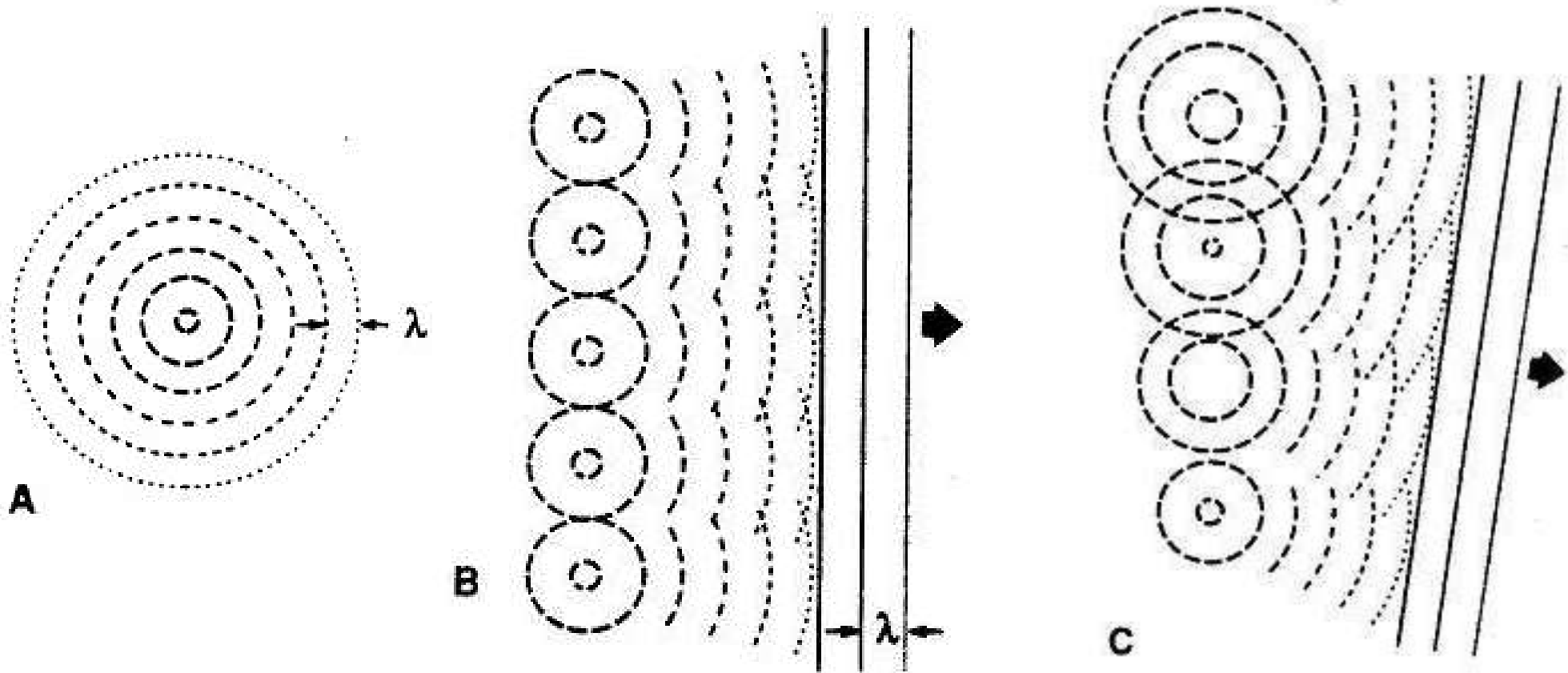
F : focal length
 $2R$: diameter of the transducer
 λ : wavelength

Focusing of the beam



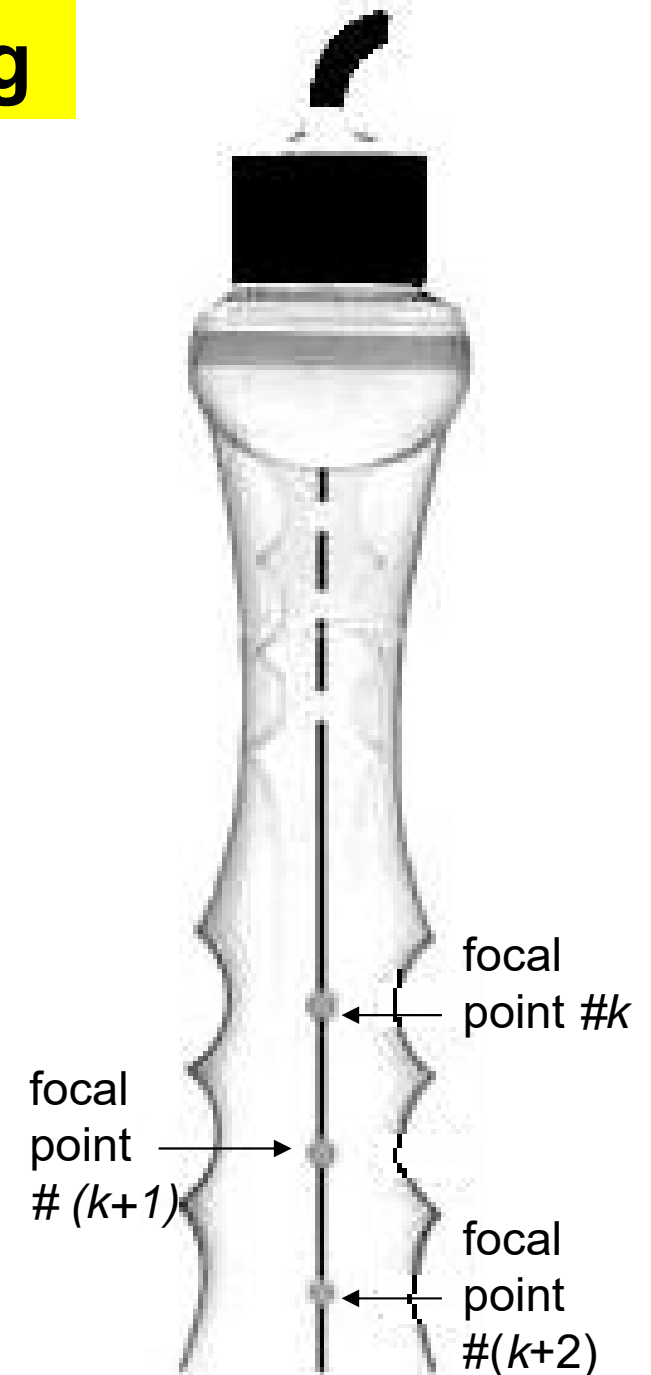
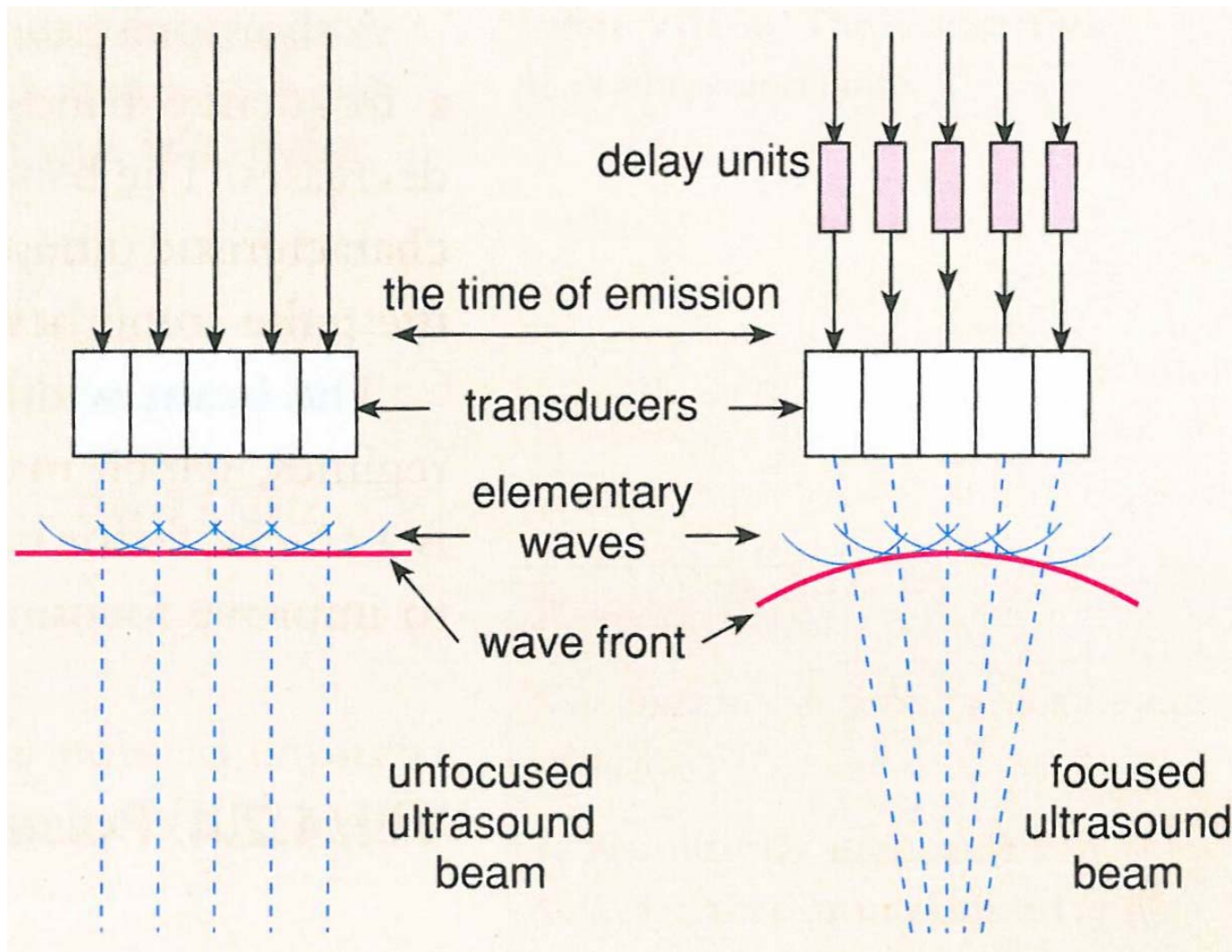
Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

Huygens' principle



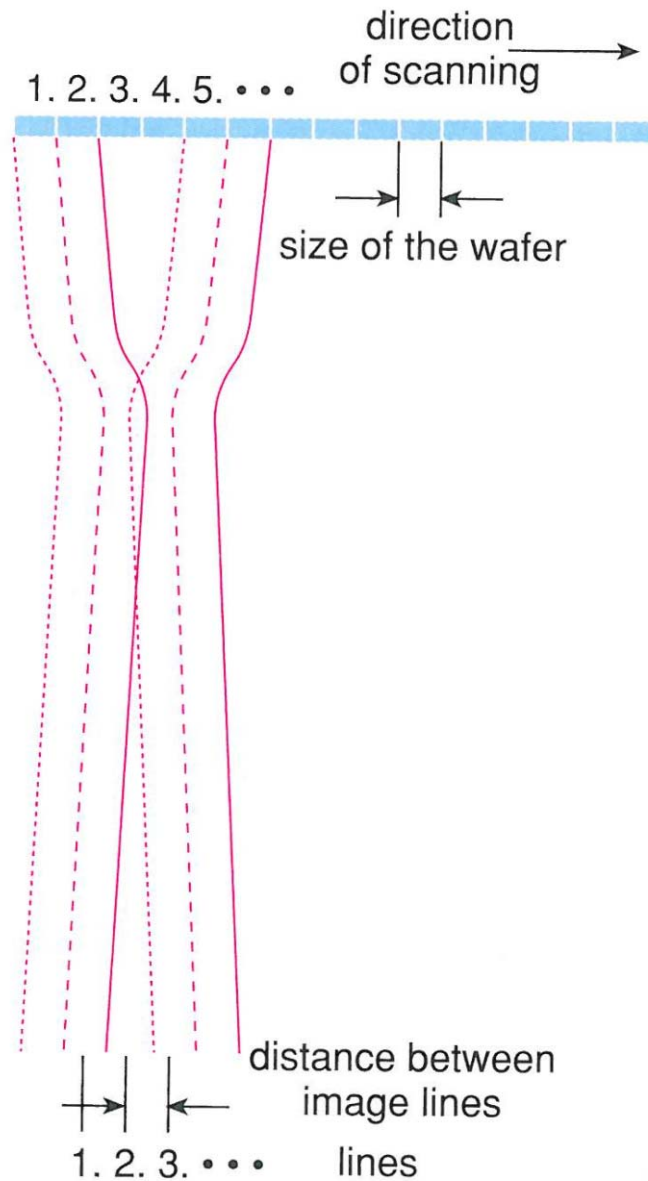
Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.

Electronic focusing

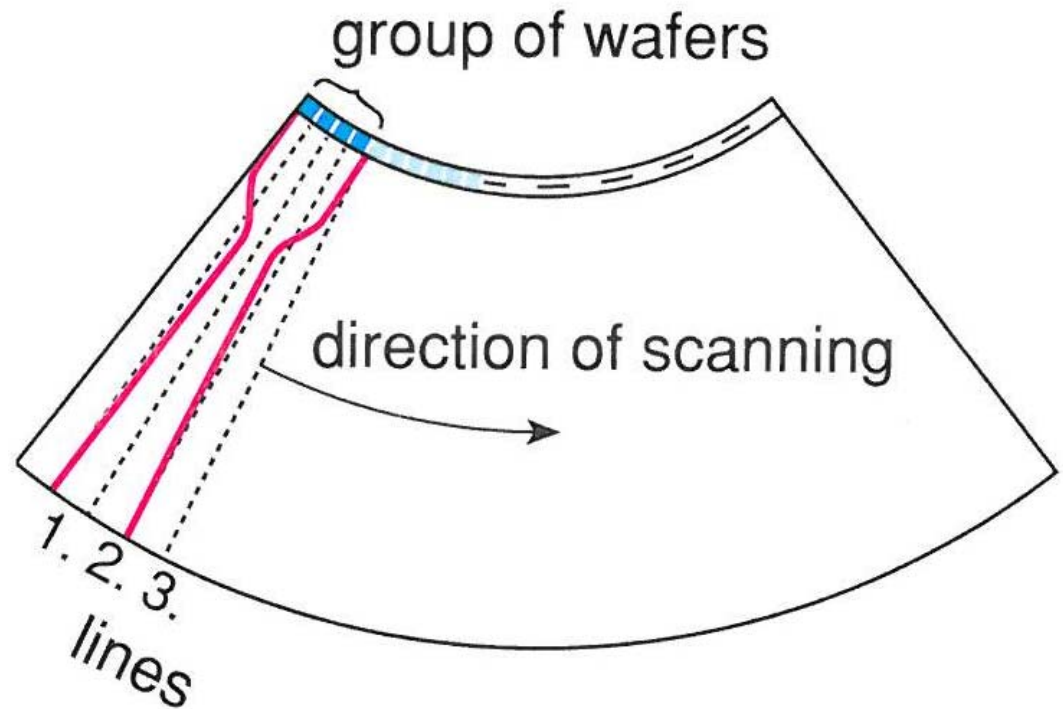


Scanning

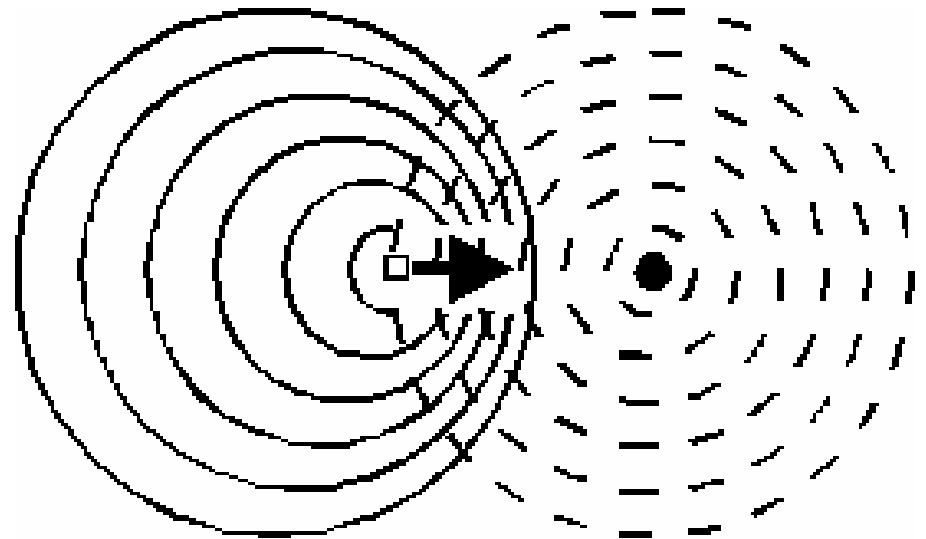
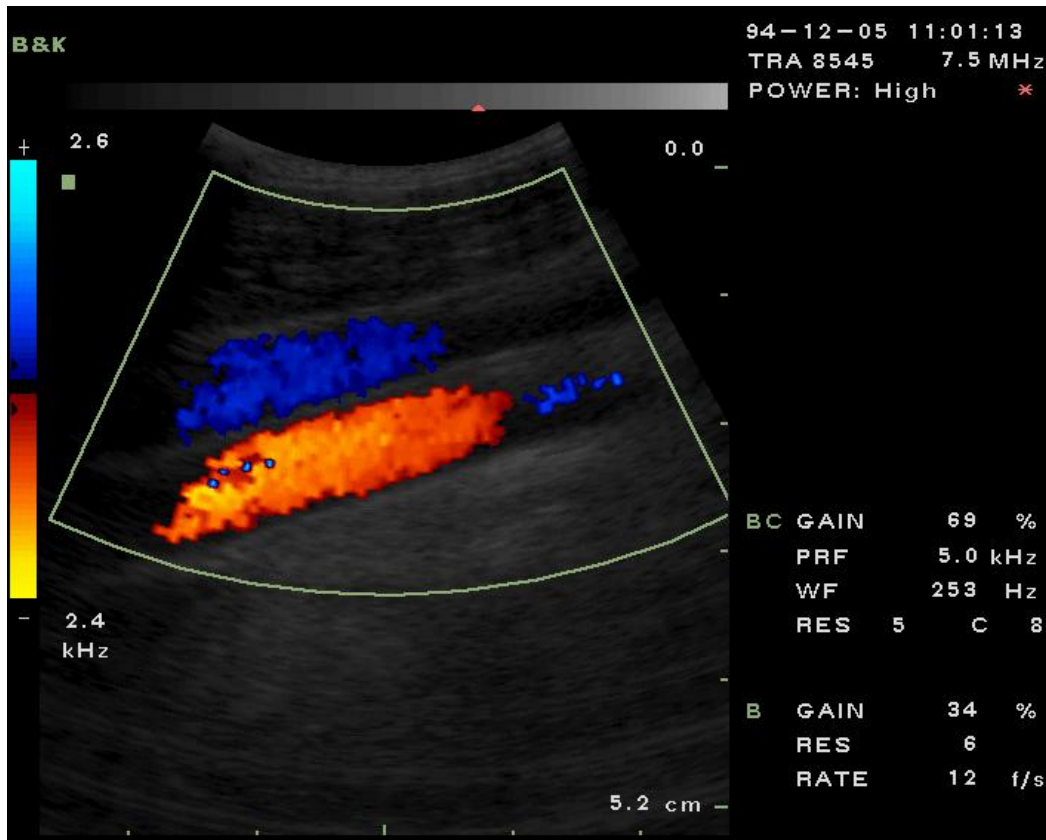
multi unit linear array



multi unit curved array



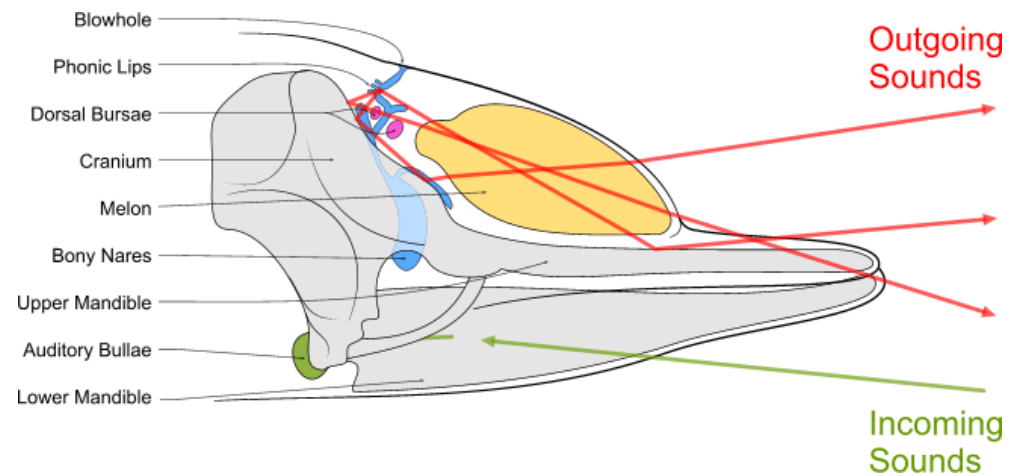
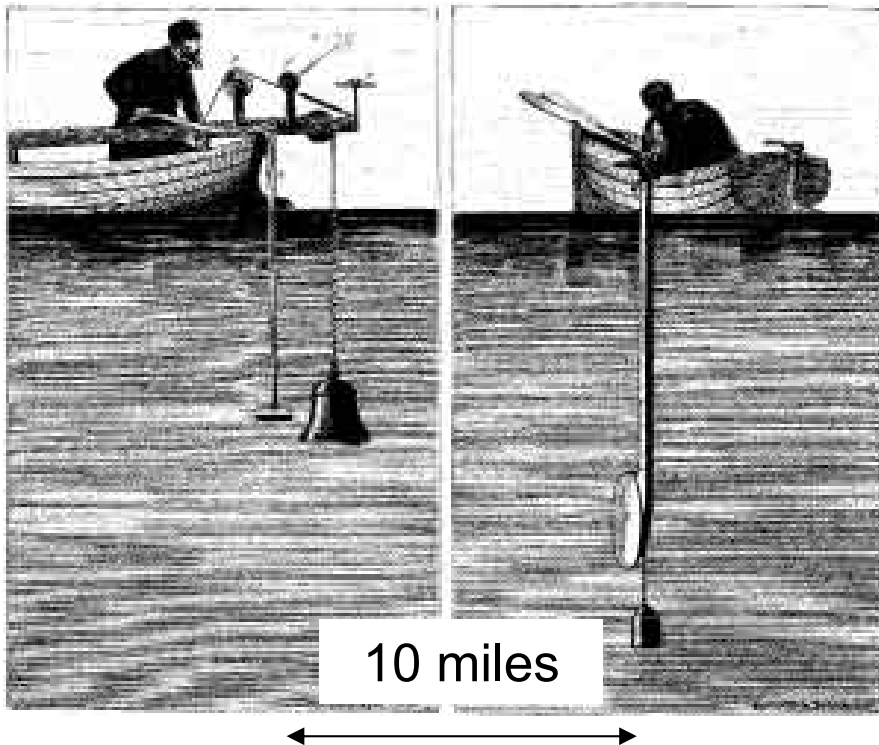
US imaging. Modes of sonography. Doppler-echo.



Echo principle

1794 Spallanzani:
bat's navigation

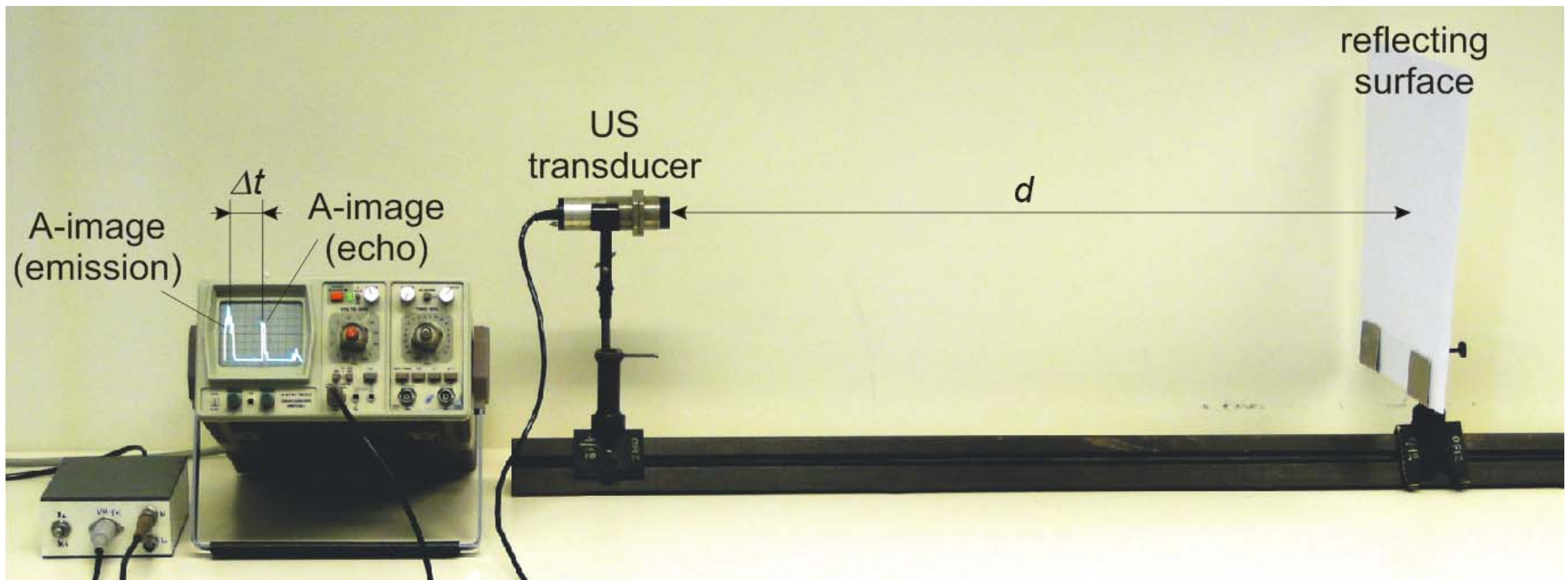
1822 Colladen
measured the speed of
sound in water



bottlenose dolphin

Echo principle

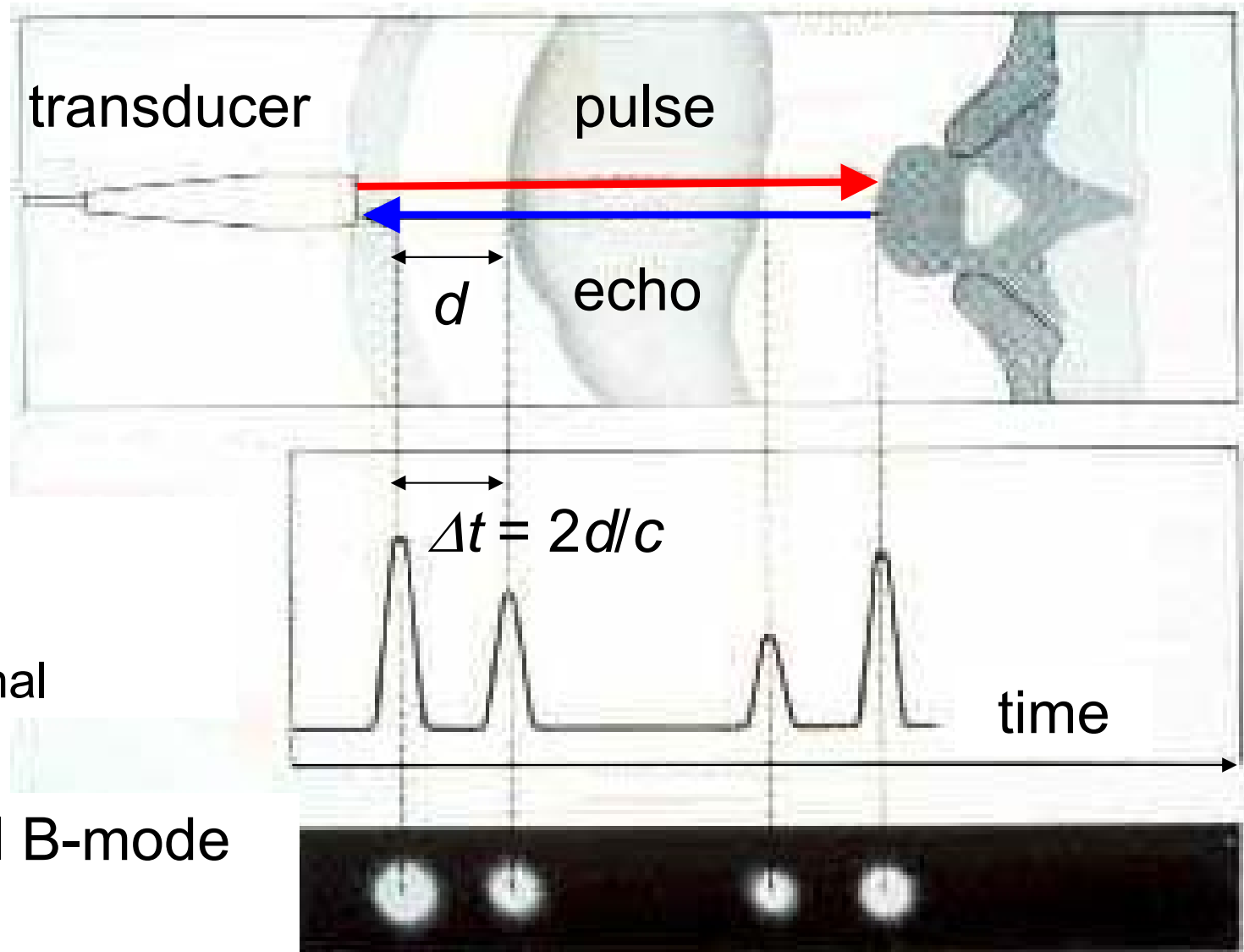
using a special US-head, short pulses are emitted in the air towards a reflecting surface, and the same US-head detects the echo signal



$$c\Delta t = d + d = 2d$$



Receiving the echos

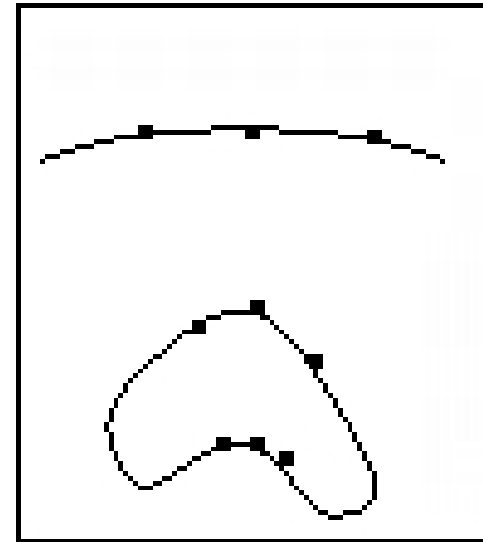
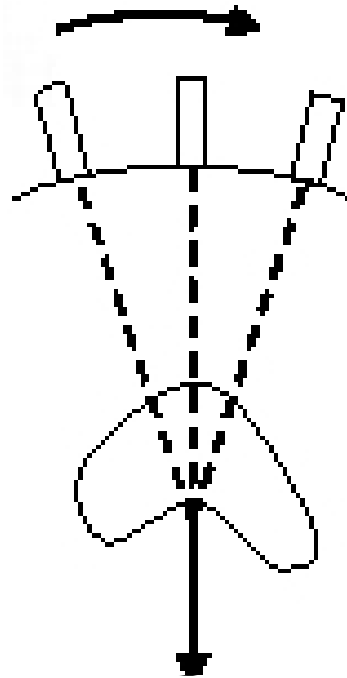


A-mode
(**A**mplitude)
only 1-dimensional

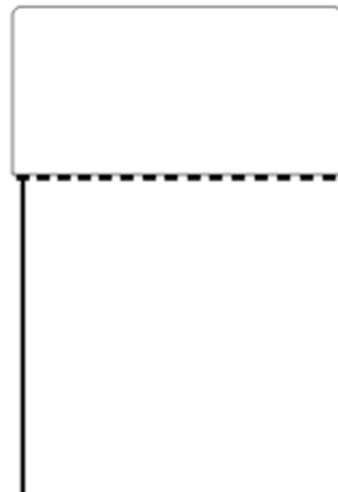
1-dimensional B-mode
(**B**rightness)

2-dimensional B-mode

moving
transducer



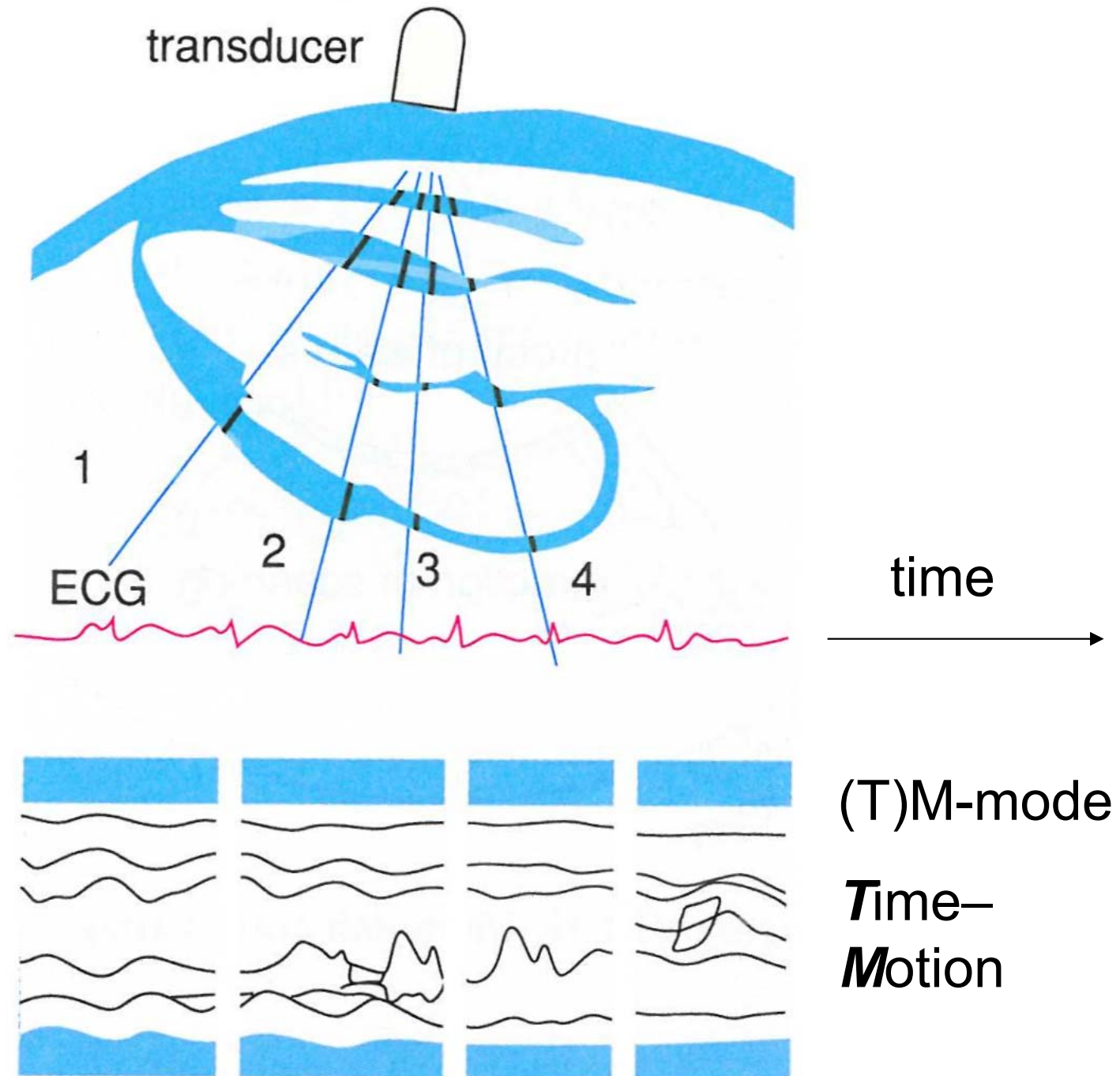
B-mode
display



TM-mode

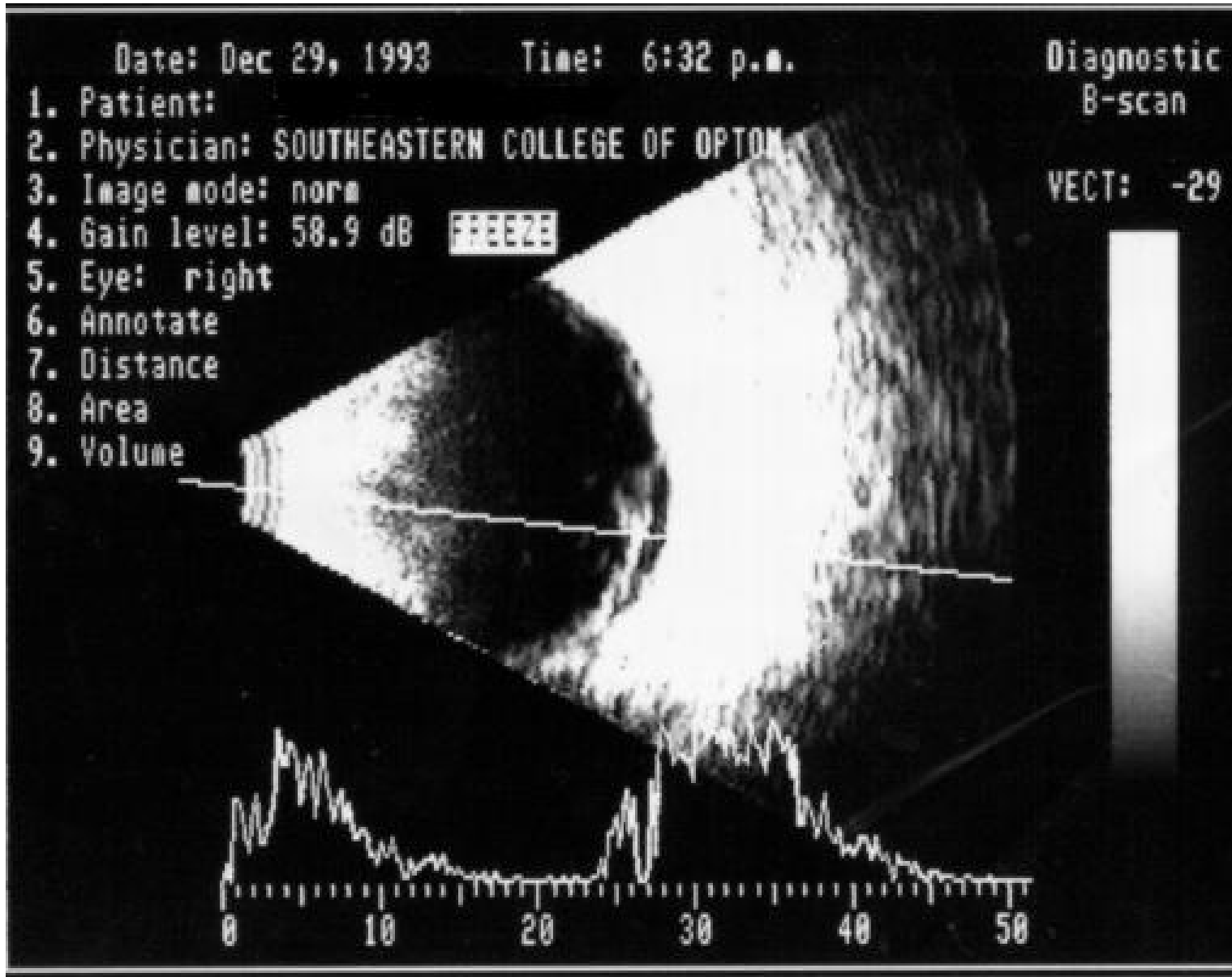
ECG signal
for reference

(vertical)
time-dependent
1-dimensional
B-mode



(T)M-mode
***Time–
Motion***

2-dimensional B-mode and A-mode (used in ophthalmology)



real speed of propagation for the accurate determination of distances:

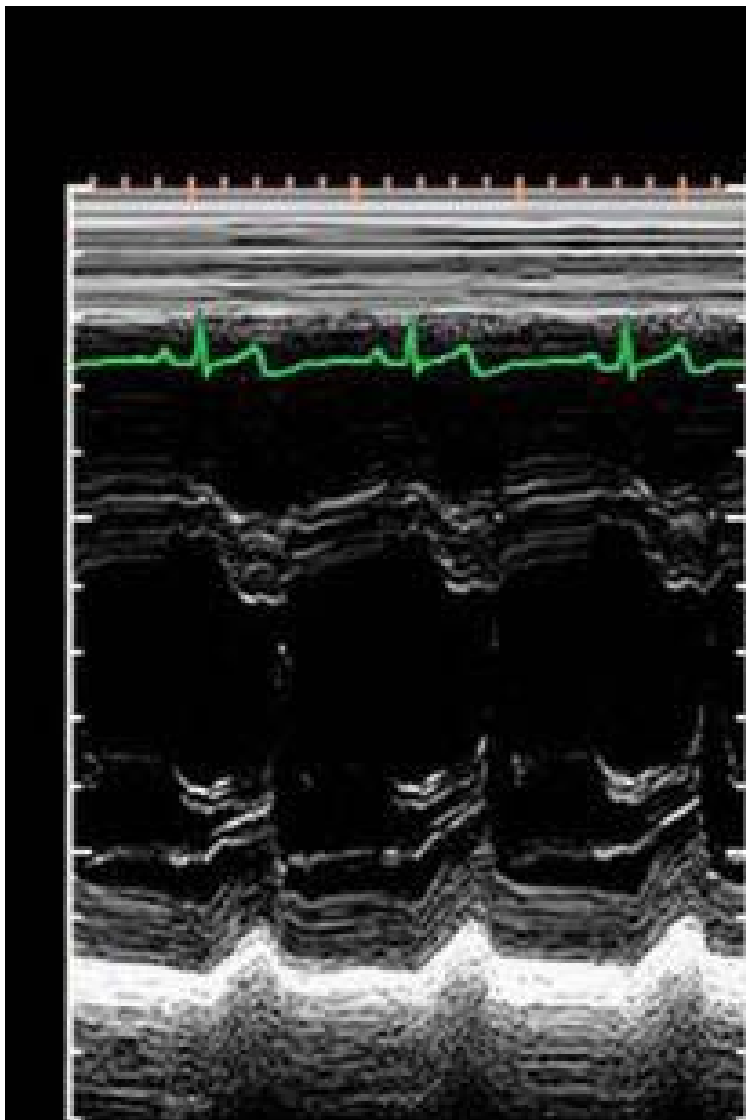
cornea: 1641 m/s

aqueous humour:
1532 m/s

crystalline lens:
1641 m/s

vitreous body:
1532 m/s

TM-mode

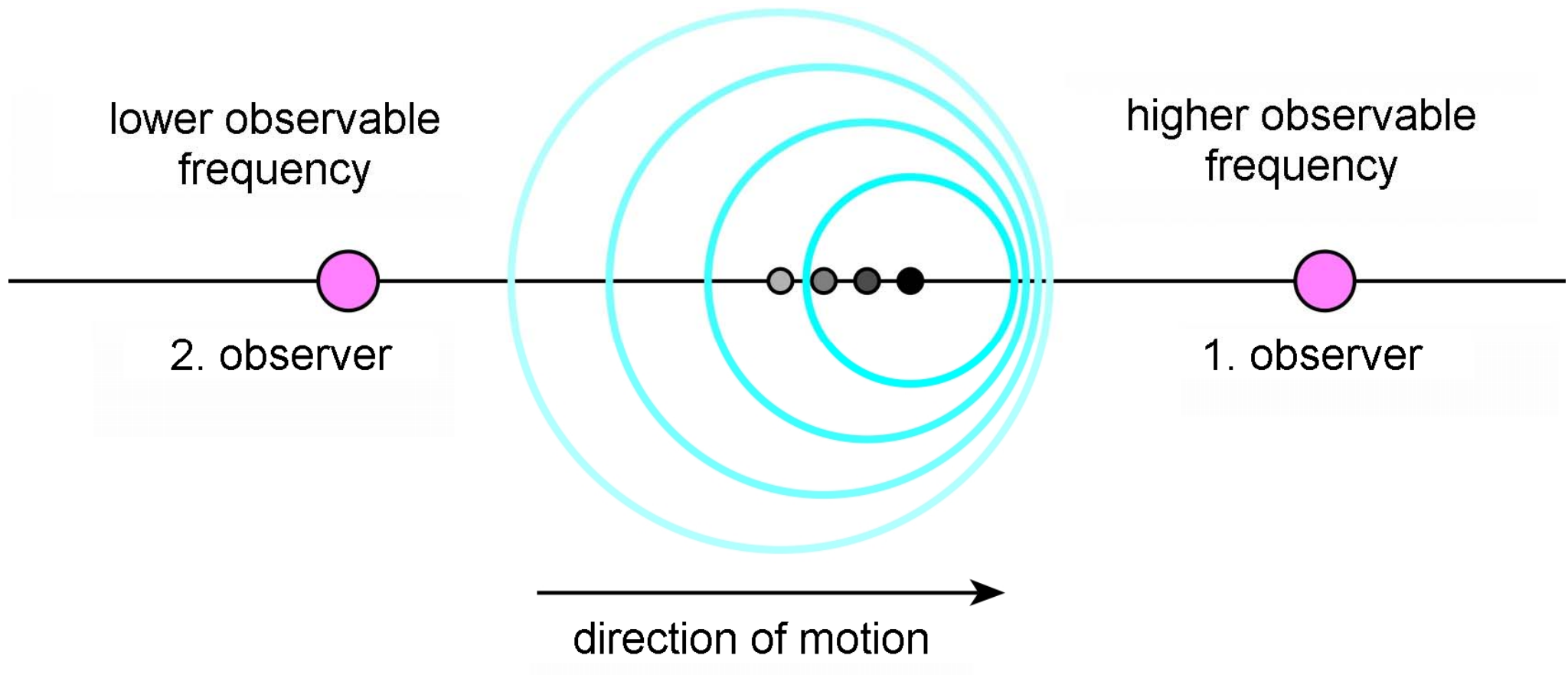


B-mode



Doppler phenomenon

„The pitch of a train whistle seems to get higher as it approaches, then seems to lower as the train whistle moves away.” (C. Doppler, 1842)



f' : **observed frequency**, f : original frequency

(a) standing source and moving observer (v_o)

+: observer approaches the source

–: observer moves away from the source

$$f' = f \left(1 \pm \frac{v_o}{c} \right)$$

(b) moving source and standing observer

(if $v_s \ll c$, then „same” as (a))

$$f' = \frac{f}{1 \mp \frac{v_s}{c}}$$

(c) moving source and moving observer

$$f' = f \frac{1 \pm \frac{v_o}{c}}{1 \mp \frac{v_s}{c}}$$

(d) moving reflecting object (surface),

(if $v_R \ll c$)

$$f' = f \left(1 \pm \frac{2v_R}{c} \right)$$

Doppler frequency = frequency change = frequency shift

if $v_i, v_R \ll c$ (i= S or O)

rearranging equation (a)

moving source or observer:

$$\Delta f = f_D = \pm \frac{v_i}{c} f$$

rearranging equation (d)

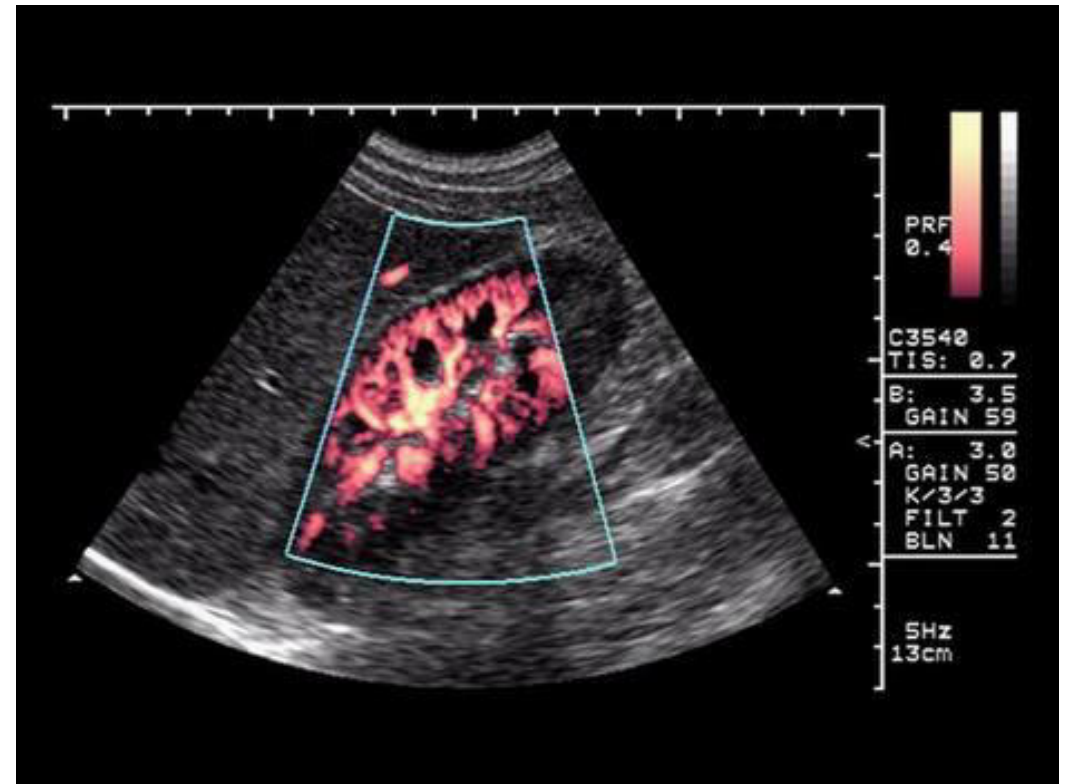
**moving reflecting object
or surface:**

$$\Delta f = f_D = \pm 2 \frac{v_R}{c} f$$

if v and c are not parallel, then $v \cos \theta$ should be used instead of v (remark: if $\theta = 90^\circ$, $f_D = 0$)

Colour coding

towards the transducer: warm colours
away from the transducer: cold colours



BART: **Blue** Away **Red** Towards

power Doppler

1-dimensional CW Doppler apparatus for measuring average flow velocity. Red blood cells as sound scatterers

CW: continuous wave

source and detector are separated

$$|f_D| = 2 \frac{v_R \cos \theta}{c} f$$

e.g. $f=8000$ kHz

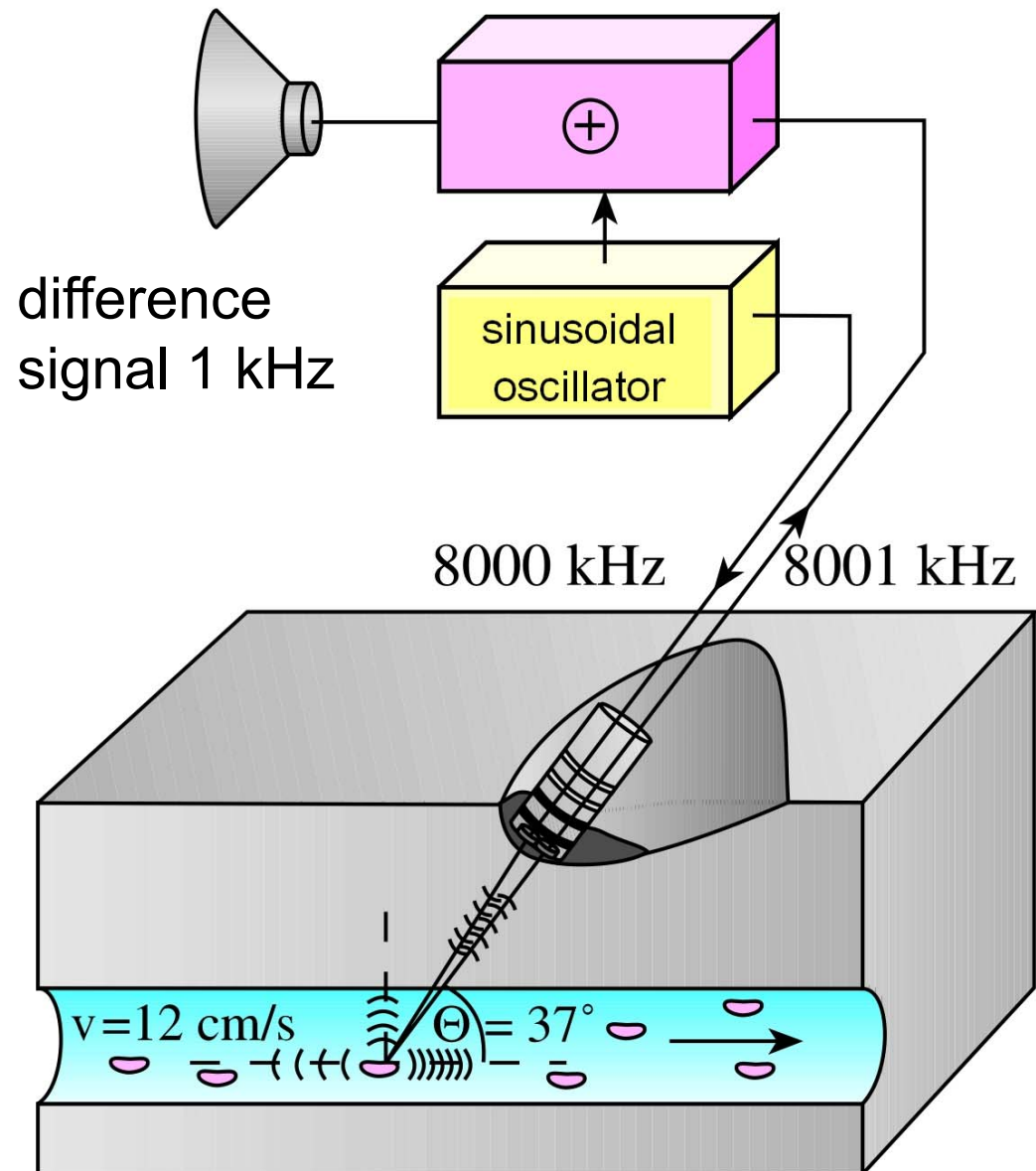
$v=12$ cm/s

$c=1600$ m/s

$\theta = 37^\circ$

$\Rightarrow f_D = 1$ kHz

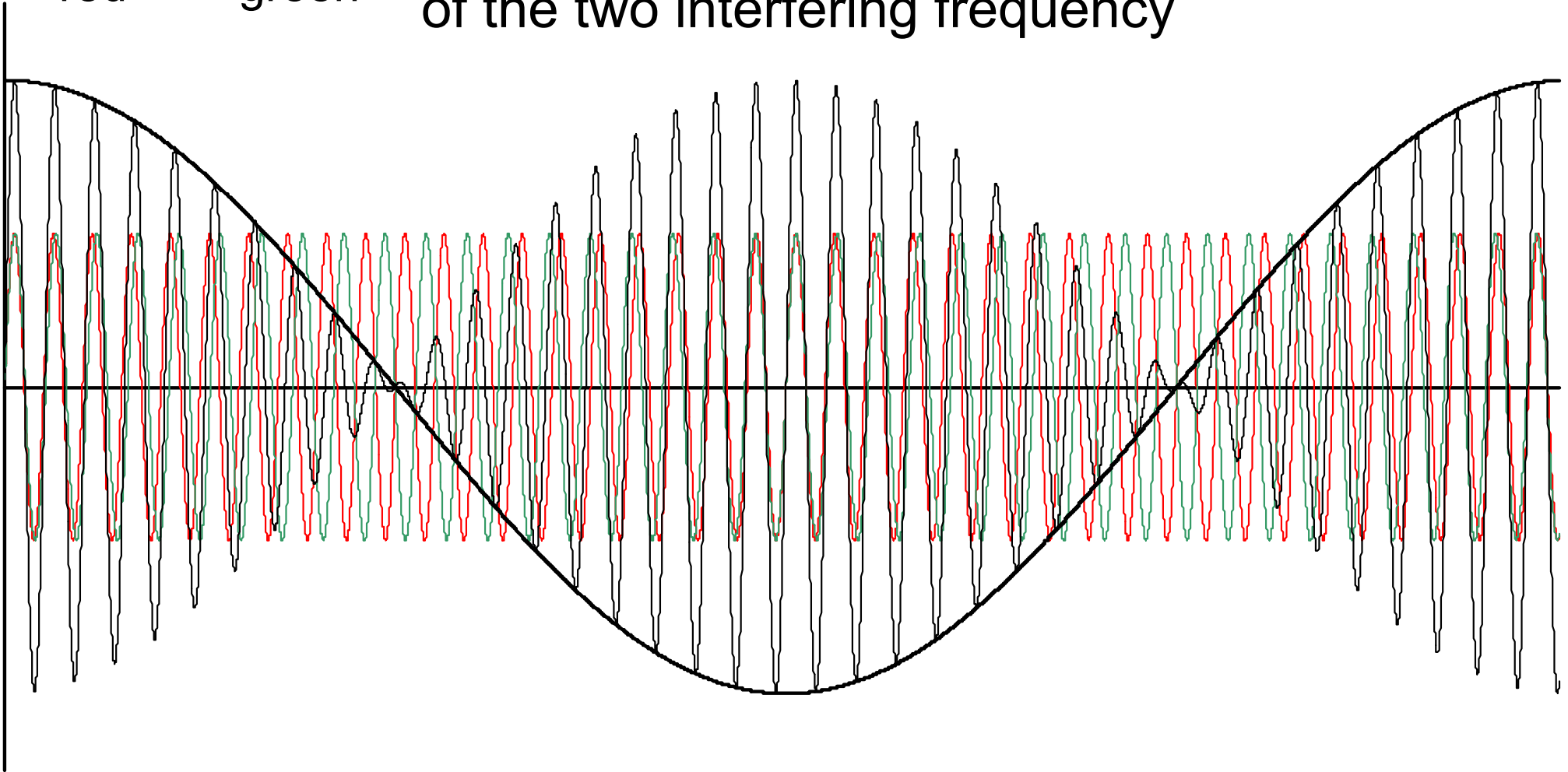
(beating phenomenon)



Beating phenomenon

$$f_{\text{red}} \geq f_{\text{green}}$$

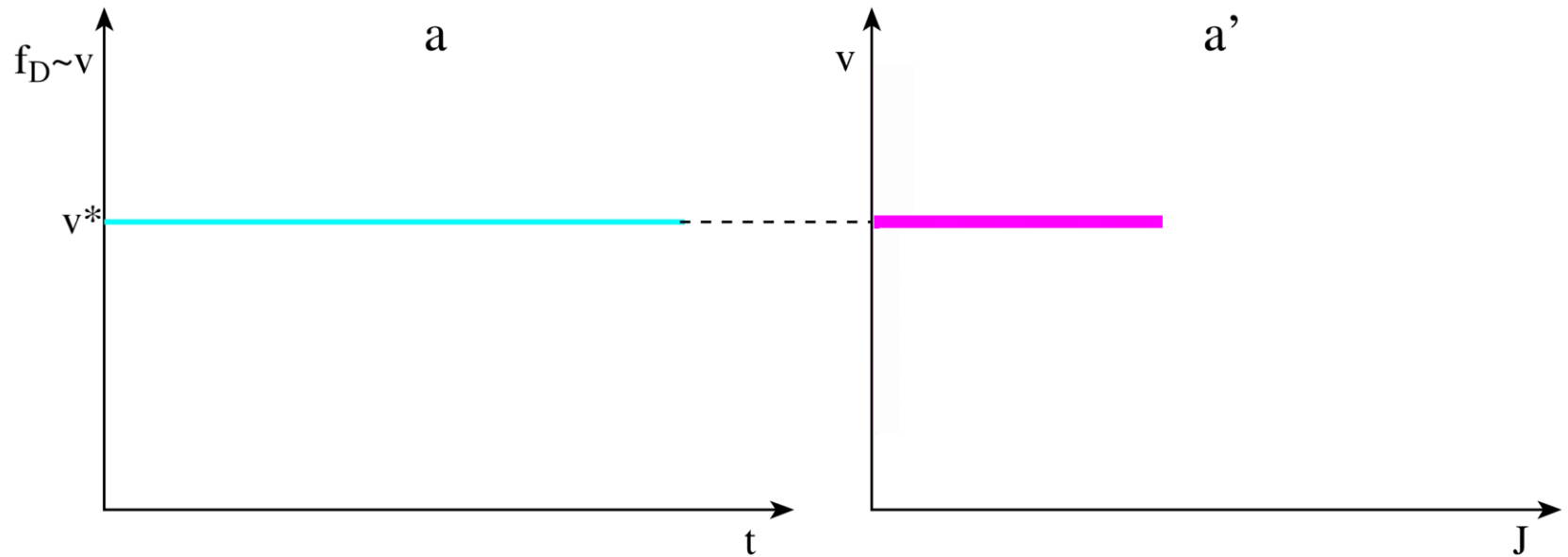
the beating frequency equals to the difference of the two interfering frequency



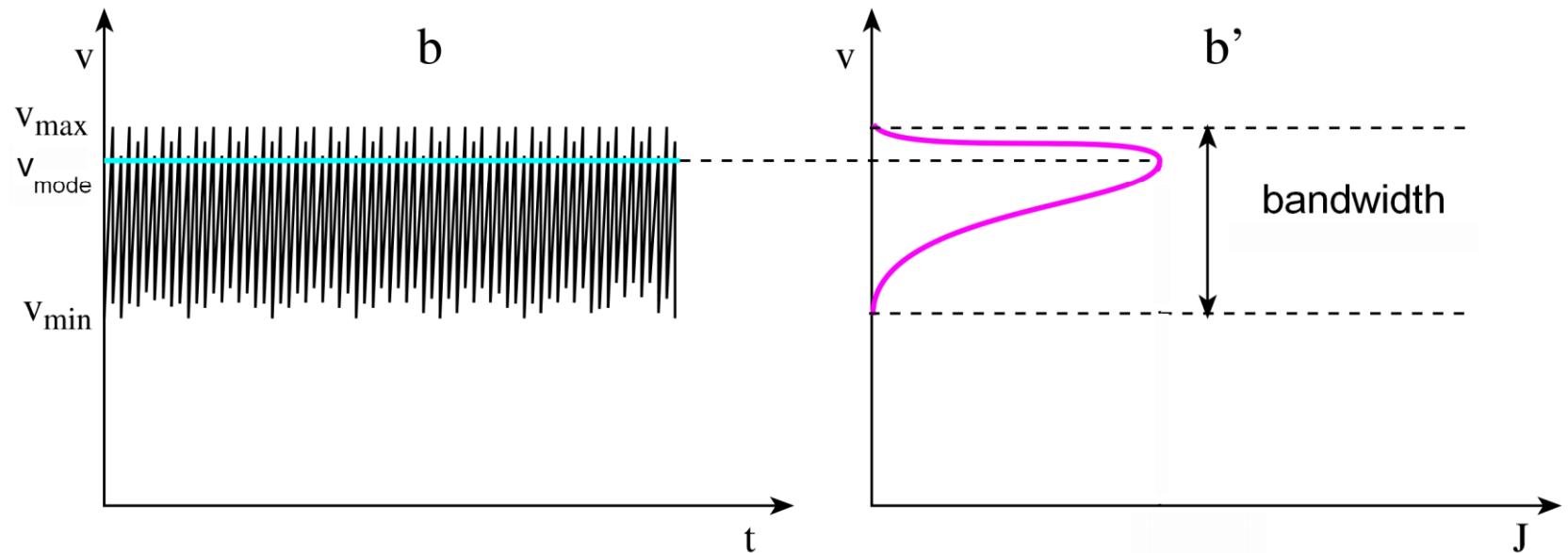
reminder: $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$

Doppler curves

one
constant
velocity (v^*)



frequency
distribution
(with v_{mode})

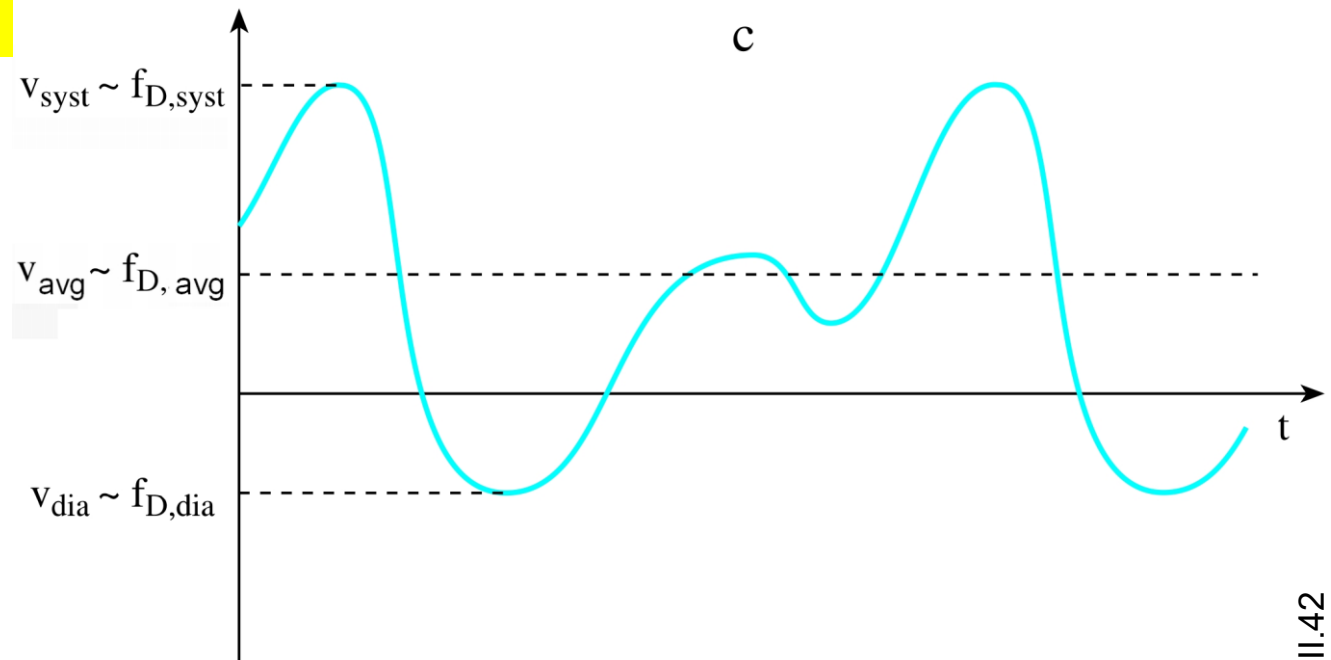


velocity distribution in
TM-mode

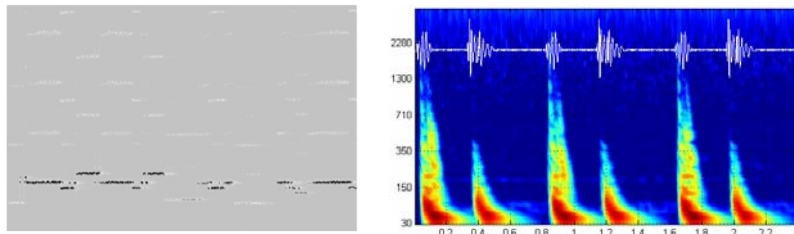
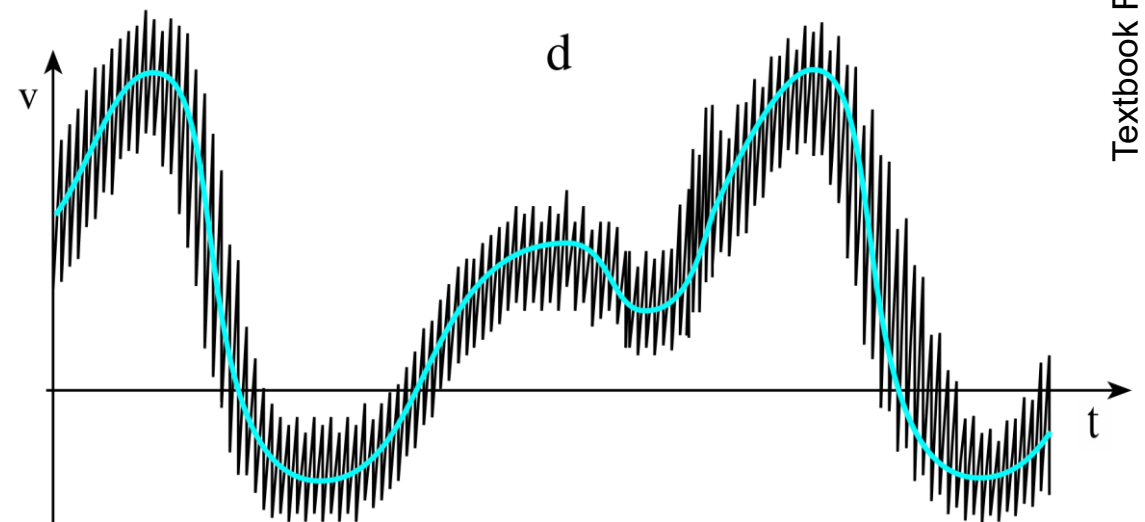
velocity distribution at
a certain time

Doppler curves

flow can be represented by one velocity in each moment



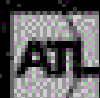
flow can be represented by a velocity distribution in each moment



cf. voiceprint, music/heart beats
in time-frequency representation

velocity distribution in TM-mode

Textbook Fig. VIII.42



PIG 695

University Hospital

LAM

CB-5 PVasc/Ven

27 Jun 00

5:25:42 pm

TIs 0.2

F# 73

MI 0.2

3.0 cm

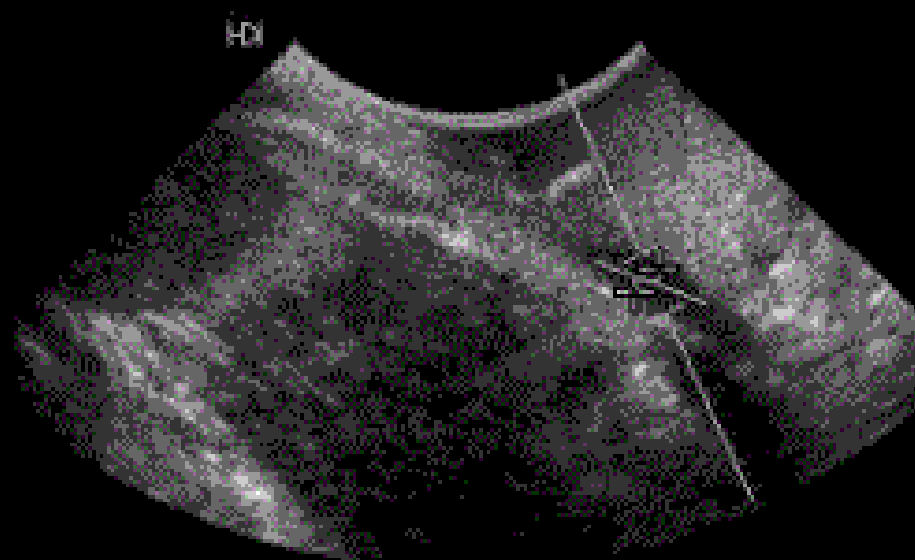
Map 8

DynRg 50dB

Persist Med

Fr Rate Med

2D Opt:Res

SV Angle -46°

Dep 1.5 cm

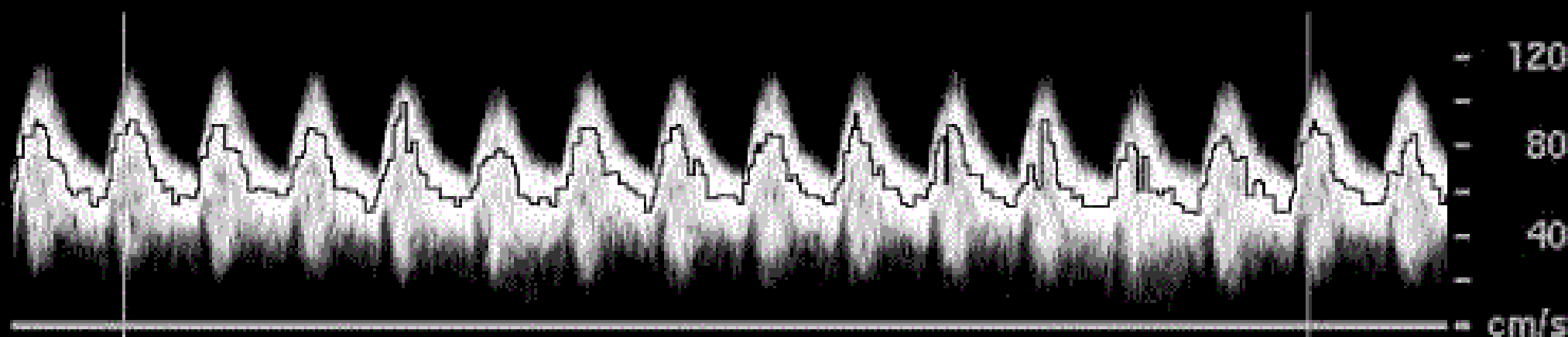
Size 4.0 mm

Freq 5.0 MHz

WF Low

Dop 68% Map 2

PRF 10000Hz



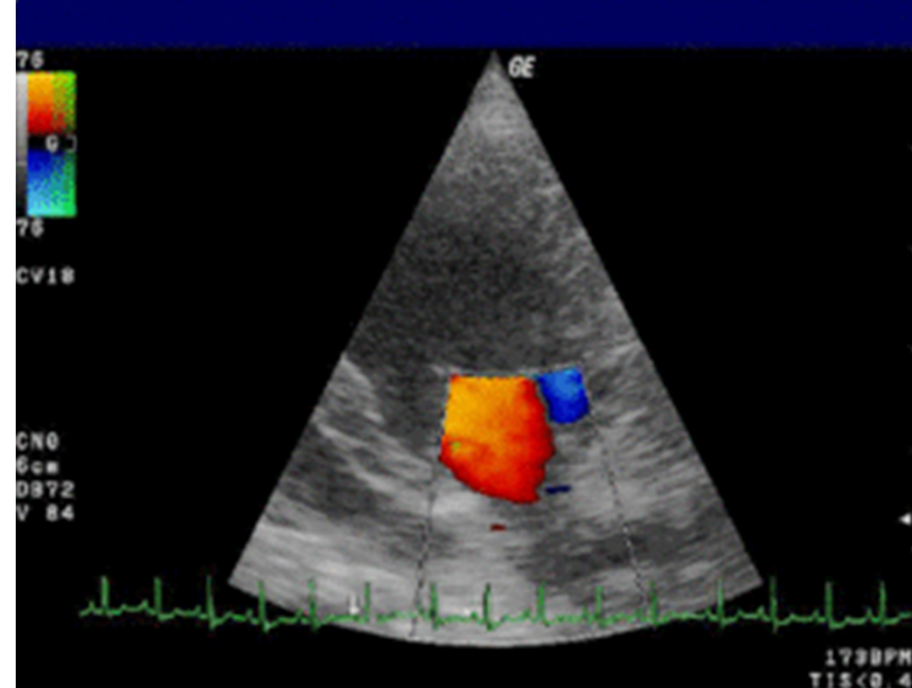
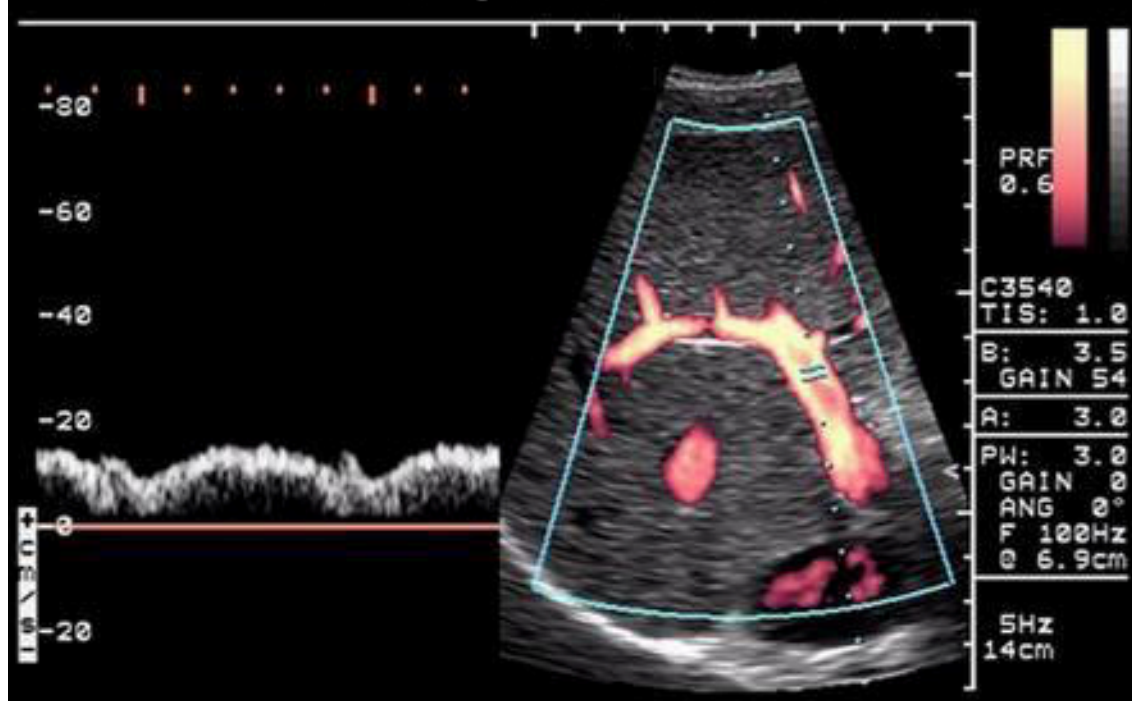
0.21 cm

0.03 cm²

TAM 65.8 cm/s

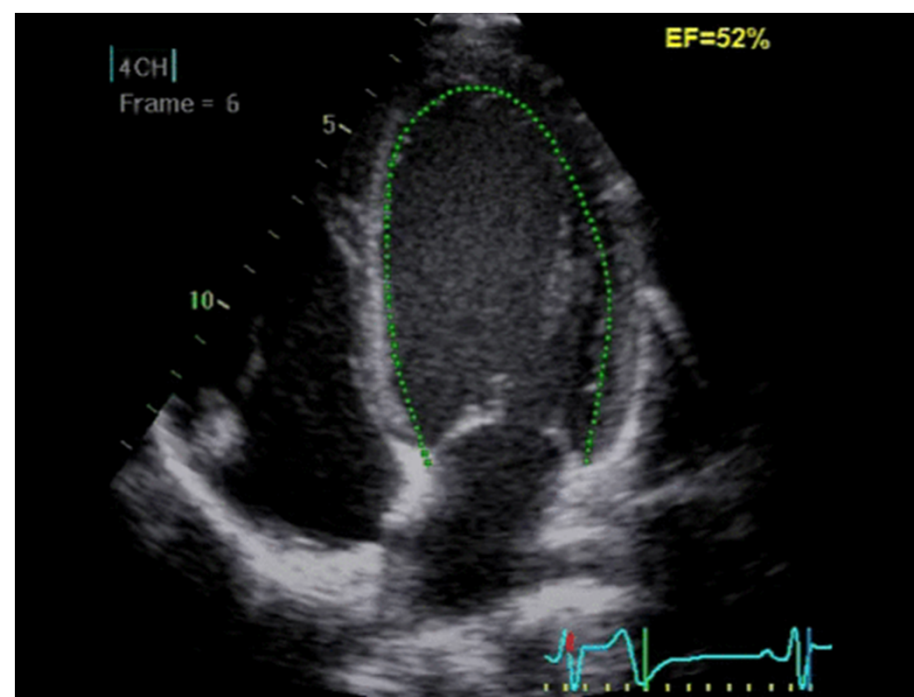
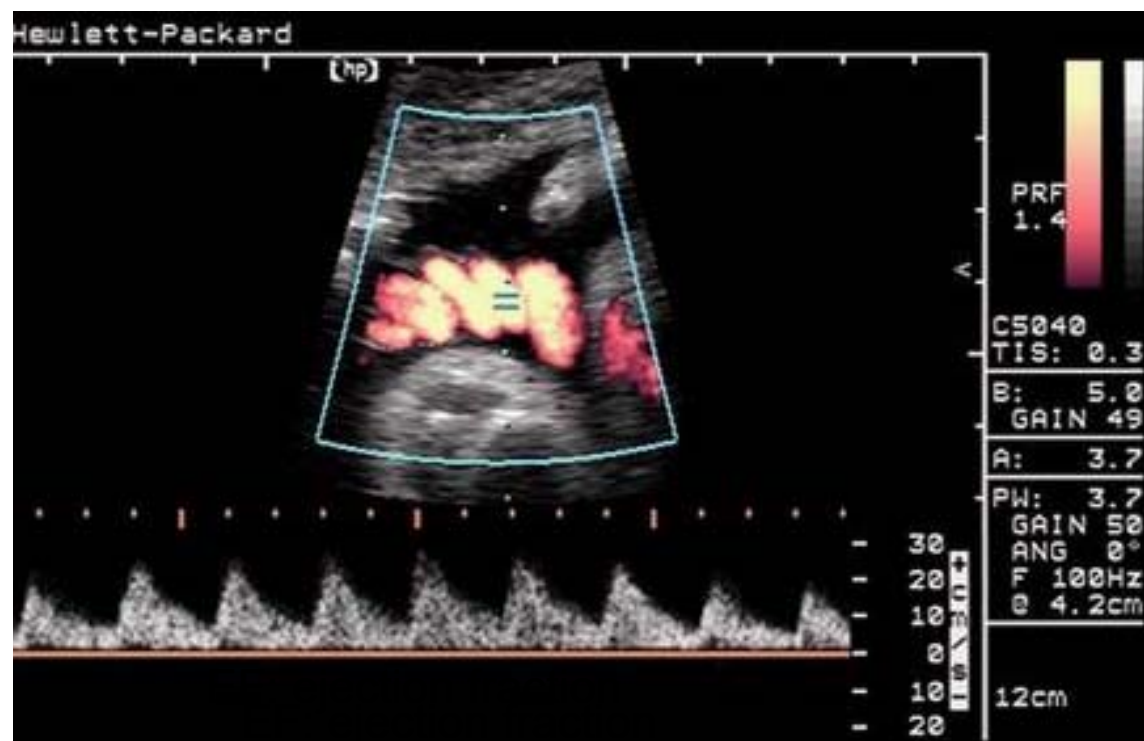
VF 118.4 ml/min

Normal portal vein flow



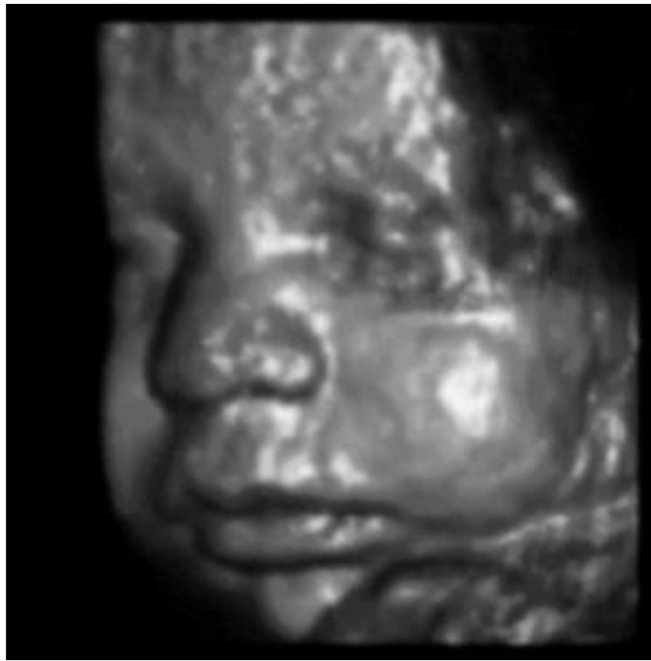
Mitral valve

EF: ejection fraction

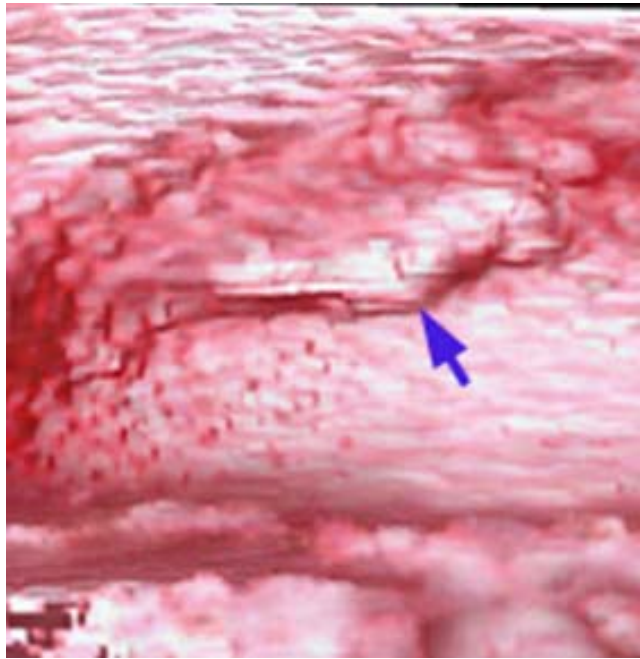


3D reconstruction

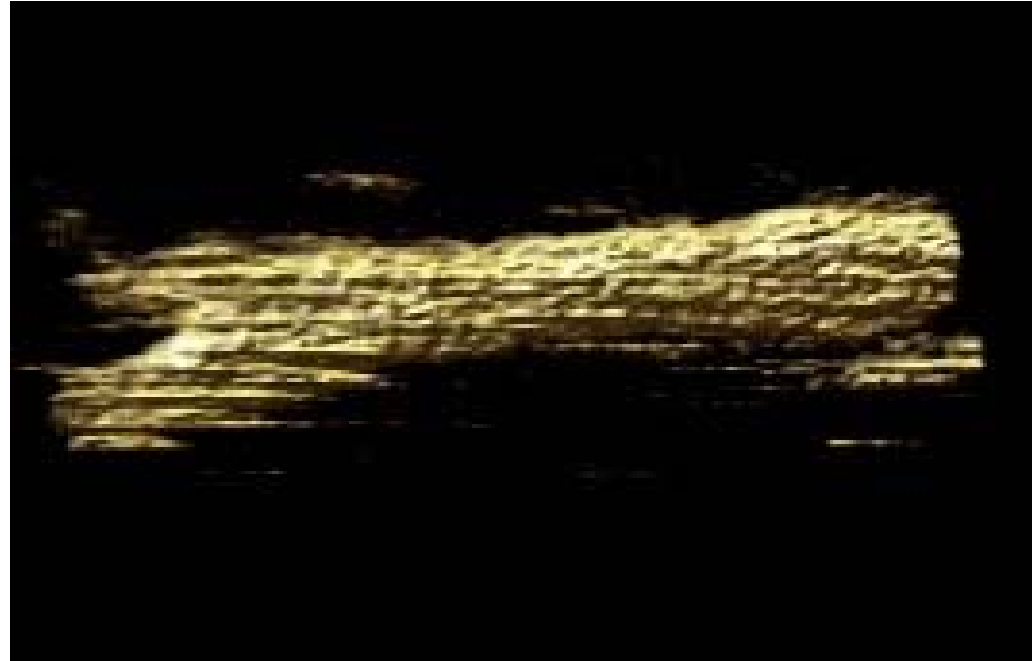
face of a fetus



bladder



carotis



Safety

in the diagnostics:

$$10 \text{ mW/cm}^2 = 100 \text{ W/m}^2$$

cf. pain threshold: 10 W/m^2

in the therapy: 1 W/cm^2

spatial average temporal
average (SATA) intensity;
spatial peak temporal peak
(SPTP) intensity;
spatial peak temporal average
(SPTA) intensity;
spatial peak pulse average
(SPPA) intensity
spatial average pulse average
(SAPA) intensity

