Medical Biophysics II.

4th lecture: Transport processes I.
Diffusion, Brownian motion, Osmosis
5th February 2024.
Dániel Veres

Diffusion?

Why?

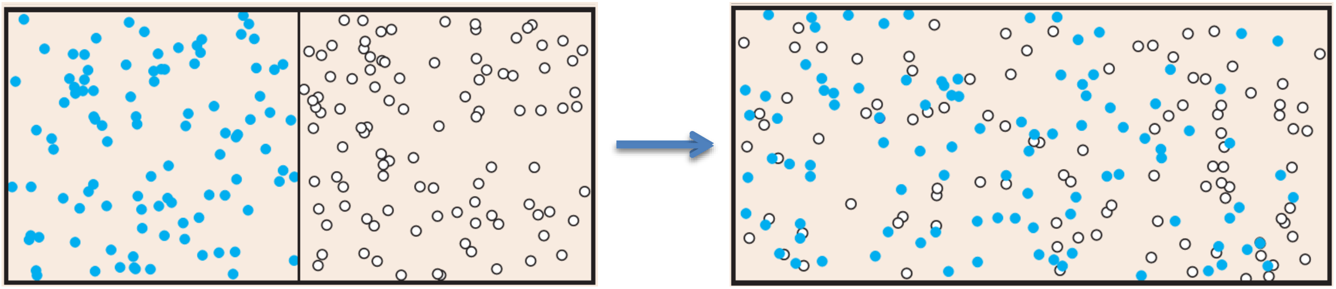
- physiology: cell function – ion diffusion…
- disorders: fibrosis, oedema, vasculitis, ascites…
- diagnostics: DWI MRI…
- therapy: dialysis, physiological saline….
- drug delivery: transdermal (liposomal), inhaled…

…….

Diffusion?

The change in the spatial distribution of particles because of **random thermal motion.**

In microscopic level with **net matter transport**.



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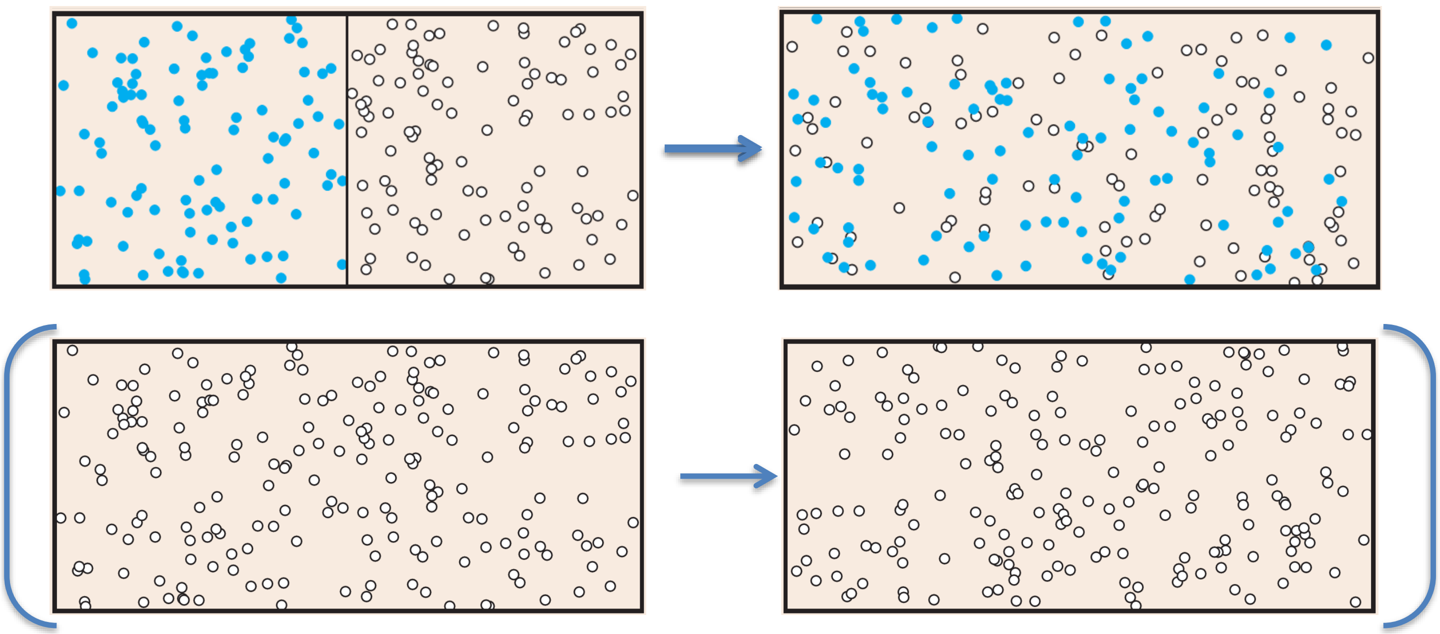
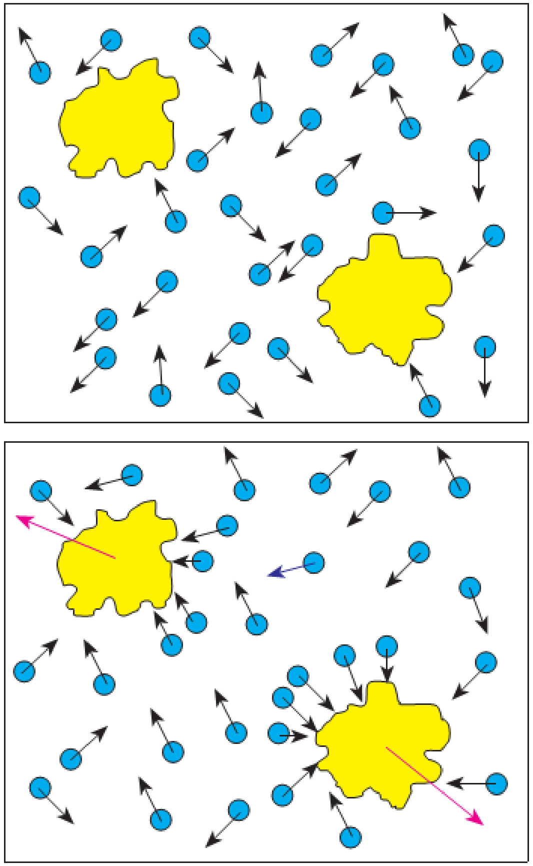


Relevant: NET transport of substance „A” in „B”.

Brownian motion

The „random walk” of a particles resulting from their collision with other particles.



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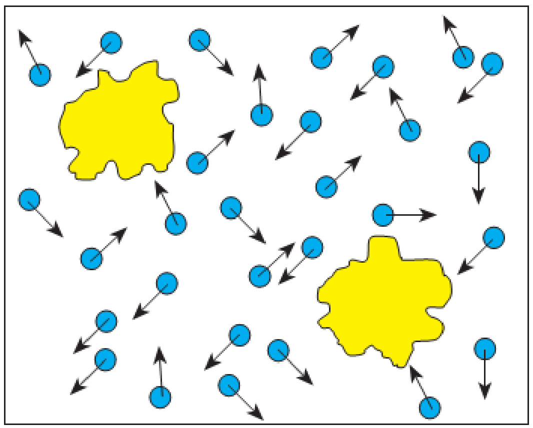
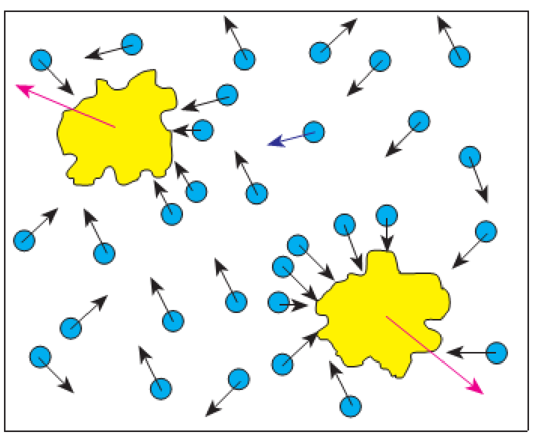
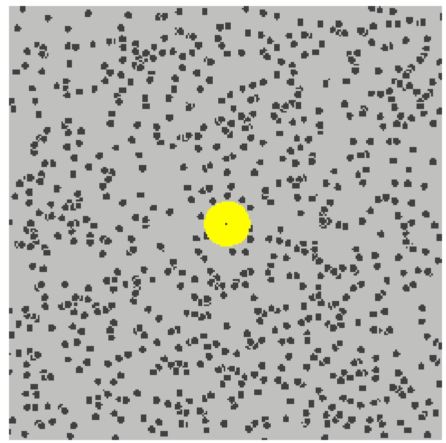
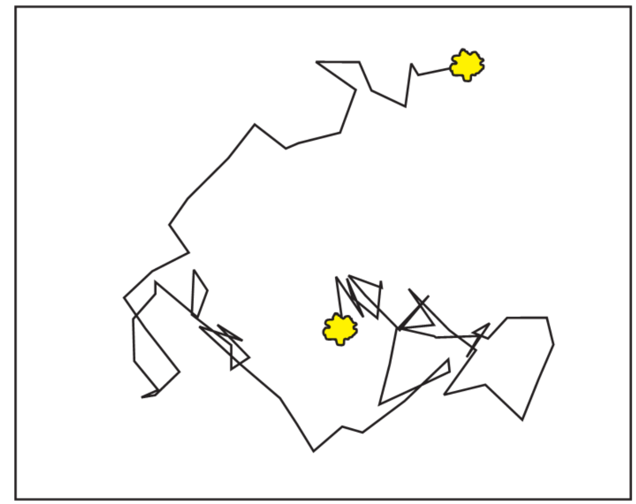


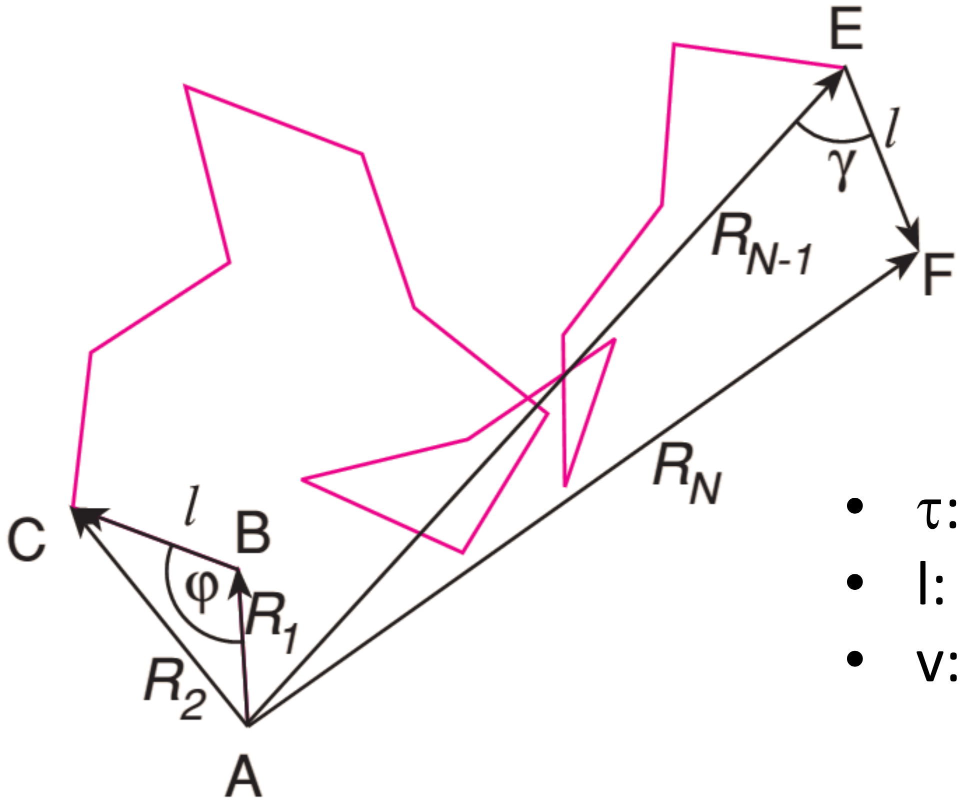






- τ: mean time between collisions
- l: mean free path
- v: mean speed of particles

How far reaches a $\overline{\text{particle}}$?



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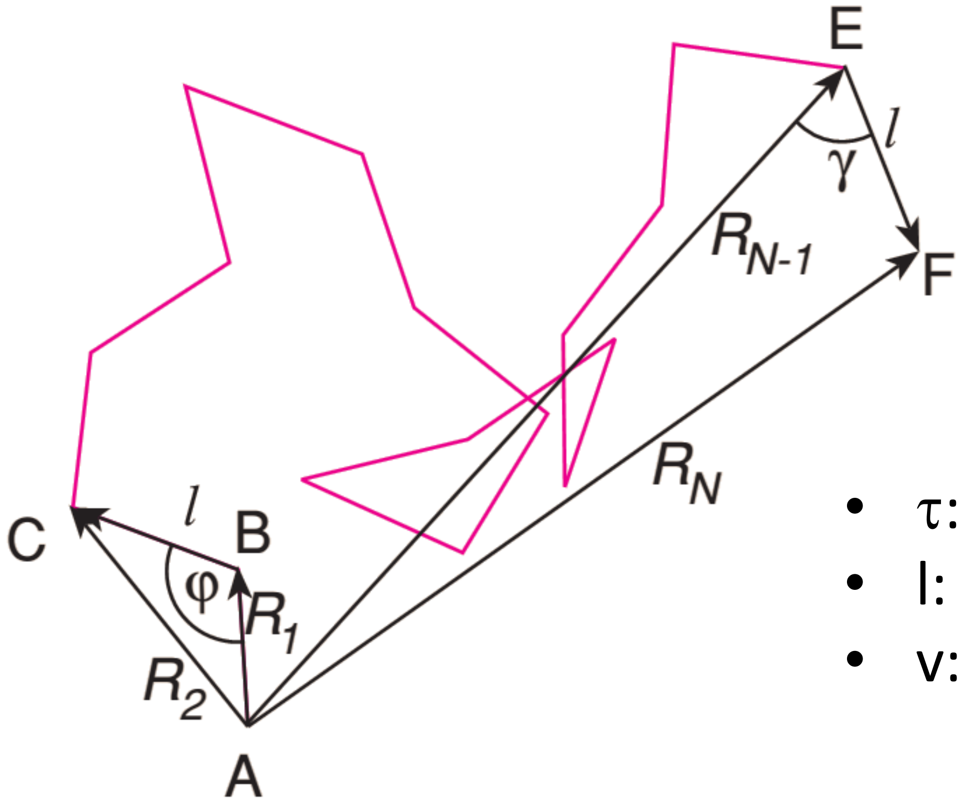

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One particle: ${R_2}^2 = {R_1}^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi$

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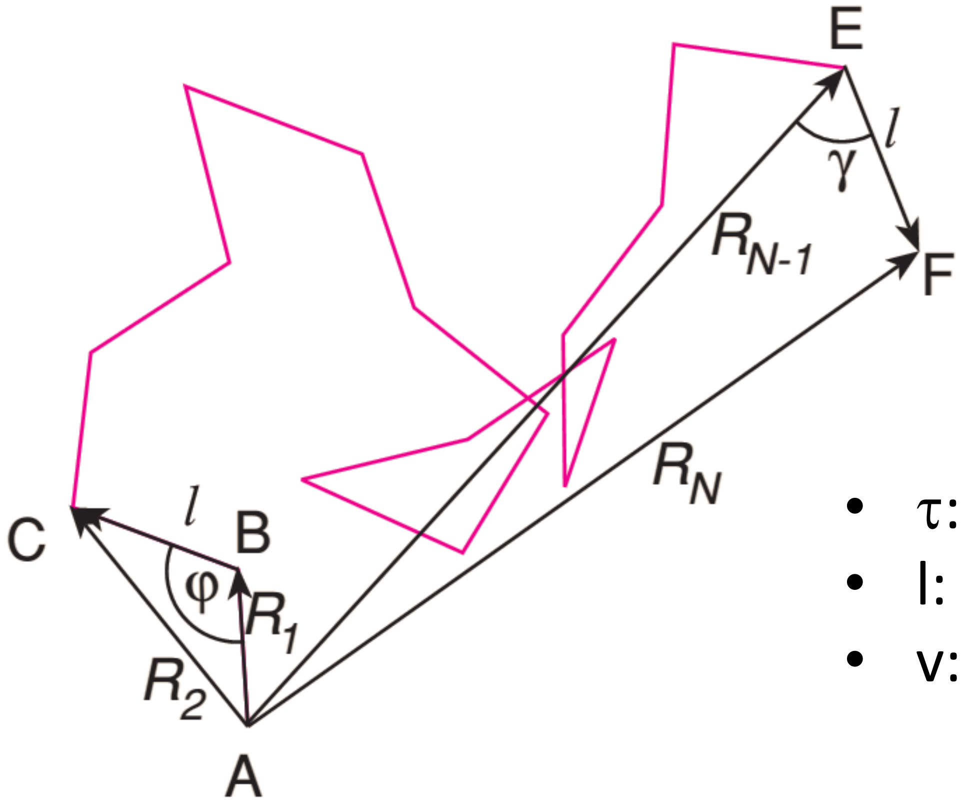

- τ: mean time between collisions
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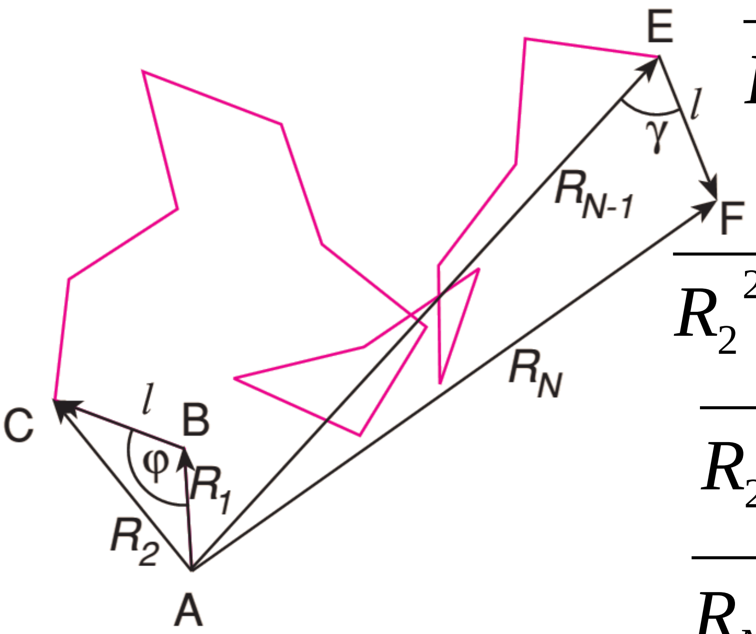
One particle: $R_2^{\,2} = R_1^{\,2} + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi$

A „mean” particle:
(mean of *n* particles): $$\overline{R_2^{\,2}} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(R_1^{\,2} + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi_i \right)$$

How far reaches a particle?



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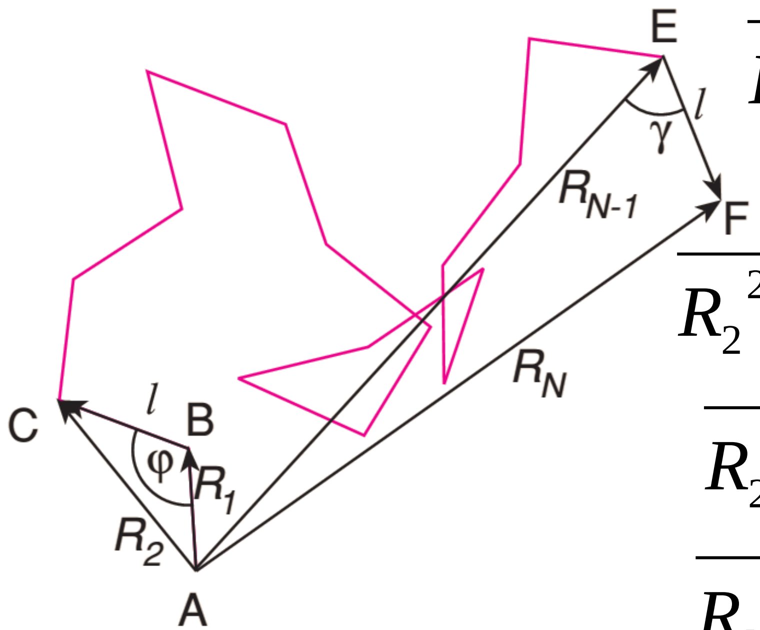
$$\overline{R_2^{\,2}} = \frac{1}{n} \cdot (n \cdot \left(R_1^{\,2} + l^2 \right) - 2 \cdot R_1 \cdot l \cdot \sum_{i=1}^{n} (\cos\varphi_i))$$

$$\overline{R_2^{\,2}} = R_1^{\,2} + l^2 = l^2 + l^2 = 2 \cdot l^2$$

$$\overline{R_N^{\,2}} = N \cdot l^2$$

How far reaches a particle?



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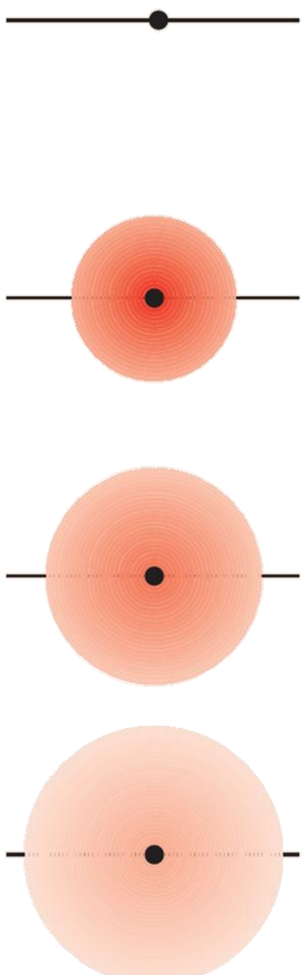
$$\overline{R_2^{\,2}} = R_1^{\,2} + l^2 = l^2 + l^2 = 2\cdot l^2$$

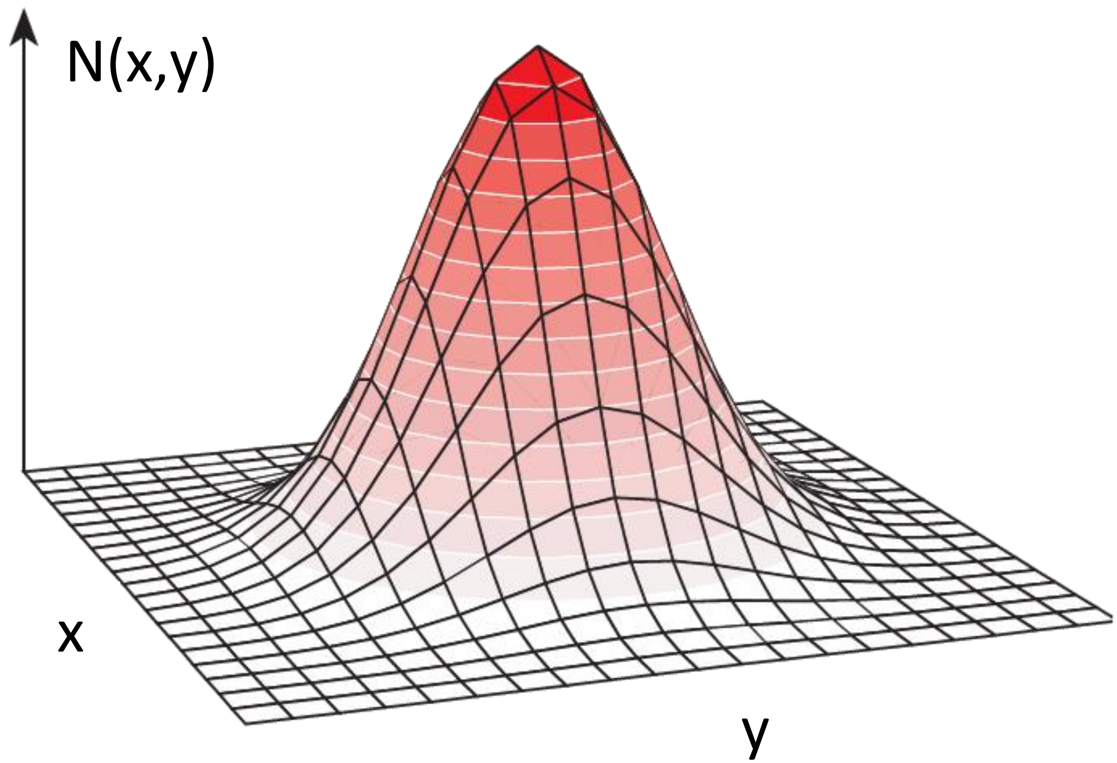
$$\overline{R_N^{\,2}} = N\cdot l^2$$

$$\overline{R_t} = \sqrt{N\cdot l^2} = \sqrt{\frac{t}{\tau}\cdot l\cdot l} = \sqrt{t\cdot v\cdot l} = \sqrt{3\cdot D\cdot t}$$

$$\frac{v\cdot l}{3} = D$$

Experiment – 2D distribution





$$\sigma \sim \sqrt{D \cdot t}$$

Matter transport - flow

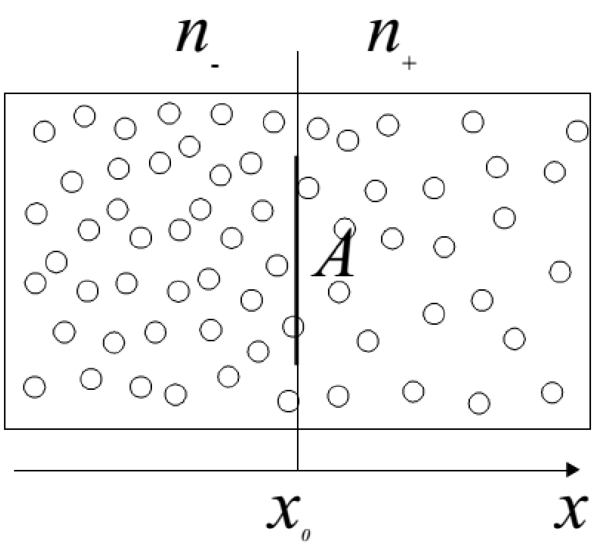
Particle flow rate: $$I_N = \frac{\Delta N}{\Delta t}; \left[\frac{1}{s} \right]$$

Particle flow density (flux): $$J_N = \frac{\Delta I_N}{\Delta A}; \left[\frac{1}{m^2 \cdot s} \right]$$

Matter flow rate: $$I_\nu = \frac{\Delta \nu}{\Delta t}; \left[\frac{mol}{s} \right]$$

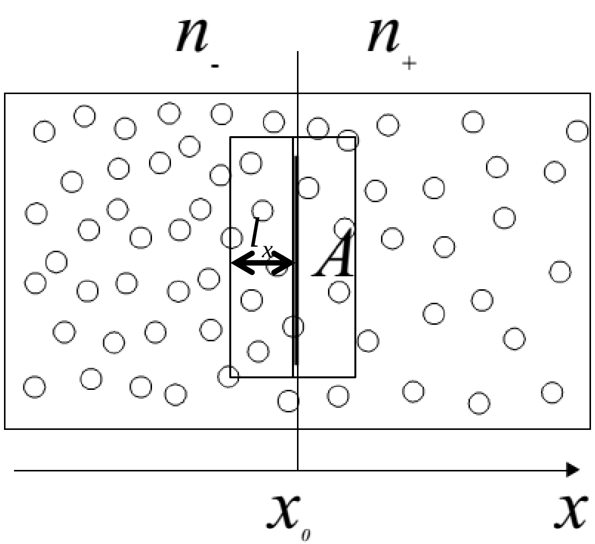
Matter flow density (flux): $$J_\nu = \frac{\Delta I_\nu}{\Delta A}; \left[\frac{mol}{m^2 \cdot s} \right]$$

Fick’s first law



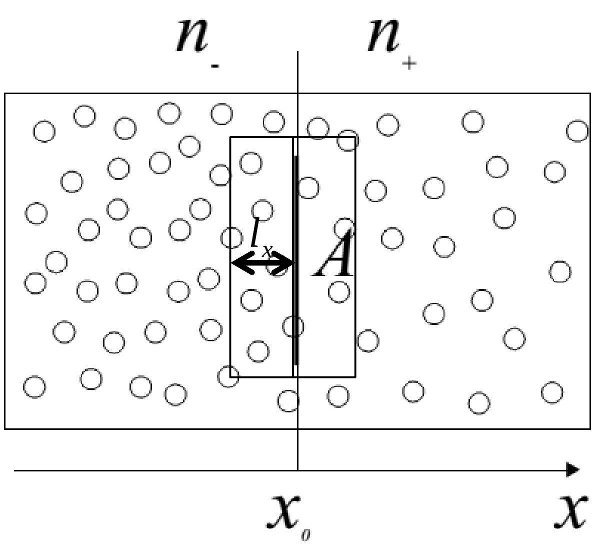
$$\Delta N = N_- - N_+$$

Fick's first law



$$\Delta N = N_{-} - N_{+} = \frac{1}{2} \cdot V_t \cdot n_{-} - \frac{1}{2} \cdot V_t \cdot n_{+} = \frac{1}{2} \cdot V_t \cdot (n_{-} - n_{+})$$

Fick's first law



$$\Delta N = N_- - N_+ = \frac{1}{2}\cdot V_t \cdot (n_- - n_+) = \frac{1}{2}\cdot \overbrace{v_x \cdot \Delta t}^{l_x} \cdot A \cdot (n_- - n_+)$$

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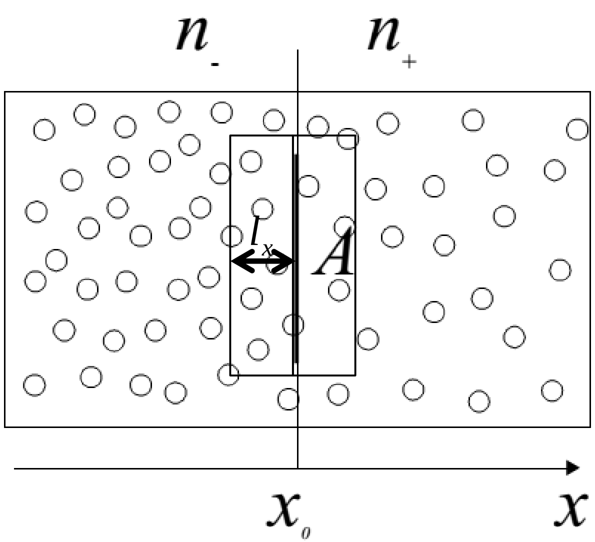

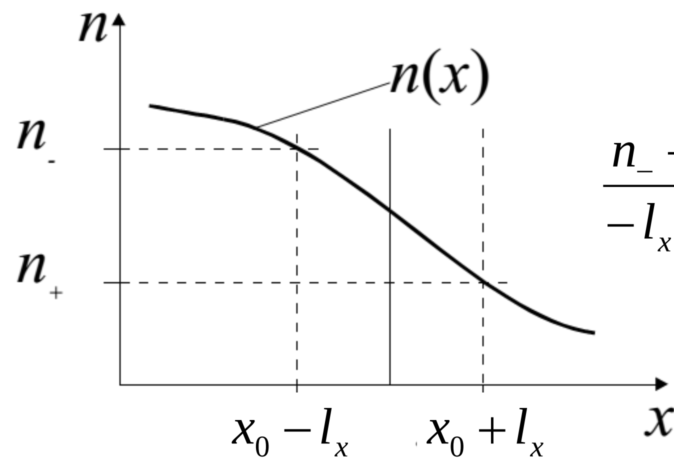

$$\frac{n_- - n_+}{-l_x - l_x} = \frac{\Delta n}{\Delta x}$$

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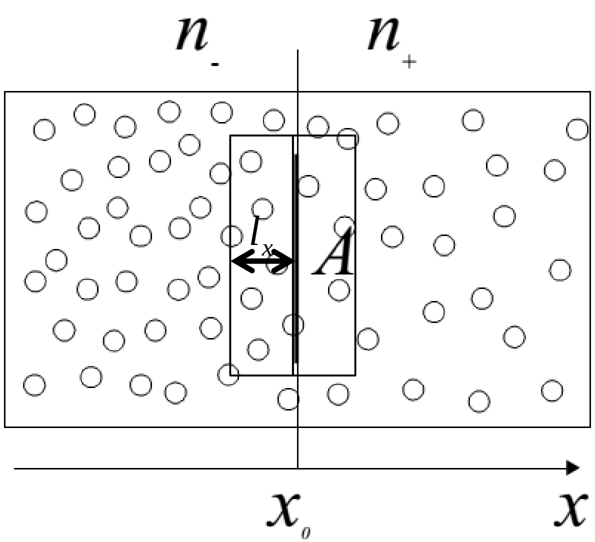

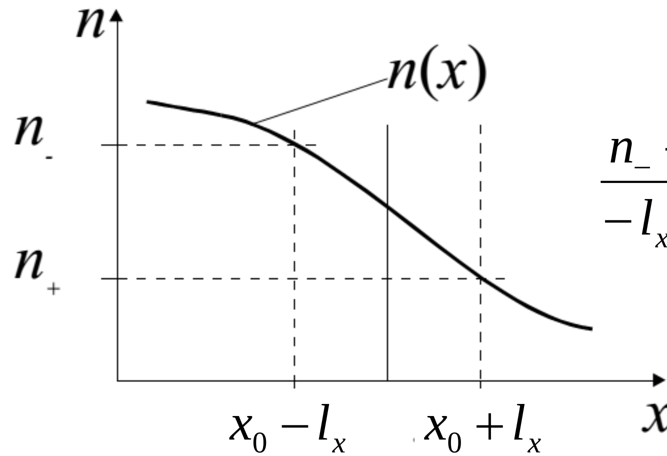

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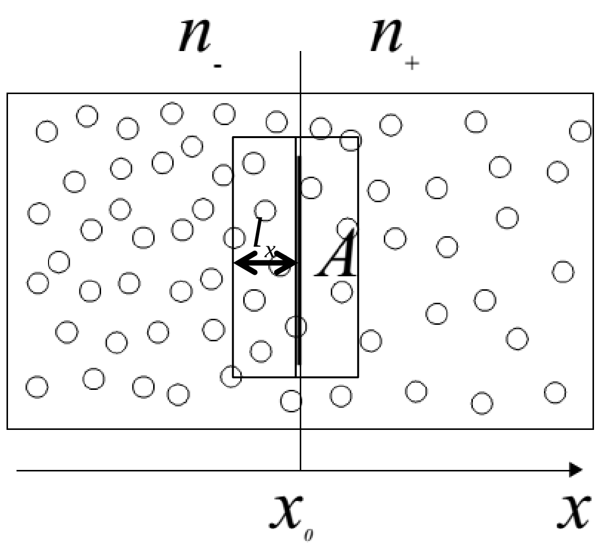

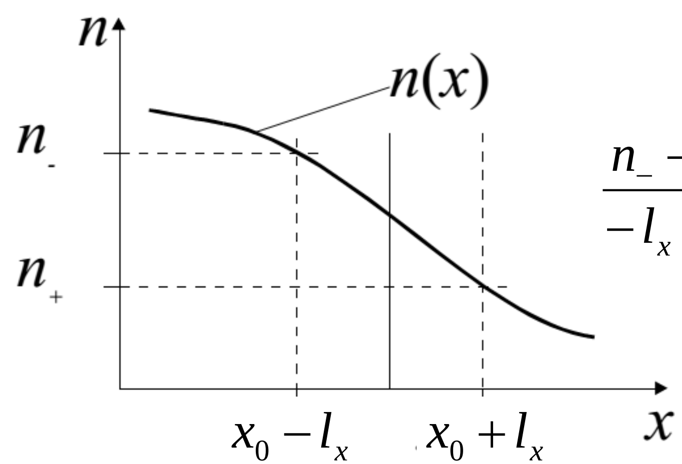

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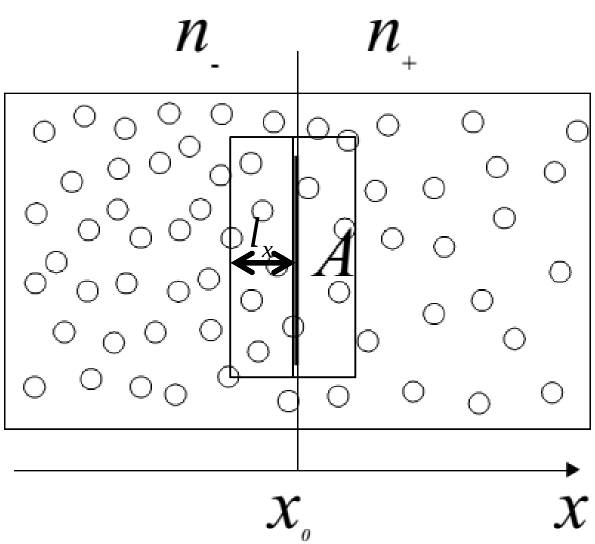
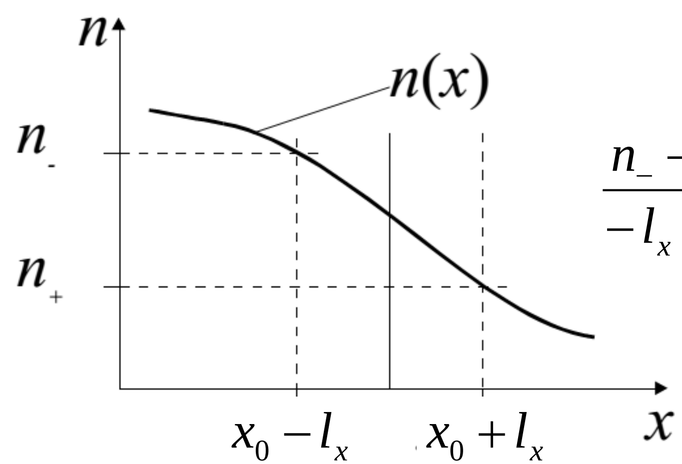
$$v_x \cdot l = D$$

$$J_{Nx} = \frac{1}{2}\cdot v_x \cdot 2 \cdot l_x \cdot - \frac{\Delta n}{\Delta x} = -D \cdot \frac{\Delta n}{\Delta x}$$

Fick's first law





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$$J_v = -D \cdot \frac{\Delta c}{\Delta x}$$

BUT!: Δc is NOT the „driving force"!

Diffusion coefficient

D gives the amount of matter diffused across a unit area in a unit time in a case of unit concentration gradient.

$$D = \frac{v \cdot l}{3}; \left[\frac{m^2}{s} \right]$$

$$D = u \cdot k \cdot T$$

Einstein-Stokes
(spheres)

$$D = \frac{k \cdot T}{6 \cdot \pi \cdot \eta \cdot r}$$

BUT!
Not directly proportional with T!

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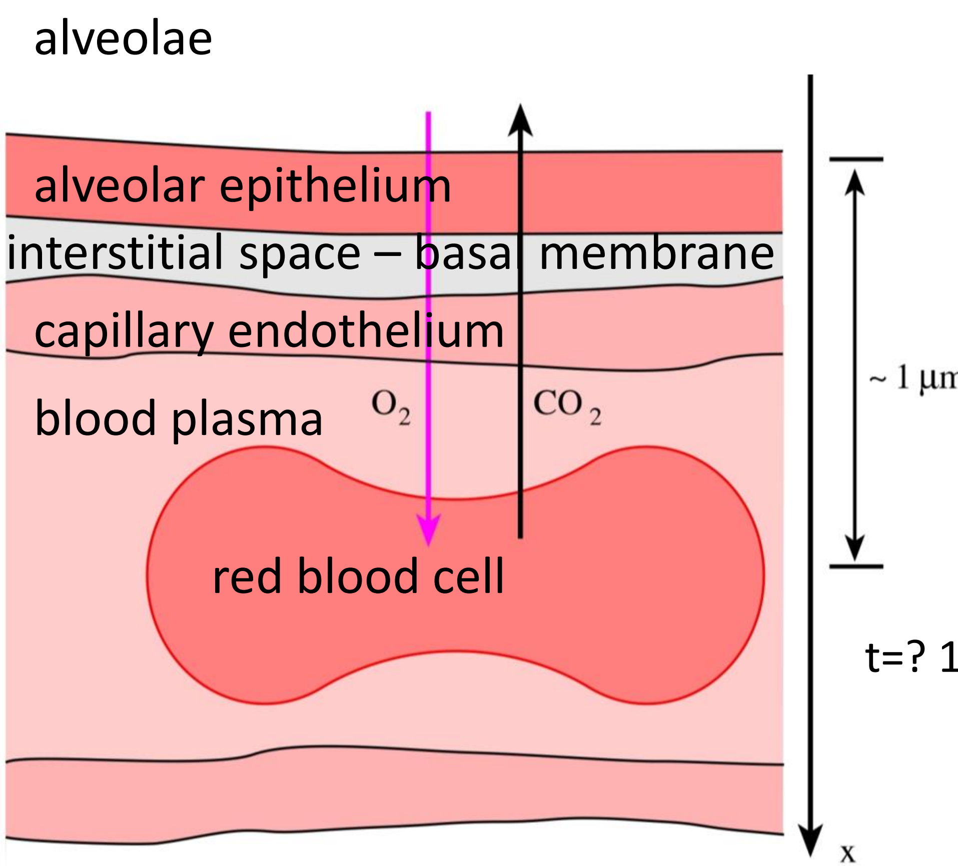


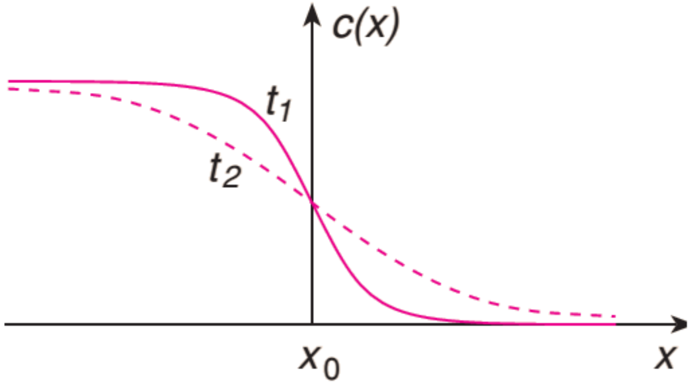
$1{,}9 \cdot 10^{-9} \frac{m^2}{s}$

$t \approx 500 \mu s$

t=? 1m distance

Fick's second law

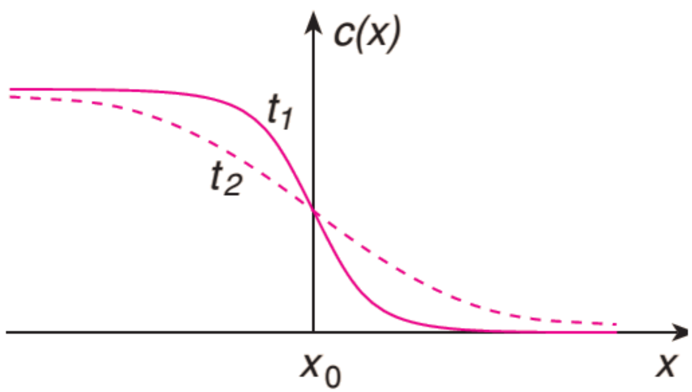
Fick II: change of the concentration gradient in time



$$c(t+\Delta t) = c(t) + D \cdot \Delta t \cdot \frac{\Delta\left(\frac{\Delta c}{\Delta x}\right)}{\Delta x}$$

Fick's second law

Fick II: change of the concentration gradient in time



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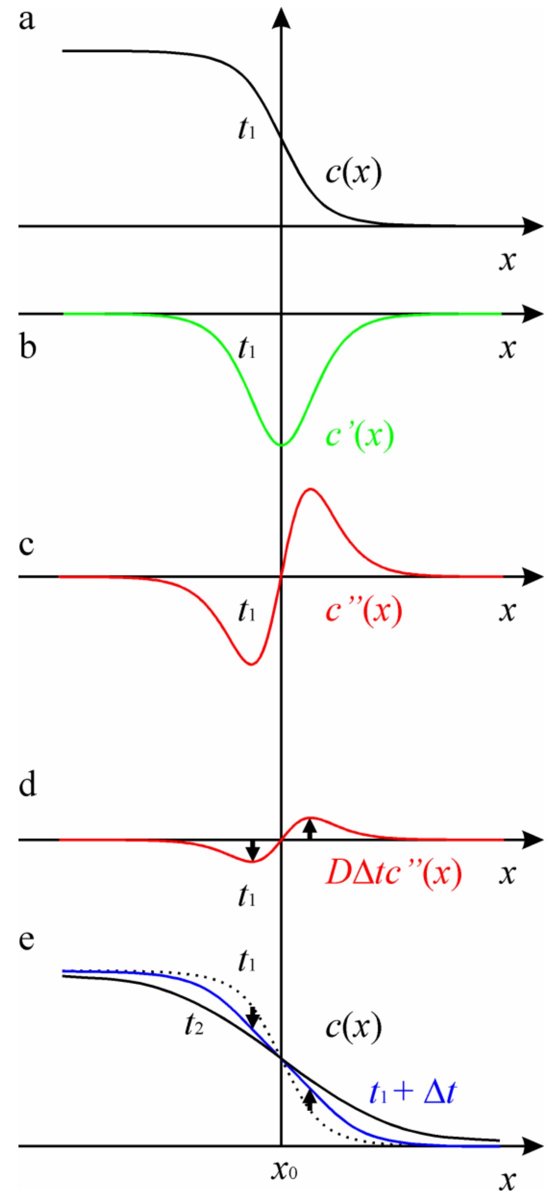
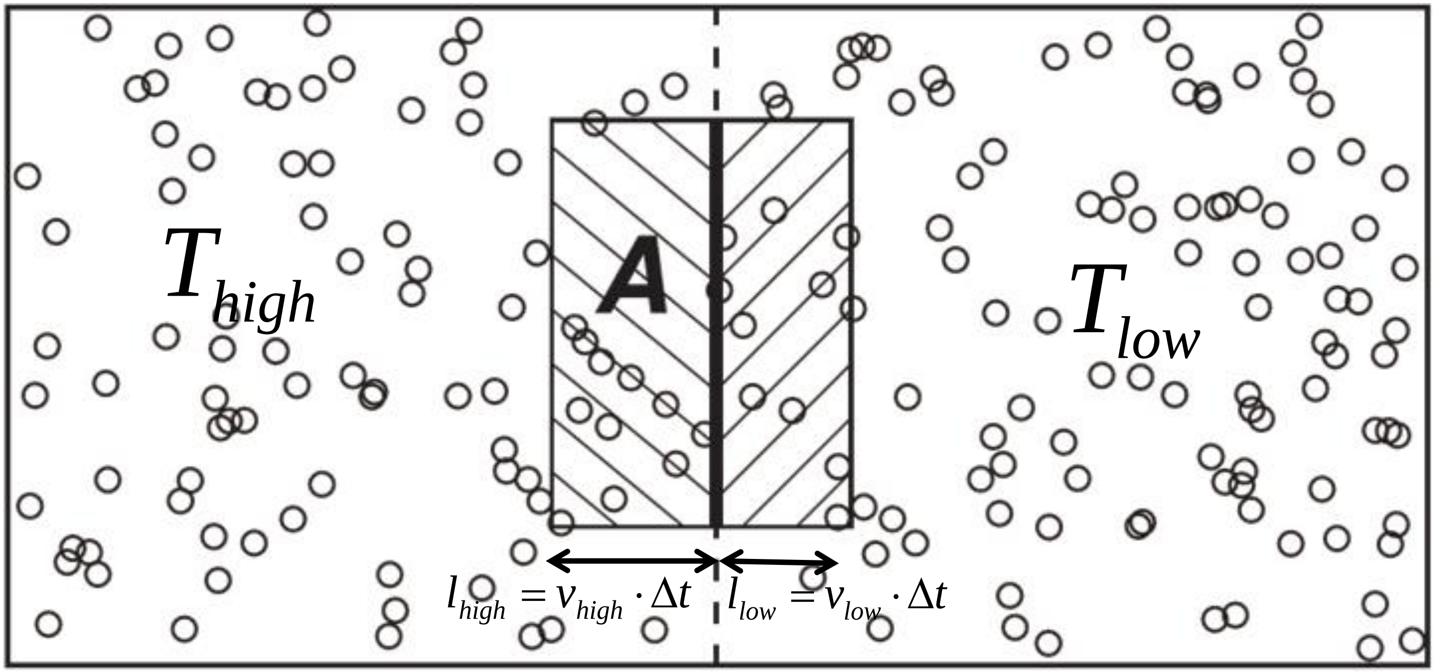


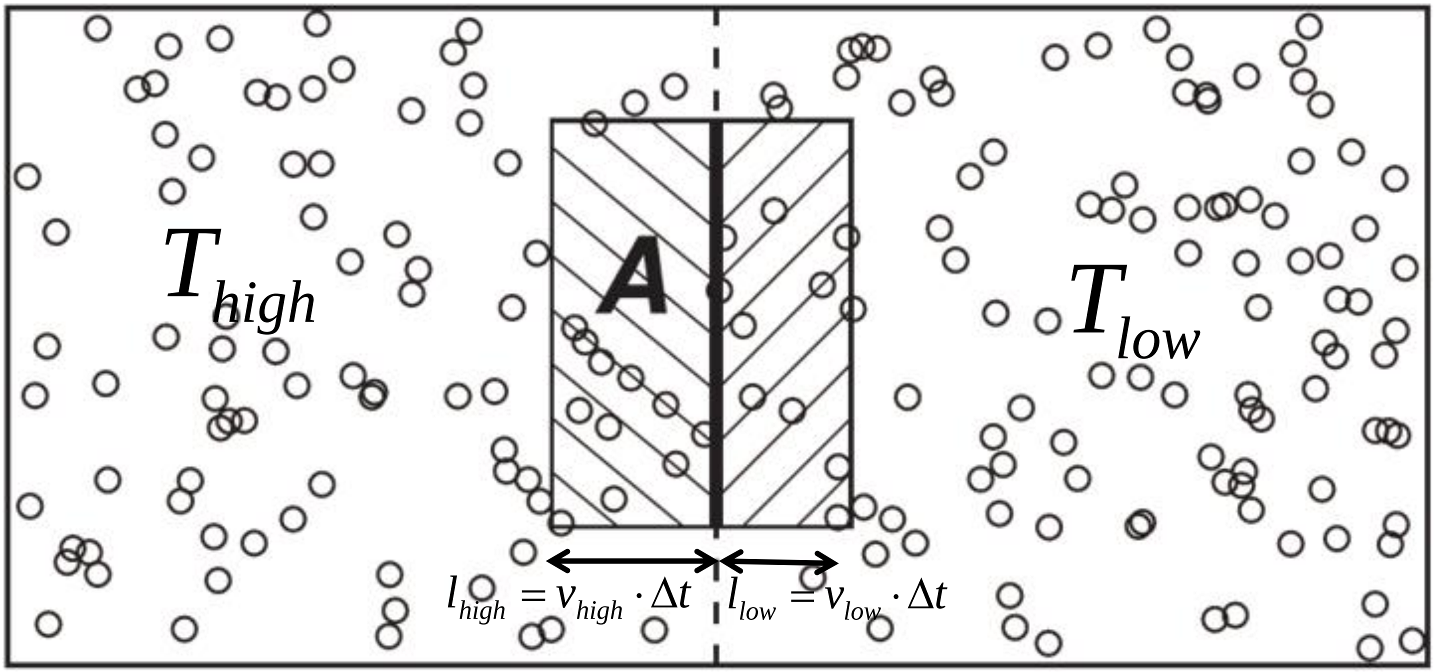
Thermodiffusion



$T_{high} > T_{low}$

$$\Delta N = N_{high} - N_{low} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot v_{high} - \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot v_{low}$$

Thermodiffusion



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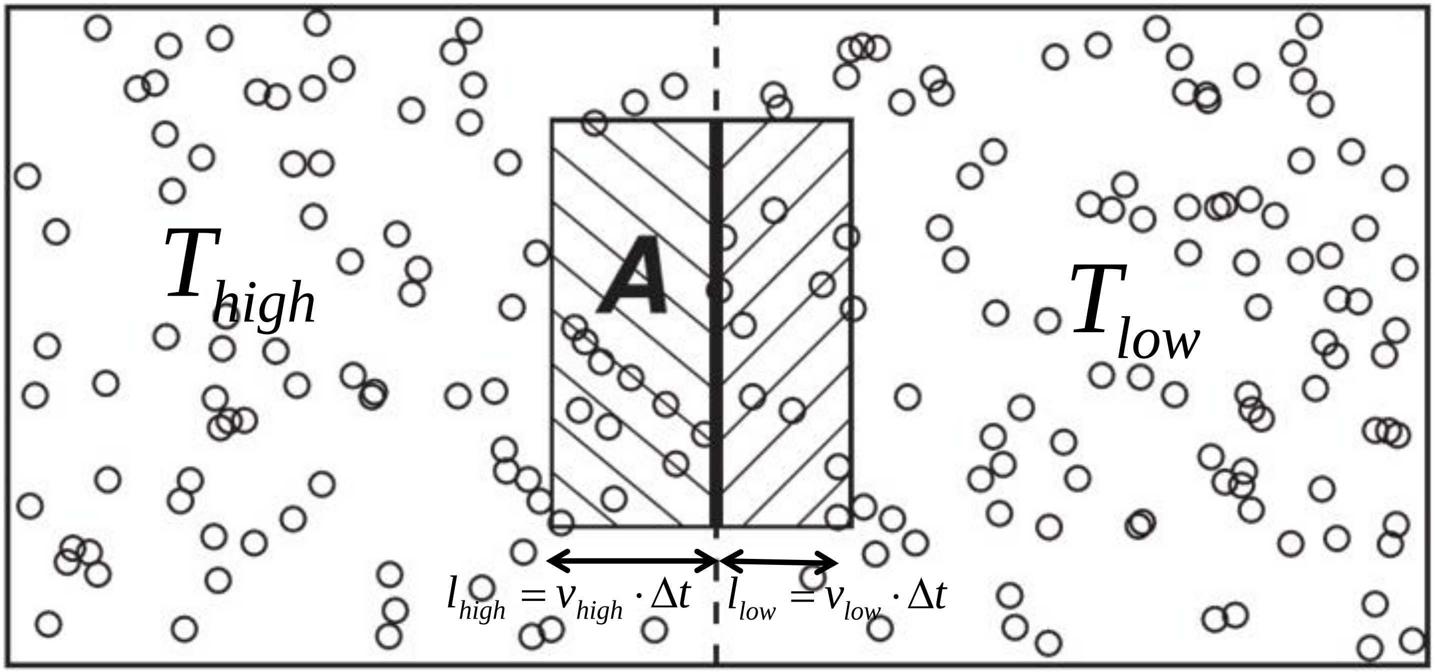
Thermodiffusion

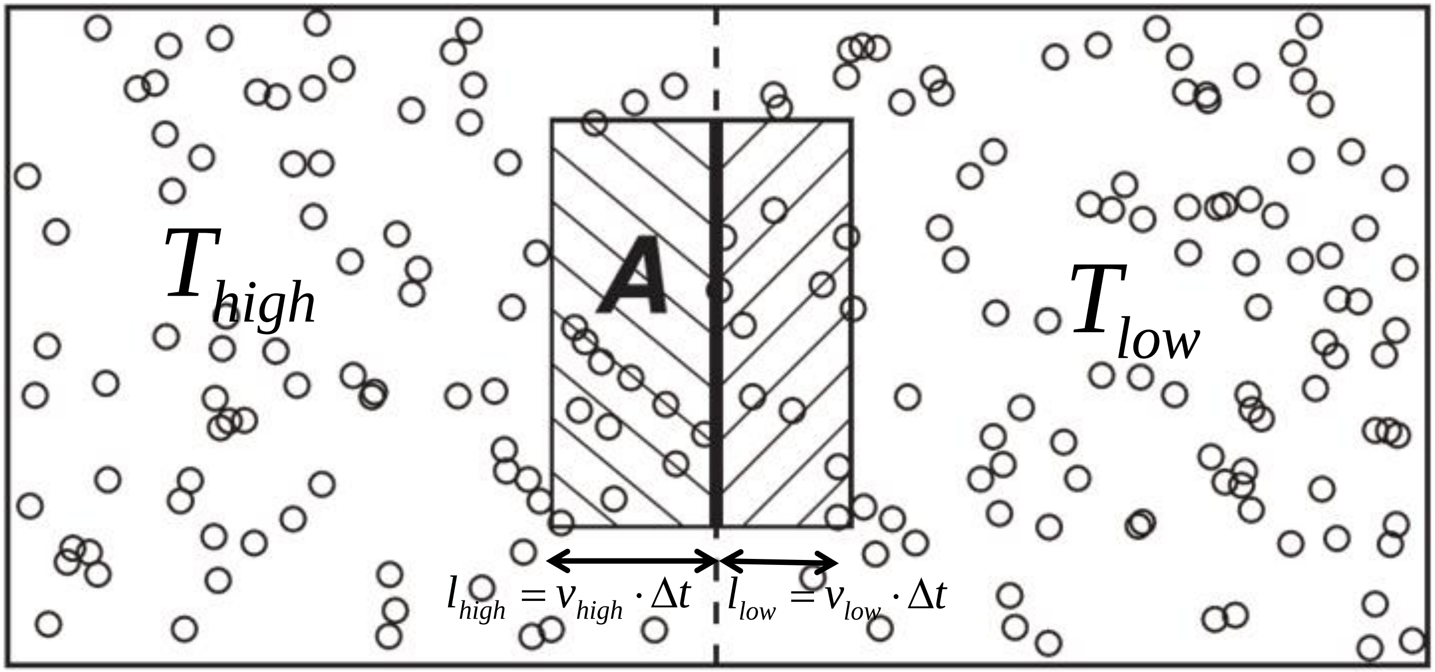


$$T_{high} > T_{low}$$

$$v = \sqrt{3 \cdot \frac{k \cdot T}{m}}$$

$$\frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{(v_{high}^2 - v_{low}^2)}{(v_{high} + v_{low})} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{\left(\frac{3 \cdot k \cdot T_{high}}{m} - \frac{3 \cdot k \cdot T_{low}}{m}\right)}{2 \cdot v_{mean}}$$

Thermodiffusion



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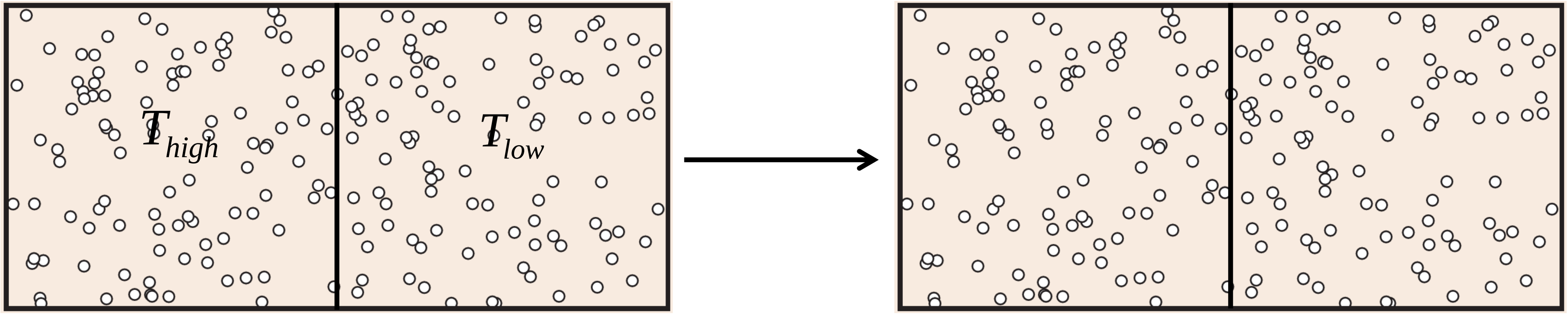
$$\frac{1}{2} \cdot \left(\frac{n \cdot \Delta t \cdot A \cdot 3 \cdot k}{m \cdot 2 \cdot v_{mean}} \right) \cdot (T_{high} - T_{low})$$

$$J_{\nu} = -L_T \cdot \frac{\Delta T}{\Delta x}$$ (Ludwig-Soret effect)

Heat conduction



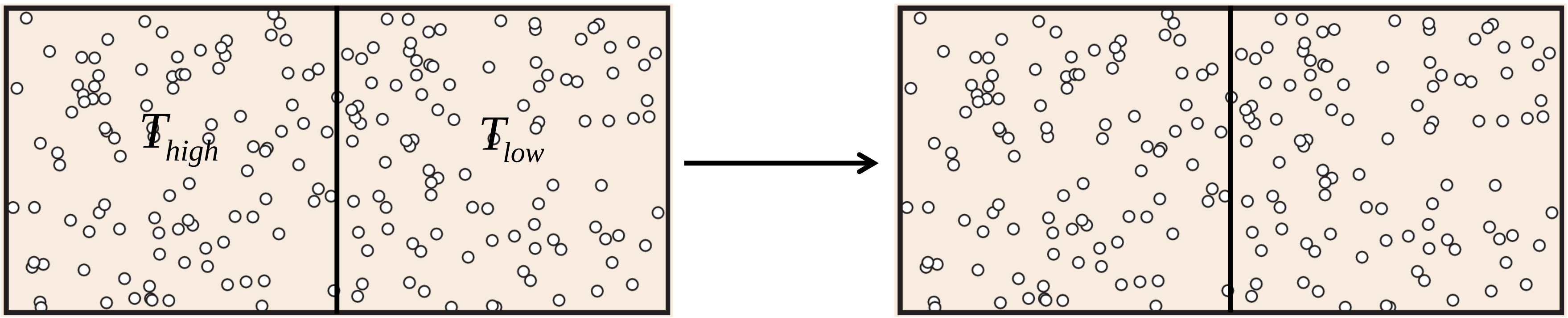
$$T_{high} > T_{low}$$

$$\Delta N = N_{high} - N_{low} = 0$$

$$N_{high} = N_{low}$$

$$\bar{\varepsilon} = \frac{3}{2} \cdot k \cdot T$$

Heat conduction



$$T_{high} > T_{low}$$

$$\Delta N = N_{high} - N_{low} = 0$$
$$N_{high} = N_{low}$$

$$\bar{\varepsilon} = \frac{3}{2} \cdot k \cdot T$$

Energy flow density

$$J_E = \frac{\Delta E}{A \cdot \Delta t} = \frac{N_{high} \cdot \frac{3}{2} \cdot k \cdot \left(T_{high} - T_{low}\right)}{A \cdot \Delta t} = -\lambda \cdot \frac{\Delta T}{\Delta x}$$

(Fourier law for heat conduction)

Generalization

Onsager-relation: $J_{ext.} = L_{cond} * X_{int\_grad}$

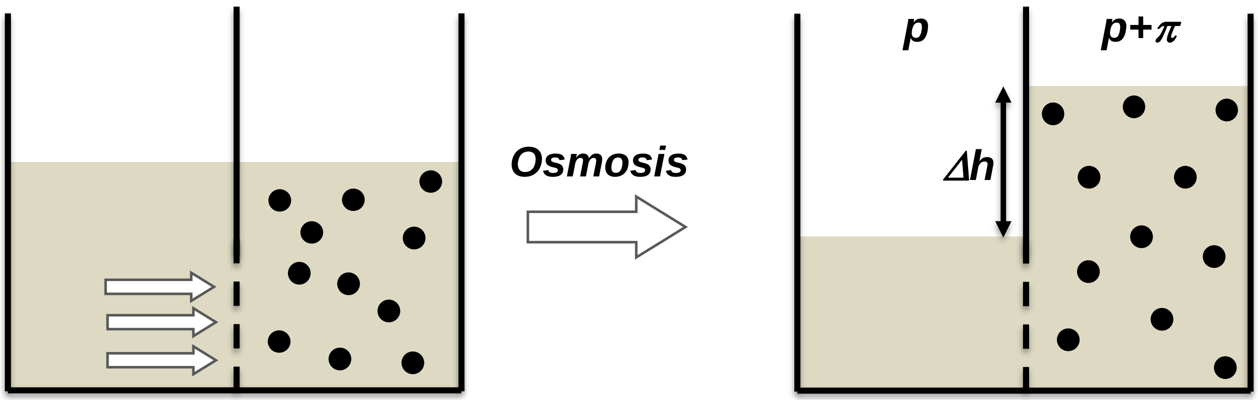
$J_{ext.}$: $flow\ density\ of\ extensive\ quantity$ (eg. J_{matter})

X_{int_grad}: $gradient\ of\ intensive\ quantity\ (eg. \frac{\Delta c}{\Delta x})$

L_{cond}: $conductivity\ coefficient\ (eg. D)$

Osmosis

One-way diffusion of the SOLVENT. (permeable membrane only for *water*)



difference in the concentration of the SOLVENT

Hydrostatic pressure difference (osmotic pressure)

$$p_{osm} = \pi = c_{solute} \cdot R \cdot T \quad \text{(Van 't Hoff law)}$$

Osmotic concentration (equivalent osmotic pressure, „ozmolarity”, „ozmolality”): The concentration of a solution that keeps balance with a heterogeneous solution. Derived units: *mOsm(/L)*, mmol/L, mmol/kg

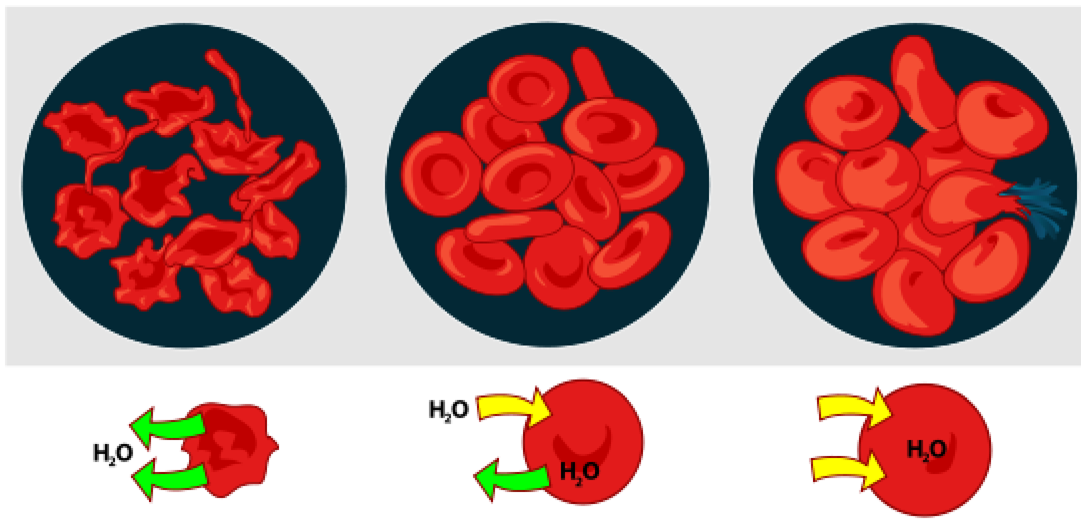
More precisely: correction needed for dissociation

Medical practice

Tonicity: „effective” osmolarity

membrane: *given cell membrane* (permeable not only for water)

non-permeable ions/molecules are important for tonicity



Hypertonic **Isotonic** **hypotonic**

„Isosmotic”, „Physiological”, „isotonic”, „normal” solutions:

Physiological/Normal/Isotonic saline: 0,9% (w/v) NaCl (isotonic)

d5W: 5% (w/v) glucose (hypotonic)

Ringer, Ringer’s lactate (isotonic)

Isosmotic not equal Isotonic!

Osmotic concentration of the blood plasma: about 300mOsm/L

OMHV