

# Physical basis of medical ultrasound

Topics :

Sound as a mechanical wave

Frequency ranges - ultrasound

Generation of ultrasound

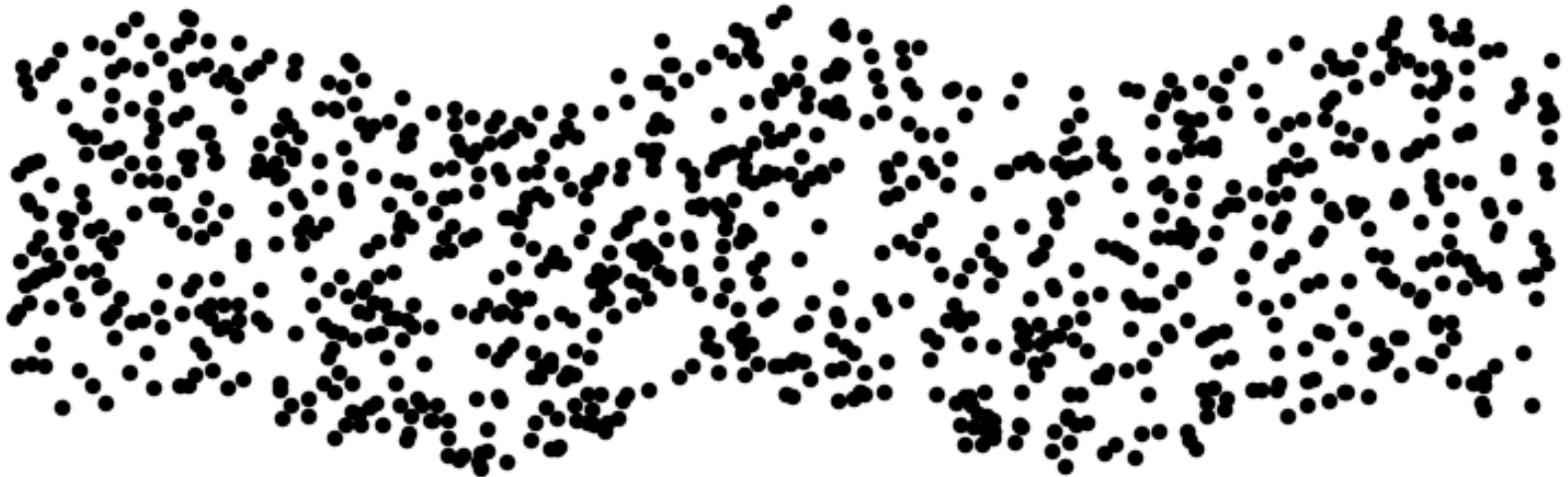
Ultrasound transducers – technical questions

Imaging by ultrasound

Doppler method

Medical imaging

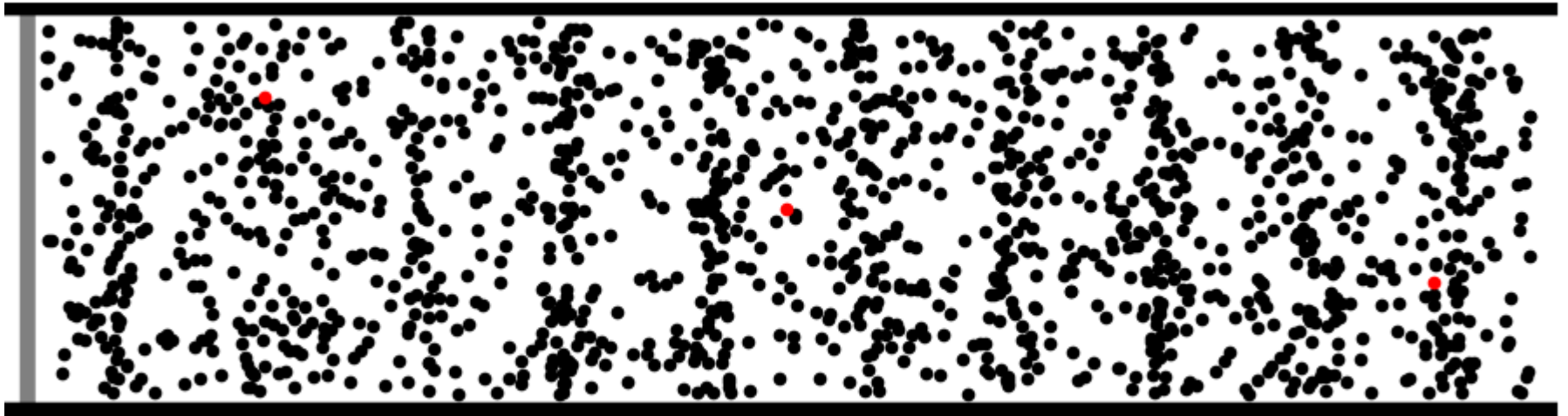
Transversal wave – such as light, or sound in some cases in solids



Transversal: wave propagation is perpendicular to the “motion”



Longitudinal waves:  
propagation direction is parallel to the “motion”

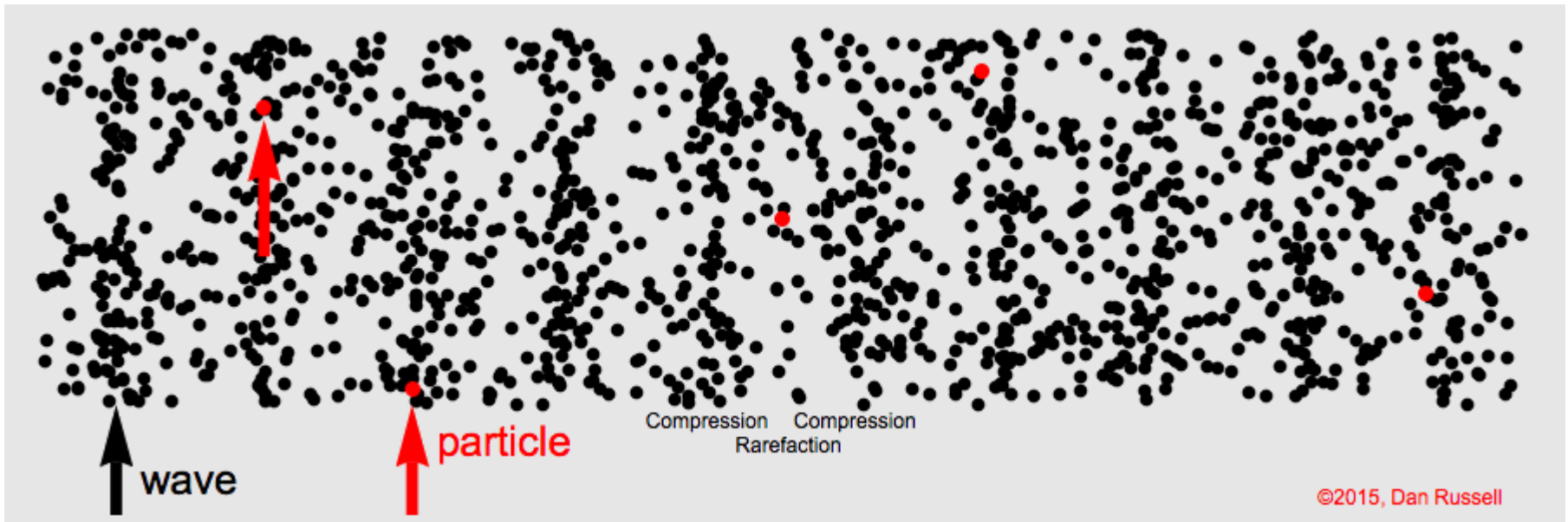


©2011. Dan Russell

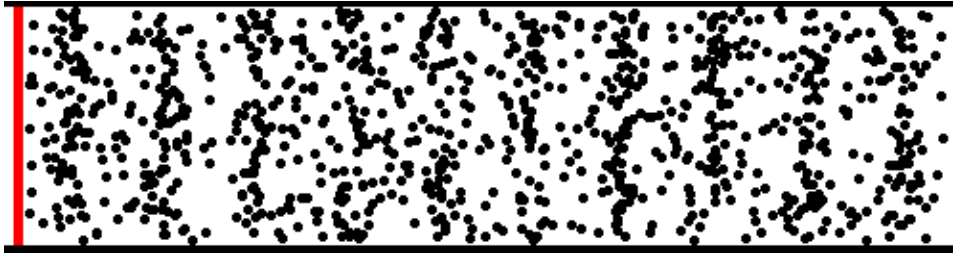
Moving surface (wave “source”)

Compression: pressure increase, density increase

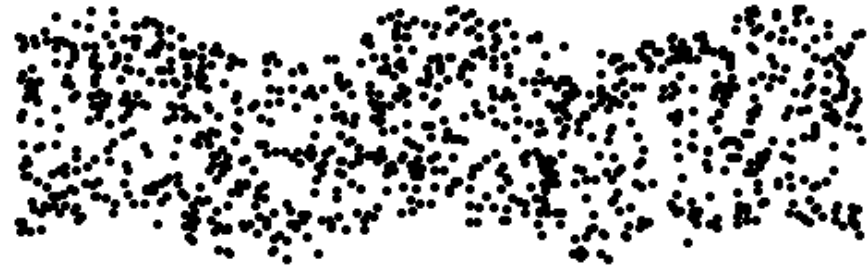
Rarefaction: pressure drop, density drop



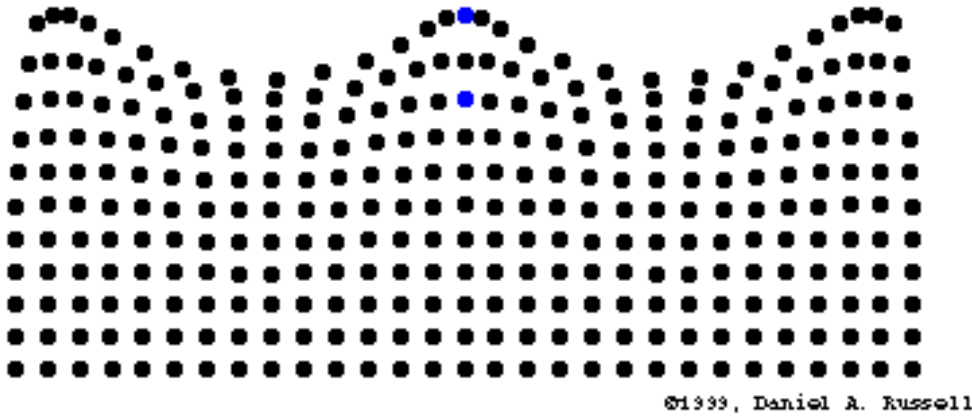
Longitudinal wave



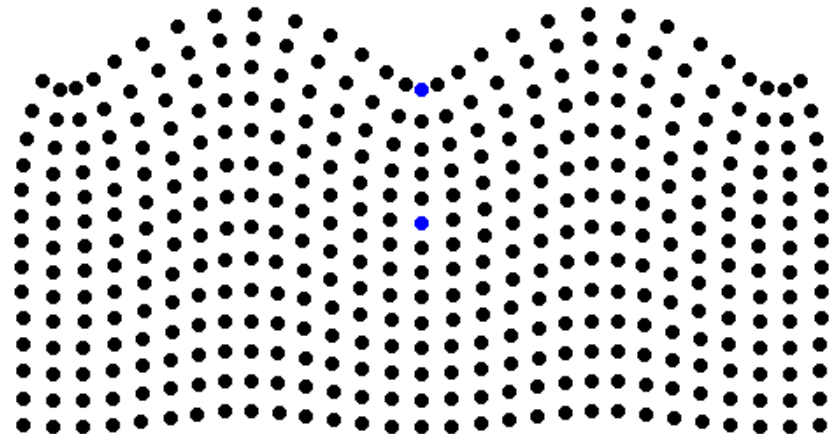
transversal wave



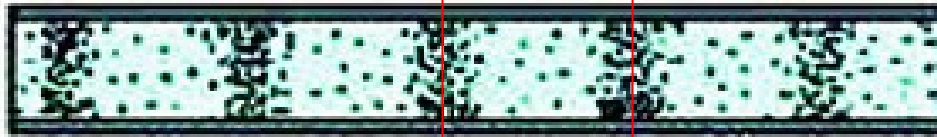
Surface wave



Rayleigh wave

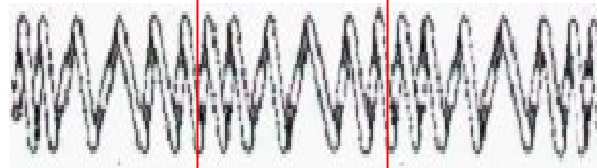


whistle

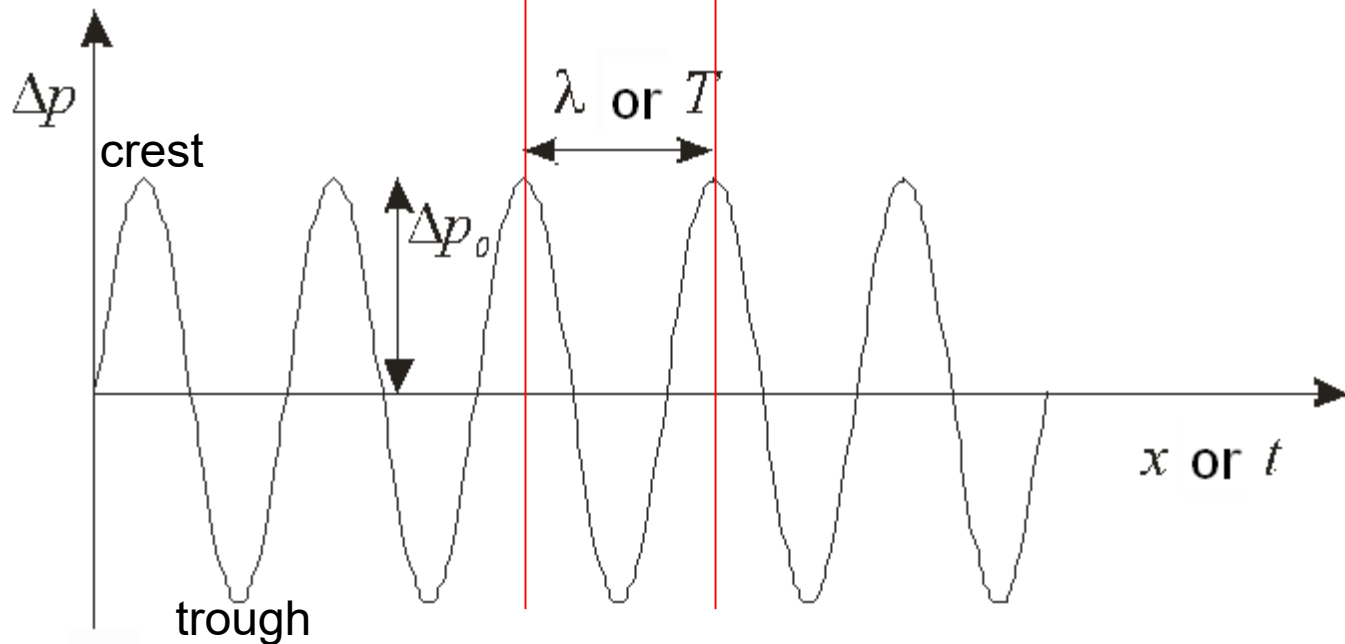


spring

compression  
rarefaction



function



hydrostatic  
pressure

pressure change,  
sound pressure

$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

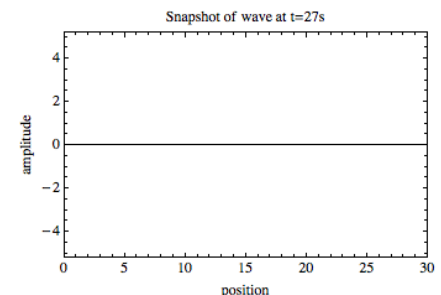
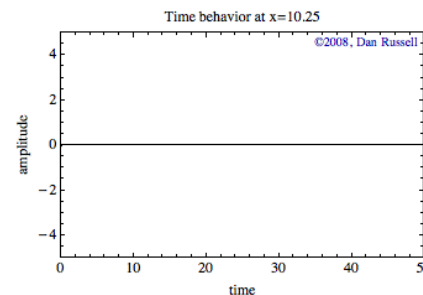
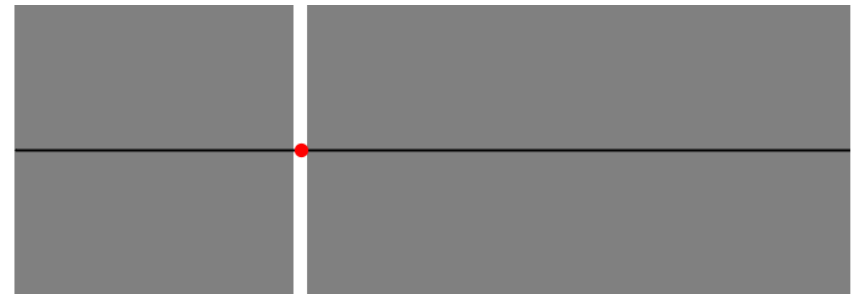
pressure DC + AC amplitude phase

$$\Delta p(t, x) = \Delta p_{\text{max}} \sin \left[ 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right) \right]$$

$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$

(see electrical analogy:  
DC=direct current,  
AC=alternating current)

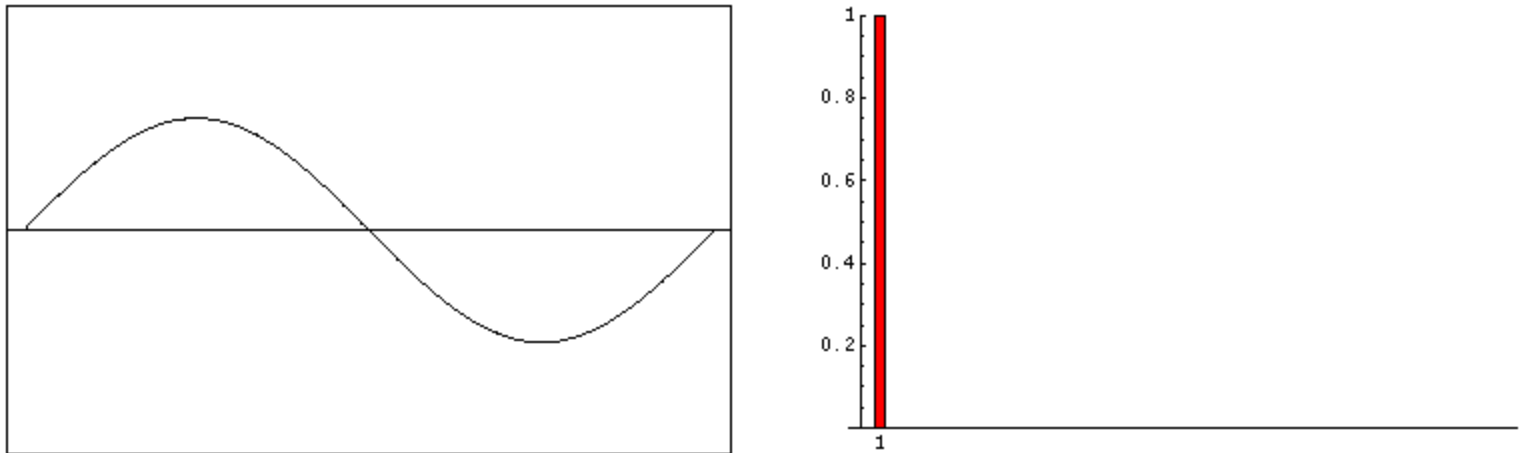
Usually  $p_{\text{hydrostat}} \gg \Delta p$





## Frequency ranges – Fourier theorem

$$Signal(t) \leftrightarrow \sum_i A_i \cdot \sin(\omega_i t) + B_i \cos(\omega_i t)$$



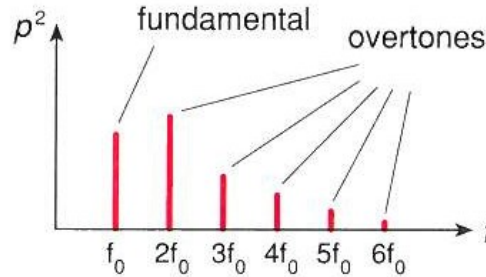
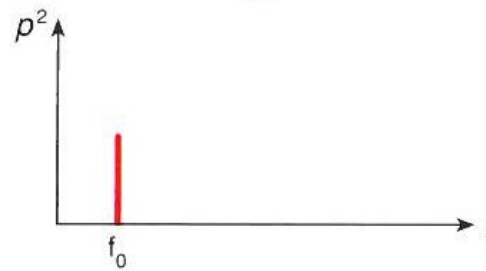
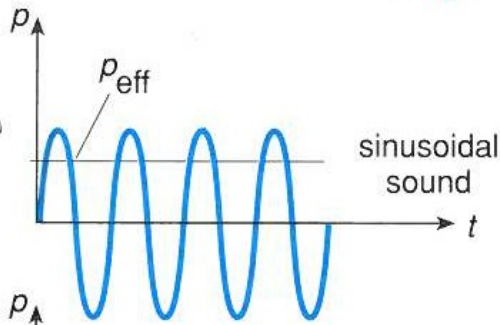
The more sin/cos signals we add, the better the accuracy

FOURIER - ANALYSIS

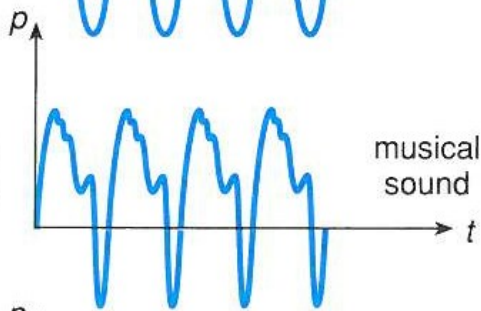
FOURIER - SYNTHESIS

TIME FUNCTION

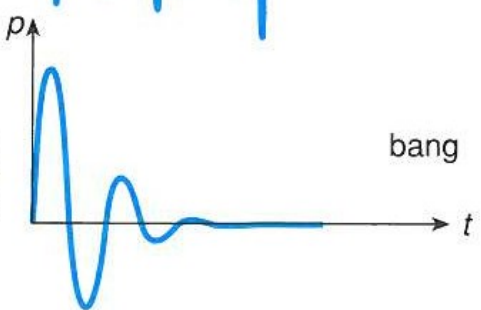
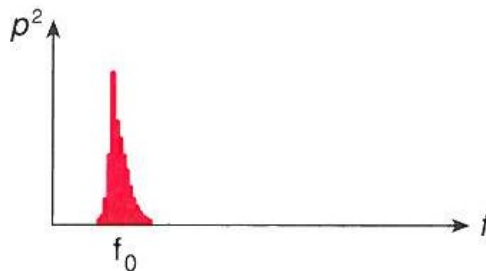
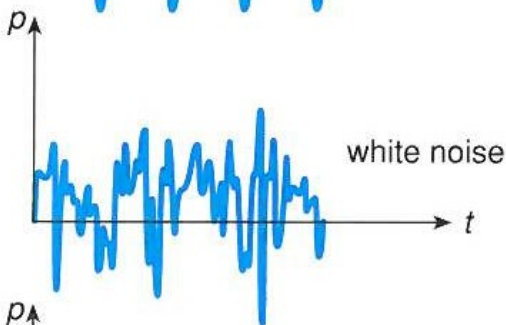
SPECTRUM



DISCRETE SPECTRA

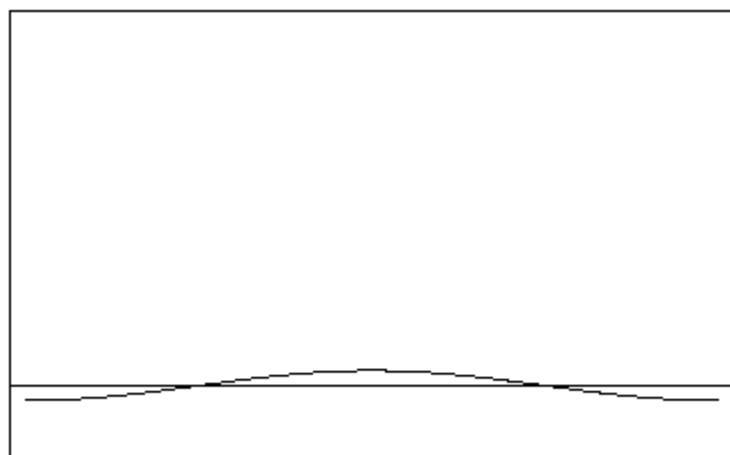
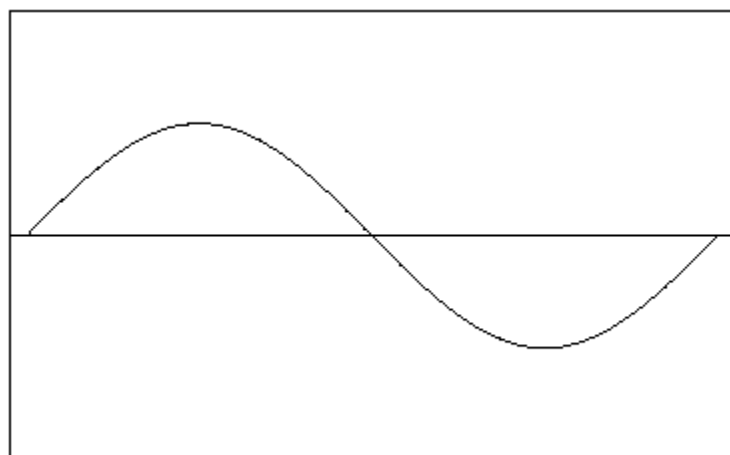
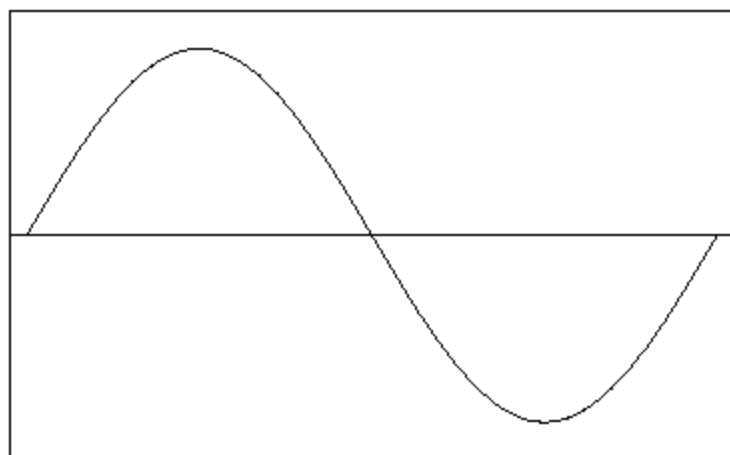


CONTINUOUS SPECTRA



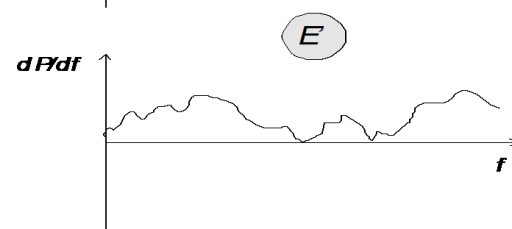
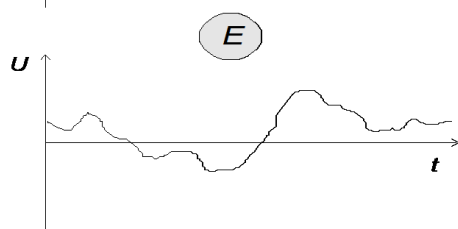
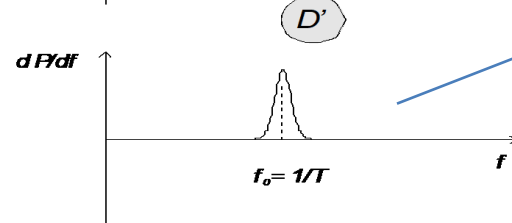
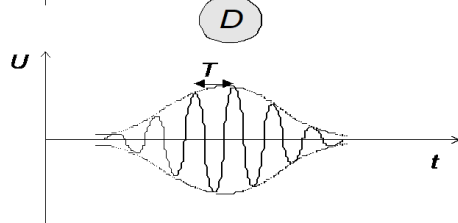
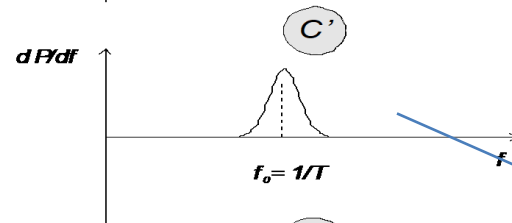
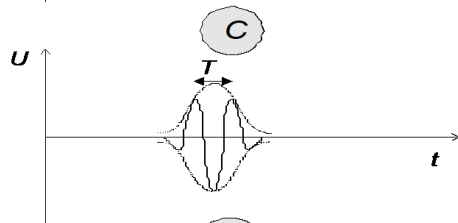
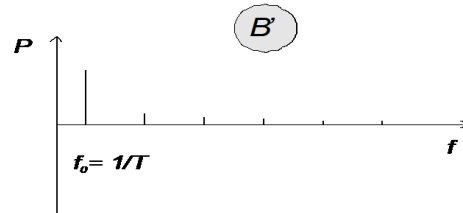
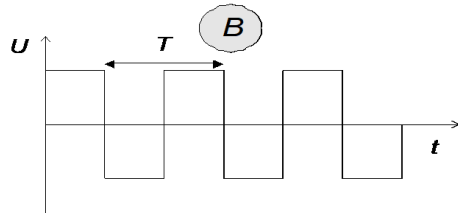
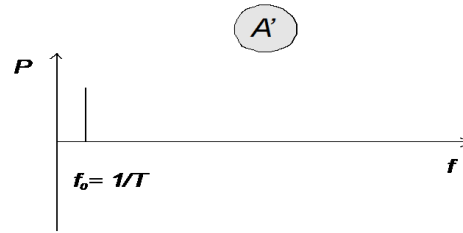
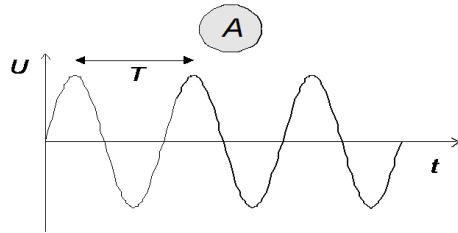
**pitch:**  
frequency of the  
fundamental

**timbre** (tone colour):  
relative strengths of  
overtones/harmonics  
(spectrum)

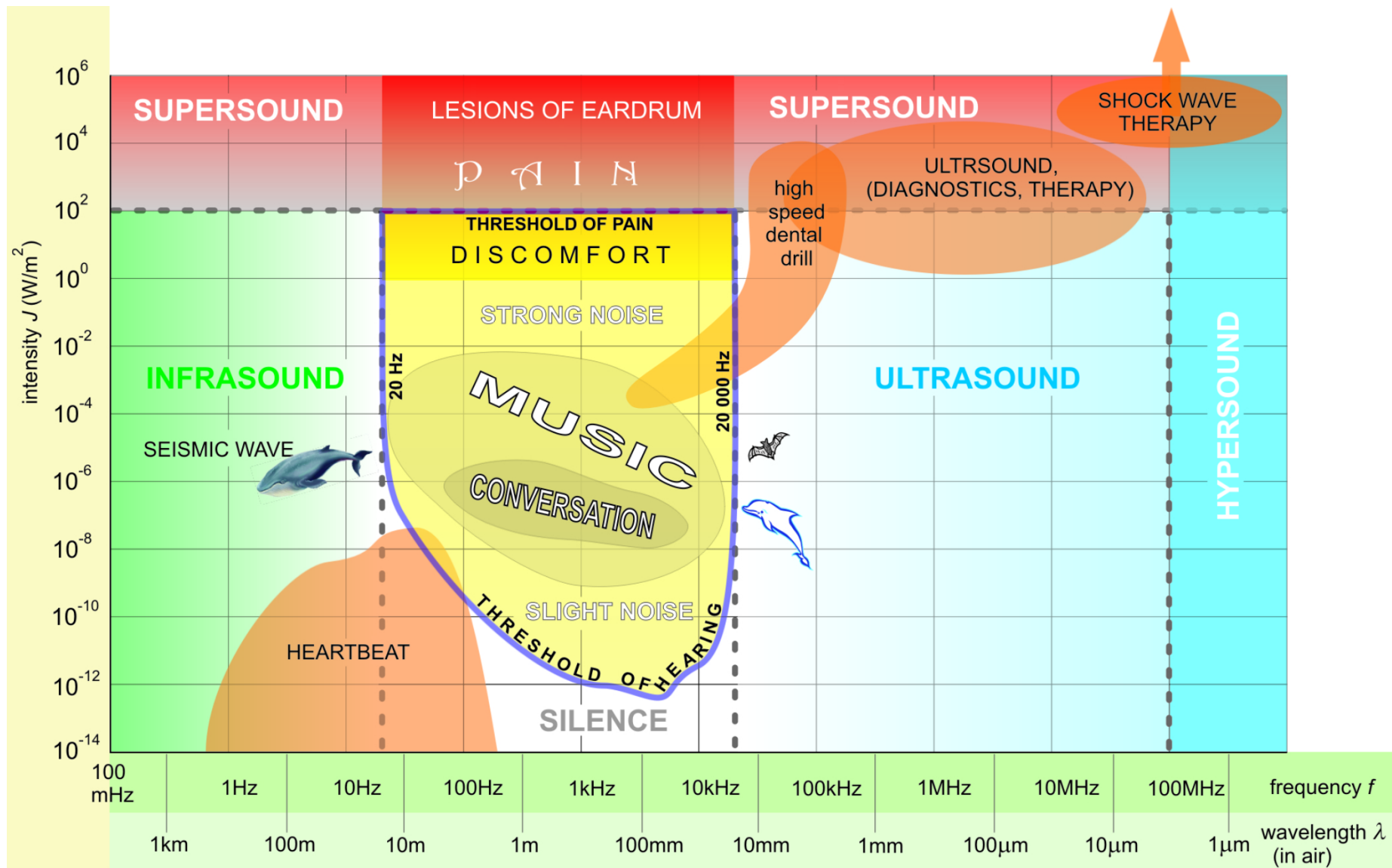


$$F(\omega) = \frac{1}{\sqrt{(2\pi)}} \cdot \int_{-\infty}^{+\infty} f(t) e^{i\omega t} dt$$

If the signal is non-periodic, we have an integral instead of the summation



If the pulse gets shorter, then the frequency spectrum spans a broader range!



## Propagation of sound waves

$$\Delta p(t, x) = \Delta p_{\max} \sin \left[ 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right) \right]$$

$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$

$$c = \sqrt{\frac{E}{\rho}} = \frac{1}{\sqrt{K\rho}}$$

$E$  is called the elastic (or Young's) modulus of the material and is a measure of the stiffness of the material. (See Hooke's law!)

Some important equations – the role of the elastic medium

$$\kappa = \frac{-\frac{\Delta V}{V}}{\Delta p}$$

**compressibility**  
relative volume decrease  
over pressure

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

**speed** of sound

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

acoustic **impedance**  
(definition)

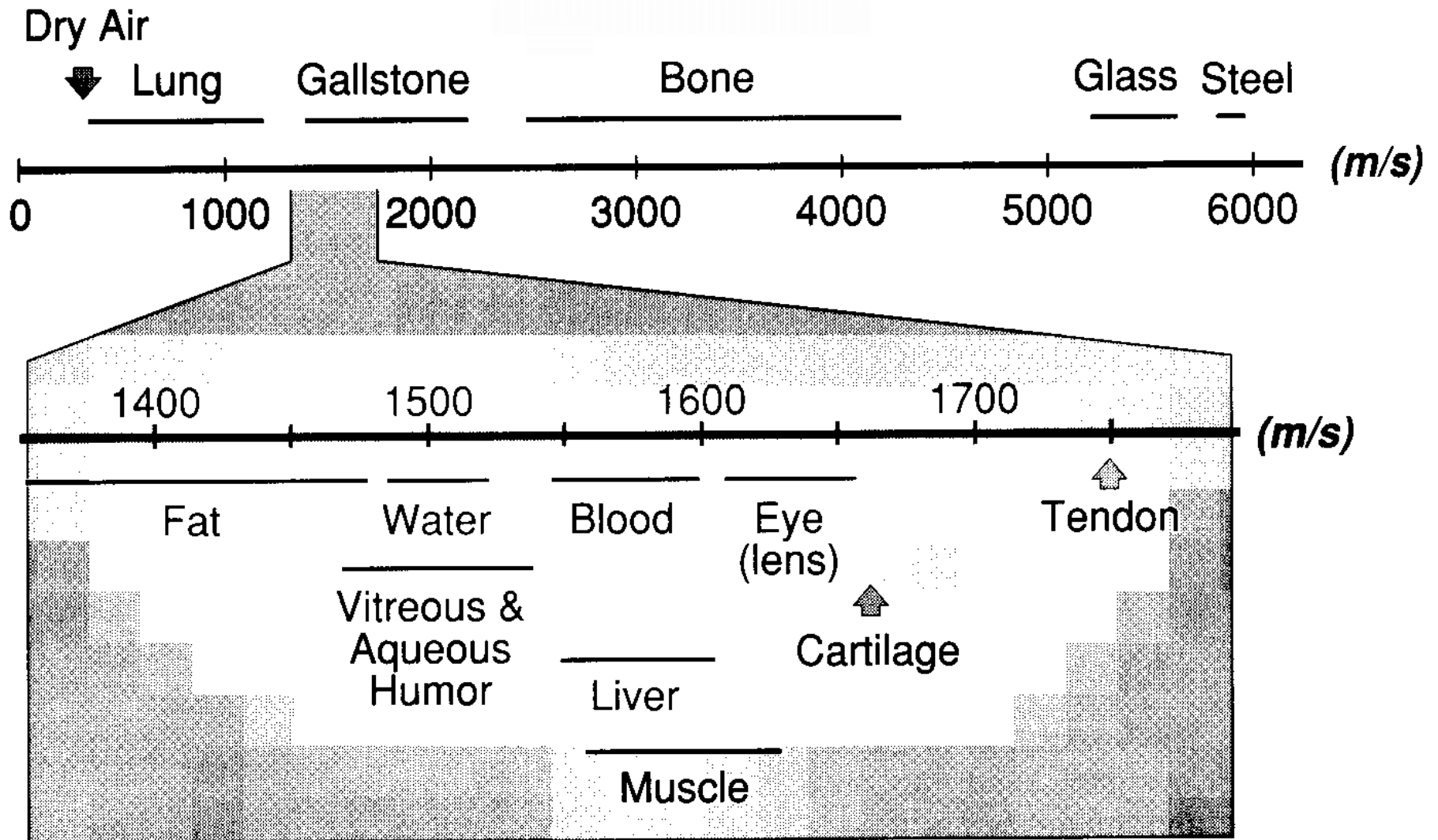
Here v is the volume flow, not the speed!

$$Z_{\text{el}} = \frac{U}{I}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance**  
(useful form)

Speed of ultrasound in various materials.  
The soft tissue median is 1540 m/s

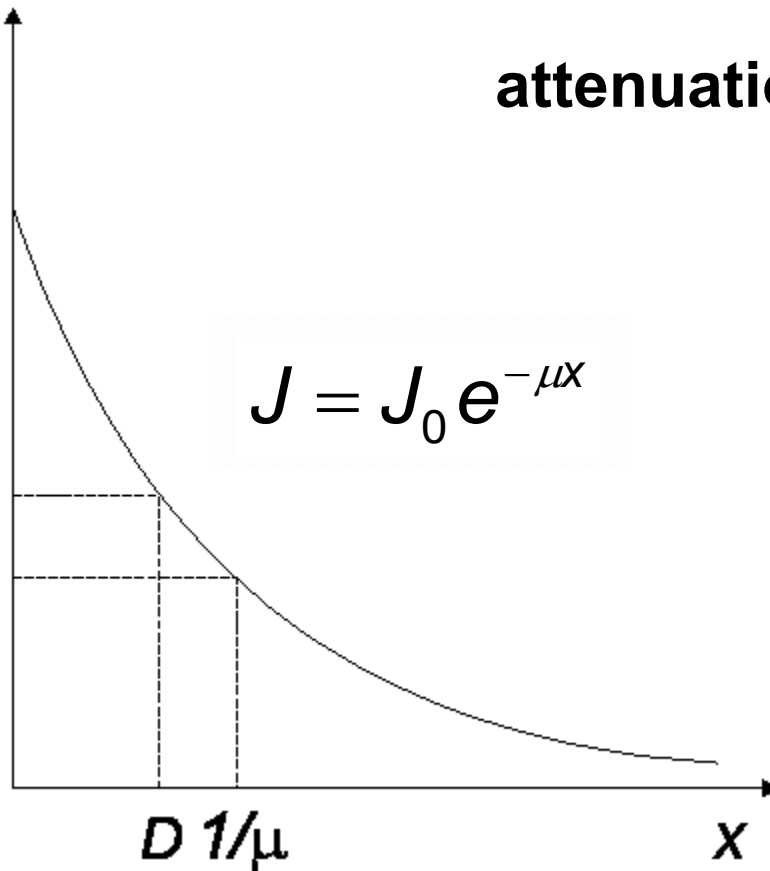




$$J = \frac{1}{Z} \Delta p_{\text{eff}}^2$$

intensity = energy-current density

**Intensity obeys the absorption law, such as any wave**



**attenuation:**  $\alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

$\mu$  is proportional to frequency in the diagnostic range!

$\mu$  is proportional  
to frequency in  
the diagnostic  
range

$$\mu \sim f^k, \quad k \sim 1 (?)$$

$$\log \mu \sim k \log f$$

double-log graph: if the graph  
is a linear, the power function  
approximation is valid

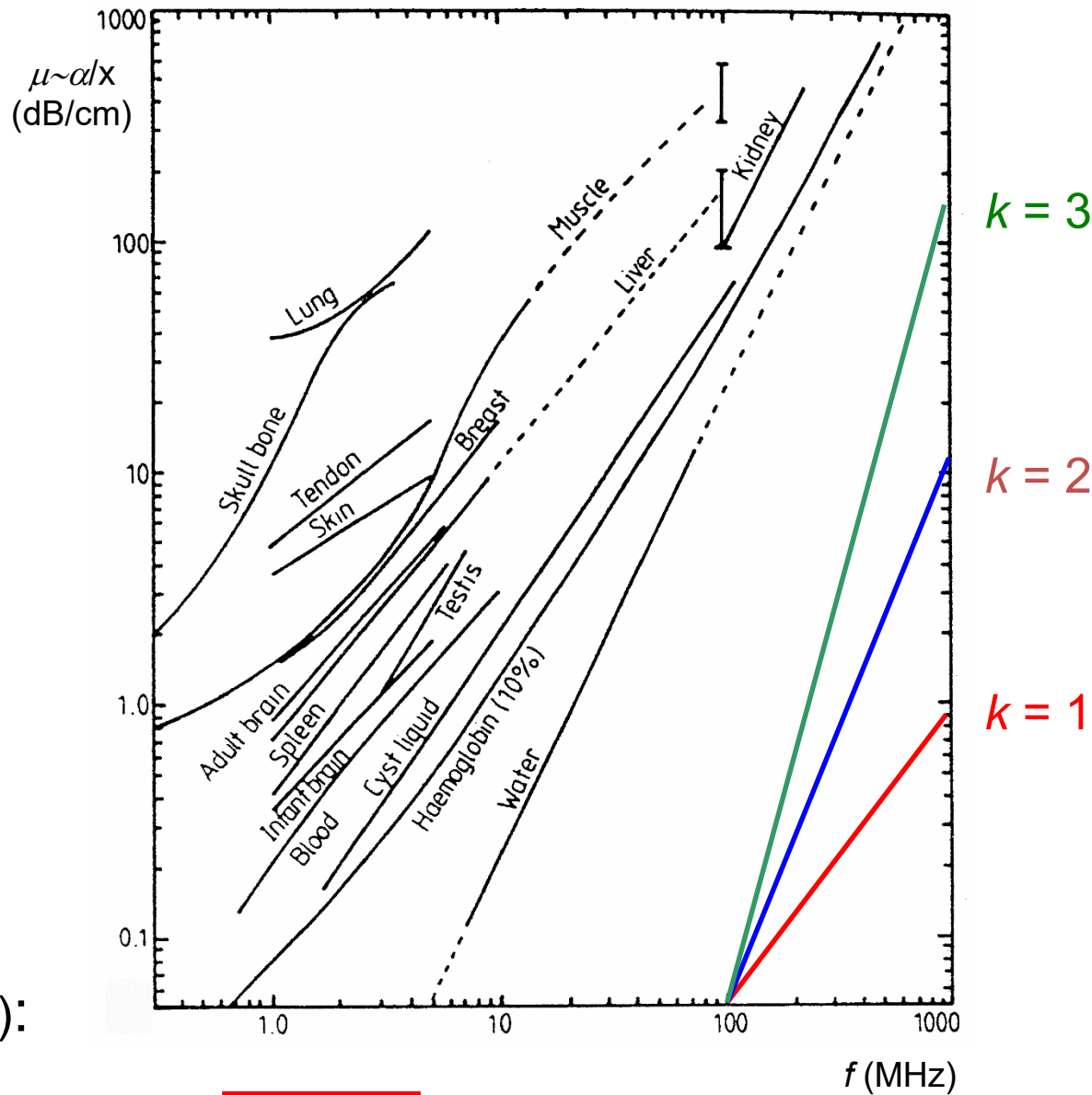
specific attenuation  
for soft tissues

(homogeneous tissue model):

$$\frac{\alpha}{f \cdot x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$

**specific  
attenuation:**

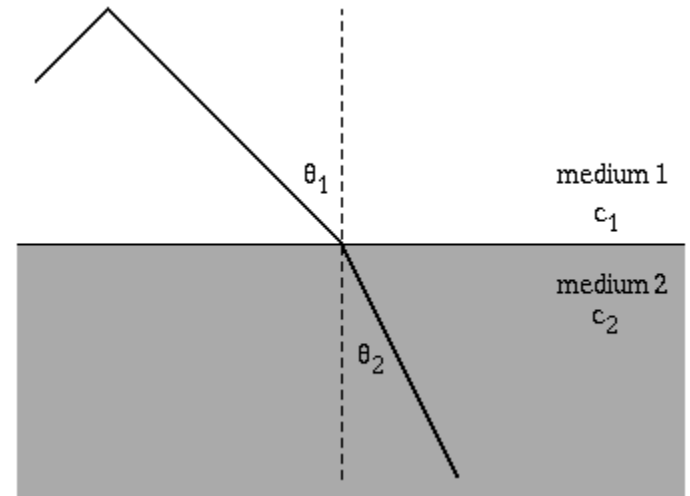
$$\frac{\alpha}{f \cdot x}$$



Reflection and refraction – again at the boundaries (as with light)

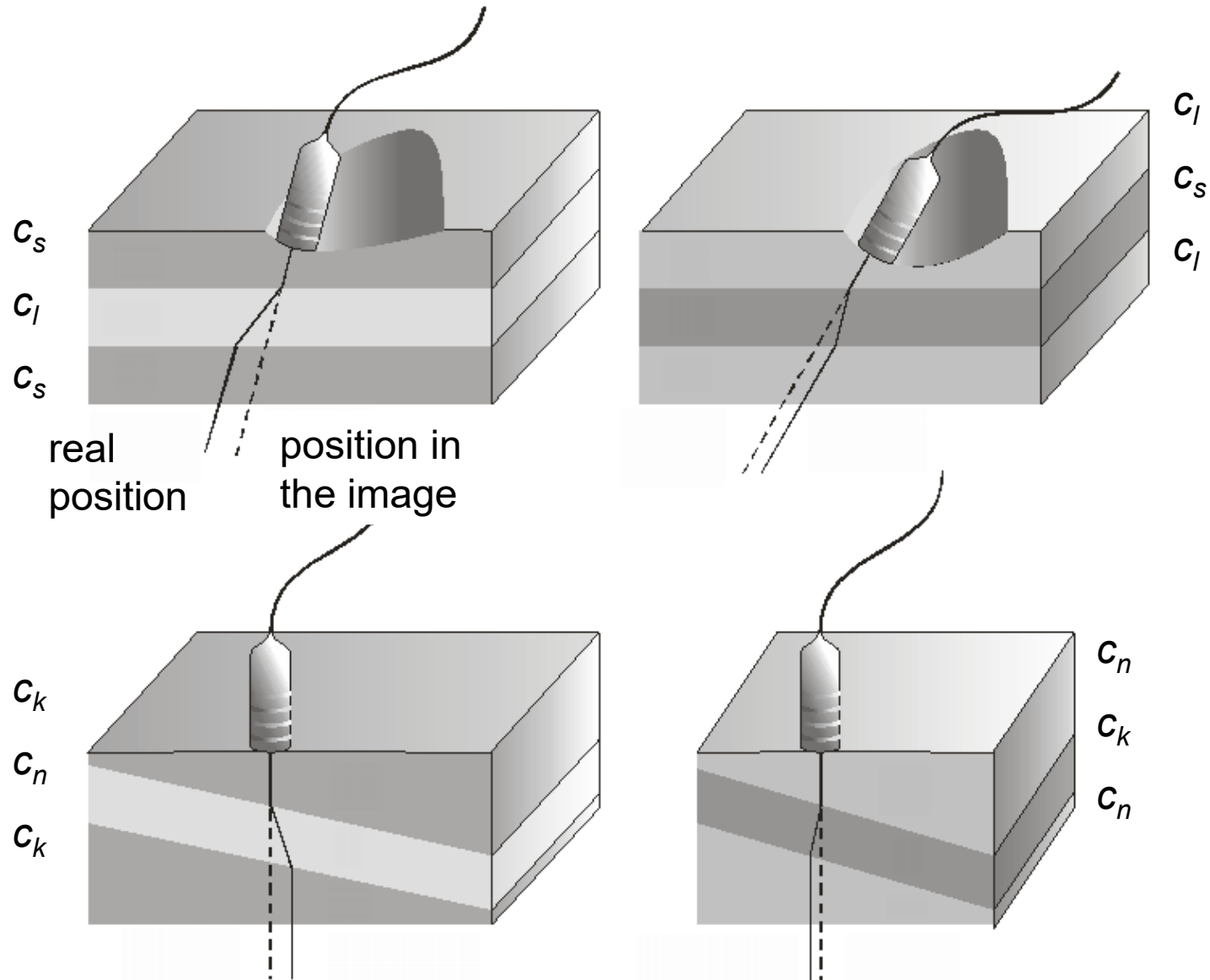
$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

Snellius-Descartes



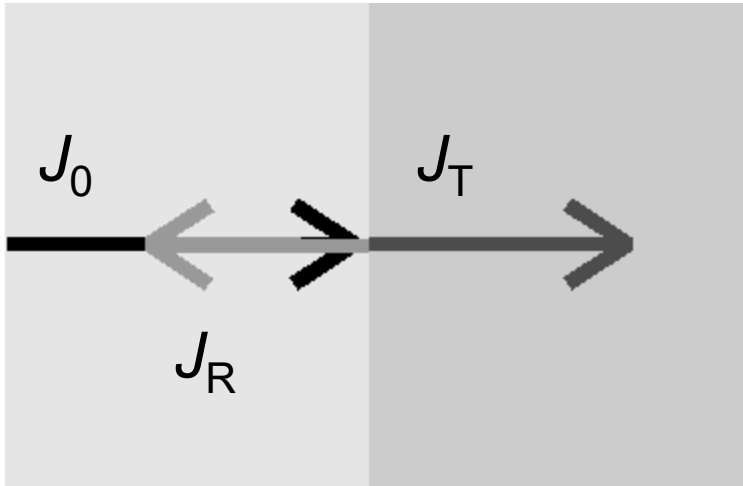
Frequency remains constant!

Ultrasound “beams” will change direction by refraction – just as light does



We need to take this into account in the imaging!

## Reflection of ultrasound (normal incidence)



$$J_0 = J_R + J_T$$

reflection and transmission  
(penetration)

**reflectivity:**

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left( \frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

Analogy to light: Z stands here  
instead of refractive index

<i>boundary surface</i>	<i>R</i>
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

“full” reflection:

$$Z_1 \ll Z_2, \quad R \approx 1$$

optimal coupling:

$$Z_{\text{connecting}} \approx \sqrt{Z_{\text{source}} Z_{\text{skin}}}$$



Coupling medium is required for medical ultrasound imaging!

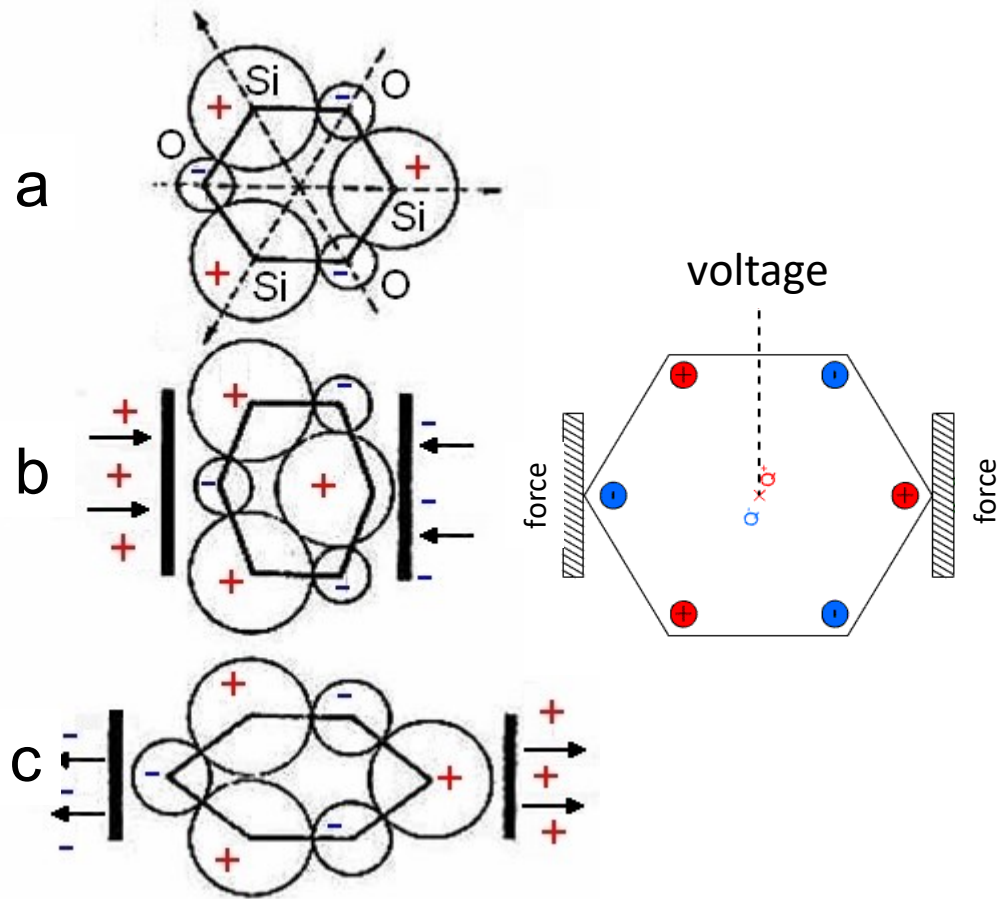
## Generation of ultrasound - Piezoelectric effect

source of electric signal  
(sine wave oscillator)

+  
transducer (piezo-crystal)

(a) Center of charge of positive and negative charges coincides.

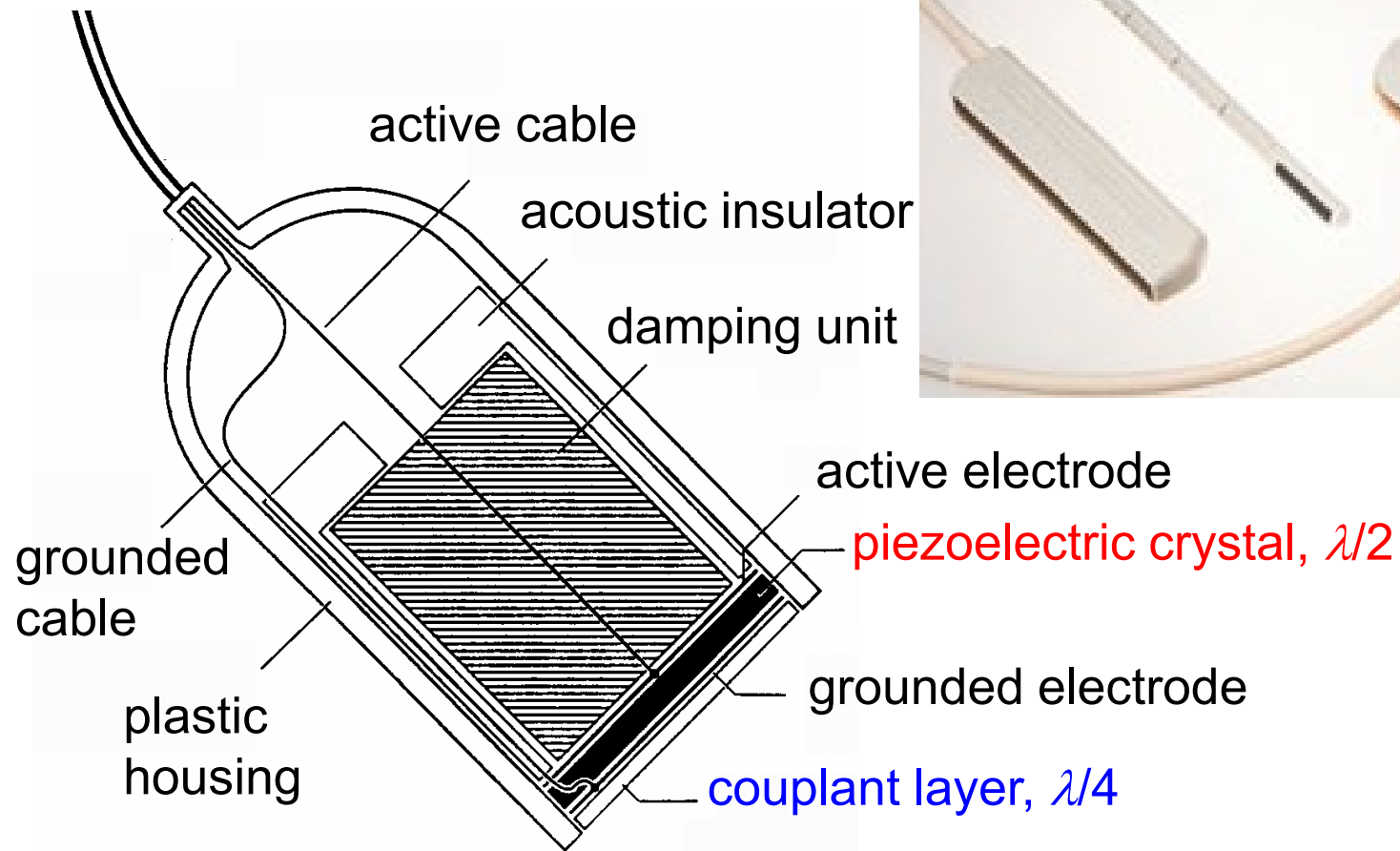
(b) and (c) As a result of pressure, the charge centers are separated, i.e. a potential difference arises (direct ~).  
The crystal is deformed when voltage is applied (inverse ~).



at home:  
gas igniter



tweeter

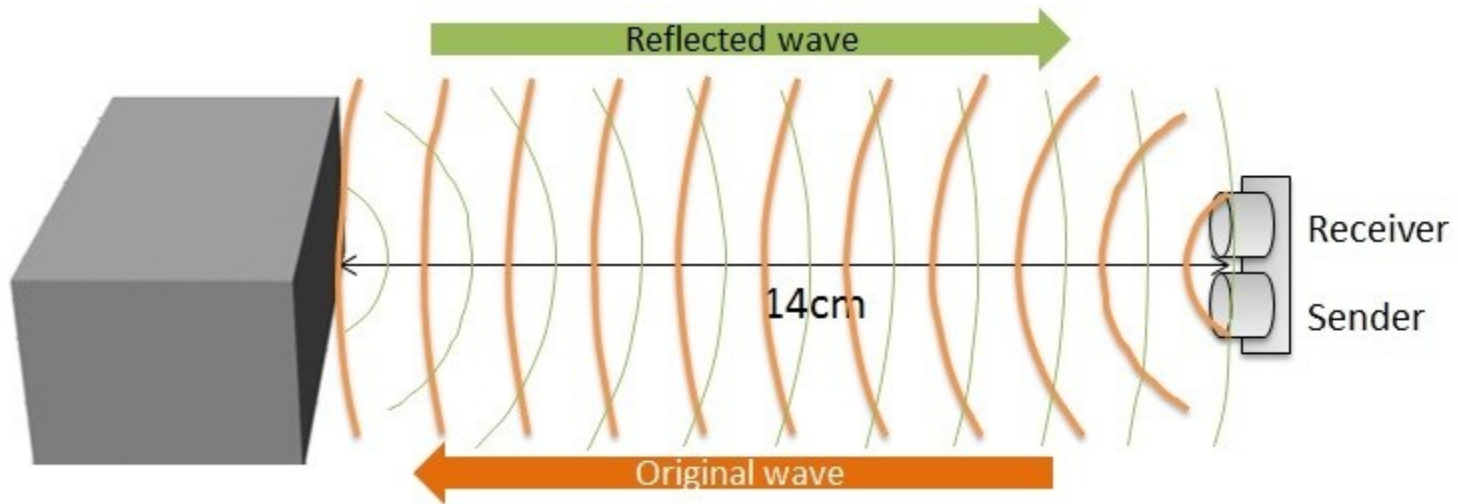


It acts as both receiver and transmitter of ultrasound pulses



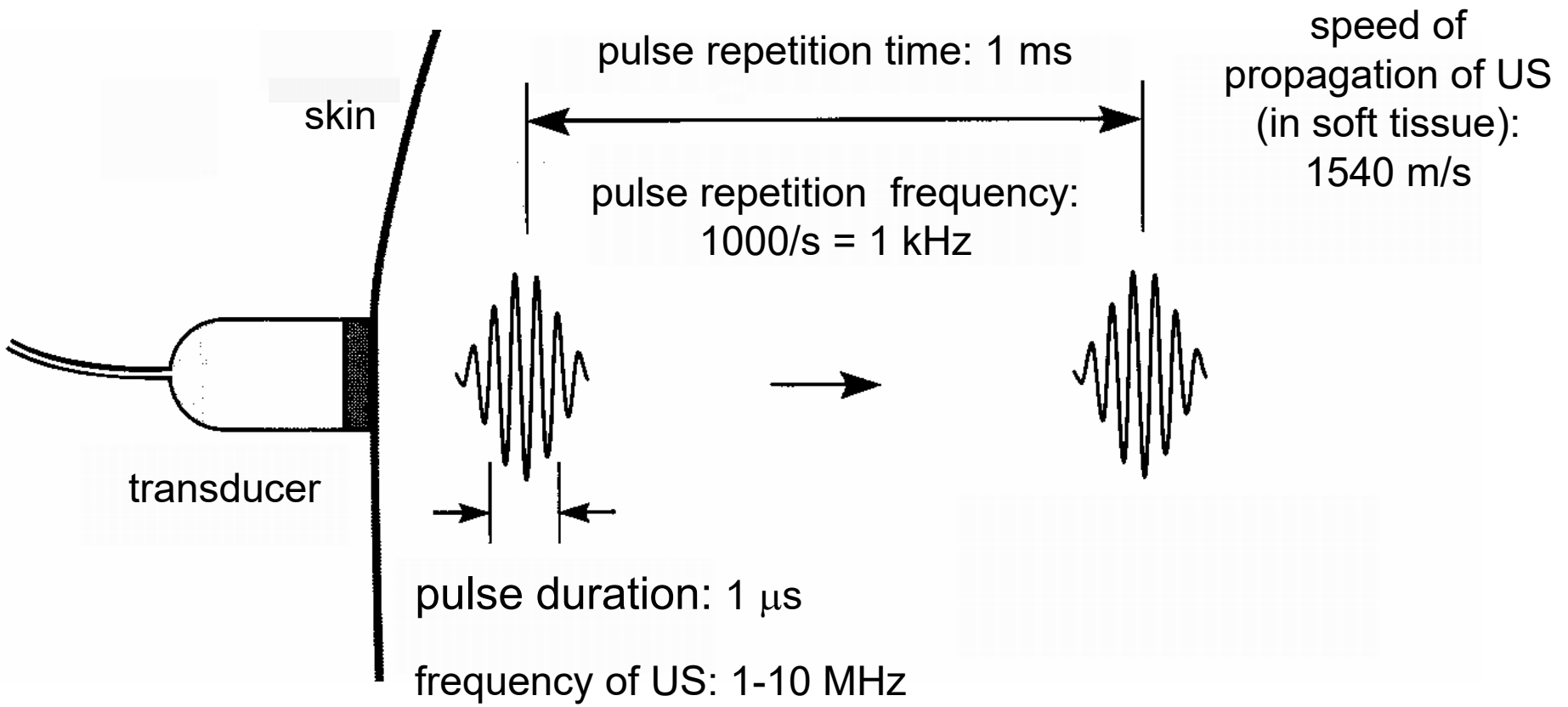
Principle of ultrasound imaging:

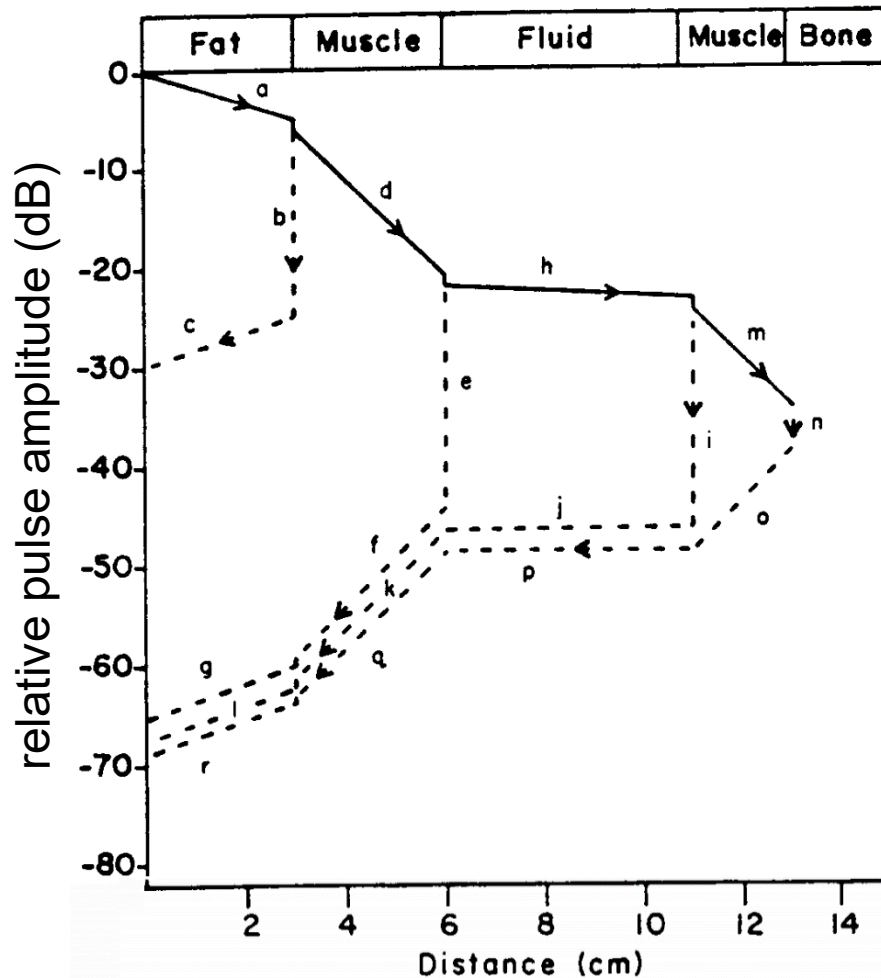
We detect the **reflections from various surfaces**



**time sharing** mode: **pulses** instead of continuous wave ultrasound

This enables the usage of the same transducer, and improves resolution





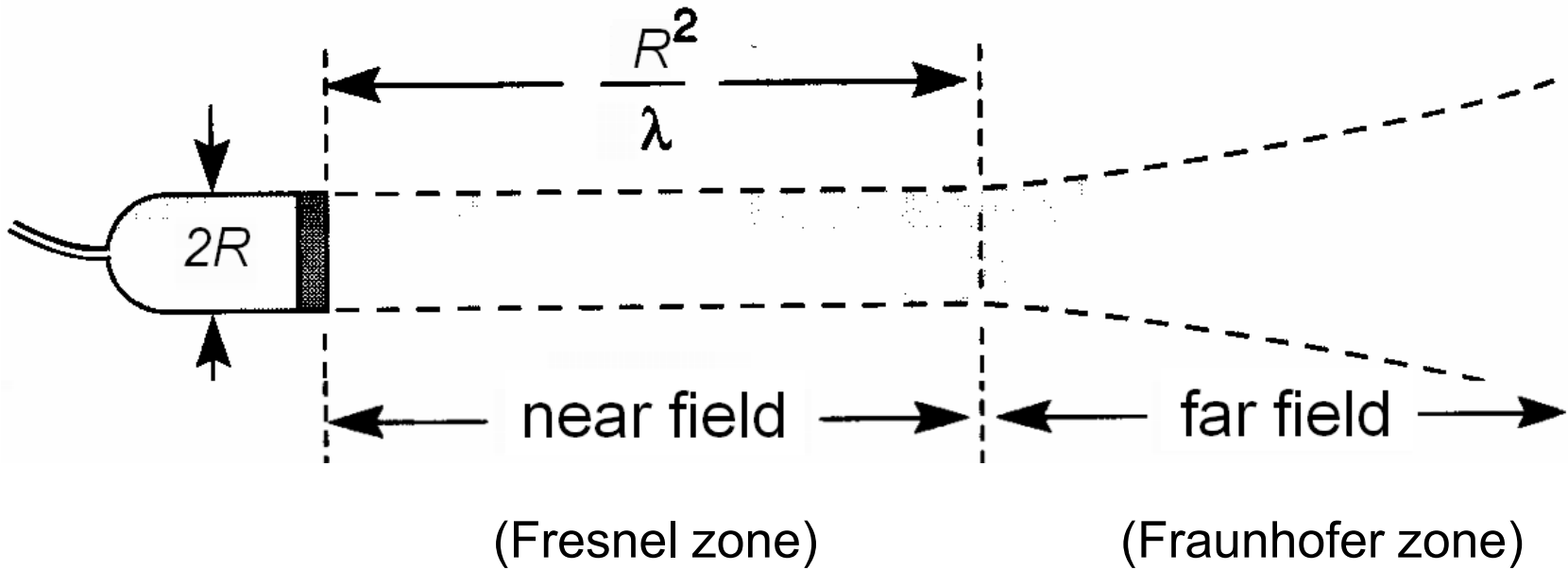
We need to deal with absorption:  
the deeper the reflection comes from,  
the weaker it will be

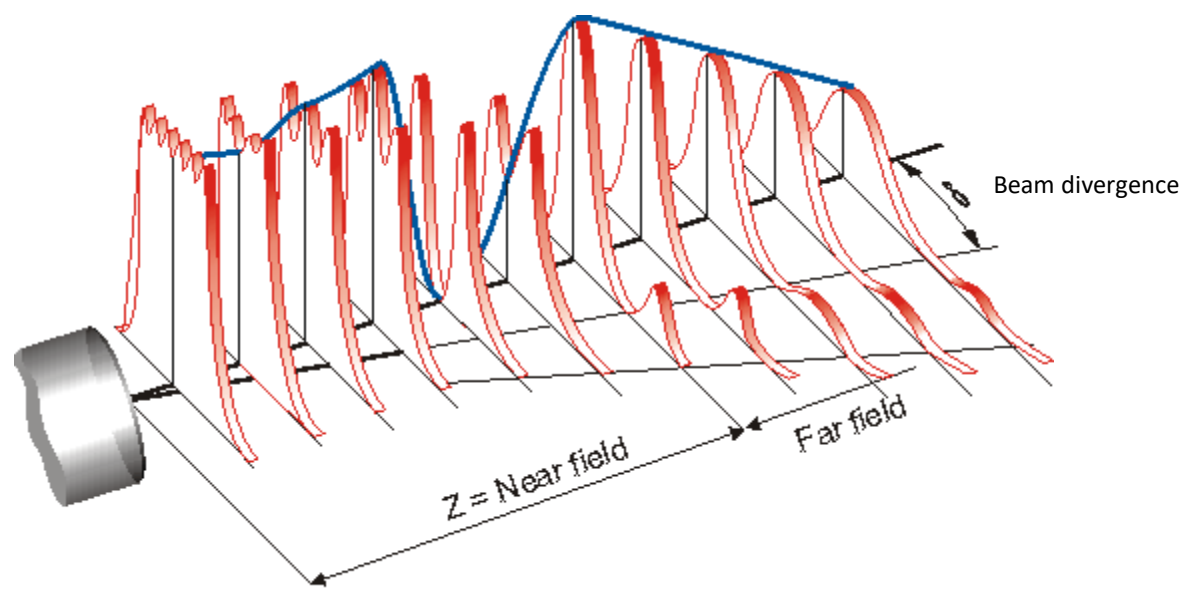
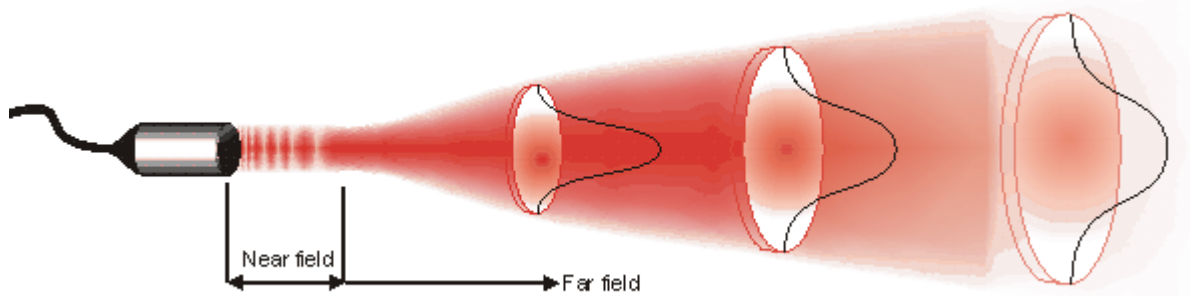
TGC: time gain compensation

DGC: depth gain control

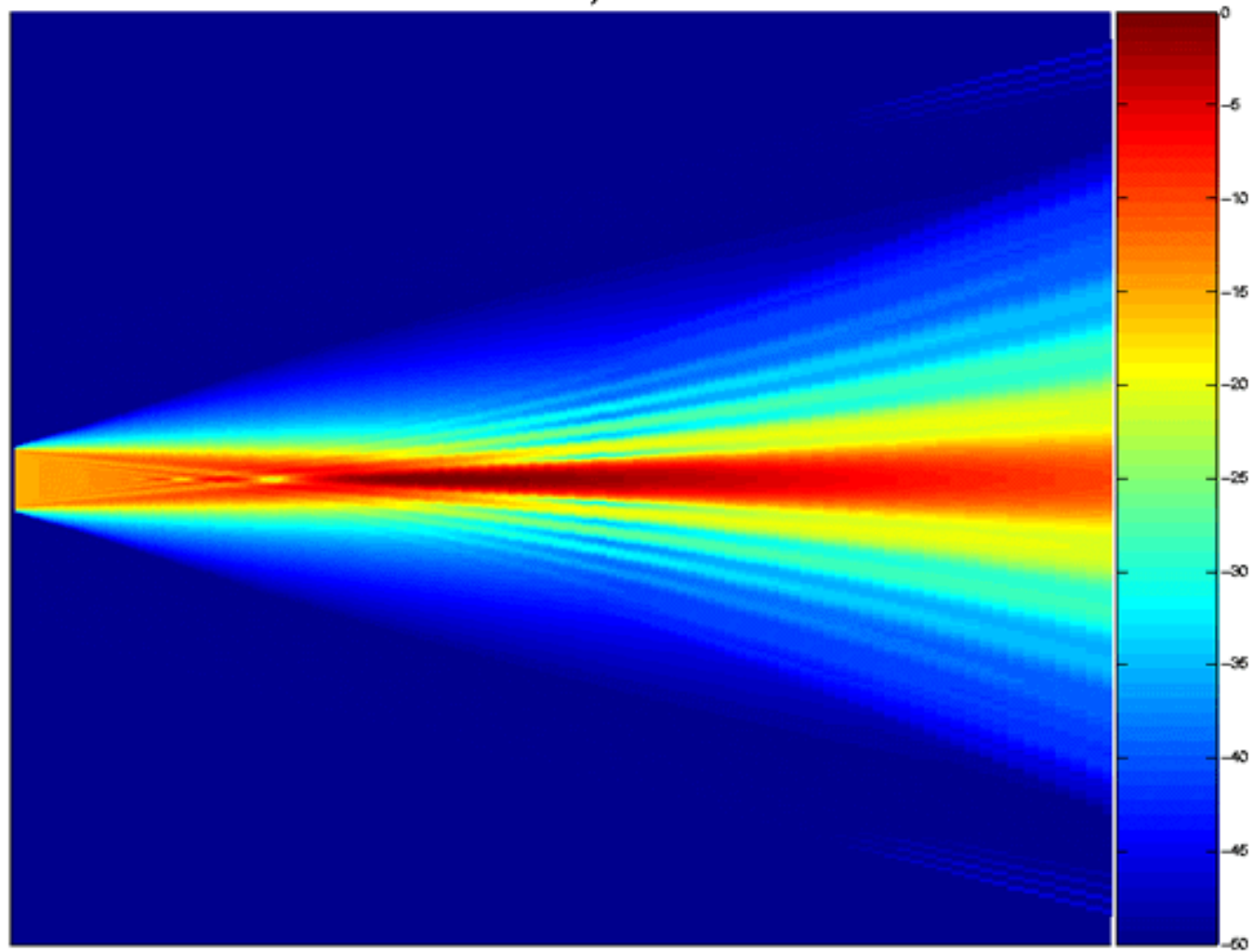
<i>boundary surface</i>	$R$	$10\lg R$ (dB)	$T$	$10\lg T$ (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

Technical details – beam shape, resolution, etc.





Fundamental, dB scale



**Resolving limit** is the distance between two object details which can be just resolved as distinct objects (the smaller the better).

**Resolution (resolving power):** the reciprocal of the resolving limit (the greater the better)

**Axial resolving limit** depends on the pulse length. Pulse length is inversely proportional to the frequency.

**Lateral** resolving limit is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

**Typical values**

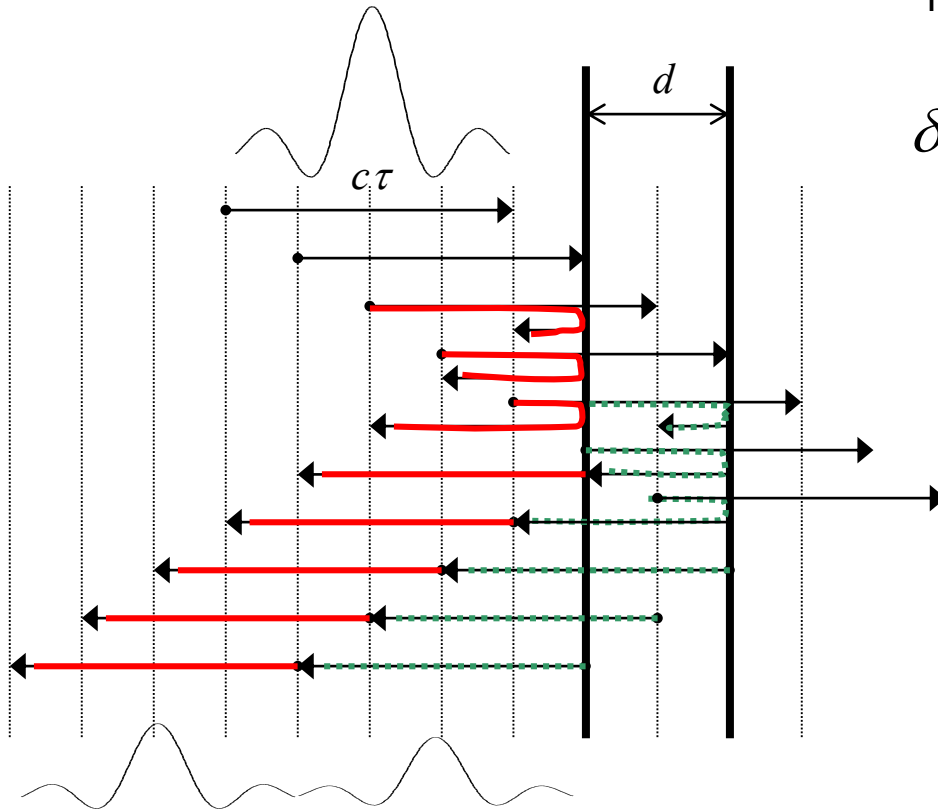
frequency (MHz):	2	15
wavelength (in muscle) (mm):	0.78	0.1
penetration depth (cm):	12	1.6
lateral resolving limit (mm):	3.0	0.4
axial resolving limit (mm):	0.8	0.15

Axial resolving limit – depends on the pulse shape

$\tau$  : pulse duration

$c_1\tau \cong c_2\tau = c\tau$  pulse length

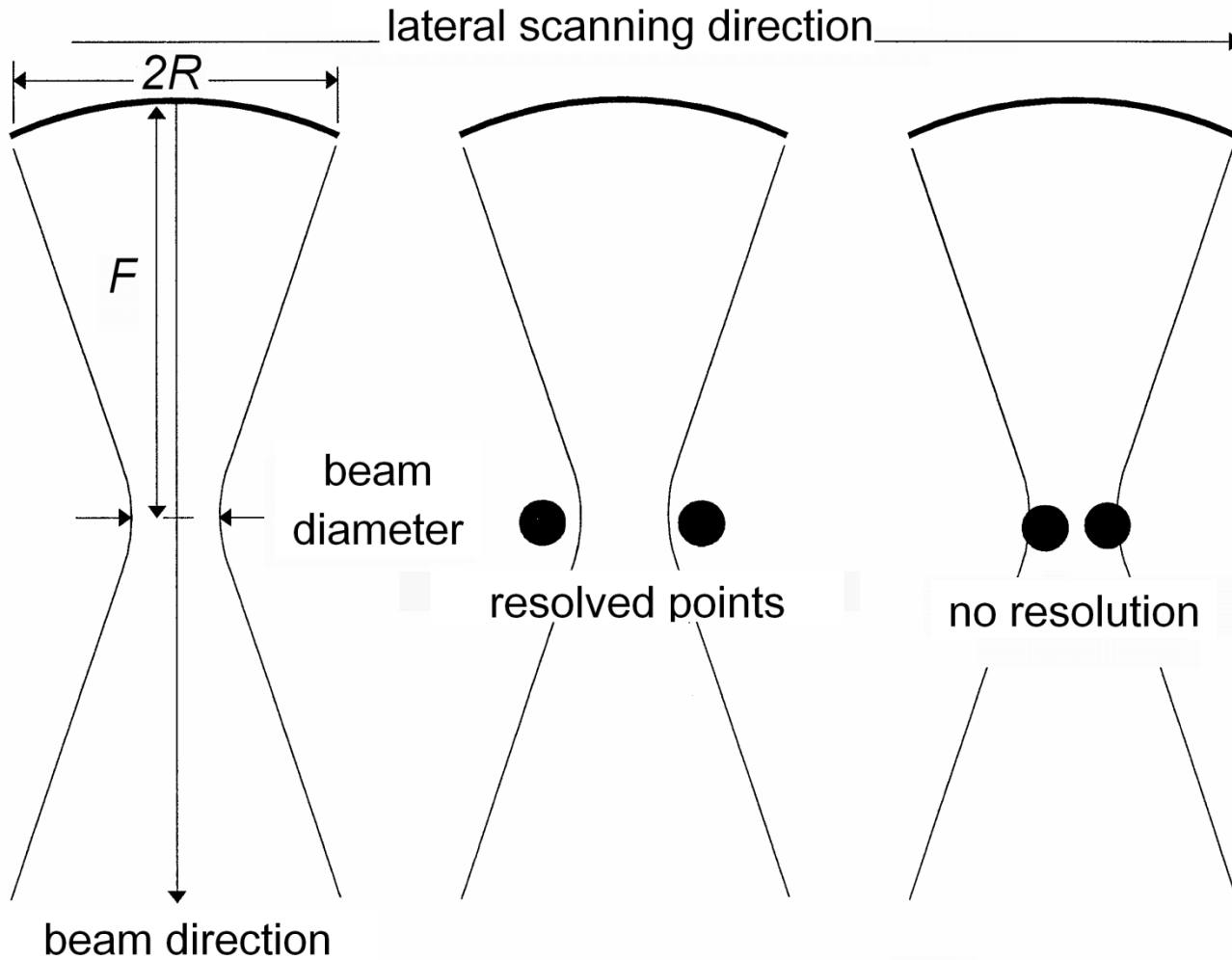
$\delta_{ax} = d = \frac{c\tau}{2}$  resolving limit



$$\tau \sim T = \frac{1}{f}$$

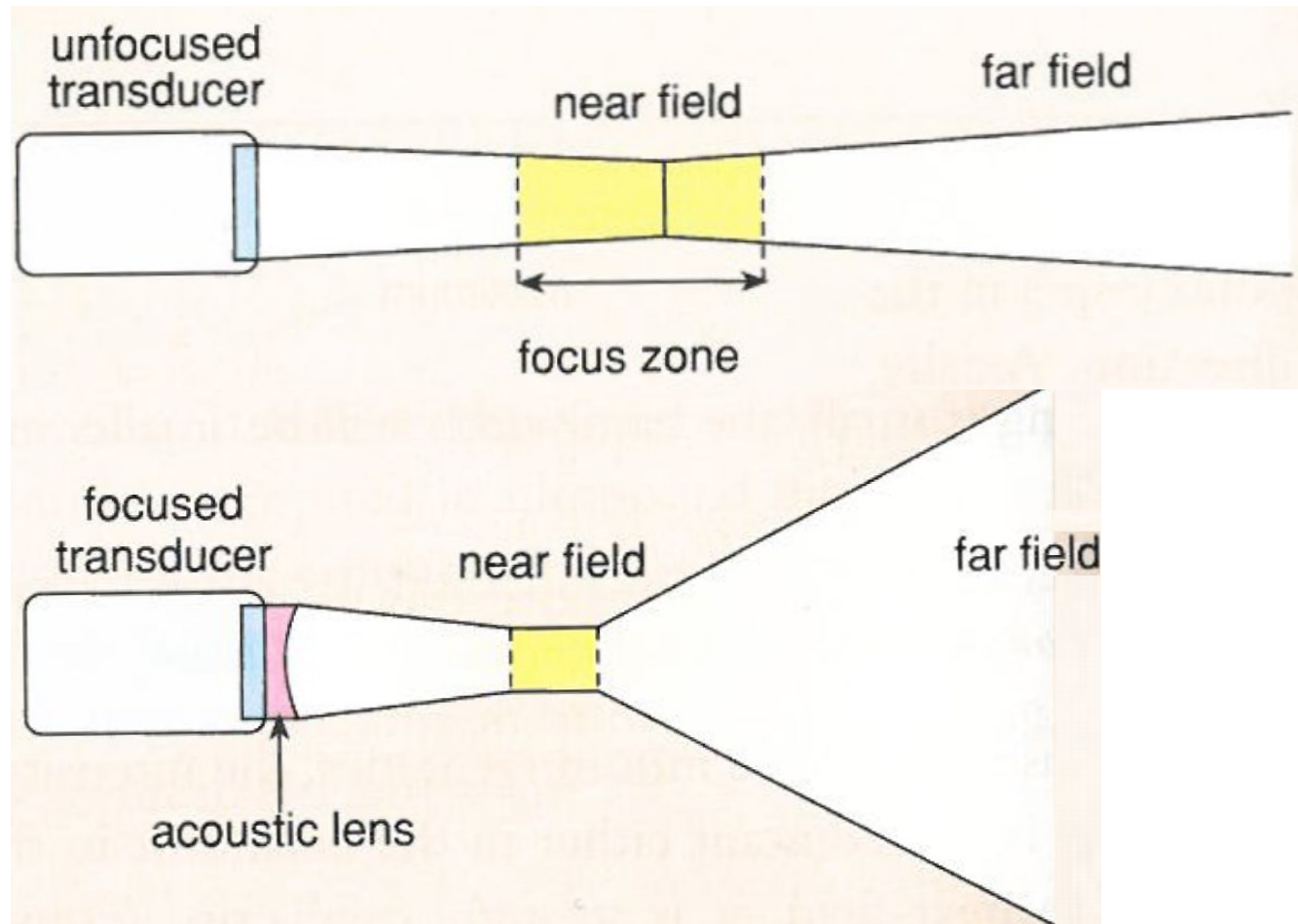


Lateral resolution – depend on the beam profile, or beam shape



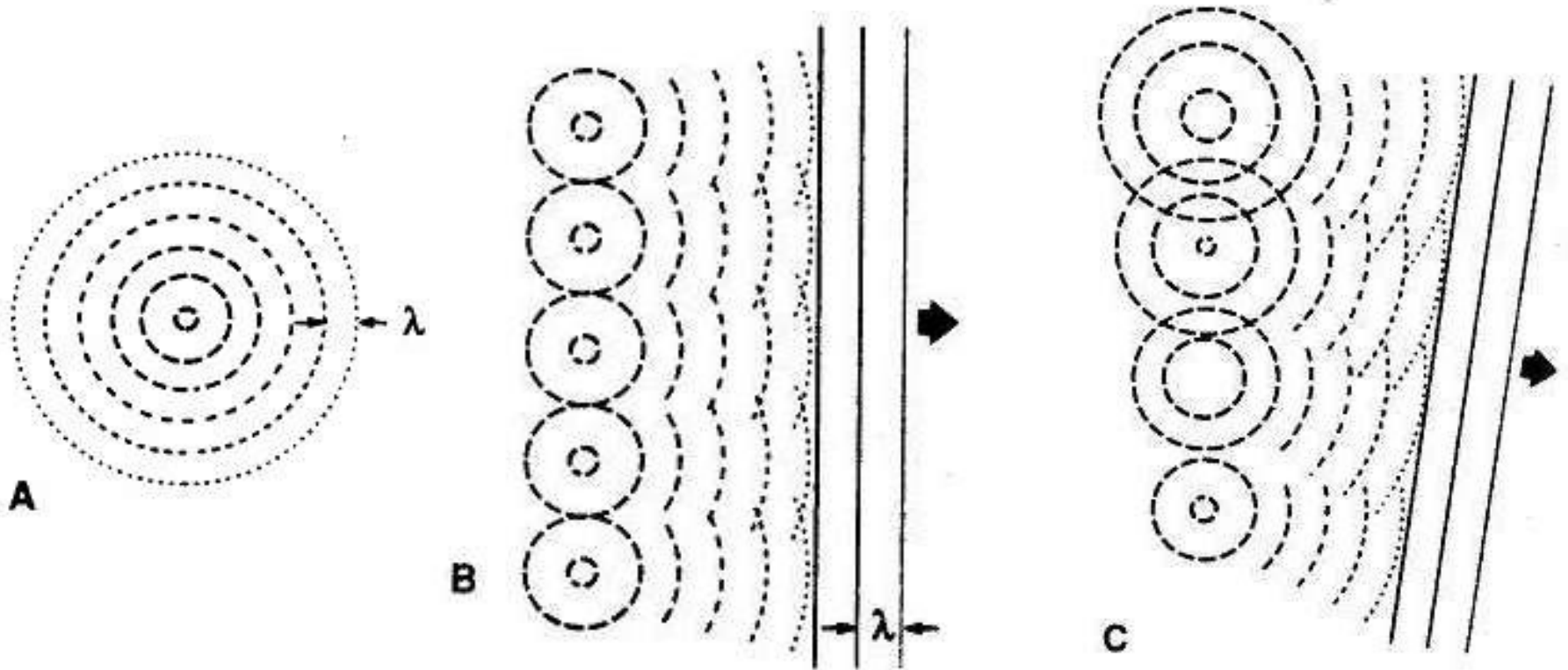
$$\left( \delta_{\text{lat}} \sim \frac{F}{2R} \cdot \lambda \right)$$

$F$ : focal length  
 $2R$ : diameter of the transducer  
 $\lambda$ : wavelength



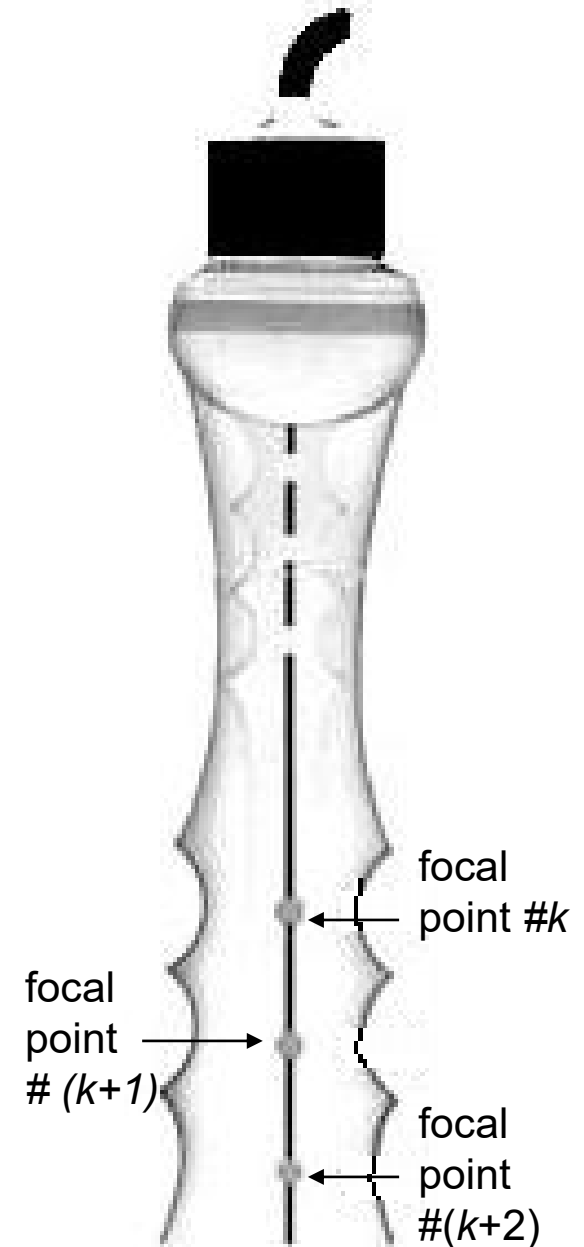
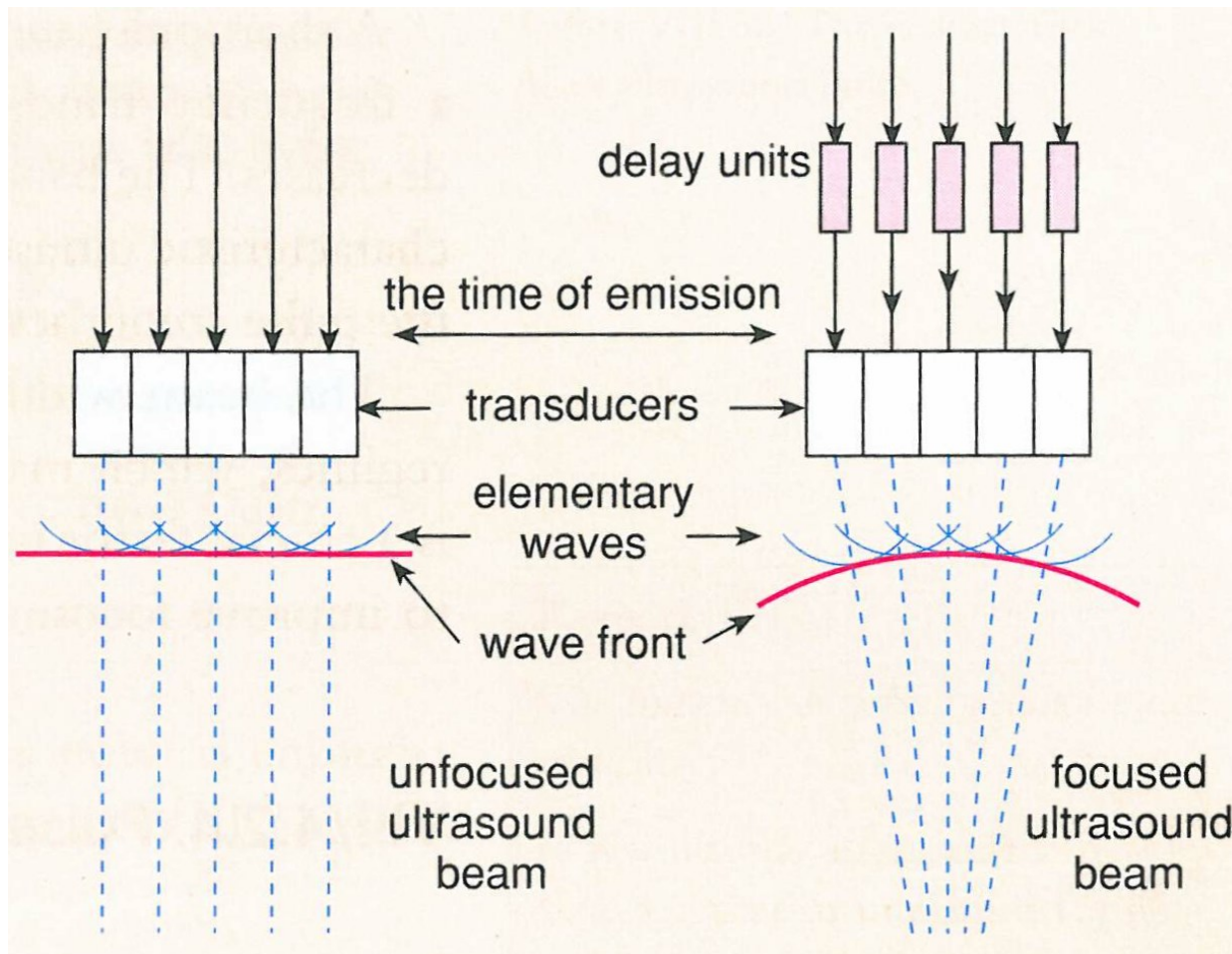
Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

## Point sources and Huygens' principle



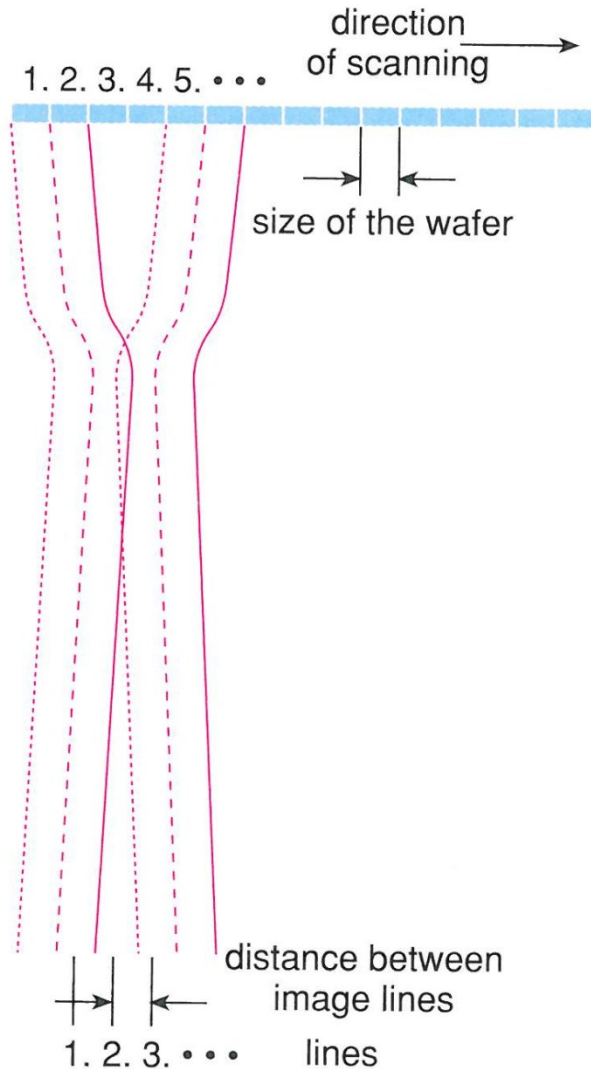
Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.

## Electronic focusing – using Huygens

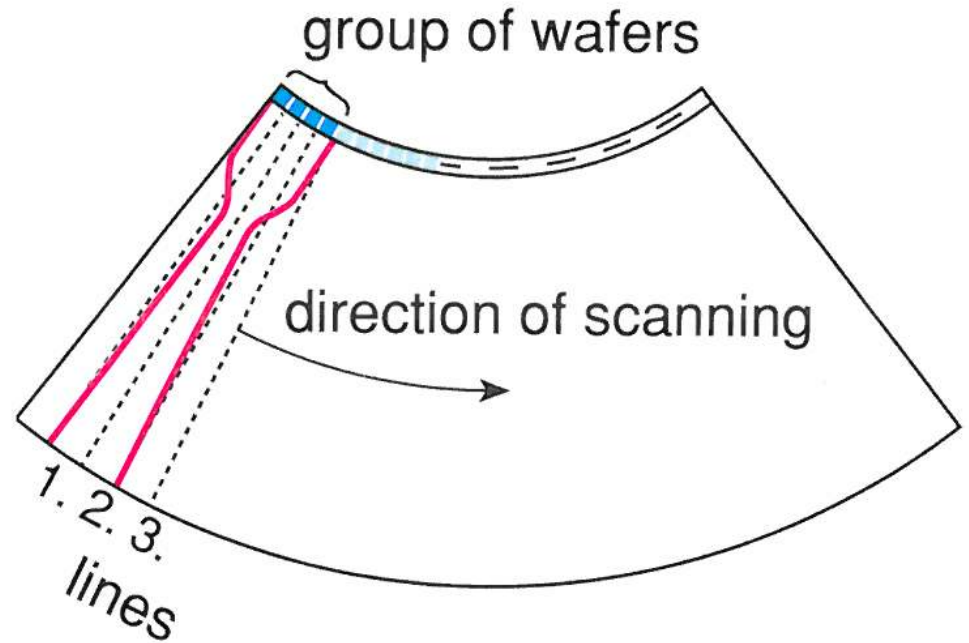


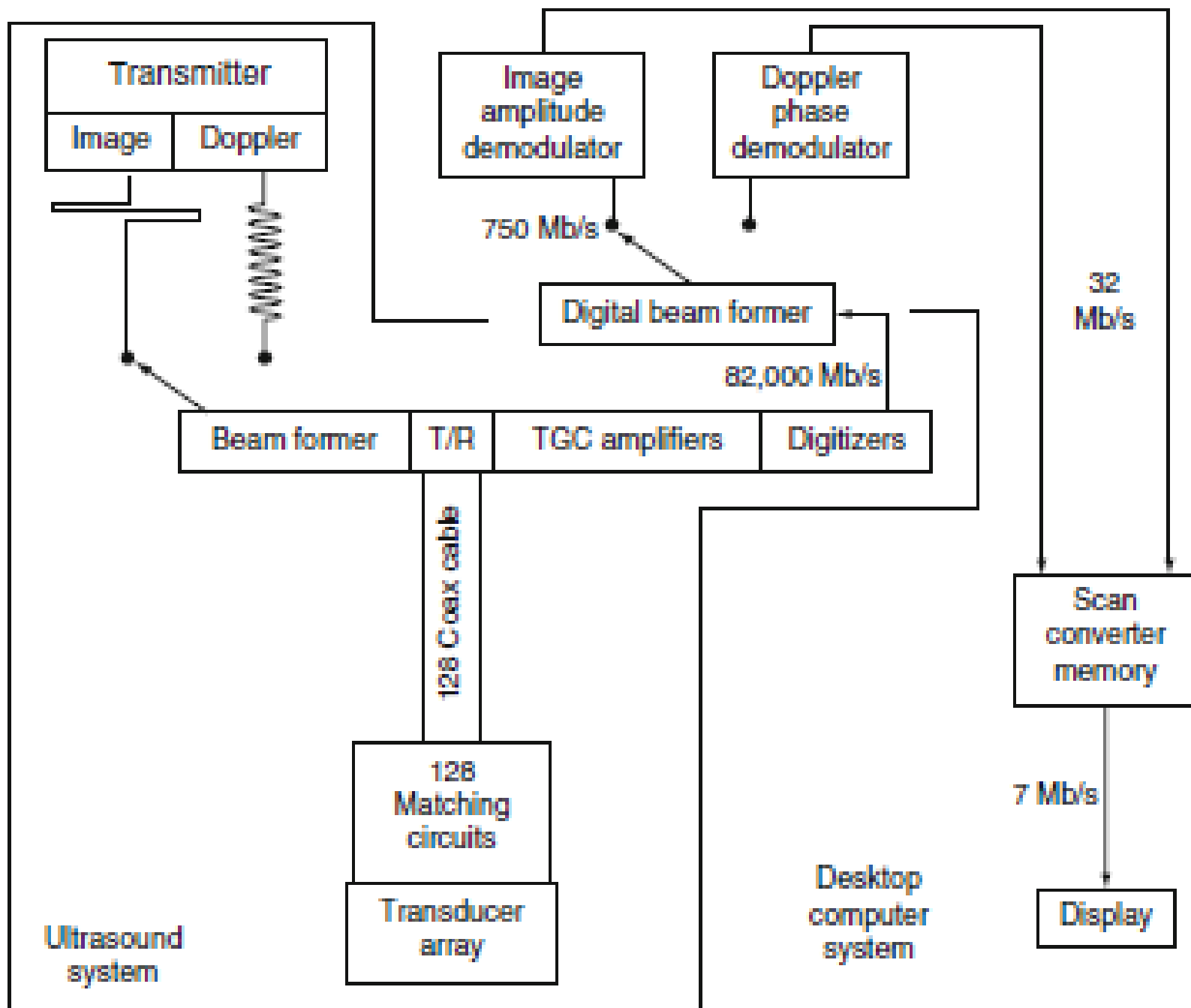
Scanning, moving the beam direction

## multi unit linear array



## multi unit curved array





## Doppler-effect : how to measure velocity of targets



Standing source  
frequency is constant  
 $f$



Moving source  
frequency changes  
("observed freq.")  
 $f'$

If the target, which reflects ultrasound is moving, then it also acts as a moving source...

$f'$ : **observed frequency**,  $f$ : original frequency

(a) standing source and moving observer ( $v_o$ )

+: observer approaches the source

–: observer moves away from the source

$$f' = f \left( 1 \pm \frac{v_o}{c} \right)$$

(b) moving source and standing observer

(if  $v_s \ll c$ , then „same” as (a))

$$f' = \frac{f}{1 \mp \frac{v_s}{c}}$$

(c) moving source and moving observer

$$f' = f \frac{1 \pm \frac{v_o}{c}}{1 \mp \frac{v_s}{c}}$$

(d) moving reflecting object (surface),

(if  $v_R \ll c$ )

$$f' = f \left( 1 \pm \frac{2v_R}{c} \right)$$



**Doppler frequency** = frequency change = frequency shift

if  $v_i, v_R \ll c$  (i= S or O)

rearranging equation (a)

**moving source or observer:**

$$\Delta f = f_D = \pm \frac{v_i}{c} f$$

rearranging equation (d)

**moving reflecting object  
or surface:**

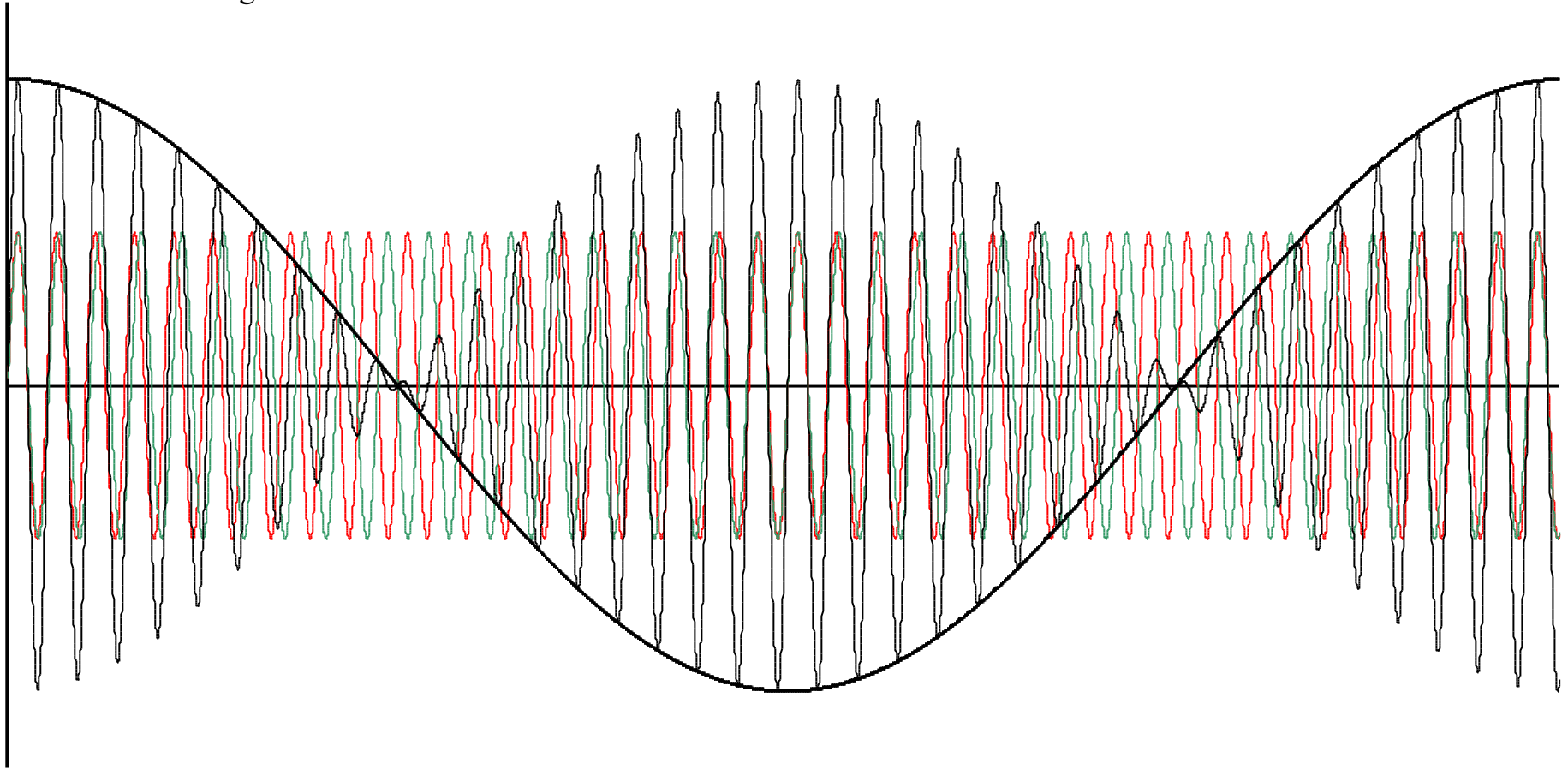
$$\Delta f = f_D = \pm 2 \frac{v_R}{c} f$$

if  $v$  and  $c$  are not parallel, then  $v \cos \theta$  should be used instead of  $v$  (remark: if  $\theta = 90^\circ$ ,  $f_D = 0$ )

# Beating phenomenon

the beating frequency equals to the difference of the two interfering frequency

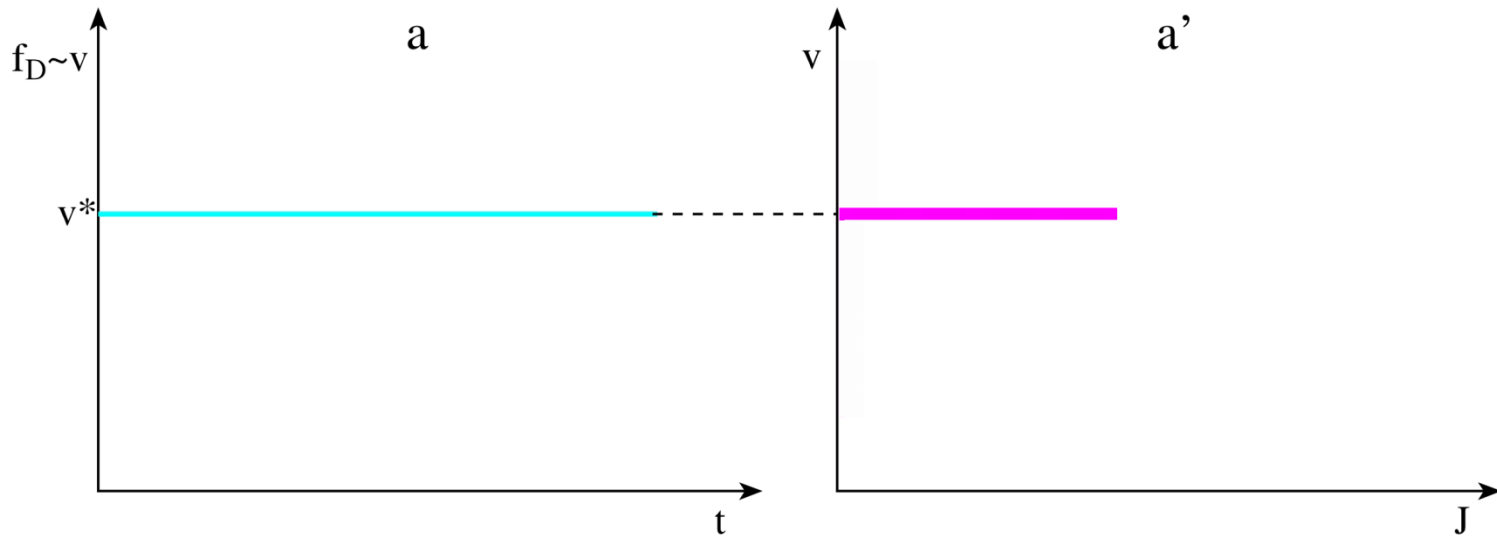
$$f_{\text{red}} \geq f_{\text{green}}$$



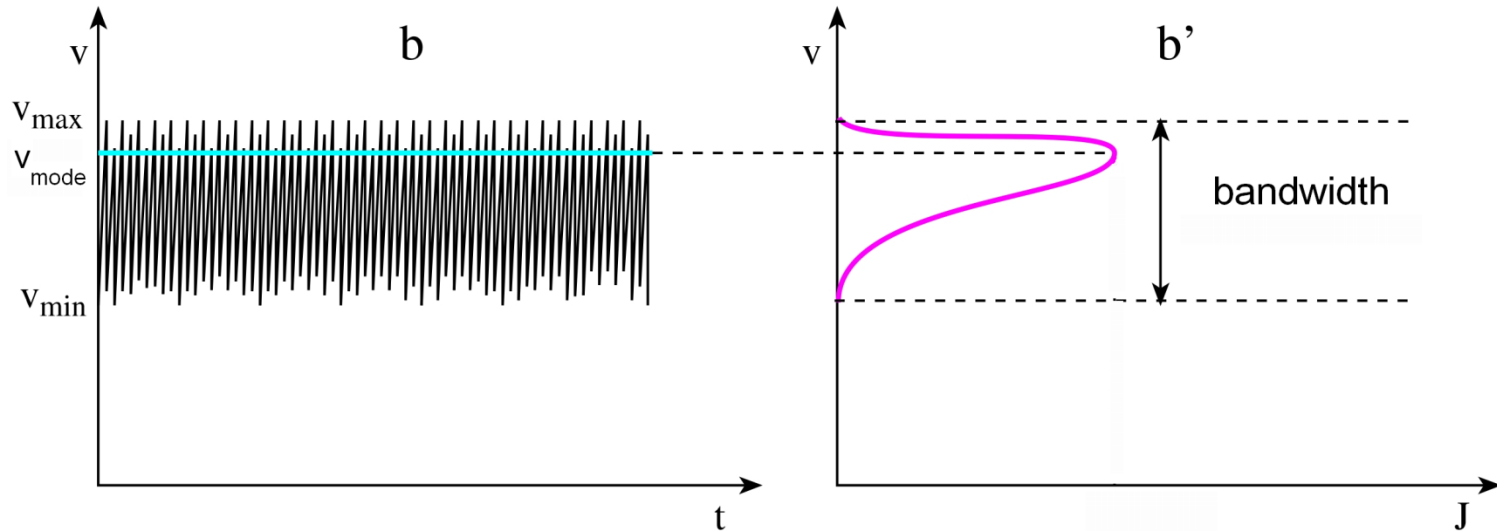
reminder:  $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$

# Doppler curves

one  
constant  
velocity ( $v^*$ )



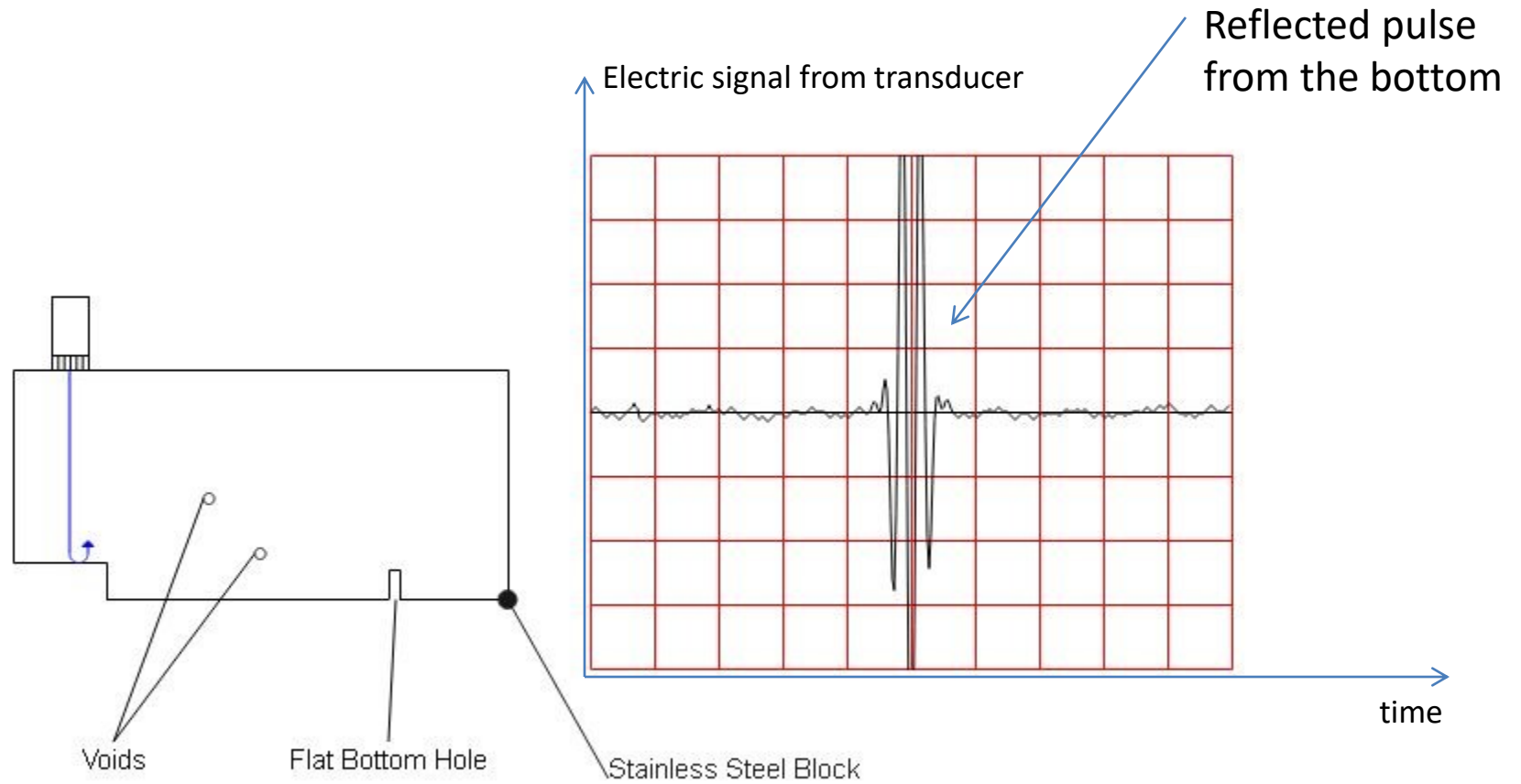
frequency  
distribution  
(with  $v_{\text{mode}}$ )

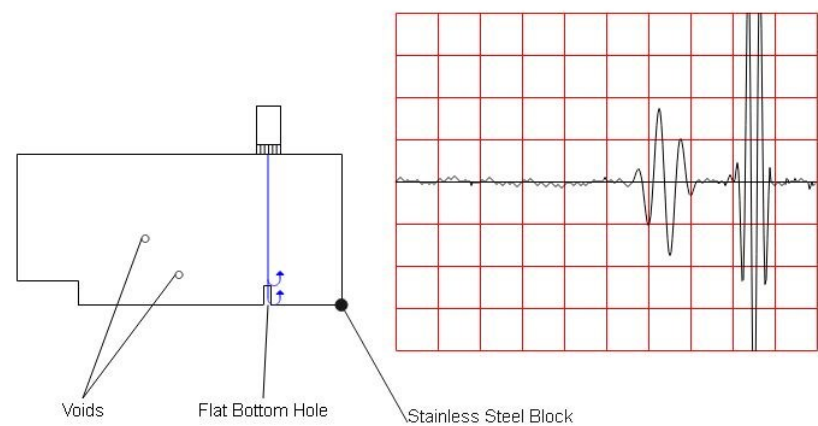
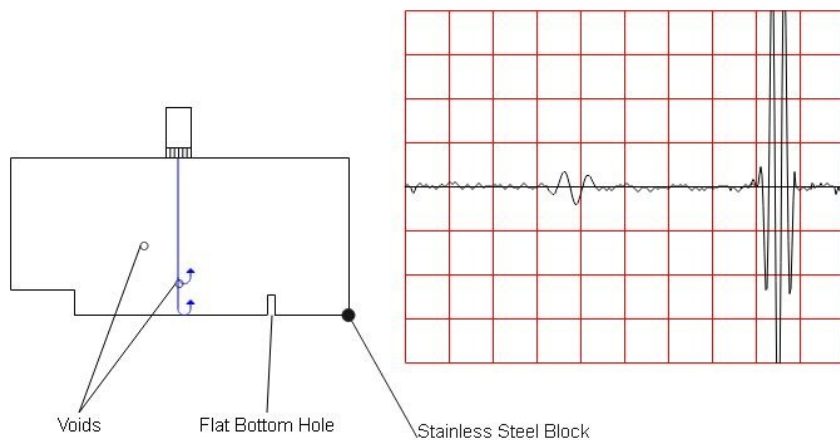
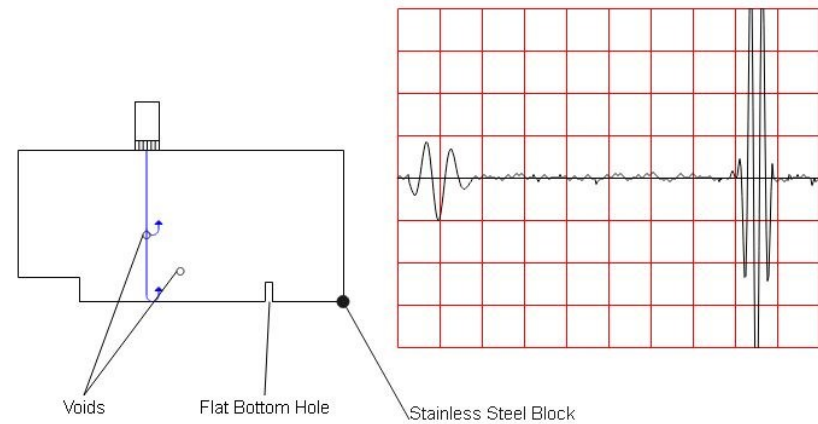
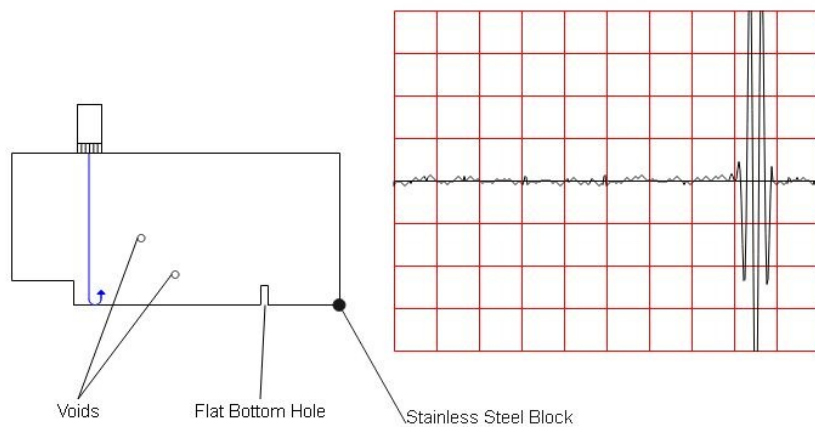


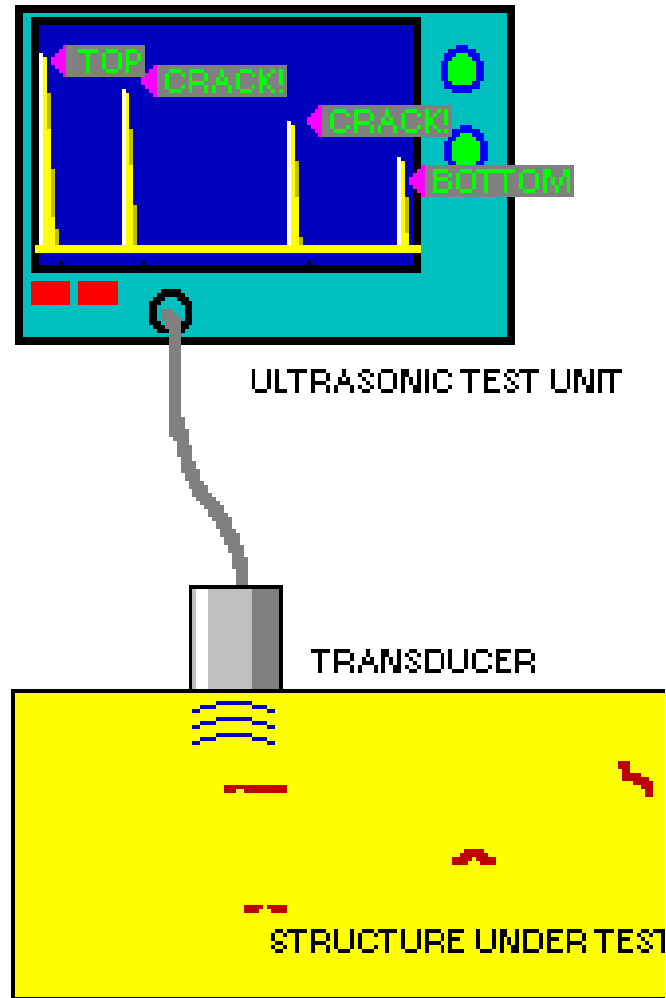
velocity distribution

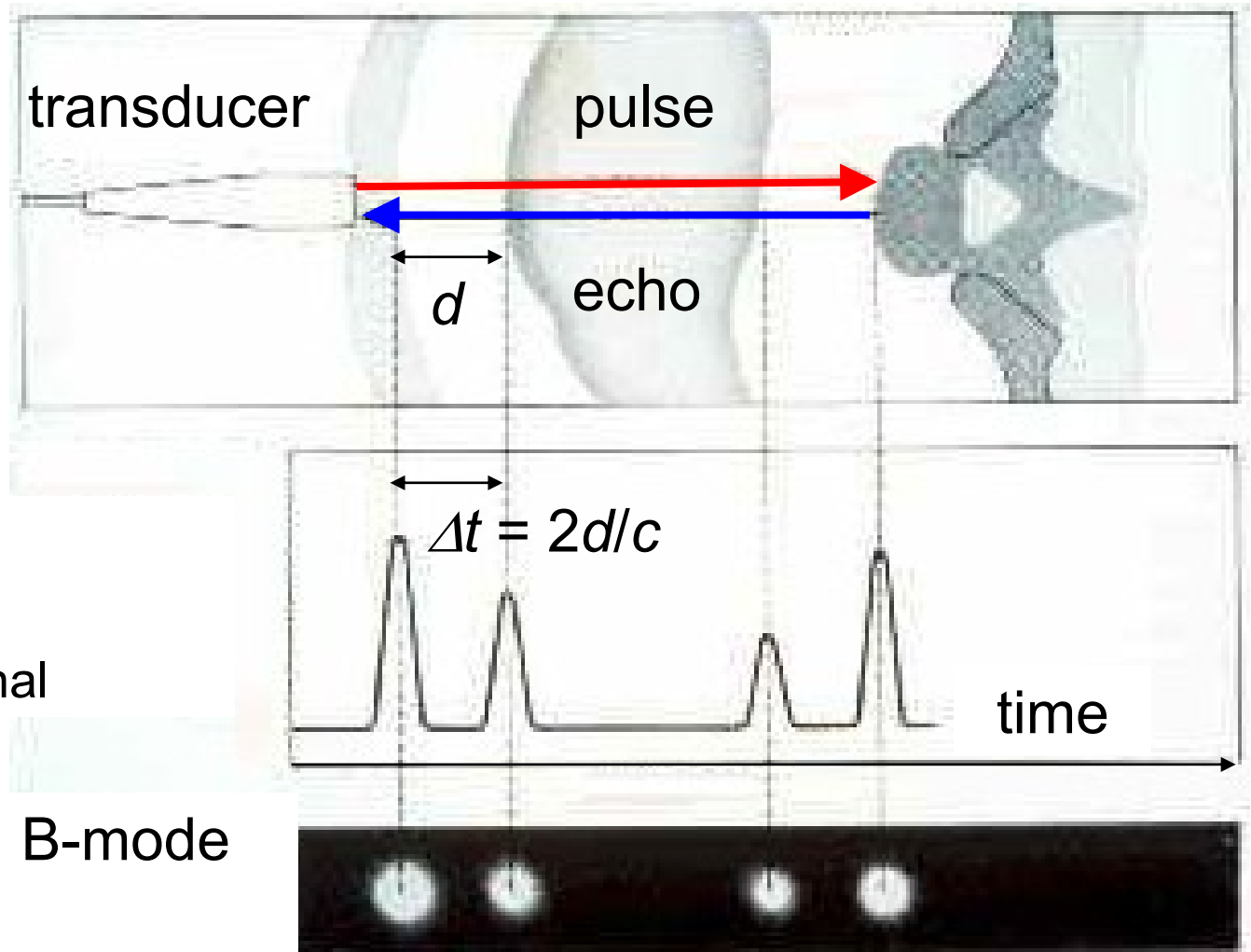
velocity distribution at  
a certain time

## Imaging surfaces by observing pulse echoes







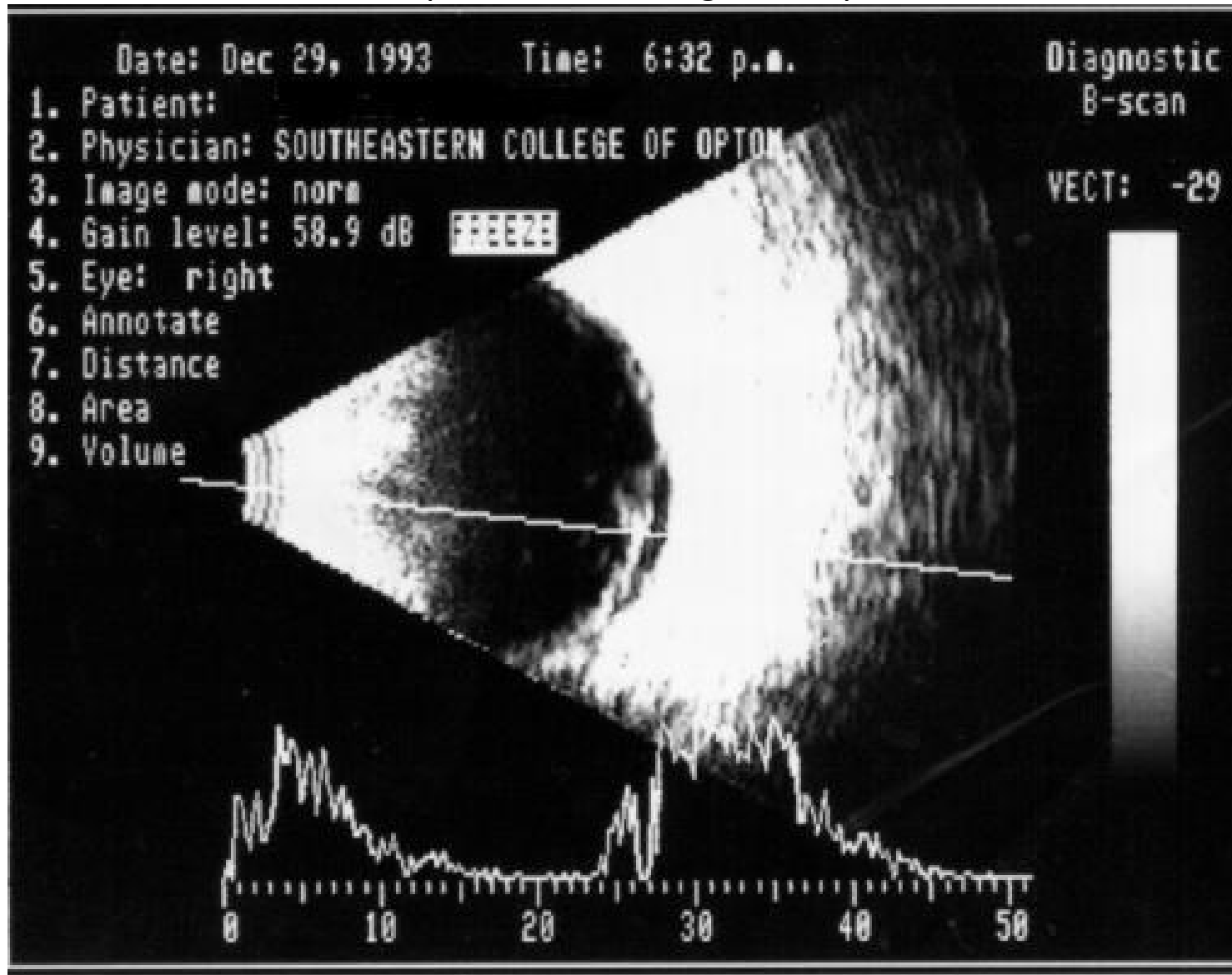


A-mode  
(**A**mplitude)  
only 1-dimensional

1-dimensional B-mode  
(**B**rightness)

A-mode and B-mode images of the same structure

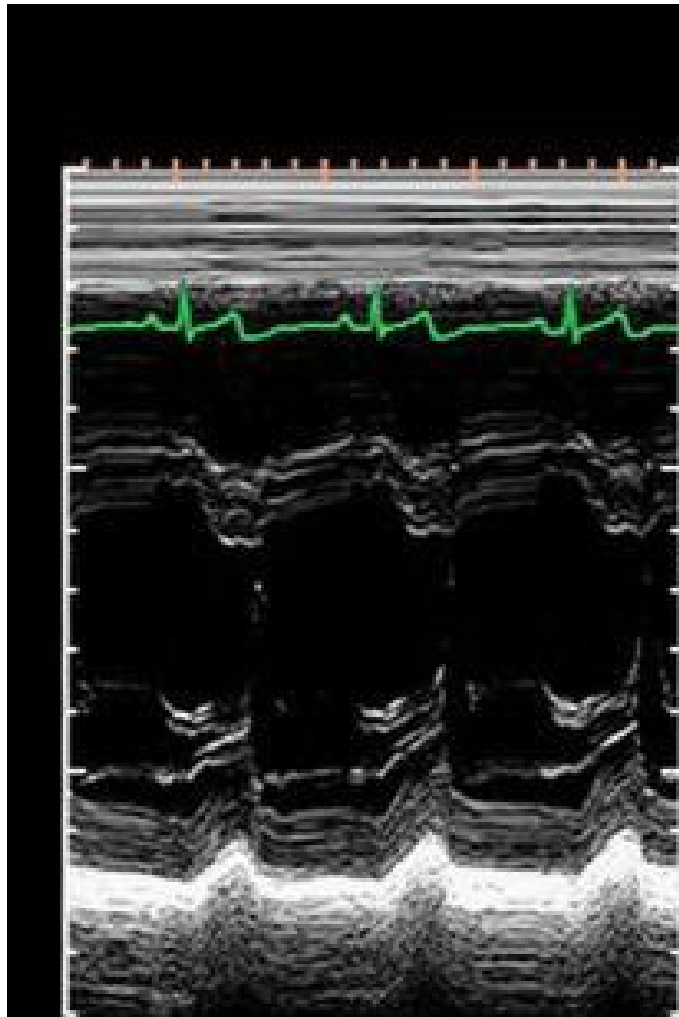
B-mode scan produces 2D images, composed of a series of 1D B-mode data





## TM-mode

Time-Motion mode: multiple 1D lines as the time goes on.



## B-mode



Doppler modes: calculate velocity from the freq. change.



BART: **Blue** Away **Red** Towards

Color-code the velocity calculated from  $f'$  or from the beating



power Doppler

Show the intensity of the doppler signal

In the doppler method **the angle is important!**

Example: 1D CW doppler with beating detection

CW: continuous wave

source and detector are separated

$$|f_D| = 2 \frac{v_R \cos \theta}{c} f$$

e.g.  $f=8000$  kHz

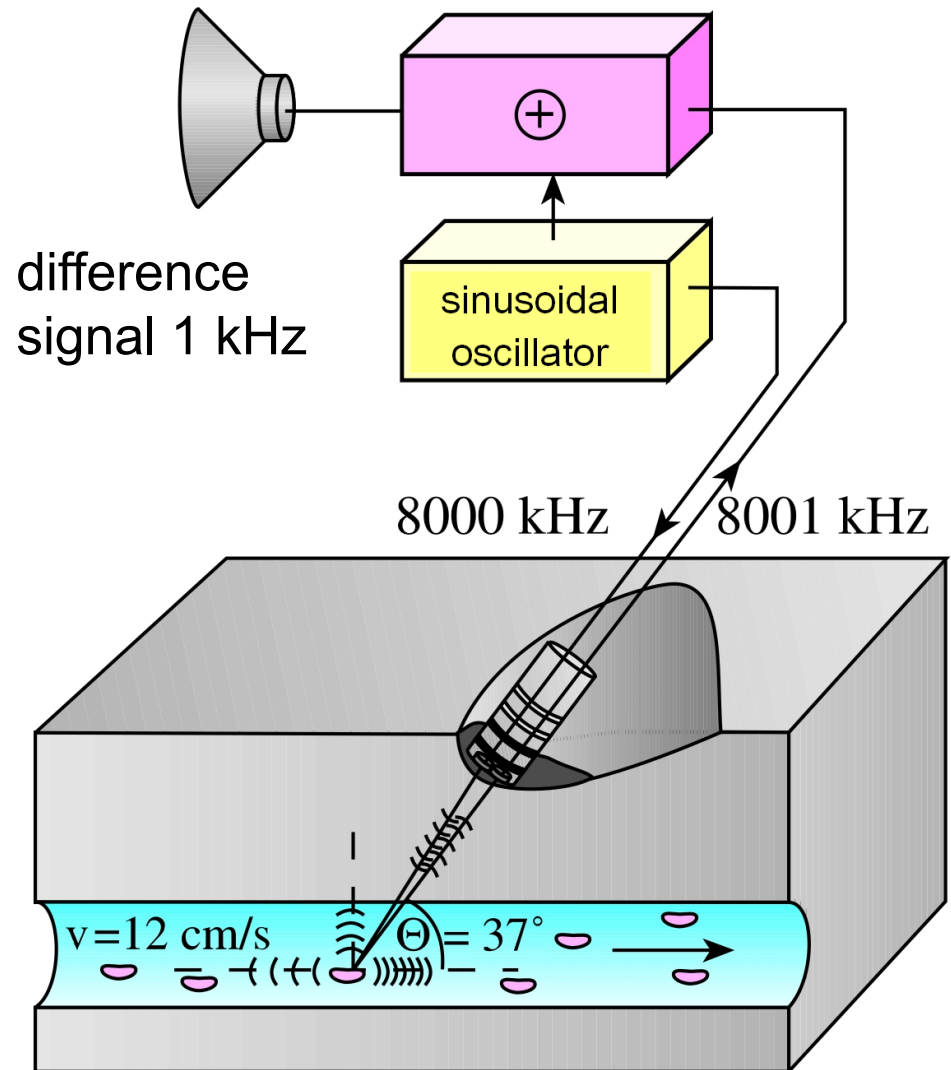
$v=12$  cm/s

$c=1600$  m/s

$\theta = 37^\circ$

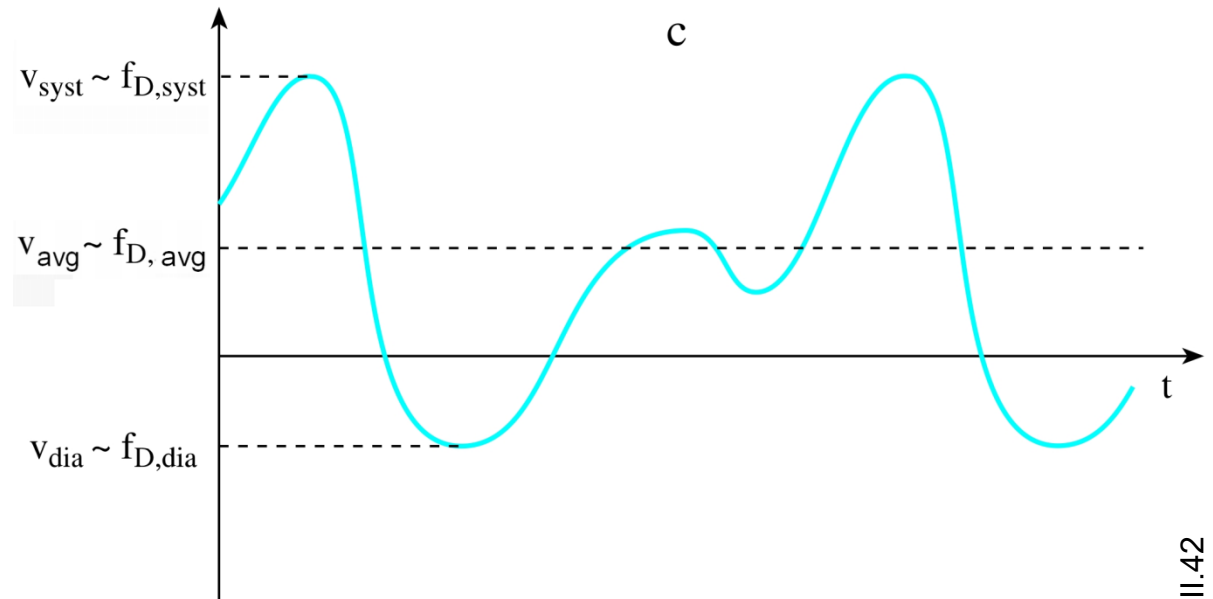
$\Rightarrow f_D = 1$  kHz

(beating phenomenon)

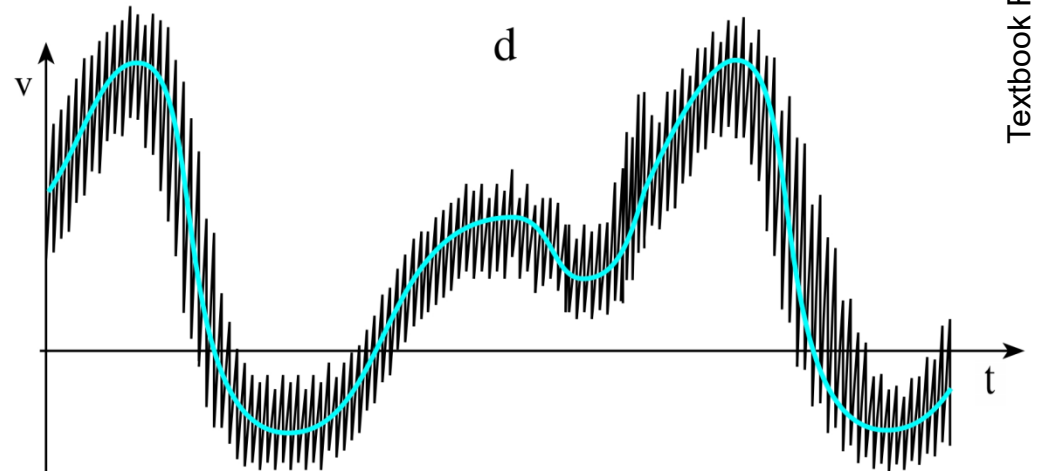


# Doppler curves

flow can be represented by one velocity in each moment



flow can be represented by a velocity distribution in each moment



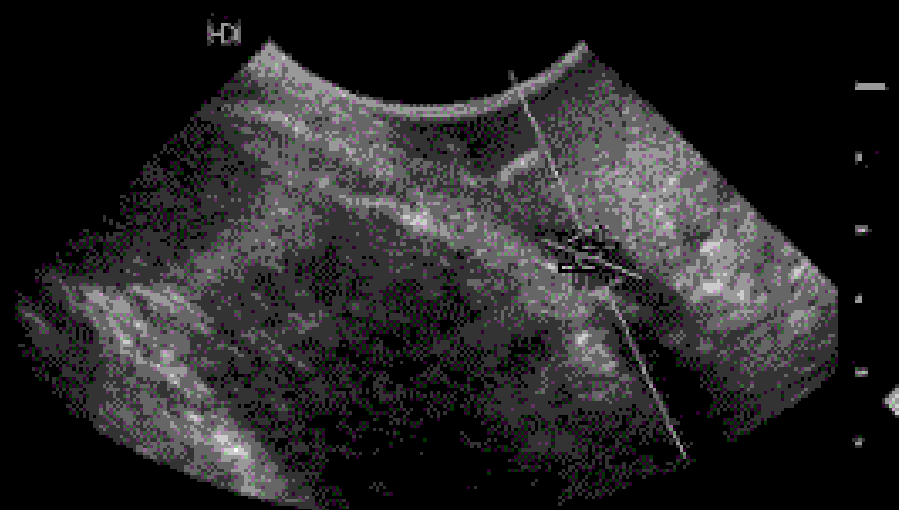
Textbook Fig. VIII.42

velocity distribution in TM-mode

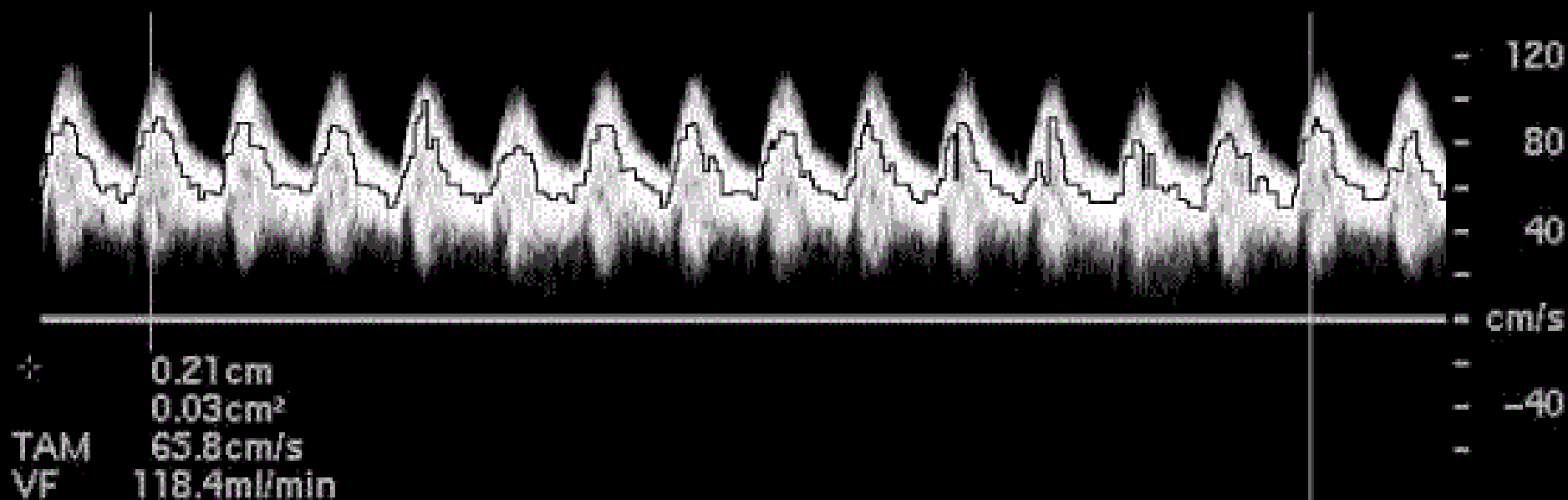
AT PIG 695 LAM  
University Hospital CB-5 PVasc/Ven

27 Jun 00 TIs 0.2 MI 0.2  
5:25:42 pm F# 73 3.0 cm

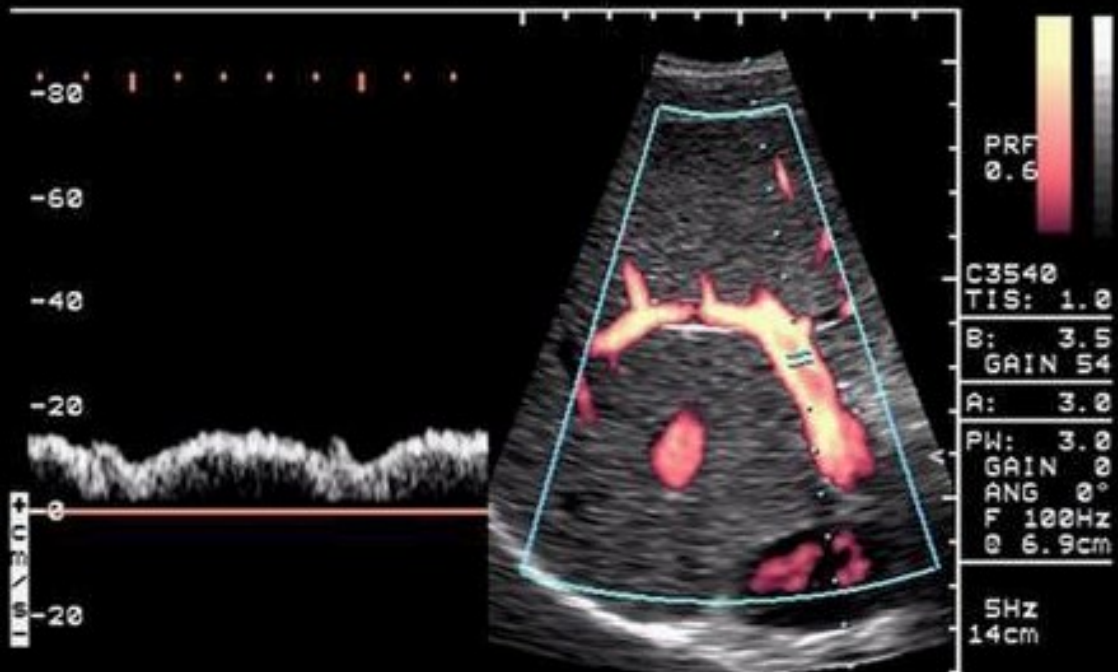
Map 8  
DynRg 50dB  
Persist Med  
Fr Rate Med  
2D Opt:Res



SV Angle  $-46^{\circ}$   
Dep 1.5 cm  
Size 4.0 mm  
Freq 5.0 MHz  
WF Low  
Dop 68% Map 2  
PRF 10000Hz



# Normal portal vein flow

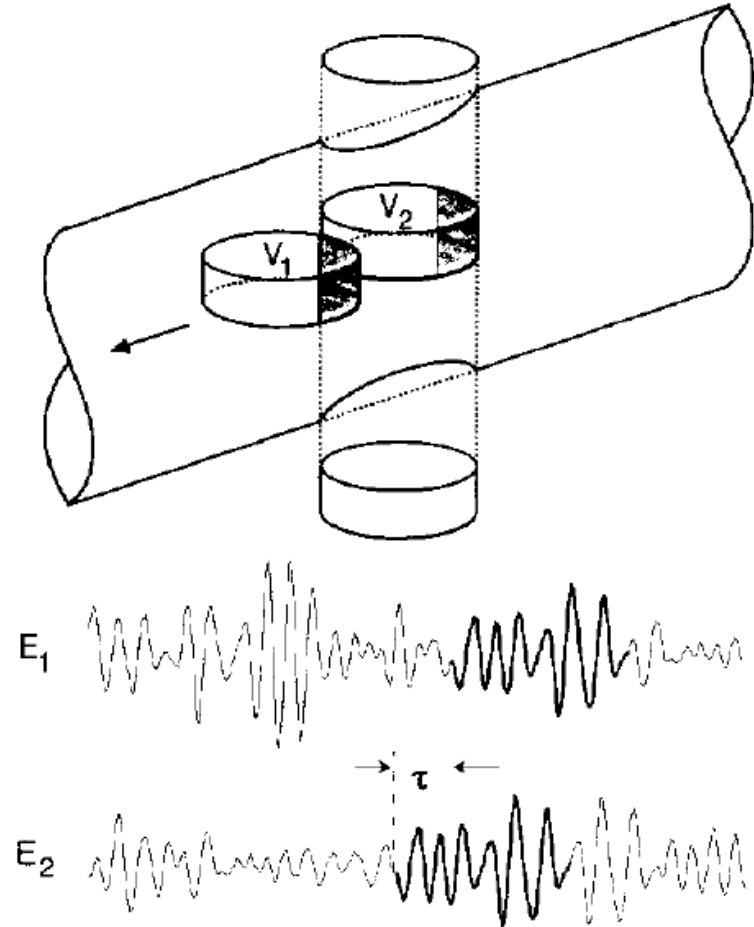


## CVI method

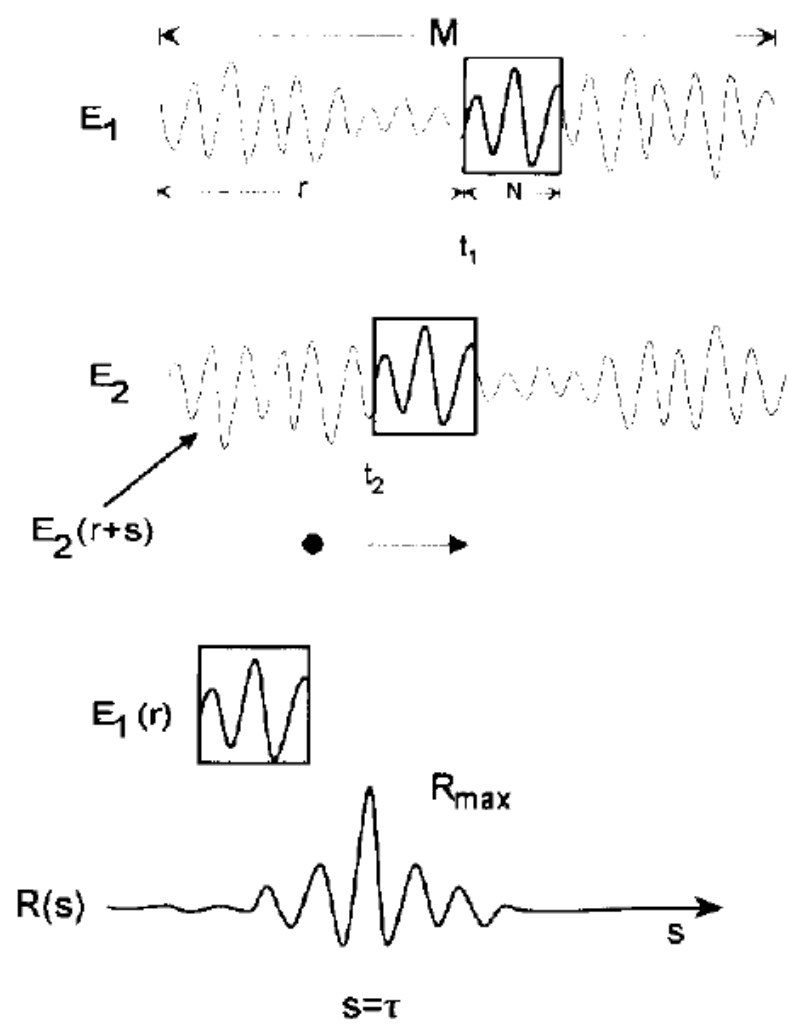
(time domain correlation method, CVI = color velocity imaging)

If the reflecting surface/scattering center moves, the detected ultrasound signal changes over time, if we set the location.

However, we detect a similar-patterned ultrasound signal a little further away in space (where the reflecting surface/scattering center has moved).



Extension material



We cut out an N-long sequence of the echo signal  $E_1$

Correlate it with the  $E_2$  echo signal at different delay positions ( $s$ )

check where the correlation is maximal, here the delay =  $\tau$

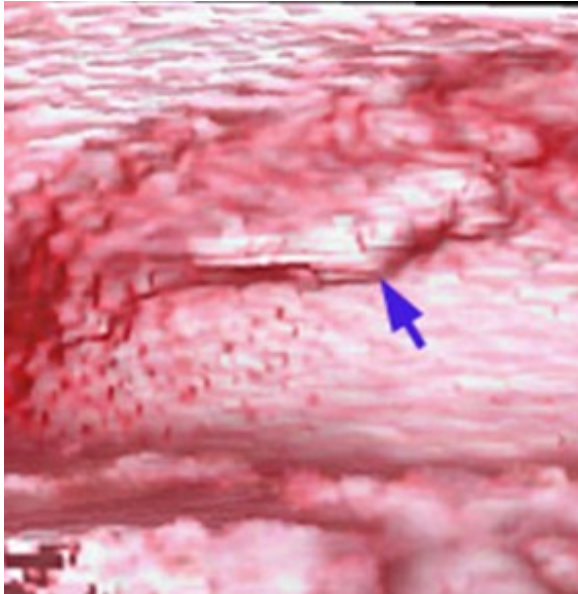


# 3D reconstruction

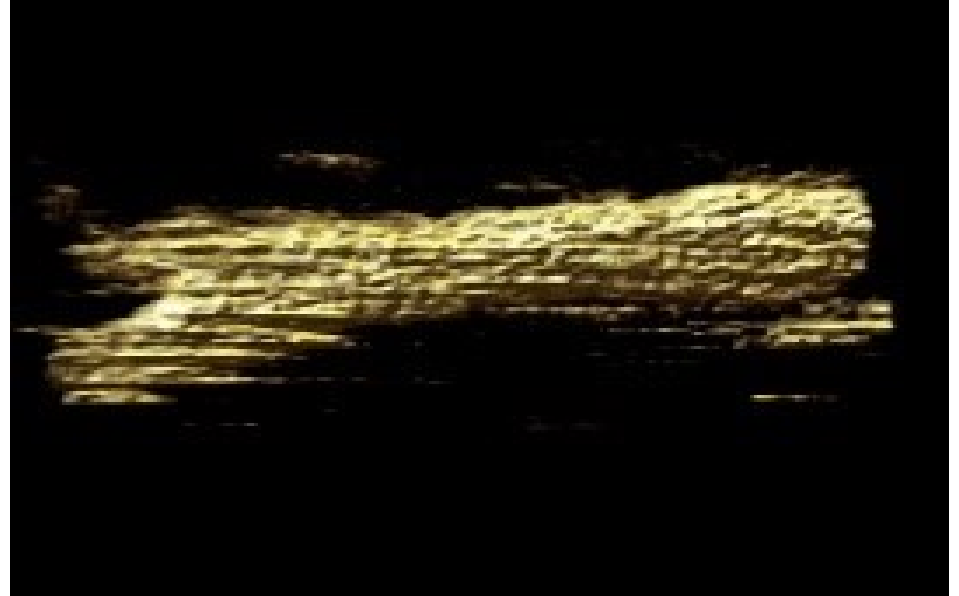
face of a fetus



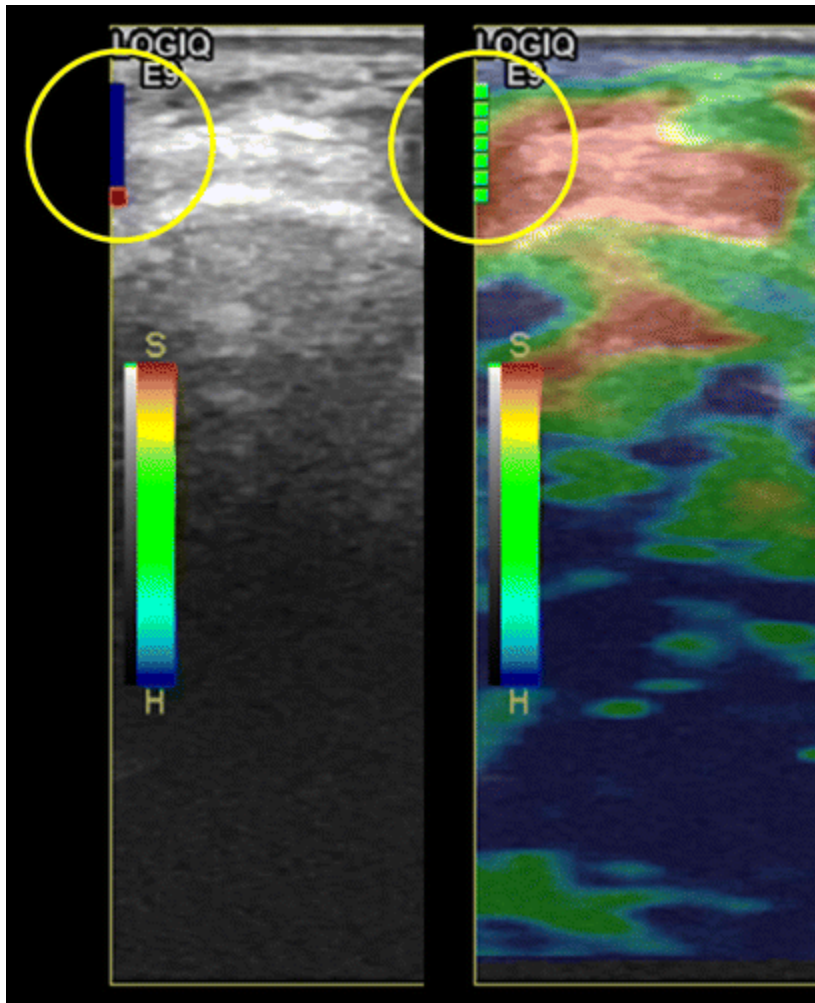
bladder



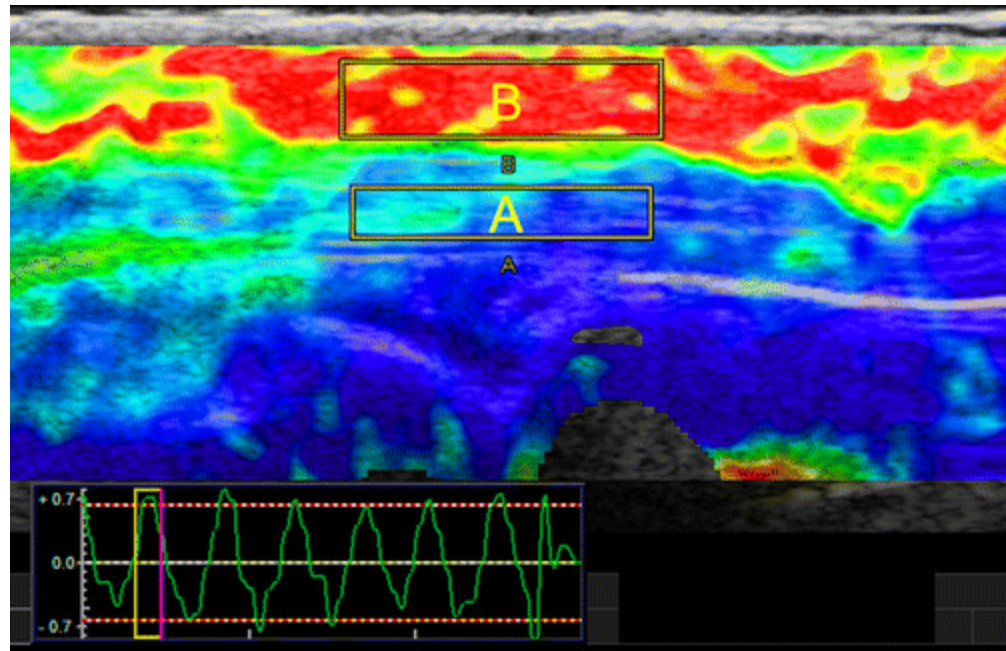
carotis

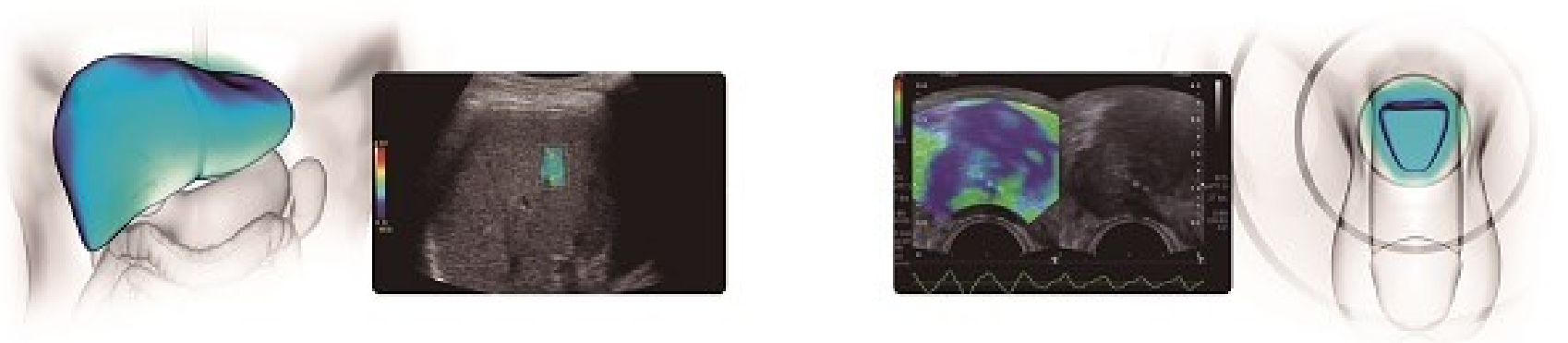


# Sonoelastography

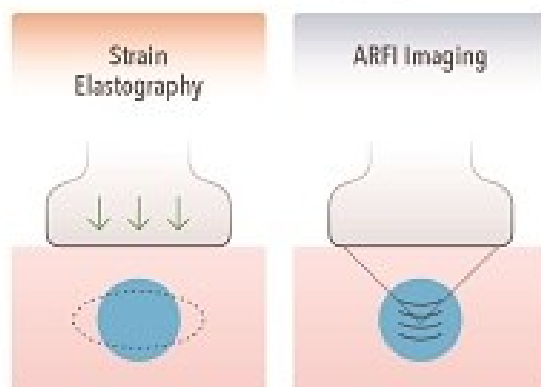


$$\frac{F}{A} = E \frac{\Delta L}{L}$$

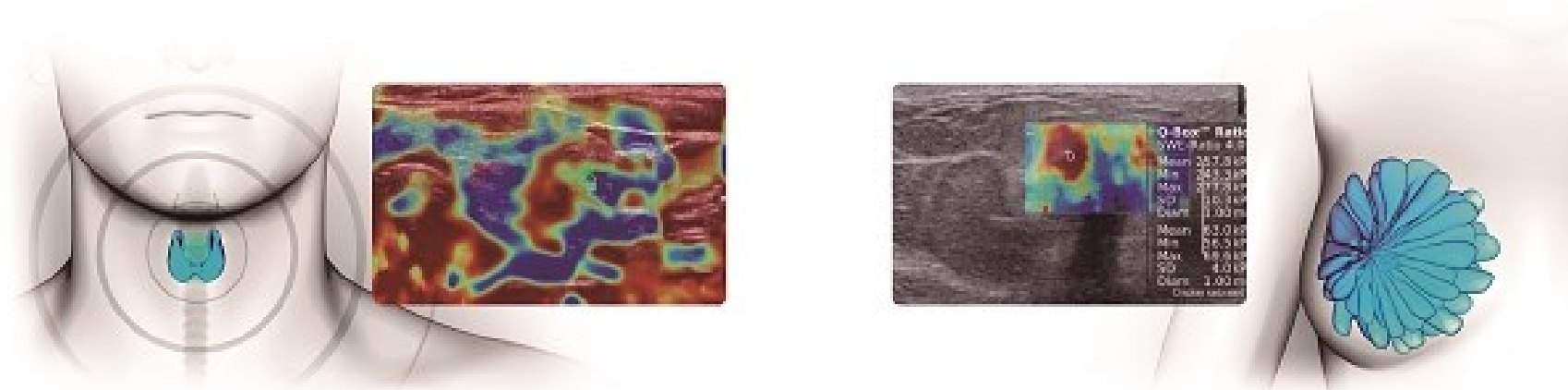
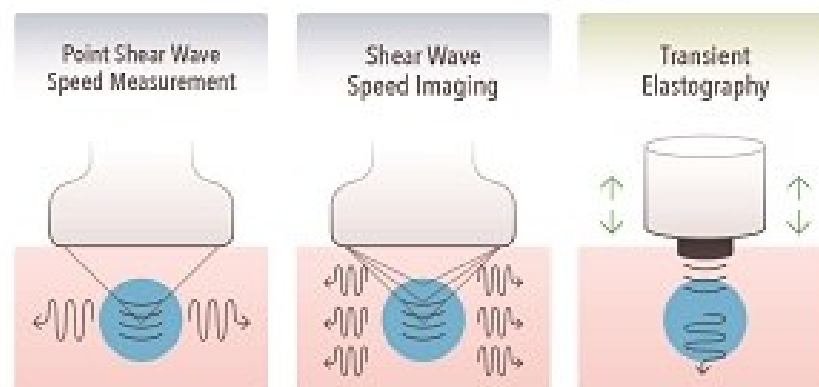




## Strain Imaging



## Shear Wave Imaging



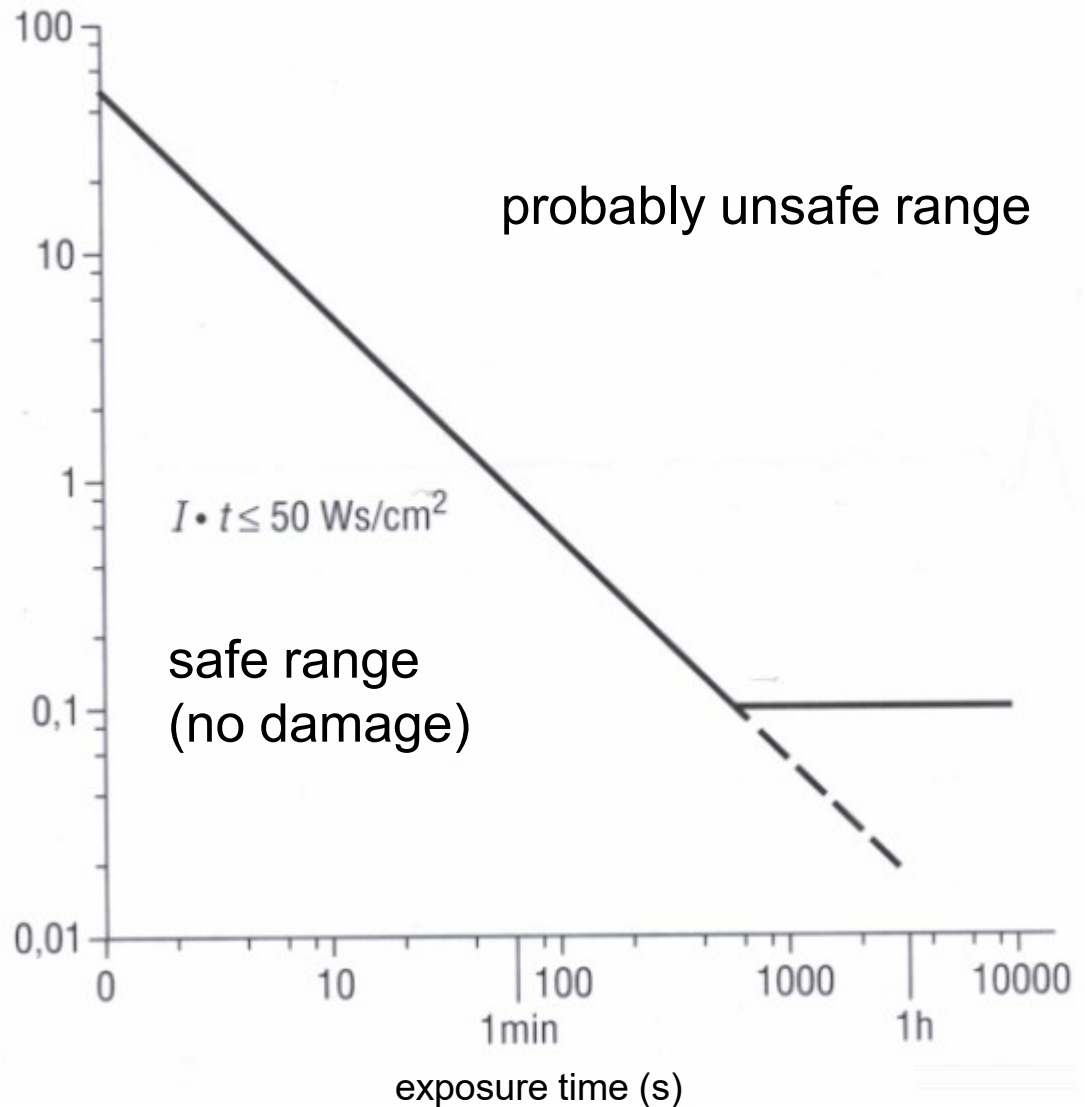
# Safety

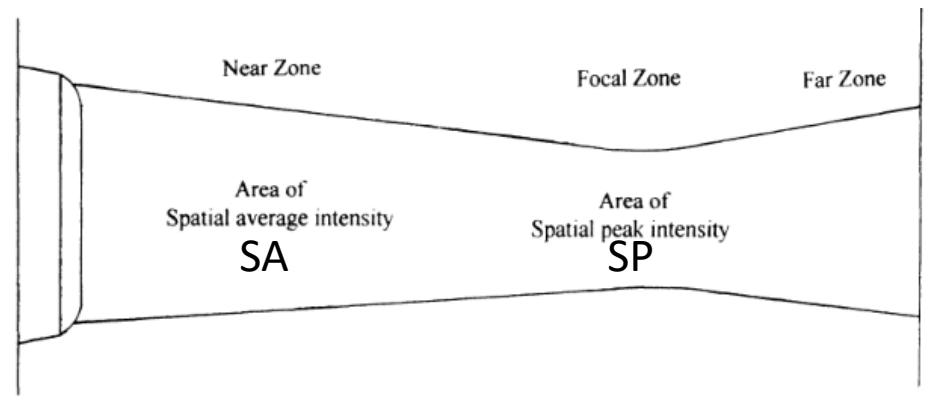
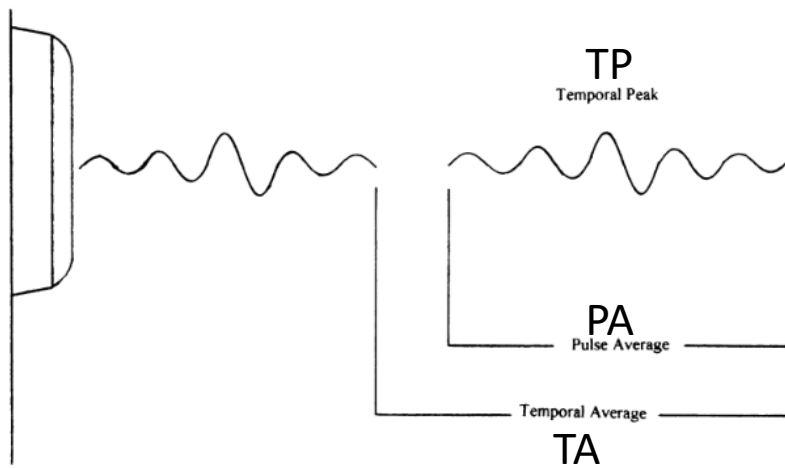
in the diagnostics:

$$10 \text{ mW/cm}^2 = 100 \text{ W/m}^2$$

spatial average temporal  
average (SATA) intensity;  
spatial peak temporal peak  
(SPTP) intensity;  
spatial peak temporal average  
(SPTA) intensity;  
spatial peak pulse average  
(SPPA) intensity  
spatial average pulse average  
(SAPA) intensity

intensity  
(W/cm<sup>2</sup>)





JDMS 10:278-282

spatial average temporal average (SATA) intensity; spatial peak temporal peak (SPTP) intensity; spatial peak temporal average (SPTA) intensity; spatial peak pulse average (SPPA) intensity ; spatial average pulse average (SAPA) intensity

Mechanical index = peak negative pressure /  
SQRT(center frequency of the US beam)

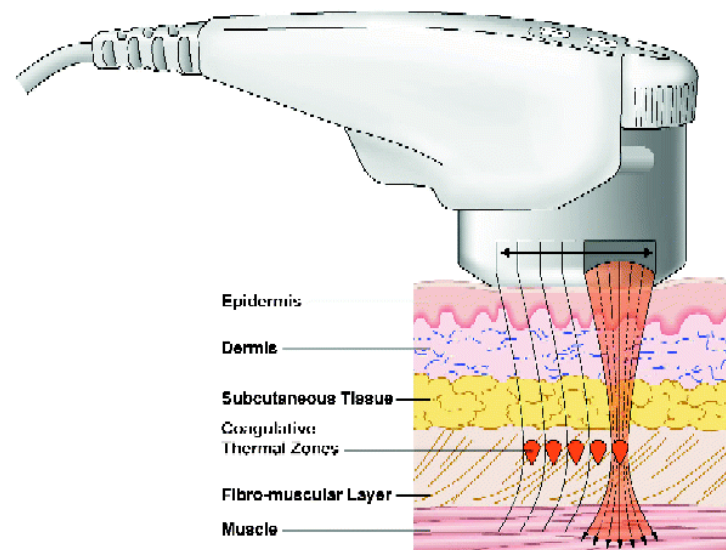
$$MI = \frac{\text{peak negative pressure}}{\sqrt{\text{center frequency of US beam}}}$$

**Thermal index =  $W_p / W_{deg}$**

$W_p$ : relevent (attenuated) acoustic power at the depth of interest

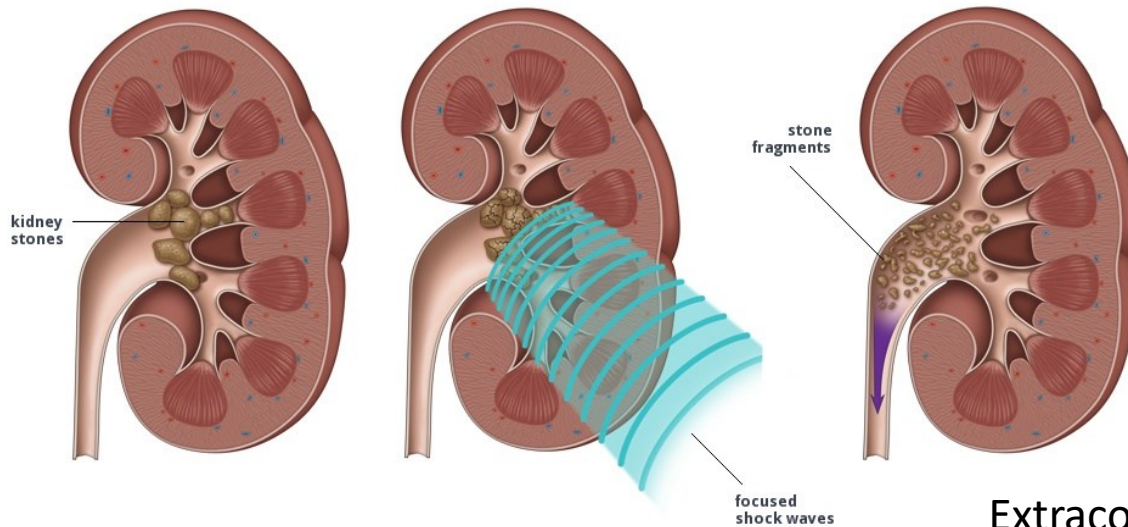
$W_{deg}$ : estimated power necessary to raise the tissue equilibrium temperature by 1 °C

TEMPERATURE INCREASES	INDICATIONS
Non-thermal	Acute injury/Tissue healing
Mild thermal (1° C)	Sub-acute injury/Tissue healing
Moderate thermal (2-3° C)	Chronic inflammation, pain, trigger points
Vigorous heating ( $\geq 4^{\circ}$ C)	Stretch collagen
Data from Draper et al. 2013 <sup>1</sup>	





in the therapy:  $1 \text{ W/cm}^2$



## Extracorporeal Shockwave Therapy

Shockwave therapy is not „genuine US”

