Mathematical and Physical Basis of Medical Biophysics

Supplementary material for the
„Medical Biophysics” and „Biophysics” courses

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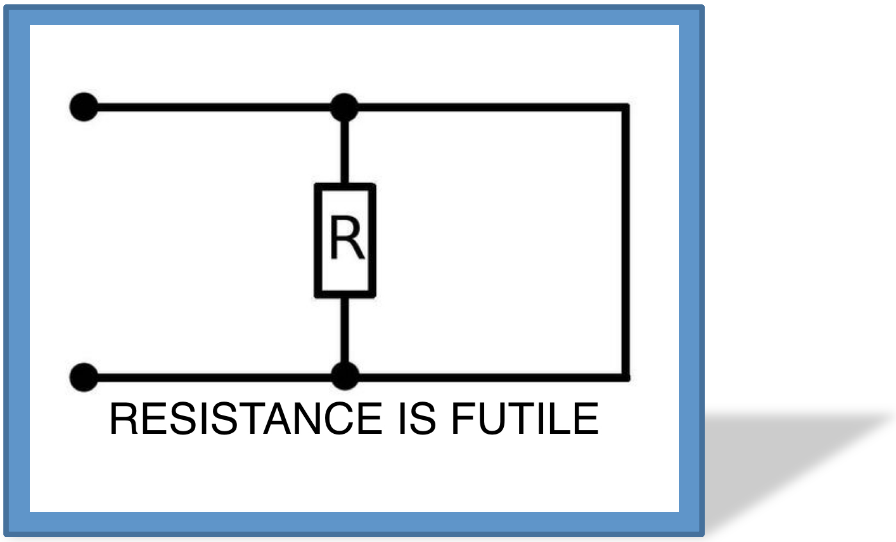

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Foreword

The subjects of „Medical biophysics” and „Biophysics” require that the students possess some basic knowledge in physics. We cannot, and should not, build knowledge on weak or completely missing foundations. The aim of this material is to give a short and concise summary of the knowledge required for the mentioned courses. However, this knowledge will be useful in other subjects as well, such as chemistry, biochemistry, physiology, etc.

Please, be aware that the problems listed in this supplementary material will be part of the exam of the above-mentioned courses.

This material contains only the foundations of mechanics, thermodynamics and electronics, since the basis of all the other disciplines (i.e. optics) can be found in the „Medical Biophysics” or „Biophysics” course materials. In all the chapters the terms and laws are listed with bold characters similarly to an encyclopedia but in a logical rather than alphabetical order. Every chapter starts with a short introduction that points out the medical relevance of the field. At the end of the chapters you will find problems with solutions that help understand the material and improve the application of the knowledge. (You will find detailed solutions at some of the problems, which are indicated with the following symbol: )

Budapest, 12th August 2016 Ferenc Tölgyesi

This material is a translation of the Hungarian version. Any comments and remarks on the language, grammar and the technical terms used in this material are greatly appreciated. Please, send your comments directly to the translators:
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1. Mathematical Aid

Mathematics is essential in physics, but it is also important and widely used in chemistry, biology and all other sciences. Here we will only give a short summary of the basic mathematics that is frequently used during the biophysics courses. Since this chapter is focusing on math, the physical terms found in the problems will be explained in later chapters.

Powers of ten: a power of 10 is any of the integer power n (exponent) of the number ten, written as 10^n.
Some examples:

- $n = 0$: $10^0 = 1$
- n positive: $10^1 = 10$ $10^2 = 100$ $10^3 = 1000$ $10^4 = 10\,000$ $10^5 = 100\,000, ...$
- n negative: $10^{-1} = 0.1$ $10^{-2} = 0.01$ $10^{-3} = 0.001$ $10^{-4} = 0.000\,1$ $10^{-5} = 0.000\,01, ...$

Note: we are using comma as the decimal mark throughout this book

Exponential identities (by the example of powers of ten):

- $10^n \cdot 10^m = 10^{n+m}$ e.g.: $10^8 \cdot 10^{-2} = 10^6$
- $\frac{10^n}{10^m} = 10^{n-m}$ e.g.: $\frac{10^5}{10^{-5}} = 10^{5-(-5)} = 10^{10}$
- $\left(10^n\right)^m = 10^{n \cdot m}$ e.g.: $\left(10^3\right)^3 = 10^9$

Scientific notation: Scientific notation or normal form is a product, in which the first factor is called the coefficient (m) which is a number greater than or equal to one and smaller than ten ($1 \leq m < 10$), and the second factor is an integer power (n) of ten:

$$m \cdot 10^n$$

For example, the scientific notation of the number 325 000 is given as $3.25 \cdot 10^5$. Further examples:

- $5\,300\,000 = 5.3 \cdot 10^6$
- $105\,000\,000 = 1.05 \cdot 10^8$
- $0.000\,000\,5 = 5 \cdot 10^{-7}$
- $0.000\,000\,006\,6 = 6.6 \cdot 10^{-9}$

In case of numbers greater than one the scientific notation will have a positive exponent, in case of fractions the exponent is negative.
With the scientific notations very large and very small numbers can be written in a more compact form, which is often useful in biophysics, for example:

- speed of light in vacuum: about 300 000 000 m/s = $3 \cdot 10^8$ m/s
- elementary charge: 0.000 000 000 000 000 000 16 C = $1.6 \cdot 10^{-19}$ C (Coulomb)
- number of red blood cells in one liter of blood: about 5 000 000 000 000 = $5 \cdot 10^{12}$
- thickness of the cell membrane: about 0.000 000 01 m = $1 \cdot 10^{-8}$ m

Rounding: The result given by a calculator often contains an unnecessary number of digits. For example, let us calculate the average body density from the measured body mass ($m = 72.5$ kg) and body volume ($V = 69.5$ Liters = 0.069 5 m^3). Density is the ratio of mass to volume, thus we divide 72.5 by 0.069 5 with a calculator. The result that appears on the display is 1043.165468 (in the unit of kg/m^3). The six digits after the decimal mark are not only unnecessary for a medical doctor, but considering the precision of mass and volume measurement they are misleading as well. Therefore, the result should be rounded – but how?
In the biophysics courses we will always round to **three significant digits!** Starting from the left, the first digit that is not zero is the first significant digit. From here we count out the third significant digit and round it depending on

the value of the fourth digit. If the fourth digit is between 0 and 4, then you leave the third significant place as it is; if, however, it is between 5 and 9, you round the third significant place up by one. According to this rule the rounded value of the above calculated density is 1 043.165 468 kg/m^3 ≈ 1 040 kg/m^3.
Further examples:

- 128 845 = 129 000
- 25.910 78 = 25.9
- 1.929 856 = 1.93
- 0.002 385 555 = 0.002 39
- 0.010 998 589 = 0.0110

It will not be a mistake in the test if you use the equals sign (=) instead of the approximately equals sign (≈) when giving the rounded result of a calculation.

Common logarithm: (a.k.a. decadic logarithm, decimal logarithm, ten-based logarithm) Logarithm is the inverse mathematical operation of exponentiation. Common logarithm is the inverse of the ten-based exponentiation (powers of ten). The symbol of common logarithm can be „$\log_{10}$" or „log". In the biophysics courses we will mostly use the symbol „lg". When we want to calculate the common logarithm of any number (*a*), we are looking for the power of 10 which will give *a* as a result, thus:

$$\lg a = x \quad \Leftrightarrow \quad 10^x = a\,.$$

Examples:

$$\lg 1000 = 3 \quad \Leftrightarrow \quad 10^3 = 1000$$

$$\lg 1 = 0 \quad \Leftrightarrow \quad 10^0 = 1$$

$$\lg 0.01 = -2 \quad \Leftrightarrow \quad 10^{-2} = 0.01$$

In other words, we can say that finding the logarithm of number *a* means to solve the equation $a = 10^x$ for *x*.

We can see even better that the logarithm and the exponentiation are inverse operations if we write the above equations in the following form:

$$10^{\lg a} = a \quad \text{or} \quad \lg\left(10^x\right) = x$$

In other words, 10 raised to the power of the logarithm of a number is equal to this number. *Vice versa*, the logarithm of 10 raised to the power of a number is equal to this number. Thus, logarithm and exponentiation cancel each other out.

Natural logarithm: Similar to common logarithm, but the base number is *e* (Euler's number, *e* = 2.71828...) and its symbol is „ln" (*logarithmus naturalis*). The solution for „ln *a*" gives the power of *e* that will give *a*:

$$\ln a = x \quad \Leftrightarrow \quad e^x = a$$

The inverse operation applies to natural logarithm, too:

$$e^{\ln a} = a \quad \text{or} \quad \ln\left(e^x\right) = x$$

Logarithmic identities (by the example of common logarithm):

- $\lg(a \cdot b) = \lg a + \lg b$ e.g.: $\lg 25 + \lg 4 = \lg(25 \cdot 4) = \lg 100 = 2$
- $\lg\left(\frac{a}{b}\right) = \lg a - \lg b$ e.g.: $\lg 20 - \lg 200 = \lg\left(\frac{20}{200}\right) = \lg 0.1 = -1$
- $\lg\left(a^n\right) = n \cdot \lg a$ e.g.: $2 \cdot \lg 5 + 2 \cdot \lg 2 = \lg\left(5^2\right) + \lg\left(2^2\right) = \lg 25 + \lg 4 = \lg 100 = 2$

Equations: The laws of physics give the correlation between different quantities, which in practice can be used to determine an unknown quantity if we know the other quantities. The physical law is treated as a mathematical equation, from which the unknown quantity (usual symbol: *x*) can be found. During the biophysics courses you will use different types of equations, most frequently the linear, quadratic, trigonometric and exponential equations.

Linear equation with one variable: The equation only contains one variable x on the power of one. Example:

$$4x + 5 = 33$$

When solving these equations for x, first we subtract 5 from both sides:

$$4x = 28$$

then we divide both sides by 4:

$$x = \frac{28}{4} = 7$$

A physical example: During a drag race in 2015, a race car moved a distance of $s = 201$ m with constant acceleration during the time interval of $t = 6.51$ s. Calculate the acceleration of the race car. The physical law describing the distance-time correlation in case of constant acceleration is:

$$s = \frac{1}{2}at^2$$

in which a is acceleration, which is the unknown. To solve the equation for a, first we multiply both sides by 2, then we divide both sides by t^2:

$$2s = at^2$$

$$\frac{2s}{t^2} = a$$

After substituting the values and rounding the result to three significant places we get: $402/6.51^2 = 9.49$ m/s^2. Note that when solving equations it is enough to give the units only with the final answer.

Quadratic equation with one variable: The equation contains x to the power of two. For example:

$$5x^2 - 26x = 24$$

The general form of the quadratic equation with parameters a, b and c:

$$a \cdot x^2 + b \cdot x + c = 0$$

The solution of the equation is called the quadratic formula:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

which gives two mathematically possible solutions x_1 and x_2 (together: $x_{1,2}$). To solve the example equation, first we rearrange it into the general form:

$$5x^2 - 26x - 24 = 0$$

Now the coefficients can be identified: $a = 5$, $b = -26$ and $c = -24$. Substitute them into the quadratic formula:

$$x_{1,2} = \frac{+26 \pm \sqrt{676 + 480}}{10} = \frac{26 \pm 34}{10}$$

The two roots (solutions) are:

$$x_1 = \frac{26 + 34}{10} = 6$$

$$x_2 = \frac{26 - 34}{10} = -0.8$$

A physical example: Another racecar on the drag race moved with an acceleration of $a = 6$ m/s^2. Calculate the amount of time needed the run a distance of $s = 201$ m. In this case, t is the unknown in the physical law:

$$s = \frac{1}{2}at^2$$

Substituting the values and rearrange into the general form:

$$201 = \frac{1}{2}6t^2$$

$$3t^2 + 0t - 201 = 0$$

Using the quadratic formula:

$$t_{1,2} = \frac{0 \pm \sqrt{0 + 2412}}{6} = \frac{\pm 49.11}{6} = \pm 8.19 \text{ s}$$

From the two solutions only +8.19 s is relevant in the physical context and –8.19 s is meaningless. However, in case of quadratic equations, where coefficient b equals zero, the following simplified solution can be used:

$$t = \pm\sqrt{\frac{2s}{a}} = \pm\sqrt{\frac{402}{6}} = \pm 8.19 \text{ s}$$

Linear system with two variables: Two linear equations with two unknown variables (x and y). Example:

$$3x + 2y = 14$$
$$x + 4y = 8$$

First we express one of the variables from one of the equations and substitute it into the other equation. Finally, we will get a linear equation with one variable. In our example we will express x from the second equation, then substitute it into the first one:

$$x = 8 - 4y$$
$$3(8 - 4y) + 2y = 14$$
$$24 - 12y + 2y = 14$$
$$-10y = -10$$
$$y = 1$$

Now that we know the value of y we can get x:

$$x = 8 - 4y = 8 - 4 = 4$$

A physical example: We would like to project a sharp and real image on the wall so that the object is located precisely 125 cm away from the wall, by using a lens with a focal distance of $f = 30$ cm. Where should we place the lens? To solve this problem, we will use the lens equation as the physical law:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

where o is the object distance and i is the image distance. Both of them are unknown variables. To determine them, we need to construct two equations. The first one is the lens equation itself and the second one is that the sum of the object and image distances is 125 cm (for this we have to know that the real image of an object appears always on the opposite side of the lens):

$$\frac{1}{30} = \frac{1}{o} + \frac{1}{i}$$
$$o + i = 125$$

Let us express i from the second equation and plug it in to the first one:

$$i = 125 - o$$
$$\frac{1}{30} = \frac{1}{o} + \frac{1}{125 - o}$$

By rearranging the equation, we get a quadratic equation:

$$o(125 - o) = 30(125 - o) + 30o$$
$$125o - o^2 = 3750 - 30o + 30o$$
$$0 = o^2 - 125o + 3750$$

Solving it with the quadratic formula:

$$o_{1,2} = \frac{+125 \pm \sqrt{15625 - 15000}}{2} = \frac{125 \pm 25}{2}$$

The two solutions are:

$$o_1 = \frac{125 + 25}{2} = 75 \text{ cm}$$
$$o_2 = \frac{125 - 25}{2} = 50 \text{ cm}$$

We substitute both solutions of o into the $i = 125 - o$ expression:

$$i_1 = 125 - o_1 = 125 - 75 = 50 \text{ cm}$$
$$i_2 = 125 - o_2 = 125 - 50 = 75 \text{ cm}$$

Both solutions are physically relevant. Please, note that if we require the image to be magnified, then only the o_2 = 50 cm and i_2 = 75 cm solutions would be correct.

Trigonometric equations: The x variable is in the argument of a trigonometric function. For example:

$$\sin x = 0.5$$

This equation can be easily solved with the inverse sine function. On nearly all types of calculators this function has a symbol of „$\sin^{-1}$". This notation is rather unfortunate because in mathematics the superscript (–1) denotes the reciprocal and not the inverse function. You can find the inverse sine function on the calculator by using the „INV + SIN" or „2ndF + SIN" button combinations. Solving the equation for x, you get:

$$x = \sin^{-1} 0.5 = 30°$$

if your calculator was set to measure the angle in degrees (D or DEG symbol appears on display). If the calculator is set to measure angles in radians (R or RAD appears on display), then you will see this result:

$$x = \sin^{-1} 0.5 = 0.524$$

This result is given in radians (the rad unit symbol is often neglected) and of course 30° = 0.524 rad. (See the measurement of angles later.)

A physical example: In case of a harmonic oscillation, the displacement of the body from equilibrium position is described with the sine function:

$$y = A \cdot \sin(2\pi \cdot f \cdot t)$$

where A is the amplitude (maximum displacement) and f is the frequency (number of oscillation in a unit time). Let us consider Foucault's pendulum which is a lead weight hung from the dome of the Pantheon in Paris with the help of a 67-m-long wire. The amplitude of the pendulum motion is 3 m and its frequency is 0.061 Hz (Hz means hertz, the SI unit for frequency, and is defined as one cycle per second: Hz = 1/s). Let us consider as starting point the instant when the pendulum swings through its equilibrium (vertical) position. Calculate the time interval in which the pendulum reaches a bowling pin placed 2 m away from the starting point. In this example the unknown variable is t, so we have to solve a trigonometric equation. We will get the result if the equation is rearranged and the inverse sine function is used:

$$\frac{y}{A} = \sin(2\pi \cdot f \cdot t)$$

$$\sin^{-1}\left(\frac{y}{A}\right) = 2\pi \cdot f \cdot t$$

$$t = \frac{\sin^{-1}\left(\frac{y}{A}\right)}{2\pi \cdot f} = \frac{\sin^{-1}\left(\frac{2}{3}\right)}{2 \cdot 3.14 \cdot 0.061} = \frac{0.7297}{0.3831} = 1.9 \text{ s}$$

Note that radian unit must be used in this equation, because the unit of frequency is 1/s and not °/s!

Exponential function: The x variable is in the exponent. For example:

$$2^x = 5$$

To solve the equation, we take the common logarithm of both sides of the equation, then rearrange it for x:

$$\lg\left(2^x\right) = \lg 5$$

$$x \cdot \lg 2 = \lg 5$$

$$x = \frac{\lg 5}{\lg 2} = \frac{0.699}{0.301} = 2.32.$$

We obtain the same result if we use the natural logarithm:

$$\ln\left(2^x\right) = \ln 5$$

$$x \cdot \ln 2 = \ln 5$$

$$x = \frac{\ln 5}{\ln 2} = \frac{1.609}{0.6931} = 2.32$$

A physical example: Because of the spontaneous decay of radioactive atomic nuclei, the activity (A) of a radioactive sample decreases exponentially with time. The radioactive decay law is:

$$A = A_0 \cdot e^{-\lambda \cdot t}$$

where A_0 is the activity of the sample at $t = 0$, A is the activity after the time interval t and λ is the decay constant of the radioactive isotope. In a medical test the patient receives an injection of radioactive isotopes with an activity of $A_0 = 200\,000$ Bq (Bq stands for Becquerel, the SI unit of activity). Calculate the time during which the activity of this isotope will decrease to $A = 25\,000$ Bq. The decay constant of the isotope is 0.005 1/min. By rearrangement and logarithmic transformation of the decay law, we get:

$$\frac{A}{A_0} = e^{-\lambda \cdot t}$$

$$\ln\left(\frac{A}{A_0}\right) = \ln\left(e^{-\lambda \cdot t}\right)$$

$$\ln\left(\frac{A}{A_0}\right) = -\lambda \cdot t$$

$$\frac{\ln\left(\frac{A}{A_0}\right)}{-\lambda} = t$$

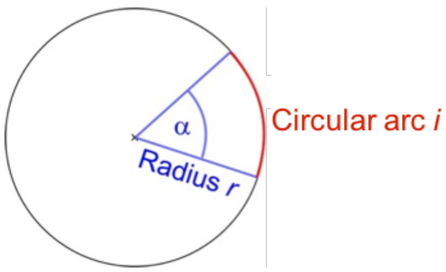
After substitution we will get the time in the unit of minutes, because the unit of the decay constant was 1/min:

$$t = \frac{\ln\left(\frac{25\,000}{200\,000}\right)}{-0.005} = \frac{\ln 0.125}{-0.005} = \frac{-2.079}{-0.005} = 416 \text{ minutes} = 6 \text{ hours and } 56 \text{ minutes}$$

Some geometric shapes:

- *Circle* (with radius r): Circumference: $C = 2 \cdot r \cdot \pi$ Surface area: $A = r^2 \cdot \pi$
- *Sphere* (with radius r): Surface area: $A = 4 \cdot r^2 \cdot \pi$ Volume: $V = \frac{4}{3} \cdot r^3 \cdot \pi$

Measuring angles: An angle can be measured in the unit of degrees (°) or in radians (rad). The degree is 1/360 of a turn, so one turn is 360°. One degree can be further divided into angular minutes (symbol: ′,; 1° = 60′), and one minute is further divided into angular seconds (symbol:′′,; 1′ = 60′′).



The definition of the radian is:

$$\alpha = \frac{i}{r}$$

where i is the length of the circular arc that the α angle subtends, and r is the radius. The arc length of a full angle equals the circumference of the circle; thus, $i = 2r\pi$, which means that 360° equals 2π radians:

$$360° = 2\pi \text{ rad} = 6.28 \text{ rad}$$

From this we get:

$$1° = \frac{2\pi}{360} = 0.01745 \text{ (rad), and } 1 \text{ rad} = \frac{360}{2\pi} = 57.3°$$

The symbol rad is often not displayed in calculations.

Problems:

1. Give the value of 10^{-3}!

2. Give the value of the expression without using a calculator: $\frac{\left(10^3\right)^2 \cdot 10^4}{10^{-2}} \cdot 10^{-10}$!

3. Give the scientific notation of 390 000 000!

4. Round the number to three significant digits: 0.004 099 099!

5. Give the value of the expression without using a calculator: $2 \cdot \lg 5 + 2 \cdot \lg 20$!

6. Solve the linear system: $\begin{matrix} x \cdot y = 3 \cdot y \\ x + y = 3 \end{matrix}$!

7. Solve the equation: $\sin x = 0.72 \cdot \sin 30°$!

8. Solve the equation: $10000 = 2 \cdot e^{0.51 \cdot x}$!

9. Calculate the radius of the sphere with a volume of 1 m^3!

10. Convert 80° into radians!

Solutions:

1. 0.001

2. 100

3. $3.9 \cdot 10^8$

4. 0.0041

5. 4

6. $x = 3$ and $y = 0$

7. 21.1º

8. 16.7

9. 62 cm

10. 1.4

2. Physical Quantities and their Units

Physics, like all natural sciences, is based on observations. The conclusions of these observations have to be quantitative, because (1) this way we can verify them, (2) they may form the basis of scientific models or theories and finally (3) they can be effectively used in practice. The quantitative observation in physics is the measurement in which we operate with well-defined physical quantities. The precise knowledge of physical quantities is essential in understanding physics and its medical contributions.

Physical quantity: The definition of a physical quantity is a measurement instruction on how to determine that quantity (often given as a formula). For simplicity, the symbol of quantities is most often one letter. The physical quantity is the arithmetical product of a numerical value and a unit:

$$\textit{Physical quantity} = \text{numerical value} \cdot \text{unit.}$$

For example, let us use the symbol m to denote the body mass of a patient. In this case, the result of the body mass measurement is given in the form of $m = 84$ kg. Note that we usually omit the multiplication sign between the value and the unit. There is no rule on which letter to use as a symbol of a physical quantity. Mass usually has the symbol of m, but other letters such as M or μ are used as well. The letters denoting a physical quantity are always written in *italic* style.
Physical quantities can be grouped in different ways: e.g., scalar and vector quantities or base and derived quantities.

Scalar (scalar quantity): a quantity **without** spatial direction, e.g., body temperature. The expression $T = 37$ °C fully describes the quantity.

Vector (vector quantity): a quantity **with** spatial direction, e.g. the speed of an aircraft. To fully describe the quantity it is not enough to give the magnitude and unit of speed (e.g. 900 km/h), but the direction of movement must also be provided.

Base quantities: arbitrarily chosen quantities from which all other quantities are originated (see table).

Unit of measurement: an arbitrarily chosen amount of a quantity that is used as a standard for measurement of that quantity. During measurement we compare the measured quantity with the unit and give the result as a product of a numerical value (x) and the unit of that measurement (e.g., x meters, if the quantity was length). The symbols used to denote units are letters, e.g., „m” for meter. In contrast to physical quantities, units have strict regulations on which letters to use as symbols. The meter can only be denoted with letter „m”. One way to decide whether the letter „m” denotes the physical quantity mass or the unit of length meter is that symbols of physical quantities are written in italic, while those of units are not.

Base units: They are the units of the base quantities (see table). All other units can be derived from them.

International System of Units (*Système International d'Unités*, abbr. SI): The coherent system of units of measurement of the seven base quantities.

International System of Units (SI)

base quantity		SI base unit	
name	usual symbol	name	symbol
length	l	meter	m
mass	m	kilogram	kg
time	t	second	s
electric current	I	ampere	A
thermodynamic temperature	T	kelvin	K
amount of substance	ν [nu] (or n)	mole	mol
luminous intensity	I	candela	cd

The determination of base units is an important and interesting problem, but it is not essential for a medical doctor who will only use them. Thus, we will not deal with this problem here.

Derived quantities and their units: Derived quantities and their units are defined by using the base quantities and base units. They are usually defined by a formula. For example, velocity (v) is defined as the distance travelled by an object (Δs) in an interval of time (Δt) divided by the duration of that interval ($v = \frac{\Delta s}{\Delta t}$). According to this formula, the SI unit of velocity is meter/second (m/s), although in practice the unit kilometer/hour (km/h) is used as well.

Change of a physical quantity: The change of a physical quantity is denoted with the upper-case letter Δ (greek „delta“). For example, the change of volume is denoted as ΔV. To calculate the change, you should always subtract the final value from the initial value. This way we get a positive result if the volume of the object increased (e.g., due to heat expansion) and negative result if the volume of the body decreased (e.g., due to cooling).

SI prefixes (metric prefixes): The prefix is a multiplication factor preceding the unit of measurement. The SI prefixes are certain powers of ten, with special names that are used to simplify the writing of very large or very small quantities. For example, the prefix kilo (written as „k”) means thousand (10^3). Accordingly, 200 000 m, for example, may be written in the more convenient form of 200 km. The SI prefixes are usually those powers of ten in which the exponent is an integer multiple of three, but there are some exceptions. The prefixes necessary for your studies can be found in the table below.

Greek alphabet symbols: Besides Latin letters we also use Greek letters as symbols for physical quantities (e.g., λ for wavelength), SI prefixes (μ for micro) or other physical phenomena (i.e. α-radiation). The Greek letters are found in the table below (those similar to Latin letters are seldom used).

SI prefixes

prefix		multiple	
name	symbol	exponent	numeral
exa	E	10^{18}	quintillion
peta	P	10^{15}	quadrillion
tera	T	10^{12}	trillion
giga	G	10^{9}	billion
mega	M	10^{6}	million
kilo	k	10^{3}	thousand
hecto	h	10^{2}	hundred
deca	da	10	ten
deci	d	10^{-1}	tenth
centi	c	10^{-2}	hundredth
milli	m	10^{-3}	thousandth
micro	μ	10^{-6}	millionth
nano	n	10^{-9}	billionth
pico	p	10^{-12}	trillionth
femto	f	10^{-15}	quadrillionth
atto	a	10^{-18}	quintillionth

The Greek alphabet

name	upper case	lower case
alpha	Α	α
beta	Β	β
gamma	Γ	γ
delta	Δ	δ
epsilon	Ε	ε
zeta	Ζ	ζ
eta	Η	η
theta	Θ	θ or ϑ
iota	Ι	ι
kappa	Κ	κ
lambda	Λ	λ
mu	Μ	μ
nu	Ν	ν
xi	Ξ	ξ
omicron	Ο	ο
pi	Π	π
rho	Ρ	ρ
sigma	Σ	σ
tau	Τ	τ
upsilon	Υ	υ
phi	Φ	φ or ϕ
chi	Χ	χ
psi	Ψ	ψ
omega	Ω	ω

Problems:

1. Give the scientific form of these quantities rounded to three significant digits without prefixes!

a) 0.004 996 PJ =

b) 32.88 fmol =

c) 1 198.7 km =

2. Give the scientific form of these quantities rounded to three significant digits without prefixes!

a) 0.245 5 μm =

b) 3.298 2 MJ =

c) 123.5 aJ =

3. Use prefixes to give the shortest possible form of these quantities:

a) 0.002 5 m =

b) $0.033 \cdot 10^{8}$ W =

c) $0.003 \cdot 10^{-6}$ mol =

d) $2\,000 \cdot 10^{10}$ Hz =

4. Use prefixes to give the shortest possible form of these quantities:

a) $5.2 \cdot 10^{-8}$ s =

b) 0.003 mol =

c) $8\,750 \cdot 10^{4}$ J =

5. Convert the units!
a) $5 \cdot 10^{6}$ fmol = ……............. nmol

b) 300 cm^2 = …........……… m^2

c) 12 dm^3 = ...….........…... cm^3

d) 25 m/s = ……..........…… km/h

6. Convert the units!
a) 0.3 GW = ……................. MW

b) 0.3 m^2 = …........……..… cm^2

c) 1 000 cm^3 = ...…............ dm^3

d) 72 km/h = ……..........….. m/s

Solutions:

1. First do the rounding, then convert to scientific form, then replace the prefix with the multiplication factor, finally multiply the powers of ten.

a) 0.004 996 PJ = 0,005 00 PJ = $5.00 \cdot 10^{-3}$ PJ = $5.00 \cdot 10^{-3} \cdot 10^{15}$ J = $5.00 \cdot 10^{12}$ J

b) 32.88 fmol = 32.9 fmol = $3.29 \cdot 10^{1}$ fmol =$\cdot 3.29 \cdot 10^{1} \cdot 10^{-15}$ mol = $3.29 \cdot 10^{-14}$ mol

c) 1 198.7 km = 1 200 km = $1.20 \cdot 10^{3}$ km = $1.20 \cdot 10^{3} \cdot 10^{3}$ m = $1.20 \cdot 10^{6}$ m

2. a) $2.46 \cdot 10^{-7}$ m
b) $3.30 \cdot 10^{6}$ J
c) $1.24 \cdot 10^{-16}$ J

3. First convert the number to scientific form, then find the prefix from the table corresponding to the power of ten. If the power of ten does not have a prefix (see task d), then use the closest prefix which will result in the shortest form of the quantity:

a) 0.002 5 m = $2.5 \cdot 10^{-3}$ m = 2.5 mm

b) $0.033 \cdot 10^{8}$ W = $3.3 \cdot 10^{6}$ W = 3.3 MW

c) $0.003 \cdot 10^{-6}$ mol = $3 \cdot 10^{-9}$ mol = 3 nmol

d) $2\,000 \cdot 10^{10}$ Hz = $2 \cdot 10^{13}$ Hz = $20 \cdot 10^{12}$ Hz = 20 THz, or
= $0.02 \cdot 10^{15}$ Hz = 0.02 PHz
From the two possibilities 20 THz is the shorter form.

4. a) 52 ns
b) 3 mmol
c) 87.5 MJ

5. a) $5 \cdot 10^{6}$ fmol = $5 \cdot 10^{6} \cdot 10^{-15}$ mol = $5 \cdot 10^{-9}$ mol = 5 nmol

b) 300 cm^2 = 300$\cdot$(cm$\cdot$cm) = 300$\cdot$(10^{-2} m$\cdot 10^{-2}$ m) = $300 \cdot 10^{-4}$ m^2 = 0.03 m^2

c) 12 dm^3 = $12 \cdot 10^{-3}$ m^3 = $12 \cdot 10^{-3} \cdot 10^{6}$ cm^3 = $12 \cdot 10^{3}$ cm^3 = 12 000 cm^3

d) $25\,\frac{\mathrm{m}}{\mathrm{s}} = 25\,\frac{0{,}001\ \mathrm{km}}{\frac{1}{3600}\mathrm{h}} = 25 \cdot 3\,600 \cdot 0.001\ \frac{\mathrm{km}}{\mathrm{h}}$ = 90 km/h

6. a) 300 MW
b) 3 000 cm^2
c) 1 dm^3
d) 20 m/s

3. Mechanics — Kinematics

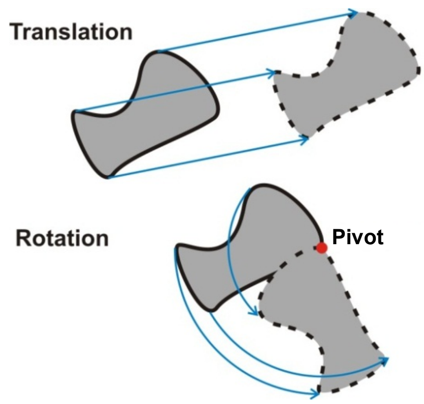
Kinematics is the branch of mechanics which describes the *motion* of bodies. The terms and quantities of kinematics are used in nearly all branches of physics and also in other natural sciences. In medical sciences, kinematics is mostly applied in the fields of biomechanics and sports medicine.

The concept of motion is relative. To decide if a body moves or not, we need to correlate it to another object or to some reference. For example, a man traveling in a bus is standing still compared to the moving bus, but moves together with the bus compared to the road and does a different kind of movement compared to the Sun.

Frame of reference: an arbitrarily chosen body (or bodies) that we use to correlate the motion of other bodies with. In order to describe the motion of bodies quantitatively, we use an abstract coordinate system as the chosen reference body, for example a rectangular (a.k.a. Cartesian) coordinate system.

Any complex motion of a **rigid body** (a solid body which retains its shape) can be described by the combination of two simple types of movement: *translation* and *rotation*.

Translation: If a body is moved from one position to another, and if the lines joining the initial and final points of each of the points of the body are a set of parallel straight lines, so that the orientation of the body in space is unaltered, then the displacement is called a translation parallel to the direction of the lines (see figure). In this case the velocity of all the points of the body has the same magnitude and direction. For example, the initial flying part of a ski jumper just after leaving the hill is approximately a pure translation.



Rotation: In this type of motion, all the points of the body move along concentric circular paths around a center point of rotation (pivot). The pirouette of a skater is approximately a pure rotation.

Translational movement

If we neglect the rotational movement of a real body and only consider its translational movement, then we can treat the body as a point-like object for simplicity. Of course, the object still has a mass, thus we can call it a point mass. The following quantities are used to describe the translational movement of a point mass: *velocity, acceleration, period, frequency and angular velocity.*

Velocity (usual symbol v): it is the distance traveled (Δs) divided by the time elapsed (Δt):

$$v = \frac{\Delta s}{\Delta t}$$

In order to get the precise instantaneous velocity of an object, the time interval Δt has to be minimized. Of course, this is relative, too. That is, Δt needs to be small enough so that the change in the velocity is negligible, otherwise we will only get the average velocity for the Δt time interval. The SI unit of velocity is m/s. The velocity gives both the direction and the magnitude of the motion, thus it is a vector quantity. The scalar absolute value (magnitude) of velocity is called speed.

Linear motion with uniform velocity: motion along a straight line with constant velocity. For this type of motion, the distance traveled (s) is a linear function of the time elapsed (t):

$$s = v \cdot t$$

which means the distance traveled is directly proportional to time. The s vs. t graph is linear (see figures below).

Acceleration (usual symbol a): the change in velocity (Δv) divided by the elapsed time (Δt):

$$a = \frac{\Delta v}{\Delta t}$$

Again, to get the instantaneous acceleration Δt needs to be small enough so that the change in acceleration is negligible. The SI unit of acceleration is m/s^2. Acceleration gives the rate of change of velocity of an object. For example, the acceleration $a = 3$ m/s^2 means that the velocity increases by 3 m/s in every second. Acceleration is also a vector quantity. If the velocity of a linearly accelerating object in increasing, then the acceleration is positive; if however the velocity is decreasing, then Δv hence the acceleration are negative.

Linear motion with uniform acceleration: motion along a straight line with constant acceleration. For this type of motion velocity (v) is a linear function of the time elapsed (t):

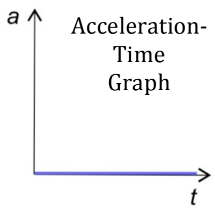
$$v = a \cdot t + v_0$$

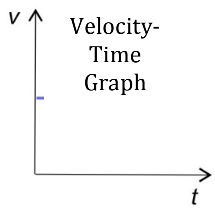
where v_0 is the velocity at time t = 0 (initial velocity). Constant acceleration means that the velocity change is directly proportional to time elapsed from the initial velocity. The v vs. t graph is linear (see figures below).

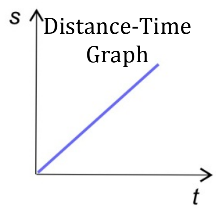
The simplest way to calculate the distance traveled during a time interval of t is to use the average velocity: $s = \bar{v} \cdot t$. If velocity changes linearly with time, then we can simply calculate $\bar{v}$ as the arithmetic mean of the initial and final velocities: $\bar{v} = (v_0 + v)/2$. An example for linear motion with uniform acceleration is **free fall**, where gravity is the only force acting on the body and all other actions such as air resistance are negligible.

Gravitational acceleration (usual symbol g): the constant acceleration of a free-falling body caused by force of gravitation. Its value varies a little at different points on Earth: it is greater at the equator and smaller at the poles. Its mean value is 9.81 m/s^2. This means that the velocity of a free falling object increases by 9.81 m/s every second. This acceleration is also true for the elevation of an object (e.g., a rock thrown up), in which case its velocity is decreasing by 9.81 m/s every second.

Graphs for linear motion with uniform (positive) *velocity:*

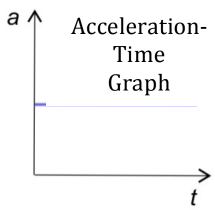


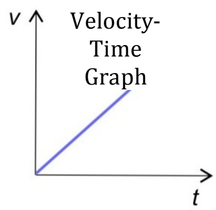


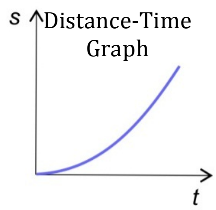


The acceleration of the body is zero at any time because its velocity is constant. A distance traveled s is increasing linearly with time according to the function $s = v \cdot t$. The slope of the line equals v velocity.

Graphs for linear motion with uniform (positive) *acceleration*:







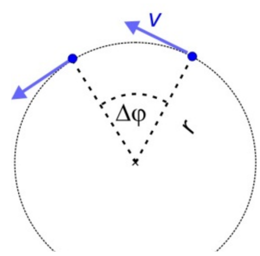
The acceleration of the body is constant in time. The velocity is increasing linearly with time according to the function $v = a \cdot t$ (in this example v_0 initial velocity is zero). The slope of the line equals a acceleration. The distance traveled is increasing non-linearly since the velocity is increasing with time.

Circular motion: movement along a circular path. (Caution, this is translation, not rotation!)

Angular velocity (usual symbol ω): It is the angular displacement ($\Delta\varphi$) divided by the time elapsed (Δt):

$$\omega = \frac{\Delta\varphi}{\Delta t}$$

Again, the time interval Δt has to be small enough so that the movement remains unchanged. The SI unit of angular velocity is 1/s. This actually means rad/s, but we usually omit rad. Similarly to velocity and speed, angular velocity is a vector, while *angular frequency* is the scalar absolute value of angular velocity.



Uniform circular motion: circular motion with constant angular velocity. The angular displacement is directly proportional to time:

$$\varphi = \omega \cdot t$$

In the case of uniform circular motion, the magnitude of tangential (?) velocity remains constant, but the direction of velocity is constantly changing since it is always tangent to the circular orbit (see figure). The velocity and angular velocity of the body depend on each other. If the angular velocity is greater, then the velocity will be greater too, but the velocity also depends on the radius of the circular orbit. With the same angular velocity but greater radius the body has to run a longer circular path in the same time, thus its velocity will become greater. By definition, we can calculate the velocity of an orbiting object by dividing the circular path length (Δs) by Δt. We can get Δs as the length of circular arc from the definition or radian as $\Delta s = r \cdot \Delta\varphi$. After substitution we will get:

$$v = \frac{\Delta s}{\Delta t} = \frac{r \cdot \Delta\varphi}{\Delta t} = r \cdot \frac{\Delta\varphi}{\Delta t} = r \cdot \omega .$$

Uniform circular motion is a periodic, repetitive motion, so every circle is a repetition of the previous. The following quantities can be used with all kinds of periodic motion:

Period (usual symbol T): duration of one cycle in a repeating event (i.e., time of one revolution). Its base unit is second (s).

Frequency (usual symbol f): number of occurrences of a repeating event per unit time (i.e., number of revolutions in a unit time). The shorter the period, the greater the number of revolutions the body will cover in a unit time, thus the frequency will be greater:

$$f = \frac{1}{T}$$

The SI unit of frequency is **hertz** (Hz; 1 Hz = 1/s, the number of periods in one second).

Angular velocity and frequency are proportional to each other. We can easily demonstrate this if we write the definition of angular velocity for one rotation. Because the time duration of one revolution is T and the corresponding angular displacement is 2π, thus:

$$\omega = \frac{2\pi}{T} = 2\pi \cdot \frac{1}{T} = 2\pi f$$

This equation explains why ω is often called angular frequency.

Again we would like to emphasize that the above-mentioned quantities can be used with any kind of periodic motion and not only for circular motion. For example, they are used to describe rotation, oscillation, wave motion and even with periodic biological motions, such as the changes of pressure, volume or electric potential during the cardiac cycle.

Rotation

The points of a rotating rigid body are moving along circular paths. The center points of these circles fall on the same line, the axis of rotation. Every point moves with the same angular velocity, but their velocity depends on their distance from the axis of rotation. The quantities used for describing circular motion can also be used to describe rotation.

Problems:

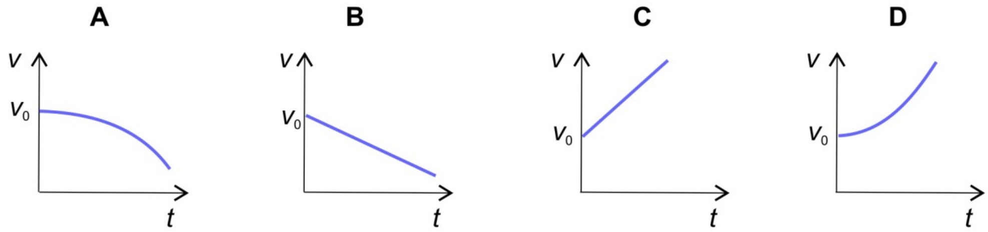
1. The distance between Budapest and Munich is 675 km. It takes 6 hours and 15 minutes to drive from Budapest to Munich on the highway. Calculate the average velocity of movement in units of km/h and m/s!

2. Calculate the time gained during a trip of 118 km on the highway, if you drive your car at the speed limit of 130 km/h instead of the safer and more economical 110 km/h.

3. During an ultrasound examination, a probe placed on the body surface transmits a very short sound pulse into the interior of the body. The sound pulse gets reflected from the surface of an organ and returns to the probe 80 μs later. Calculate the distance of the organ boundary to the body surface if we know that the speed of sound in the body is 1 500 m/s.

4. During a thunderstorm you hear the sound of thunder 5 seconds after you saw the flash of lightning. Calculate how far away the lighting struck, if we suppose that light is infinitely fast and the speed of sound is 330 m/s.

5. You throw a rock up in the sky with an initial velocity of v_0. Let us suppose that gravitation is the only force acting on it (i.e., there is no air resistance). Which figure shows the correct change in the stone's velocity during its elevation?



6. An apple falls from the tree, and after 0.8 s it lands on the head of a man sleeping under the tree. Let us assume that the apple's motion was free fall.
a) What is the velocity of the apple at the time of impact (give the result in km/h)?
b) How high was the branch from the ground?

7. You throw a rock up in the sky with an initial velocity 36 km/h. Assume that air resistance is negligible. a) After how long a time will the stone reach its highest elevation? b) How high will this rock fly?

8. A satellite is orbiting Earth at an altitude of 1 670 km. It orbits our planet every 2 hours. (The average radius of the Earth is 6 370 km). Calculate:
a) the period,
b) the frequency,
c) the angular velocity,
d) the velocity in km/h
e) the number of revolutions per week!

9. Mary sits in a carousel 8 m from the axis. The carousel runs for three and a half minutes during which it makes 20 revolutions. Calculate:
a) the period,
b) the frequency (in the unit of Hz),
c) the angular velocity
d) the velocity.

Solutions:

1. 15 min = 0.25 h. The total time is 6.25 h. Average velocity is:
$v = \frac{\Delta s}{\Delta t} = \frac{675}{6.25} = 108\ \frac{\text{km}}{\text{h}}$
Conversion to m/s unit: $108\ \frac{\text{km}}{\text{h}} = 108 \frac{1000\ \text{m}}{3600\ \text{s}} = \frac{108}{3.6}\frac{\text{m}}{\text{s}} = 30\frac{\text{m}}{\text{s}}$

2. 9 minutes and 54 seconds

3. The movement of the US pulse is assumed to be linear motion with uniform velocity of v = 1 500 m/s. During the time interval t = 80 μs = $8 \cdot 10^{-5}$ s the total distance traveled (back and forth) is:
$s = v \cdot t = 1\,500 \cdot 8 \cdot 10^{-5} = 0.12$ m
Thus, the distance of the organ boundary from the body surface is at 0.06 m = 6 cm.

4. 1.65 km

5. B

6. a) The apple is an object starting its motion from rest (initial velocity $v_0 = 0$) and moves linearly with uniform acceleration. Its acceleration is $a = g = 9.81$ m/s^2. Its final velocity after the $t = 0.8$ s time interval is:
$v = a \cdot t + v_0 = g \cdot t + 0 = 9.81 \cdot 0.8 = 7.85\ \text{m/s} = 28.3\ \text{km/h}$.
Of course, this is only true in case of free fall. In reality, the apple is slowed down by air resistance.

b) Since the velocity of the apple increases linearly with time, the average velocity can be calculated as the arithmetic mean of the initial and final velocities:
$\bar{v} = (v_0 + v)/2 = (0 + 7.85)/2 = 3.93\ \text{m/s}$. With this average velocity the distance traveled during the time interval of $t = 0.8$ s is $s = \bar{v} \cdot t = 3.93 \cdot 0.8 = 3.14$ m.

7. a) 1.02 s; b) 5.1 m

8. The satellite orbits around the center of Earth, so the radius of its orbit is the sum of its elevation and the average radius of Earth: r = 6 370 km + 1 670 km = 8 040 km = $8.04 \cdot 10^6$ m.
a) The period is given by the problem: T = 2 hours.
b) Frequency is the reciprocal of period: $f = \frac{1}{T} = \frac{1}{2 \cdot 3\,600} = 1.39 \cdot 10^{-4}$ Hz.
c) Angular velocity is proportional to frequency: $\omega = 2\pi \cdot f = 6.28 \cdot 1.39 \cdot 10^{-4} = 8.73 \cdot 10^{-4}\ \text{s}^{-1}$.
d) Velocity is proportional to angular velocity: $v = r \cdot \omega = 8.04 \cdot 10^6 \cdot 8.73 \cdot 10^{-4} = 7020\ \frac{\text{m}}{\text{s}} = 25\,300\ \frac{\text{km}}{\text{h}}$.
e) One week is 7·24 = 168 hours. If one revolution requires 2 hours, then during 168 hours the satellite makes 84 revolutions.

9. a) 10.5 s; b) 0.095 2 Hz; c) 0.598 1/s; d) 4.79 m/s

4. Mechanics — Statics and Dynamics

Dynamics is a branch of mechanics concerned with the study of forces acting between bodies and their effect on their motion. Statics however deals with a special case only, when the forces acting on a body are in equilibrium, thus the body is at rest.
In medical practice, statics and dynamics have relevance in the fields of sports medicine, orthopedics and physical therapy. Statics and dynamics are essential in understanding the working principles of the musculoskeletal system. For example, we can calculate the forces stretching the tendons during various movements or the forces that compress the vertebral disks in different body positions. Statics and dynamics are used in other biomedical related fields too, for example to understand the sound amplifying function of the lever system of auditory ossicles in the middle ear, or to determine the force of mastication created by the lever system of the lower jaw and the masticatory muscles.

Depending on their properties, different types of *interactions* can occur between bodies. These interactions include gravitation, friction, electric attraction/repulsion, magnetic attraction/repulsion, nuclear forces, etc. An interaction between two objects means that they exert *forces* on each other. The physical quantity „force" is used to describe the strength of these interactions. The presence of an interaction (or force) on bodies causes a change either in their motion or in their shape. Both kinds of changes can be measured and used to define the force. Now we will use the more conventional way to describe force: change in motion (acceleration).

Force (usual symbol F): the product of mass (m) and acceleration (a) of the body exposed to the force.

$$F = m \cdot a$$

Its SI unit is **newton** (N; $1\ \mathrm{N} = 1\,\dfrac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$). Force is a vector quantity, its direction equals the direction of acceleration (which is not necessarily the same as the direction of speed or movement!). In the following, we will only consider the vector properties of force in some special cases.

There are some general laws in physics describing the relationship between force acting on the body and its acceleration. These are *Newton's laws of motion*.

Newton's first law (law of inertia): When viewed in an inertial reference frame, an object either remains at rest or continues to move linearly at a constant velocity, unless acted upon by a net force.

Every object remains at rest or moves in a straight line with uniform velocity until another object will compel it to change its motion. This means that the cause of change in the motion of an object is always an interaction with another object. We note here that this is not true in case of accelerating reference frames, but we will not deal with such special cases.

Newton's second law (fundamental law of dynamics): The acceleration of an object and the force acting on the object are proportional to each other:

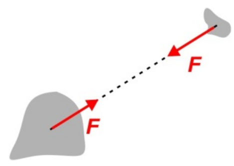
$$F = m \cdot a$$

or — if more forces act on the object at the same time —:

$$\sum F = m \cdot a$$

where ΣF represents the so called net force, which is the vectorial sum of the forces acting on the object. If there is only one force acting, then this law is practically the same as the definition of force. If there are more forces acting on the body at the same time, then the law means more: we need to sum the forces, so the acceleration will be proportional to net force.

Notably, the first law can be viewed as a special case of the second law. Namely, if the net force acting on an object is zero, then the acceleration of the object will be zero. This means that the velocity of the object remains constant or zero, therefore the object is either moving with uniform velocity or remains at rest.



Newton' third law (law of equal action and reaction): This law describes the symmetry of interactions. When object A exerts a force of F on object B, then object B exerts a force of F on object B with equal magnitude but opposite direction. This means that forces always appear in pairs. Note, however, that if you want to write the second law for object A to calculate its acceleration, you must not sum the two forces mentioned by the third law, because only one of them acts on object A!

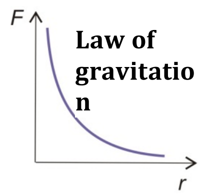
Equilibrium: An object remains in equilibrium if the net force acting on it is $\Sigma F = 0$. Consequently, its acceleration will be zero, thus it is either moving in a straight line with uniform velocity or is at rest. The latter, special case is studied by the field of *statics*.

The laws of dynamics will become useful in practice if we can determine the forces acting between two objects without knowing their acceleration, but we know their distance from each other and some other properties (e.g., mass, electric charge, etc.). The various physical laws of different types of interactions give the relationship between forces and other measurable properties of the interacting objects. With these laws we can calculate the force acting on the object, thus we can calculate its acceleration by using the second law. By knowing the acceleration we can calculate the velocity-time and distance-time graphs for that object by using the laws of kinematics, which means that we are able to predict the path of movement of the object. Altogether this is the fundamental concept of the so called mechanistic determinism.
The laws of the various interactions include the law of gravitation, Hooke's law, Coulomb's law, etc. Let us see some of these laws in detail.

Law of universal gravitation: a particle attracts every other particle in the universe with a force that is directly proportional to the product of their masses (m_1 and m_2) and inversely proportional to the square of the distance (r) between them:

$$F = G\frac{m_1 \cdot m_2}{r^2}$$

where G is the gravitational constant, $G = 6.67 \cdot 10^{-11}$ m^3/(kg·s^2). With the distance increasing the strength of the interaction decays inversely with the square of the distance (see figure).



Gravity (symbol F_{gravity}): the force that accelerates and object falling freely towards the Earth. This force is in good approximation with the gravitational force exerted by the Earth on the object (the rotation of the Earth slightly modifies this force, hence they are not equal). To calculate it, we will use the law of universal gravitation. If the object is on the surface of Earth, then for m_1 we can use the mass of the Earth and for r the average radius of Earth. From these two constants and from the gravitational constant, we will get the gravitational acceleration g (9.81 m/s^2) (Try to calculate it yourself!):

$$g = G\frac{m_{Earth}}{r_{Earth}^2}$$

The gravity of Earth acting on the object with a mass of m is given by:

$$F_{\text{gravity}} = m \cdot g$$

Of course, if we want to be precise, we need to consider that the value of g is not constant but depends on the radius of Earth. Because the Earth is not precisely spherical and its radius is greater at the equator than at the poles, g varies accordingly.

Weight (usual symbol W): Weight is the force with which an object is pressing or pulling the surface that holds it. If the object is at equilibrium and only gravity and the normal force of the surface act on it, then its weight equals gravity. However, there are cases when the weight of the object is greater or smaller than gravity (see problem 9.). If the object is not supported by any surface (i.e., it falls freely), then its weight is zero, and this case is called weightlessness.

Hooke's law: if a spring is extended, then a restoring force develops which is proportional to the extension (s):

$$F = -k \cdot s$$

where k is the spring constant (unit: N/m) that shows what force a spring can exert if we extend it by 1 meter. The value of the spring constant depends on the spring's material properties and dimensions. The minus sign in the equation is needed because the elastic restoring force developed in the spring points in a direction opposite to extension. Hooke's law is also valid for compression, but then the signs are opposite. Hooke's law can be used to describe the extension of ligaments and tendons in the human body.

***Problems*:**

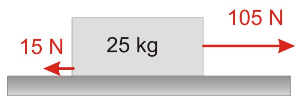
1. A bullet with a mass of 10 g moving at 200 m/s strikes the wall and stops after 0.002 s. Assuming that the deceleration of the bullet is uniform, calculate:
a) The braking force acting on the bullet.
b) How deep the bullet penetrated the wall?

2. A racecar (m=1 500 kg) accelerates from rest with uniform acceleration. It reaches the 100 km/h velocity in 3.1 s.
a) Calculate the force accelerating the car.
b) Calculate the distance in which the car reaches the 100 km/h velocity.

3. In a particle accelerator an ion with a mass of $2.5 \cdot 10^{-25}$ kg is accelerated by a constant force of $1.6 \cdot 10^{-12}$ N.
a) Calculate the acceleration of the ion.
b) Calculate the increase in velocity during 10 ns.

4. The acceleration of a paratrooper (m = 70 kg) at a given moment during its fall is 0.5 m/s^2. What kind of forces act on the paratrooper at this moment? Calculate the value of these forces!

5. A father is pulling a sled for 5 seconds starting from rest with a constant force of 105 N. The mass of the sled together with the child on it is 25 kg. The friction force acting on the sled is 15 N.
a) Calculate the acceleration of the sled.
b) What will be the final velocity of the sled after 5s?
c) How far can dad pull the sled during this time?



6. A man is pulling a sled with constant velocity (m = 20 kg). The rope suddenly breaks. The sled slides on for 6,1 seconds with uniform decelaration and stops after traveling 9.2 m.
a) Calculate the velocity of the sled at the moment when the rope breaks.
b) Calculate the acceleration (in fact, deceleration) of the sled.
c) Calculate the force that slows the sled.

7. Calculate the gravitational force between the nucleus (one proton) and the electron of a hydrogen atom, if their distance is 100 pm. (Mass of proton: $1.67 \cdot 10^{-27}$ kg; the mass of electron: $9.11 \cdot 10^{-31}$ kg.)

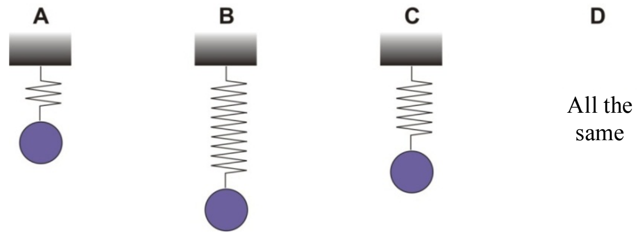
8. Calculate the gravitational force between two asteroids (masses 200 000 and 300 000 tons) when they pass by each other at a distance of 2 km.

9. A sandbag with a mass of 40 kg hangs from a rope.
a) Calculate the gravity pulling the bag.
b) Calculate the force with which the bag pulls the rope (its weight).
c) Calculate the gravity and the weight if we hang the bag in an elevator that is accelerating with 2 m/s^2 downwards.

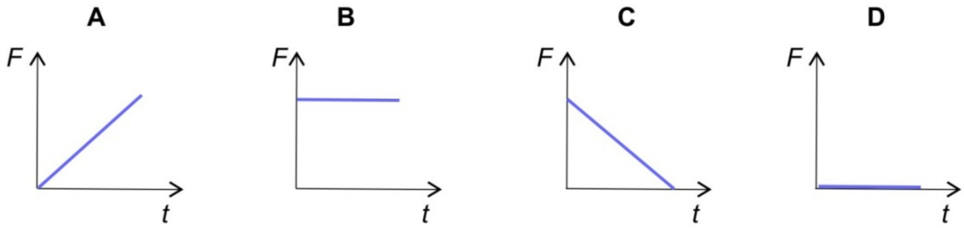
10. Let us consider the Achilles tendon as a simple spring with a spring constant of $3 \cdot 10^5$ N/m. Calculate the force needed to stretch the tendon by 2 mm.

11. We hang a mass of 2 kg on a spring. Its extension after equilibrium is 25 cm. Calculate the spring constant.

12. All the springs shown in the figures are extended by 10 % when we hang the same mass on them. Which spring has the highest spring constant? Or do all have the same spring constant?



13. The figures show the change in force as a function of time:



a) We throw a ball straight up. Which figure shows correctly the change of gravity acting on the ball?
b) We slowly compress a spring uniformly. Which figure shows correctly the change of spring force during compression?
c) A ball falls freely towards the ground. Which figure shows correctly the change in the ball's weight?

Solutions:

1. a) The acceleration of the bullet by definition is: $a = \frac{\Delta v}{\Delta t} = \frac{0-200}{0.002} = \frac{-200}{0.002} = -10^5\ \frac{\text{m}}{\text{s}^2}$.

The braking force on the bullet is: $F = m \cdot a = 0.01 \cdot \left(-10^5\right) = -1\,000\ \text{N}$. The negative sign with both values shows that the direction of acceleration is opposite to the direction of force (deceleration).

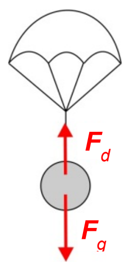
b) Average velocity of the bullet assuming uniform acceleration: $\bar{v} = \left(v_0 + v\right)/2 = \left(200 + 0\right)/2 = 100$ m/s. Distance traveled with this average velocity: $s = \bar{v} \cdot t = 100 \cdot 0.002 = 0.2\ \text{m} = 20\ \text{cm}$.

2. a) 13 400 N; b) 43.1 m

3. a) Only one force is acting on the ion, thus: $a = \frac{F}{m} = \frac{1.6 \cdot 10^{-12}}{2.5 \cdot 10^{-25}} = 6.4 \cdot 10^{12}\ \frac{\text{m}}{\text{s}^2}$.

b) The change in velocity during Δt = 10 ns = 10^{-8} s ice calculated by using the formula defining acceleration: $\Delta v = a \cdot \Delta t = 6.4 \cdot 10^{12} \cdot 10^{-8} = 6.4 \cdot 10^4\ \frac{\text{m}}{\text{s}}$.

4. Gravity (F_g) points downwards, and both the air resistance and the braking force of the parachute point upwards creating a drag force (F_d). The resultant force of these two is $\Sigma F = F_g - F_d$. According to Newton's second law:
$F_{gravity} - F_{\text{drag}} = \sum F = ma = 70 \cdot 0.5 = 35\ \text{N}$. From the given parameters we calculate gravity as $F_{\text{gravity}} = mg = 70 \cdot 9.81 = 687\ \text{N}$, thus the drag force is $F_{drag} = F_{gravity} - 35 = 687 - 35 = 652\ \text{N}$. This means that gravity is still greater than the drag force, and the paratrooper is accelerating.



5. a) 3.6 m/s^2; b) 18 m/s; c) 45 m

6. a) 3.02 m/s; b) –0.495 m/s^2; c) –9.9 N

7. In this simple model we assume that the nucleus and the electron of the hydrogen atom are particles, so we can use the universal law of gravity: $F = G\frac{m_1 \cdot m_2}{r^2} = 6.67 \cdot 10^{-11} \cdot \frac{1.67 \cdot 10^{-27} \cdot 9.11 \cdot 10^{-31}}{(100 \cdot 10^{-12})^2}$ $=1.01 \cdot 10^{-47}$ N. We note that this attractive force is so weak that it is not enough to hold the atom together. It is the much stronger electrostatic interaction that holds them together (see Chapter 10., problem 1.)

8. 1 N

9. a) Gravity is always calculated as: $F_{\text{gravity}} = mg = 40 \cdot 9.81 = 392$ N.

b) If the sandbag is at equilibrium, then the net force acting on it hence its acceleration are zero. The rope holds the bag with the same force as gravity: $F_{rope} = F_{gravity}$. According to Newton's third law, the sandbag pulls the rope with the same force as the one exerted by the rope on the bag. The weight of the bag equals the force that pulls the rope: $W = F_{\text{rope}} = F_{\text{gravity}} = 392$ N. In this problem the magnitude of weight and gravity are the same.

c) For sure the gravity is still 392 N, but the sandbag now is away equilibrium, and it accelerates together with the elevator (downwards). According to the second law: $\sum F = F_{gravity} - F_{rope} = ma$, where F_{rope} is not necessarily the same as previously. (We consider the downward direction as positive.) From here we can calculate the force on the rope: $F_{rope} = F_{gravity} - ma = 392 - 40 \cdot 2 = 392 - 80 = 312$ N. In this example the rope force is smaller than gravity, therefore the bag is pulling the rope with smaller force, so its weight is less: $W = 312$ N. Probably you have experienced this case when standing in an elevator that is starting to move downwards. For a moment you can feel your weight decreasing. If the elevator moved upwards, then the weight of the sandbag would be greater than gravity: 472 N.

10. We need the same force as the elastic force developed in the stretched tendon. According to Hooke's law:
$F = k \cdot s = 3 \cdot 10^5 \cdot 0.002 = 600$ N

11. 78.5 N/m

12. A

13. a) B; b) A; c) D

5. Mechanics — Work and Energy

Energy is one of the most important quantities. It describes the state of an object or a system. Energy and work are closely related concepts. We can demonstrate their relationships with a simple example. A crane lifts up a piece of concrete so it does work. Meanwhile the crane loses energy (burns fuel) and the piece of concrete gains (potential) energy. Work is done on an object when you transfer energy to that object. As you can see in the previous example, there are many different kinds of energies (kinetic, potential, internal, etc.). The different kinds of energies can be converted into each other, but in an isolated system the amount of total energy remains constant. The permanent conversion of energy is an inherent process of life. The energy required to sustain the physiological functions of living organisms comes from chemical energy (and photonic energy in plants). This chemical energy can be converted, for example, into the kinetic energy of blood flow or the mechanical energy produced by muscles. In case of the human body, the major part of this chemical energy is used to maintain the proper functioning of ion pumps in the cell membranes. The sodium-potassium pump by itself consumes 70 % of the cell's ATP to maintain the physiological ion concentrations.
For another example, let us see the work done by the heart's left ventricle. The blood has to be pumped up to the level of the aortic arch and also accelerated in order to maintain the proper blood flow. Altogether, the heart does about 1 joule of work during every contraction, its average power is about 1 watt. These values are miniscule, but the heart is working through a lifespan!

Work and energy are used to describe the interactions between objects, but they can be applied more widely when compared with force. For example, they can be used to describe thermal or chemical interactions, too. Now we provide an explanation using the example of mechanical interactions.

Work (usual symbol W): if a constant force F acts on an object that moves in the direction of the force along a straight line, then the force does a work of

$$W = F \cdot s$$

on the body, and s is the displacement of the object. Work describes not only the strength of the interaction but also the distance in which the interaction took place. If the force acting on the object is at an angle with the direction of movement, then we only take the projection of the force vector along the displacement. This is done by multiplying the above expression by the cosine of the angle α between the force and displacement vectors:

$$W = F \cdot s \cdot \cos\alpha$$

The SI unit of work is **joule** (J; 1 J = 1 N·m). Work is a scalar quantity, it has no direction. A special case is if the direction of force is perpendicular to the direction of displacement, that is, $\alpha = 90°$ and $\cos \alpha = 0$. In this case the work is zero. For example, if somebody carries a suitcase, then the force acting on the suitcase points in vertical direction, whereas the displacement of the suitcase is horizontal. Thus, the mechanical work is zero. In mechanics the above-mentioned definition is used for three simple but frequent cases: when an object is accelerated by force, when we lift up an object against gravity and when a spring is extended. We will describe these in detail below.

Power (usual symbol P): the rate of doing work, calculated as the work done divided by the duration (t) of work:

$$P = \frac{W}{t}$$

The SI unit of power is **watt** (W; 1 W = 1 J/s).

Energy (usual symbol E): the ability of a system to perform work. When work is done on a system its energy will either increase or decrease, thus work and energy share the same unit, joule. However, in physics, as well as in medicine, other energy units are also used: electronvolt (eV) and calorie (cal).
Conversions: 1 eV = $1.6 \cdot 10^{-19}$ J and 1 cal = 4.19 J.

Concepts of energies used in the field of mechanics are kinetic and potential.

Kinetic energy (usual symbol E_{kin}): the energy of an object due to its motion. It depends on the mass (m) and the velocity (v) of the object:

$$E_{\mathrm{kin}} = \frac{1}{2} m \cdot v^2$$

Explanation of the formula: in order to uniformly accelerate a mass of m from rest to a final velocity v in a time interval t, we have to achieve an acceleration of $a = \frac{v}{t}$, which requires a force of $F = ma = m\frac{v}{t}$, that acts throughout the displacement of $s = \bar{v}t = \frac{v}{2}t$ (since we assume uniform acceleration from rest, the average velocity $\bar{v}$ equals half of the final velocity v). The work done during acceleration comes from the definition of work:

$$W = F \cdot s = m\frac{v}{t} \cdot \frac{v}{2}t = \frac{1}{2}m \cdot v^2$$

This work is converted into the kinetic energy of the object which may be used to do work again (e.g., the kinetic energy of an accelerated ram is used by the police to break a door).

Potential energy: energy that results from a position or configuration. It is related to the position of an object in a force field. Depending on the type of the force field, it can be gravitational, electric or magnetic potential energy. An example for the configuration-dependent potential energy is the elastic energy stored in an object undergoing elastic deformation.

Gravitational potential energy (usual symbol E_{pot}): capacity for doing work as a result of the object's position in a gravitational field:

$$E_{\text{pot}} = m \cdot g \cdot h$$

where g is gravitational acceleration and h is the height above an arbitrarily chosen reference level. Explanation of the formula: in order to lift up a mass m from the reference level to a height h, we have to exert a force of mg on the object along a displacement of h, thus the work during the elevation is:

$$W = F \cdot s = mgh$$

This work will be converted into gravitational potential energy. For example, in an upright position the heart has to do a work in order to elevate the blood up to the level of the aortic arch, which will be converted into the potential energy of blood.

Elastic potential energy (usual symbol E_{elastic}): energy stored as a result of deformation of an elastic object, such as stretching a spring:

$$E_{\text{elastic}} = \frac{1}{2}k \cdot s^2$$

Explanation of the formula: to stretch the spring we have to exert a uniformly increasing force. According to Hooke's law the force needed to reach the extension of s is $k \cdot s$. Because of the uniform extension the average stretching force is half of the final force value: $\frac{1}{2}ks$. The work done during the stretch:

$$W = F \cdot s = \frac{1}{2}ks \cdot s = \frac{1}{2}ks^2$$

This work will be stored in the elastic spring as elastic potential energy. The elastic potential energy stored in the string of a drawn bow is utilized for shooting the arrow. In the human body, the elastic potential energy stored in the tendons and ligaments has an important role in human motion. When you stretch the ligaments in your body, the elastic energy stored by them can be released during motion in the opposite direction. However, if you keep the ligaments extended for a longer time, they will lose the elastic energy due to stress relaxation.

Conservation of energy: the total amount of energy in an isolated system remains constant. For the mechanics example we can write the formula:

$$E_{\text{kin}} + E_{\text{pot}} + E_{\text{elastic}} = \textit{constant}$$

The system is isolated if the objects in it interact only with each other but not with objects that outside the system. In reality, such a system does not exist, but we can usefully approximate real sytems with it. For example, a free-falling object in the gravitational force field of the Earth (if we neglect air resistance) can be considered an isolated system. In this case it is only the falling object and the Earth that are in interaction. During the fall the potential energy is transformed to kinetic energy, but their total sum remains constant.

Mass-energy equivalence: one of the result of Einstein's theory of relativity, according to which every object with mass m has a so-called rest energy *E:*

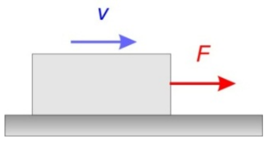
$$E = m \cdot c^2$$

where c is the speed of light in vacuum ($c = 3 \cdot 10^8$ m/s). There are some interactions in physics where the mass transforms into energy and *vice versa*. This means that mass and energy are two different forms of the same physical entity because they can be transformed into each other. For example, the medical imaging technique

Positron Emission Tomography (PET) is based on the mass-energy equivalence. In this imaging technique the positrons emitted by the radioactive isotope introduced into the patient annihilate as they interact with electrons and transform their rest energy into the electromagnetic energy of gamma photons.

Problems:

1. A father is pulling a sled with a horizontal force of $F = 30$ N, at a constant velocity of $v = 2.5$ m/s.



a) Calculate the work done by the father in 10 minutes.
b) How many chocolate bars can supply the energy need of this work, if one bar contains 535 kcal?
c) Calculate the father's power.
d) Calculate the force of friction acting on the sled.

2. A car (m=1.2 t) is uniformly accelerating from rest for 12 seconds to reach a velocity of 100 km/h.
a) Calculate the force necessary for this acceleration.
b) Calculate the distance run by the car during acceleration.
c) Calculate the work done by the accelerating force.
d) Calculate the average power of the car.
e) Calculate the kinetic energy of the car at the end of the acceleration.

3. A boy with his sled (total mass: 30 kg) arrives at the bottom of the slope with a velocity of 4 m/s. As he slows down on the horizontal section, he slides 24 m before he stops.
a) Calculate the combined kinetic energy of the boy and the sled at the start of slow-down.
b) Calculate the work done by the force of friction that stops the movement of the sled.
c) Calculate the force of friction.

4. The left ventricle of the heart pumps about 70 g of blood by one contraction into the aorta. This amount of blood reaches the aortic arch that is located approximately 15 cm above the ventricle and has a flow velocity of 30 cm/s.
Calculate:
a) the work needed to lift the blood,
b) the work needed to accelerate the blood,
c) the power of the left ventricle during a contraction that lasts for 0.2 s!

5. Someone is pulling a bucket full of water (m=12 kg, including the mass of 10 liters of water in it) to the top of an 8 m deep well, with uniform velocity of 50 cm/s. Calculate
a) the force acting on the bucket
b) the work done
c) the power
d) How many kcal work is done by a man who lifts up a total of 4.8 m^3 water from the well in a day?

6. Calculate the amount of energy stored in a spring that is extended by 5 cm. $k = 400$ N/m.

7. Calculate the amount of energy stored in the Achilles tendon with a spring constant of $3 \cdot 10^5$ N/m that is extended by 2 mm.

8. A small piece of concrete with a mass of 30 dkg loosens on the facade of an old building, falls 20 meters and becomes smashed on the street. By neglecting drag, calculate the kinetic, potential and total energy of the concrete piece
a) at start and
b) at the moment of impact.
c) What is the final velocity of the concrete piece when it smashes on the street?

9. A ball (m=0.8 kg) falls to the floor from a height of 2 m and bounces back to a height of 1.2 m. Calculate the amount of energy lost due to air drag and collision with the ground.

10. Calculate the rest energy of an electron ($m_e = 9.11 \cdot 10^{-31}$ kg)! Convert your result to eV!

Solutions:

1. a) During 10 minutes (= 600 s), the sled runs a distance of $s = v \cdot t = 2.5 \cdot 600 = 1\ 500\ \text{m}$. The work done: $W = F \cdot s = 30 \cdot 1500 = 45000\ \text{J} = 45\ \text{kJ}$.
b) Energy content of one bar is 535 kcal = 535 000·4.19 = 2 241 650 J. So the work done by the father is covered by $\frac{45\ 000}{2\ 241\ 650} = 0.02$ bar, that is only 1/50 of a bar! (Note that the efficiency of the human body when converting the chemical energy of food to work is far from 100%.)
c) $P = \frac{W}{t} = \frac{45000}{600} = 75\ \text{W}$
d) Since the sled moves with constant velocity, its acceleration is zero, thus the net force (sum of all forces) acting on it is also zero. Therefore, the force of friction, which is exactly opposite to the pulling force, is also 30 N.

2. a) 2 780 N; b) 167 m; c) 463 kJ; d) 37.8 kW; e) 463 kJ

3. a) $E_{\text{kin}} = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 30 \cdot 4^2 = 240$ J.
b) The system loses all its energy during deceleration, thus the work of the breaking force equals the kinetic energy but has a negative sign: –240 J. (It is negative, because the energy of the system has decreased.)
c) The work of the force of friction (F_{f}) is $W = F_{\text{f}} \cdot s$. We have already calculated the work, so the force is: $F_{\text{f}} = \frac{W}{s} = \frac{-240}{24} = -10$ N. The negative sign shows that this force acts opposite to the direction of movement.

4. a) It equals the change in potential gravitational energy: 0.103 J; b) It equals the change in kinetic energy: 0.003 J; c) Total work is 0.106 J, and the power during one contraction is 0.53 W. (The total work of heart is about ten times greater than this value and in calculating the average power we have to consider the circa 0.8 s diastole too. This is how we can get the values of 1 J and 1 W mentioned in the introduction.)

5. a) 118 N; b) 942 J; c) 58.9 W; d) 108 kcal

6. The elastic potential energy is: $E_{\text{elastic}} = \frac{1}{2}k \cdot s^2 = \frac{1}{2}400 \cdot 0.05^2 = 0.5\ \text{J}$.

7. 0.6 J

8. an) At start (1):
$E_{\text{kin},1} = 0$, $E_{\text{pot},1} = m \cdot g \cdot h = 0.3 \cdot 9.81 \cdot 20 = 58.86\ \text{J}$, $E_{\text{total},1} = 0 + 58.86 = 58.86\ \text{J}$,
assuming that zero altitude is the street level.

b) At the impact (2):
$E_{\text{pot},2} = 0$.
Because we neglected air drag, we can use the conservation of energy:
$E_{\text{total},2} = E_{\text{total},1} = 58.86\ \text{J}$, from this
$E_{\text{kin},2} = E_{\text{total},2} - E_{\text{pot},2} = 58.86 - 0 = 58.86\ \text{J}$.

c) We know from the previous part: $E_{\text{kin},2} = \frac{1}{2}m \cdot v^2 = 58.86\ \text{J}$. To solve for velocity:

$v = \sqrt{\frac{2E_{\text{kin},2}}{m}} = \sqrt{\frac{2 \cdot 58.86}{0.3}} = 19.8\ \frac{\text{m}}{\text{s}} = 71.3\ \frac{\text{km}}{\text{h}}$. (In real case the velocity is slower due to air drag!)

9. 6.28 J

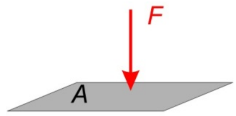
10. According to the mass-energy equivalence:
$E = m \cdot c^2 = 9.11 \cdot 10^{-31} \cdot \left(3 \cdot 10^8\right)^2 = 8.2 \cdot 10^{-14}\ \text{J} = 8.2 \cdot 10^{-14} / 1.6 \cdot 10^{-19}\ \text{eV} = 513\ \text{keV}$.

6. Mechanics — Pressure

The concept of pressure is used in many fields besides physics, for example in chemistry, physiology and also in medical routine (e.g., blood pressure measurement). Because the measurement of blood pressure is so fundamental in medicine, we will discuss it in greater detail. The total pressure of blood comes from the pressure generated by heart and the hydrostatic pressure. The latter depends strongly on the position of the body, i.e., sitting, standing or lying down. The regulation of blood pressure is a complicated process involving many different organs (i.e., heart, kidneys, blood vessels, etc.).
The blood pressure is not constant with time but fluctuates. The magnitude of fluctuation is different at the various locations of the circulatory system. The highest value is called systolic, while the lowest the diastolic pressure. Blood pressure is normally measured on the upper arm with an inflatable elastic cuff of a pressure gauge (so called sphygmomanometer). The inflated cuff compresses the brachial artery, thus blood flow is stopped. Subsequently, the cuff is slowly deflated so its pressure decreases slowly. When the pressure in the cuff drops below the systolic pressure, the artery opens and blood starts to flow with turbulence so that creates a characteristic sound (Korotkoff sound). The cuff pressure in the moment of appearance of the Korotkoff sound equals the systolic value. As we decrease the pressure further, the flow of blood will change from turbulent to more uniform, thus the Korotkoff-sound disappears. The cuff pressure in the moment of disappearance of the Korotkoff sound equals the diastolic value.

When you press with the same force on a sponge using your palm or using your finger, the deformation of the sponge will be very different. As you can see from this example, in some cases the size of the surface area on which the force is distributed plays an important role. As a consequence, we need to introduce a new physical quantity, *pressure.*

Pressure (usual symbol p): the F force applied in perpendicular to a surface A of an object:



$$p = \frac{F}{A}$$

The SI unit of pressure is **pascal** (Pa; 1 Pa = 1 N/m^2). In engineering and medical sciences other units are also used, such as bar, standard atmosphere (atm) and millimeters of mercury (mmHg) which is also called torr. The conversion of these units is as follows:

1 bar = 10^5 Pa 1 atm = $1.01 \cdot 10^5$ Pa 1 mmHg = 133 Pa

We can use pressure to describe the internal state of gases and fluids. Their pressure strongly depends on their *density*.

Density (usual symbol ρ): the mass (m) of a homogenous body divided by its volume (V):

$$\rho = \frac{m}{V}$$

If the body is inhomogenous, then the formula above gives the average density. The SI unit of density is kg/m^3, but in practice we also use kg/dm^3 and g/cm^3 units, too. The conversion of these units is as follows:

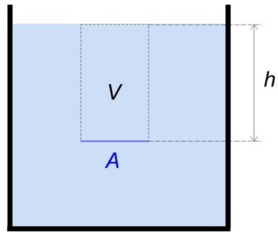
$$1 \text{ g/cm}^3 = 1 \text{ kg/dm}^3 = 1000 \text{ kg/m}^3$$

The density of different kinds of materials — especially in the case of gases —varies greatly with temperature and pressure. See the table below for examples:

Density of materials

material	ρ (g/cm^3)
air (at 0 °C and 101 kPa)	0.001 29
water (at 4 °C and 101 kPa)	1
water (at 100 °C and 101 kPa)	0.958
ice	0.92
aluminum	2.7
mercury	13.6
gold	19.3
human body (average)	1.04

Now that we know what density is, we can discuss the spontaneously arising pressure in fluids and gases caused by the gravitational field of the Earth, the so called *hydrostatic pressure*.



Hydrostatic pressure: pressure in gases and fluids arising due to gravity. Let us consider the fluid framed with dashed line in the figure as a separate object. This body with a height of h is pressing the underneath surface A with a force equal to its weight:

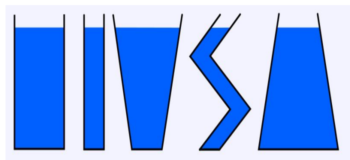
$$W = m \cdot g = \rho \cdot V \cdot g = \rho \cdot A \cdot h \cdot g$$

where ρ is the density of the fluid, V is the volume of the framed fluid and g is gravitational acceleration. According to the definition, the pressure on surface A is:

$$p = \frac{W}{A} = \frac{\rho \cdot A \cdot h \cdot g}{A} = \rho \cdot h \cdot g$$

The resulting expression shows that the pressure in fluids and gases is directly proportional to depth. You can feel the increase of hydrostatic pressure with your eardrum as you dive deeper under water. However, the direct proportionality is only true if the density of the medium is the same at every depth. While in case of fluids this can be a good approximation, it fails in case of large volumes of gases (e.g., the atmosphere). As a consequence, the dependence of the atmospheric pressure on altitude is not directly proportional.

The outdated pressure unit of millimeters of mercury is still in use in medical routine. The definition of this unit is based on hydrostatic pressure. 1 mmHg is the pressure exerted by a 1-mm-tall column of mercury. By substituting the height h = 1 mm = 0.001 m and the density of mercury (13.6 g/cm^3) into the formula of hydrostatic pressure, we will get approximately 133 Pa. This is where the previously shown conversion between Pa and mmHg comes from.



The hydrostatic paradox: hydrostatic pressure depends only on the density and on the height of the fluid and is independent on the shape of the container. The pressure at the bottom of the containers shown in the figure is the same, because the height of the fluid measured from the bottom is the same in each case. Our intuition suggests that at the bottom of the container on the right side the pressure is greater due to the larger volume, but really it is the same as in the other cases.

Finally, we discuss briefly some aspects of pressure in gases.

The pressure of gases: particles in a gas (atoms or molecules) are in constant motion so they collide with each other and with the walls of the container. The particles exert a force on the wall in every collision. However, this force is extremely tiny and lasts only for a very short time, but the number of particles is enormous. The pressure of the gas comes from the sum of these tiny forces.

Atmospheric pressure (barometric pressure): atmospheric pressure decreases exponentially with elevation. At an elevation of 5 000 m the pressure is only half of the value measured at sea level. The pressure of air at a given altitude is not constant but fluctuates according to the local temperature and weather conditions. The arbitrarily chosen *standard atmospheric pressure* is 101 kPa (1 atm).

Partial pressure: In a mixture of gases, each gas has a partial pressure which is the hypothetical pressure of that gas if it alone occupied the volume of the mixture at the same temperature. The sum of the partial pressures of each individual gas in the mixture gives the total pressure of the mixture. For example, air is a mixture of nitrogen, oxygen, carbon-dioxide, water vapor, and small quantities of many other gases. Approximately 21% of air is oxygen, thus it is responsible for 21% of the total 101 kPa standard atmospheric pressure. The partial pressure of oxygen at sea level is 21 2 kPa. Oxygen uptake of the human body depends strongly on the partial pressure of oxygen in the lungs and in the blood. If the partial pressure of oxygen is too low in the lungs (e.g., at high elevations), then the oxygen supply will be insufficient and the condition is is called hypoxia. The body is able to adapt to this condition, for example by increasing the amount of the oxygen-carrying protein hemoglobin hence the number of red blood cells.

***Problems**:*

1. A physician is compressing the chest of a patient with his palms at a force of 280 N while performing CPR. Calculate the pressure on the chest. Use certain approximation!

2. Masticatory forces in man may reach up to 100 N (by comparison, in crocodiles it may be up to 1 000 N!). When someone bites on a bone chip in the burger or on the seed of a fruit, this force is concentrated on a surface area of 1 mm^2. Calculate the pressure!

3. a) Calculate the pressure that a 70-kg-person exerts on the floor while standing. Assume that the total surface of the two soles is 200 cm^2.
b) Calculate the pressure this man exerts on the surface of ice during skating! Assume that the total surface of the blades is 4 cm^2.

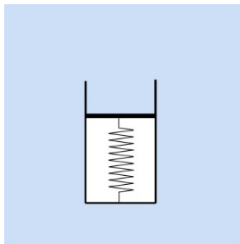
4. In an experiment we measure the body mass of Mr. Pascal: 82 kg. We determined his body volume by asking him to submerge in a cylindrical water-filled container. The cross-sectional area of the container is 1 m^2. When he is completely submerged, the water level elevated by 7.9 cm.
a) Give Mr. Pascal's body volume in the unit of cm^3, and in liters!
b) Calculate his average body density!

5. a) Calculate the mass of a gold cube with a width of 10 cm!
b) Calculate the pressure exerted by this cube on a horizontal shelf!

6. A person dives 10 meters below the surface of a pond the temperature of which is 4 °C. Calculate:
a) the hydrostatic pressure at this depth
b) the total pressure at this depth (atmospheric pressure is 101 kPa)
c) the force with which water is pressing the person's eardrum ($A_{eardrum}$= 55 mm^2).

7. Calculate the pressure (i.e., total pressure!) at 1 km below the sea surface! Let us assume that the density of sea water is $1.08 \cdot 10^3$ kg/m^3 at every depth.

8. The figure shows a device for a simple pressure measurement. The small cylinder has vacuum inside and its top is sealed with a light piston. The piston is connected to the bottom of the cylinder with a spring. If we place this device in vacuum, then the spring will be uncompressed. The cross-sectional area of the piston is 2 cm^2, and the spring constant is $4 \cdot 10^3$ N/m.



a) When this device is placed in the atmosphere, the compression of the spring is 5.1 mm. Calculate the atmospheric pressure!
b) Calculate the compression of the spring if we place the device to the bottom of a 10-m-deep pond, that has a temperature of 4 °C! Assume that the atmospheric pressure is the same as in part a).

9. Calculate the hydrostatic pressure generated by blood in the foot of a standing man. Density of blood is 1.05 g/cm^{3}and the height of the man is 170 cm.

10. Convert these units!
a) 180 mmHg = ...Pa
b) 16 kPa = ...mmHg
c) 2.5 bar = ...Pa
d) 760 mmHg = ...bar = ...atm

Solutions:

1. To calculate the pressure we need to estimate the surface area. For example, the surface area of the author's palm can be approximated by a 8 cm×17 cm rectangle that has an area of 8 cm · 17 cm = 136 cm^2. We can round this value to 140 cm^2, thus the pressure is:

$$p = \frac{F}{A} = \frac{280}{0.014} = 20\,000\text{ Pa} = 20\text{ kPa}.$$

2. 100 MPa (!), that is approximately ten times the atmospheric pressure.

3. a) 34.3 kPa; b) 1.72 MPa!

4. a) The volume of excluded water equals the volume of a cylinder that has a base area of 1 m^2 = 10 000 cm^2, and a height of 7,9 cm: $V = A \cdot h = 10\ 000 \cdot 7.9 = 79\ 000\text{ cm}^3 = 79\text{ dm}^3 = 79\text{ L}$.
b) $\rho = \frac{m}{V} = \frac{82\ 000}{79\ 000} = 1.038\ \frac{\text{g}}{\text{cm}^3} = 1\ 038\ \frac{\text{kg}}{\text{m}^3}$.

5. a) 19.3 kg; b) 18.9 kPa

6. a) First, we need to convert the density: 1 g/cm^3 = 1 000 kg/m^3.
The hydrostatic pressure is: $p = \rho \cdot h \cdot g = 1\,000 \cdot 10 \cdot 9.81 = 98\,100\text{ Pa} = 98.1\text{ kPa}$.
Pressure thus increases approximately by 1 atm every 10 meters.

b) Total pressure is the sum of the atmospheric and hydrostatic pressures, because the atmospheric pressure is distributed in the water undiminished. The total pressure is:

$$p_{total} = p_{\text{hydrostatic}} + p_{atmospheric} = 98.1\text{ kPa} + 101\text{ kPa} = 199\text{ kPa}.$$

c) The area is $55 \cdot 10^{-6}$ m^2, thus the force is: $F = p \cdot A = 199\,000 \cdot 55 \cdot 10^{-6} = 10.9\text{ N}$.

7. 10.7 MPa

8. a) $1.02 \cdot 10^5$ Pa; b) 10.1 mm

9. 17.5 kPa

10. a) 180 Hgmm = 23.9 kPa
b) 16 kPa = 120 Hgmm
c) 2.5 bar = 250 kPa
d) 760 Hgmm = 1.01 bar = 1 atm

7. Mechanics — Oscillations

Oscillations and the related physical phenomena are important, for example, in case of voice generation and hearing. The oscillation of vocal chords generate sound, while the oscillation of the tympanic membrane (eardrum) is the first step in auditory sensation. The oscillation-related phenomenon of resonance plays an important role in the process of hearing. The external auditory canal and the system of auditory ossicles have different resonance frequencies, which explains why human hearing is most sensitive the 1000 – 4000 Hz frequency range. The concept of oscillation is also used beyond the field of mechanics to describe the periodic change of any kind of quantity. For example, electric oscillation refers to the periodic change in voltage. The basic concepts of oscillations discussed in this chapter can also be applied to these non-mechanical oscillations. The human body produces many periodic or, rather, „quasi-periodic" (nearly periodic) oscillations, e.g., the change in electric potential related to heart function (ECG-signals) or the daily fluctuation of body temperature or the monthly fluctuation of certain hormone levels.

Oscillation is the periodic (repetitive) motion about a point of equilibrium. For example, the movement of a swing or a mass attached to a spring are both oscillations.

Period (usual symbol T): duration of one cycle in a repeating event; its base unit is the second (s).

Frequency (usual symbol f): number of cycles per unit time; the reciprocal of period:

$$f = \frac{1}{T}$$

The SI unit of frequency is hertz (Hz; 1 Hz = 1/s). For example, the frequency of human heart function at rest is an average of 72 1/min = 1.2 1/s, thus it is 1.2 Hz. A heart rate of 120 beat per minutes means that the frequency is 2 Hz, and the cardiac period is 0.5 s.

Angular frequency (usual symbol ω): 2π times frequency (the scalar absolute value of angular velocity):

$$\omega = 2\pi \cdot f$$

Its SI unit is 1/s.

Point of equilibrium: point around which the system oscillates. In this point the resultant force acting on the object is zero, therefore its acceleration is zero. Note that in a typical oscillating system the object swings through this point with maximal velocity (see below).

Displacement (usual symbol y): the momentary distance of the oscillating object from the point of equilibrium.

Amplitude (usual symbol A): maximal displacement.

The time course of oscillations can vary greatly, but there is a special case: *harmonic oscillation*.

Harmonic oscillation: oscillation along a straight line, in which the displacement is a sine function of time:

$$y = A \cdot \sin(\omega \cdot t + \varphi_0)$$

where A is the amplitude, ω is the angular frequency and φ_0 is the **initial** phase. The expression $\omega \cdot t + \varphi_0$ in the argument of the sine function is the **phase angle** or, in short, the **phase**. The velocity of an object experiencing harmonic oscillation is changing periodically according to the function:

$$v = \omega \cdot A \cdot \cos(\omega \cdot t + \varphi_0)$$

The system reaches its maximum velocity $v_{max} = \omega \cdot A$ when passing through the equilibrium point and hence the displacement is zero. Conversely, in the moment when the displacement is maximum the velocity is zero. Children know intuitively that jumping off a swing in its lowest position is difficult, because the swing moves with maximum velocity. However, at the highest position the swing stops for a moment, which makes it easier to jump off.

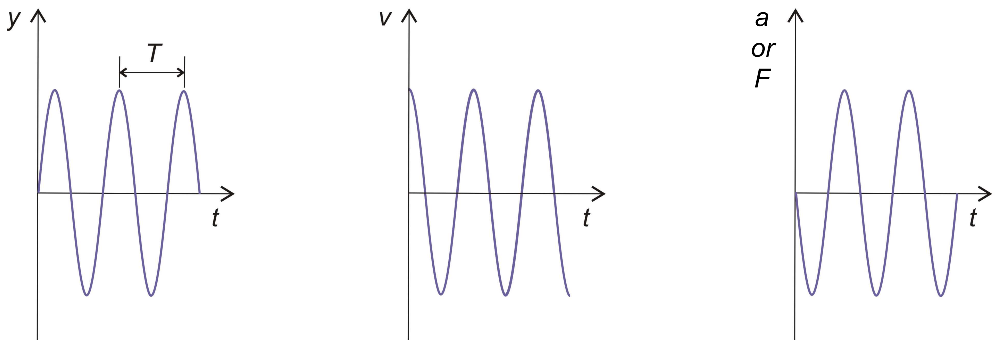
The velocity of an oscillating object changes periodically, which means that it accelerates and decelerates periodically. The acceleration of an object experiencing harmonic oscillation is changing periodically according to the function:

$$a = -\omega^2 A \cdot \sin(\omega \cdot t + \varphi_0)$$

Acceleration is zero at the point of equilibrium, while it is maximum when the displacement is maximum ($a_{max} = -\omega \cdot A$). The force acting on the object experiencing harmonic oscillation can be determined fromNewton' second law:

$$F = ma = -m\omega^2 A \cdot \sin(\omega \cdot t + \varphi_0) = -m\omega^2 y$$

As you can see, in case of harmonic oscillation the periodically changing **force acting on the object is proportional to the displacement**. Force always points at the point of equilibrium, therefore it is also called *restoring force* (i.e., it tends to restore the equilibrium from which the system has been displaced). The figures below show the displacement, velocity and force (or acceleration) *versus* time diagrams, respectively, in case of a harmonic oscillator where the initial phase angle $\varphi_0 = 0$.

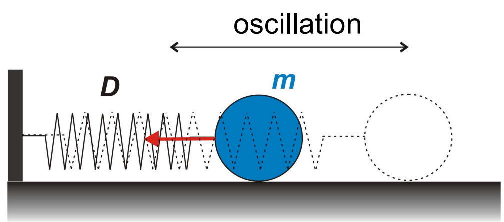


Oscillator: a physical system capable of oscillation (e.g., mass attached to a spring or a pendulum). A system is called **simple harmonic oscillator** if it undergoes simple harmonic oscillation.

The mass-on-spring or pendulum behave as simple harmonic oscillators only if there is no loss of energy during their motion, or if the loss of energy in negligible during the time course of investigation.

The *pendulum* is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its equilibrium position, it is subject to a restoring force due to gravity and the tension force of the string. This force will cause the pendulum to undergo a nearly harmonic oscillation. When released, the restoring force combined with the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth. Such a system is Foucault's pendulum that is often used to demonstrate the rotation of the Earth. Hereafter we will discuss the mass-on-spring oscillators.

Mass-on-spring oscillator: a system of mass *m* attached to a spring with spring constant *k*. When the mass is displaced from its equilibrium position and released, the elastic restoring force of the spring will cause the system to undergo simple harmonic motion spontaneously, provided that the energy loss due to friction is negligible. Such a spontaneous oscillation that proceeds on its own without external influence is called a **free oscillation**. The frequency of free oscillation is:



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

This frequency is the **natural frequency (or eigenfrequency)** of the oscillator. We get the above equation by combining the previously discussed equation of the restoring force:

$$F = -m\varpi^2 y$$

with the equation of Hooke's law on the elastic force of a spring (see chapter 4.):

$$F = -k \cdot s$$

Note that in the case of free oscillation the *y* displacement in the first equation is the same as the *s* extension in Hooke's law.

By combining the two equations (since $-y = -s$, we can remove them from the equations), we obtain:

$$k = m \cdot \varpi^2$$

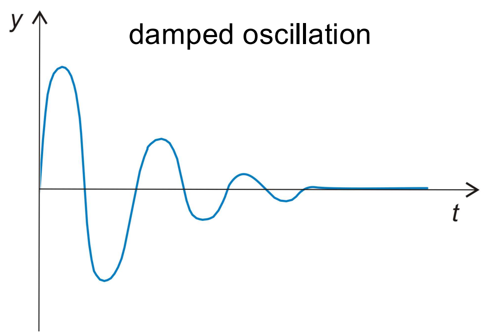
and rearranging for ω yields :

$$\varpi = \sqrt{\frac{k}{m}}$$

Finally, the natural frequency is:

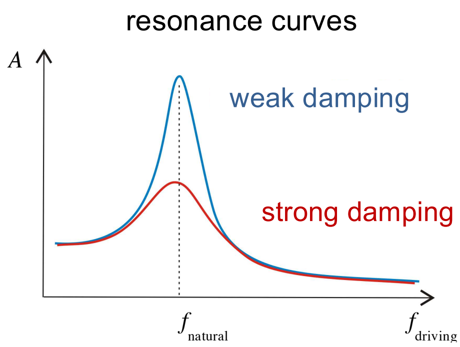
$$f = \frac{\varpi}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Damped oscillation: oscillation subjected to energy loss, thus the amplitude of the oscillation decreases over time. In fact, every free oscillation is damped, because energy loss (even if very little) is present in every real system. Since damping forces are present both in a pendulum or in the mas-on-spring systems, the description of their oscillation with the sine functions above is only an approximation. The amount of damping depends on the internal structure of the system and on its interactions with the environment. For example, if we place a mass-on-spring oscillator from air into water, then dampening will be much greater due to the greater viscosity of the surrounding medium.



Driven oscillation: an external periodic (i.e., driving) force acts on the system besides the restoring and damping forces. A swing undergoes driven oscillation if, for example, a mother keeps pushing it to keep the swinging amplitude of her child constant. By contrast, if she pulled the swing once and then released it, then the swing would undergo damped free oscillation. The oscillation of vocal chords or the tympanic membrane are driven oscillations (i.e., driven by cyclic muscle contractions or the incoming sound, respectively).

Resonance: vibrating together (e.g., driving force together with a system). If we vary the frequency of the oscillating external driving force acting on a system (f_{driving}), then the amplitude of the driven system greatly amplifies around a specific frequency value, the **resonance frequency** ($f_{\text{resonance}}$). The resonance frequency of a driven system equals its natural frequency (f_{natural}) which occurs in case of free oscillation. The graph representing the oscillation amplitude as a function of the driving frequency is called the **resonance curve** (see figure). The figure shows that the amplitude increases as we increase the driving frequency until it reaches its maximum at the natural frequency, then it decreases. The magnitude of resonance also depends on damping. If damping is weak, then oscillations with extremely large amplitudes can be produced that may damage or even destroy the system in a phenomenon called resonance catastrophe. Resonance catastrophe may occur, for example, in a bridge caused by the synchronous marching of soldiers if the rhythm of the march matches the bridge's natural frequency. Therefore, soldiers are always ordered to break stride when crossing a bridge. Resonance occurs not only in the case of mechanical vibrations. A prime example is the novel medical imaging method MRI, which stands for Magnetic Resonance Imaging (MRI). In MRI the external (driving) electromagnetic field vibrates together with the oriented magnetic spins in the human body.



***Problems*:**

1. The heart rate of a cycler during competition may reach 160 bpm.
a) Calculate the heart frequency in Hz.
b) Calculate the period of heart.

2. A child is swinging at the playground. He makes 10 swings in 15 seconds.
a) Calculate the period of the swing.
b) Calculate the frequency.

3. A pendulum makes 15 swings in one minute. Give its frequency in the unit of 1/min and Hz!

4. Hungary's oldest functioning Foucault's pendulum is found in the cathedral of Szombathely, where it was installed in 1880 and it is still in operating condition. The length of the string is 30 m, the mass of the bob is 30 kg and it takes one swing in 11 seconds.
a) Calculate its frequency!
b) Calculate the number of swings the pendulum makes in one day.

5. Calculate the period of a sound wave with 440 Hz frequency.

6. Some musicians with absolute hearing can recognize the pitch of the 1 000 Hz sound wave if it sounds only for 4 ms! How many periods of this wave appear in this short time?

7. Mobile phones use the frequency of 1 800 MHz. Calculate its period!

8. Which statement is true for harmonic oscillation?
A: The amplitude of the oscillation is increasing.
B: The amplitude of the oscillation is changing according to a sine function.
C: The restoring force is proportional to the displacement.
D: The distance run by the oscillating object is a linear function of time.

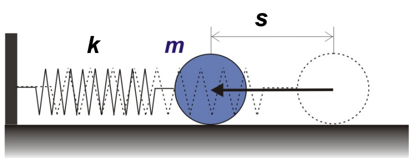
9. The amplitude and period of a harmonic oscillation are $A = 3$ cm and $T = 20$ s.
a) Write down the functions that describe the change in displacement, velocity and acceleration as a function of time!
b) Calculate the maximum velocity!
c) Calculate the maximum acceleration!
d) Calculate the displacement, the velocity and acceleration in the moment of $t = 3$ s!

10. The displacement-time function of a harmonic oscillation is: $y = 3\ \text{cm} \cdot \sin\left(0.5\frac{1}{\text{s}} \cdot t\right)$. Calculate:

a) the amplitude,
b) the angular frequency,
c) the frequency,
d) the period,
e) the maximum velocity,
c) the maximum acceleration,
d) the displacement, the velocity and the acceleration in the moment of $t = 2$ s!

11. Let us consider a swing as a harmonic oscillator! The velocity of the swing at the lowest position is 3 m/s. The acceleration at the highest point is 3 m/s^2. Write down the displacement-time function!

12. The parameters of the spring oscillator shown by the figure are: $m = 3$ kg and $k = 300$ N/m. We displace the ball to the left by $s = 10$ cm from the equilibrium position, then release it. Let us assume there is no loss of energy in the system! Calculate:
a) the natural frequency of the system,
b) the period of the oscillation,
c) the amplitude of the oscillation!

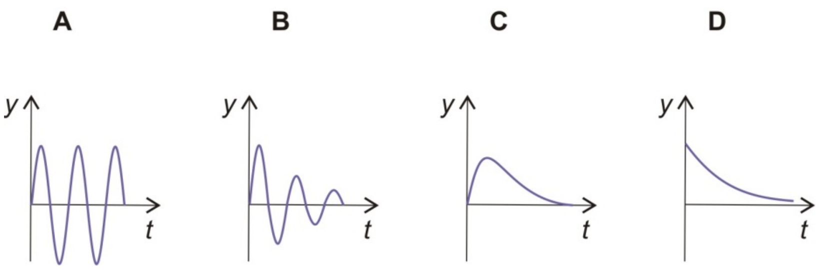


13. Upon increasing the mass of a mass-on-spring oscillator by 30 g, its period will double. Calculate the original mass!

14. The period of a mass-on-spring oscillator is 3 seconds. Upon decreasing its mass by 500 g, the period will become 2 seconds.
a) Calculate the original mass!
b) Calculate the spring constant!

15. We suspend a ball of 0.4 kg on a vertically positioned spring with a spring constant of 60 N/m. Upon releasing the ball the system undergoes harmonic oscillation.
a) Calculate the amplitude of the oscillation!
b) Calculate the period of the oscillation!

16. Which of these figures show damped oscillation?



Solutions:

1. a) 2.67 Hz; b) 0.375 s

2. a) 1.5 s; b) 0.667 Hz

3. 15 1/min = 0.25 Hz

4. a) 0.064 5 Hz; b) 5 574

5. 2.27 ms

6. 4

7. 556 ps

8. C

9. a) Besides the amplitude (A = 3 cm) we also need the angular frequency: $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{20} = 0.314\ \frac{1}{\mathrm{s}}$.
By taking the initial phase zero ($\varphi_0 = 0$):
$$y = A \cdot \sin(\omega \cdot t) = 3\ \mathrm{cm} \cdot \sin\left(0.314\ \frac{1}{\mathrm{s}} \cdot t\right)$$
$$v = A \cdot \omega \cdot \cos(\omega \cdot t) = 3\ \mathrm{cm} \cdot 0.314\ \frac{1}{\mathrm{s}} \cdot \cos\left(0.314\ \frac{1}{\mathrm{s}} \cdot t\right) = 0.942\ \frac{\mathrm{cm}}{\mathrm{s}} \cdot \cos\left(0.314\ \frac{1}{\mathrm{s}} \cdot t\right)$$
$$a = -A \cdot \omega^2 \cdot \sin(\omega \cdot t) = -3\ \mathrm{cm} \cdot 0.0986\ \frac{1}{\mathrm{s}^2} \cdot \sin\left(0.314\ \frac{1}{\mathrm{s}} \cdot t\right) = -0.296\ \frac{\mathrm{cm}}{\mathrm{s}^2} \cdot \sin\left(0.314\ \frac{1}{\mathrm{s}} \cdot t\right)$$

b) maximum velocity from the velocity-time function: 0.942 cm/s.
c) maximum acceleration from the acceleration-time function: 0.296 cm/s^2.
d) By substituting t = 3 s into the displacement-time function, we get:
$$y = 3 \cdot \sin(0.314 \cdot 3) = 3 \cdot 0.809 = 2.43\ \mathrm{cm}$$

(In these functions the angles should be in radians, so the calculator has to be in RAD mode!)

$v = 0.314 \cdot 3 \cdot \cos(0.314 \cdot 3) = 0.554 \ \frac{\text{cm}}{\text{s}}$, $a = -0.314^2 \cdot 3 \cdot \sin(0.314 \cdot 3) = -0.239 \ \frac{\text{cm}}{\text{s}^2}$.

10. a) 3 cm; b) 0:5 1/s; c) 0:079 6 Hz; d) 12.6 s; e) 1.5 cm/s; f) 0.75 cm/s^2;
g) y = 2.52 cm, v = 0.81 cm/s and a = 0.631 cm/s^2

11. The velocity is maximum at the lowest, bottom position: $A = 3$ m/s. The acceleration is maximum at the highest, turning point: $A \cdot \omega^2 = 3$ m/s^2. Let us divide the two equations to get ω: $\frac{A \cdot \omega^2}{A \cdot \omega} = \omega = \frac{3\ \text{m/s}}{3\ \text{m/s}^2} = 1\ \frac{1}{\text{s}}$. Substituting this angular velocity into the first equation, we get the amplitude: $A = 3$ m. Finally, the displacement–time function is: $y = A \cdot \sin(\omega \cdot t) = 3\ \text{m} \cdot \sin\left(1\ \frac{1}{\text{s}} \cdot t\right)$.

12. a) Natural frequency: $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{300}{3}} = 1.59$ Hz.

b) Period is reciprocal of frequency: $T = \frac{1}{f} = \frac{1}{1.59} = 0.629$ s.

c) The maximum displacement cannot be greater than the initial value, that is, 10 cm. (Otherwise the energy of the system would increase.)

13. The period of the two cases, where m denotes the original mass:

$T = 2\pi\sqrt{\frac{m}{k}}$ and $2 \cdot T = 2\pi\sqrt{\frac{m+0.03}{k}}$.

By substituting the first equation into the second, we get: $2 \cdot 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m+0.03}{k}}$.

After simplifying with 2π and squaring both sides, we get:

$$4\frac{m}{k} = \frac{m + 0.03}{k}$$

$$4m = m + 0.03$$

$$m = 0.01\ \text{kg} = 10\ \text{g}.$$

14. a) 0.9 kg; b) 3.95 N/m

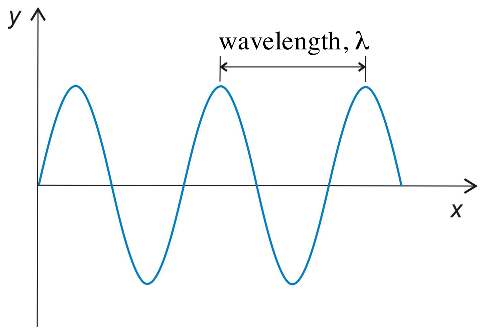
15. a) 6.54 cm; b) 0.513 s

16. B

8. Mechanics — Waves

Waves are not only interesting and eye-catching physical phenomena, but they are essential for human life. Most of the information from the environment is brought to us by two kinds of waves: light and sound waves. We communicate mainly through these waves, thus our life would be very difficult without them.
In medical practice, other types of waves also play an important role. Ultrasound (US) is used mainly for diagnostic purposes (imaging or blood flow examination with the Doppler method), but there are some therapeutic applications, too (e.g., US physiotherapy or muscle pain treatment). Electromagnetic waves are used both in therapy and diagnostics depending on their energies (e.g., microwaves, X-rays or gamma-rays). The pulse waves propagating along the blood vessel play an important role in vessel mechanics and aneurysm formation. Waves and oscillations are closely related to each other, thus the terms described in the previous chapter will be applied here as well.

Wave: propagation of oscillation. Besides the temporal periodicity (as in the case of oscillations), waves also possess spatial periodicity (e.g., periodicity of water waves). Based on their origin we distinguish mechanical, electromagnetic and matter waves. While mechanical waves (e.g., sound) require a compressible medium to propagate, electromagnetic waves (e.g., light) can propagate in vacuum (their „medium" is the electromagnetic field).



Wavelength (usual symbol λ): the distance between two points of a wave that have the same phase (e.g., distance between two neighboring crests). Base unit of wavelength is meter (m).

Velocity of propagation (usual symbol c): the velocity of the points in a wave that are in the same phase (e.g., crests). The velocity of propagation can be determined by using the definition of velocity: distance covered in one period T (this corresponds exactly to the wavelength λ) divided by the elapsed time T:

$$c = \frac{\lambda}{T} = \lambda \cdot \frac{1}{T} = \lambda \cdot f$$

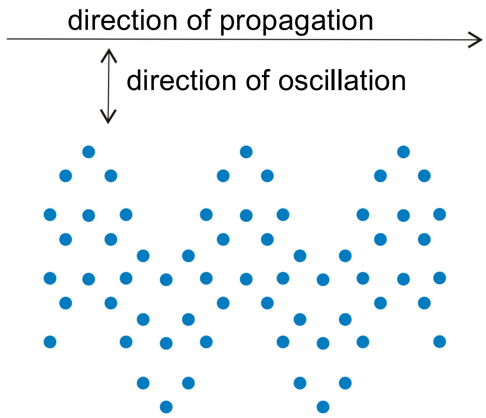
where f is the frequency of the wave. This is a general wave relationship which applies to all kinds of waves. For example, the velocity of propagation of electromagnetic waves (e.g., light) in vacuum is 300 000 000 m/s = $3 \cdot 10^8$ m/s (!). Note, however, that the speed of light is smaller in transparent materials, such as glass or water. Compared to this enormous speed, sound propagates much more slowly (see table at the end of this chapter).

It is important to emphasize that frequency is an „internal property" of a wave which remains constant even when the wave passes from one medium to another, for example, when light passes from air to water. The velocity of propagation and wavelength, by contrast, depend on the properties of the medium. Thus, when a wave slows down in the new medium its wavelength becomes shorter and *vice versa*, while its frequency remains constant.

According to the relationship between the direction of oscillation and the direction of propagation we classify the waves in two main types: *transverse* and *longitudinal* waves.

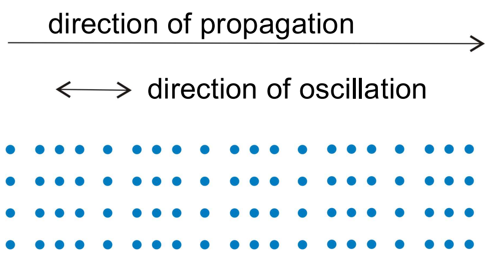
Transverse wave: the direction of oscillation is perpendicular to the direction of propagation. The surface waves in water are transverse. Electromagnetic waves (e.g., light) are also transverse waves. Importantly, this definition does not specify the plane of oscillation, because all planes that are perpendicular to the propagation fulfil the criteria.

Transverse wave:

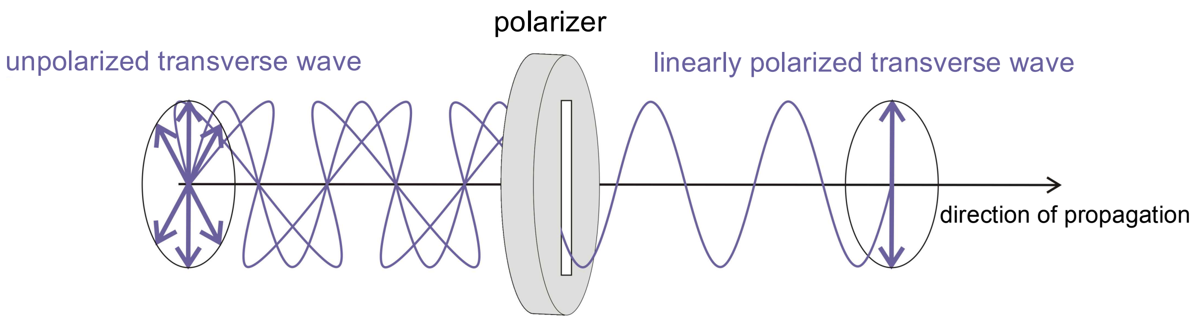


Longitudinal wave: the direction of oscillation is parallel to the direction of propagation. For example, sound waves propagating in air are longitudinal.

Longitudinal wave:



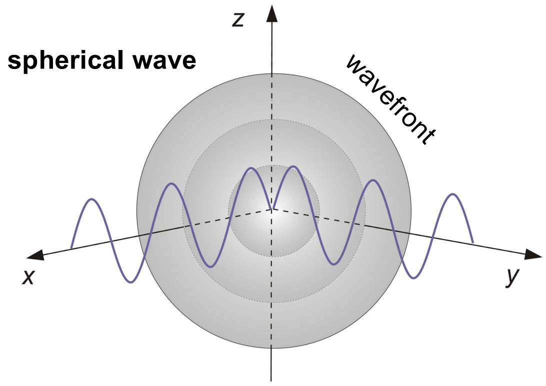
Linear polarization: As mention before, in case of transverse waves many perpendicular directions of oscillation are possible. The direction of oscillation can vary while it remains always perpendicular to the direction of propagation. In this case the wave is unpolarized. If one distinct direction of oscillation is selected, then the wave is linearly polarized. In a **linearly polarized wave** the plane of oscillation and the direction of propagation remains constant with time. The instrument capable of creating polarized wave is called a **polarizer**.

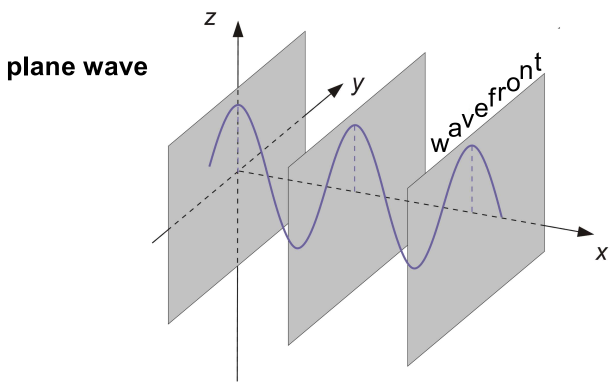


As light is a transverse wave, it can be polarized. Some insects and birds are able to sense the direction of polarization (of reflected sunlight, for example) and use it for orientating their movement. In medical practice, polarized light is used in polarization microscopy, circular dichroism spectroscopy and in some cases for therapeutic purposes.

Wavefront: the surface containing points of waves in identical phases.

Spherical wave: waves originating from a common point but propagating in every direction in space (hence they are also called spatial waves). The wavefronts are concentric spherical surfaces. For example, when you slap your hands the sound waves created are spherical waves. Its two-dimensional equivalent is the circular wave. For example, surface waves created by a pebble tossed in a pond.





Plane wave: a wave in which the wavefronts form planes that are parallel to each other but perpendicular to the direction of propagation. For example, the light beam of a laser pointer is roughly a plane wave. The two-dimensional equivalent of this type is a wave propagating along a line, i.e. the waves traveling on a rope when shaking its ends.

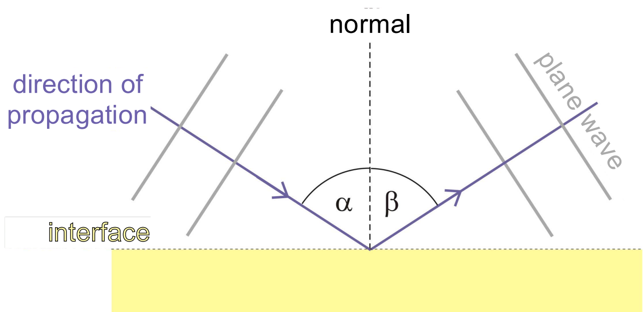
Reflection and *refraction* occur if the wave reaches an interface between two different media. At the interface one part of the wave is always reflected and the other part enters the new medium and refracts. The ratio of reflection and refraction is determined by the relative properties of the two media.

Reflection: incident wave (or part of it) "turning back" at the interface between different media. The law of reflection:

$$\alpha = \beta$$

where α is the angle of incidence and β is the angle of reflection. The angles are measured between the propagation line and the normal axis. The lines of propagation of the incident and reflected waves and the normal axis are in the same plane. (A typical mistake is that the angles of α and β are measured not from the normal axis but from the interface. Although in reflection these angles are also identical, but in refraction they are not!)

reflection



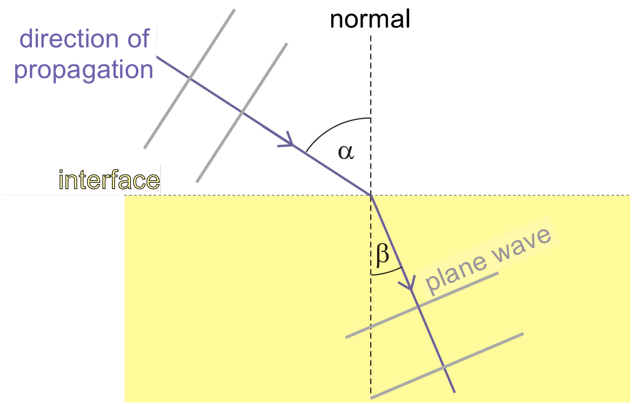
Mirrors are used in many optical devices as relective surfaces. The color of objects is determined by the reflecting properties (in case of opaque materials). Ultrasound imaging is based on the reflection of ultrasound at the boundary of two different organs or tissues.

Refraction: the change in the direction of propagation of a wave when passing through an interface between two media. The relationship between the angles before and after the interface (i.e., incident and refracted angles) is described by the **law of refraction** – or as it is called in case of light – **Snell's law:**

$$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}$$

where α is the angle of incidence, β is the angle of reflection, c_1 is the velocity of propagation in the first medium and c_2 in the second medium. The line of propagation of the incident and refracted waves and the normal are in the same plane.

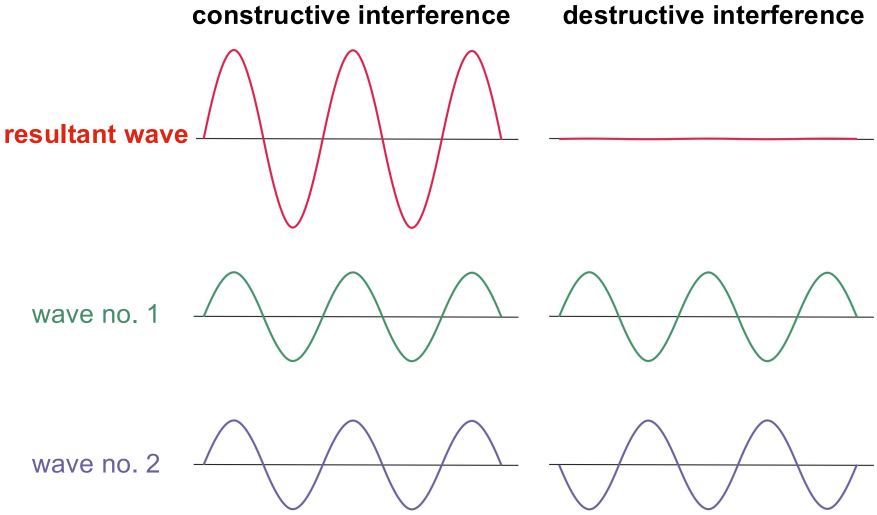
refraction



Lenses and prisms that work based on light refraction are essential parts of many medical instruments, for example, microscopes and spectrometers. Reflection of light plays an important role in the image formation of the eye. Rainbow is created when the light waves of the Sun are refracted by rain drops.

Typical wave phenomena are *interference, standing waves* and *diffraction.*

Interference: occurs when two (or more) waves meet. The requirement for interference is that the waves have identical wavelengths and their phase difference stays constant in time. In this case a well observable, nice pattern can be seen as a result of wave superposition. When two waves superpose in identical phases, the resultant wave will become larger (constructive interference). If the two waves had equal amplitudes and identical phases, then the resultant amplitude will be twice the size (see figure). When two waves superpose in precisely opposite phases, then the resultant wave will become smaller (destructive interference). If the two waves had equal amplitudes and opposite phases, the resultant amplitude will zero, thus the waves cancel out each other (see figure). If the phase difference is in between the before mentioned cases, the superposition will result in a partial construction or destruction.

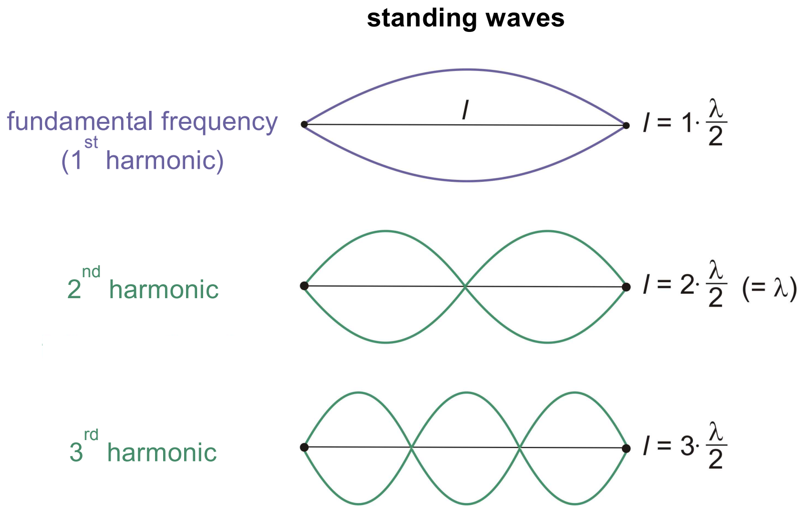


The colored pattern seen on a soap bubble or on the surface of an oil patch are the result of light interference. Optical devices based on interference (filters, monochromators, etc.) are parts of microscopes, spectrometers. Holography is also based on interference. Interference can also create so called *standing waves.*

Standing wave: resultant pattern of interference between plane waves that propagate against each other and have identical wavelengths and amplitudes. The resultant pattern appears as if the wave was standing rather than propagating. For example, in case of wave reflection a standing wave can be created as the incident and reflected waves interfere. Each point in a standing wave oscillate with different amplitudes. In the so called nodes, the amplitude is zero, while halfway between nodes (at the so called anti-nodes) the amplitude is maximum (see figure). The points between two nodes oscillate in identical phases and the points in the neighboring intermodal regions oscillate with opposite phases. The distance between two nodes equals the half of the wavelength. For example, the end points of a string fixed at both ends have to be nodes, thus the length of the string (*l)* determines the possible wavelengths of standing waves (see figure):

$$l = k \cdot \frac{\lambda}{2} \quad (k = 1, 2, 3, ...)$$

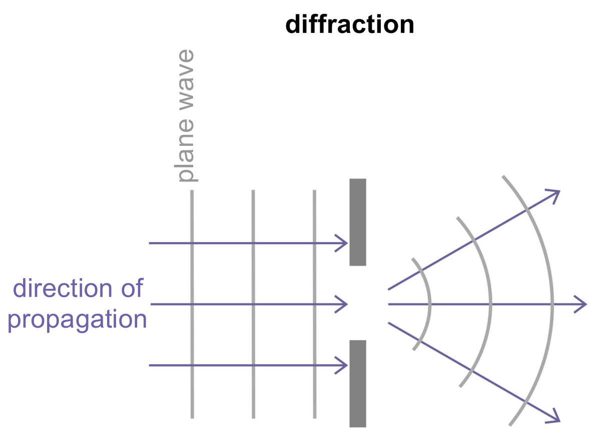
The frequencies that fulfil the criteria determined by this equation are the resonant frequencies of the string. The smallest resonant frequency (which has the longest wavelength) is called the fundamental frequency or first harmonic, and the higher frequencies are called higher harmonics or overtones.



Standing waves are produced, for example, by a vibrating guitar string during play. The equation above tells us why the pitch of the sound is increasing if we make the string shorter. If l is shorter, then λ will be shorter as well, thus the frequency (which determines pitch) increases because the product of wavelength and frequency is constant (the velocity of propagation). The guitar string vibrates at many harmonic frequencies at the same time, therefore we hear a collection of different harmonics that determines the timbre of the music played. Similarly to the guitar string, standing waves can also be created if only one end of the resonator is fixed, for example, in case of the external ear canal, where the eardrum seals one end of a tube resonator. Of course in this case the equation that gives the harmonics is different.

Diffraction is also a result of interference.

Diffraction: change in the direction of wave propagation due to an obstacle or slit in the path of the wave (not an interface!). If the size of the obstacle or slit is much greater than the wavelength, then the change in the direction is not appreciable. The smaller the size of the obstacle or slit, the greater the effect of diffraction will be. As a result, waves can propagate into regions behind the slit which are unexpected by intuitition as it would suggest a shadow. Diffraction is responsible for the finite resolution of every optical device. For example, the resolution of the microscope is limited by diffraction based Abbe's principle.



In

case of the resolution of eye diffraction plays an important role besides other biological factors (receptor density). Diffraction is explained by a model, the so called *Huygens–Fresnel principle.*

Huygens–Fresnel principle: a concept of wave propagation. According to this model every point on a wavefront acts as a source of new elementary waves. These elementary waves cannot be seen, but can be imagined as spherical waves propagating in every direction in space. The interference of these elementary waves creates the macroscopically observable wavefront at a given time. Accordingly, the new wavefront is the envelope of the elementary waves. This principle makes it easier to understand reflection, refraction and diffraction. Let us consider the simplest case of diffraction: the slit is so small that it is only a point. Accordingly, only one spherical wave will originate from this point, which will be the new wavefront since there are no other elementary waves to interfere with. This wave will propagate in every direction behind the slit, thus the diffraction is total.

Finally, let us discuss the most important mechanical wave in medicine, *sound*.

Sound: mechanical vibration that propagates as a wave in compressible materials. The human ear can only perceive a given frequency range of these vibrations. Classification of sounds is based on the human hearing:

Range of sounds

sound range	infrasound	audible sound	ultrasound	hypersound
frequency (Hz)	< 20	20–20 000	20 000–10^9	10^9 <

Sound propagates both as transverse and longitudinal wave in solid materials, but only as longitudinal wave in liquids and gases because in liquids and gases there are no shearing forces acting at rest. (However, the surface waves on water, for example, are partially transverse.) Density and pressure fluctuations occur in liquids and gases when sound propagates in them. The pitch of an audible sound is determined by its frequency, loudness is determined by the amplitude of pressure fluctuations (and the sensitivity of the ear), and timbre is determined by the collection of fundamental and higher harmonics. The speed of sound strongly depends on the properties of the medium, namely the compressibility. The easier it is to compress a material (i.e., the greater its compressibility), the farther are its component molecules from each other, hence the longer the time required for the molecules to pass on the vibration. Therefore, sound propagates most slowly in the greatly compressible gases and most rapidly in the less compressible solids. Speed of sound also depends on temperature (much less in solids) and pressure (especially in the case of gases). Speed of sound is independent of frequency, which means that every sound propagates with the same speed in a given medium (in contrast to light waves the speed of which depends on their color, a phenomenon called dispersion that leads to rainbow formation). The table below lists the speed of sound in different media.

Speed of sound in various media

medium	c_{sound} (m/s)
air (0 ºC, 101 kPa)	330
helium gas (0 ºC, 101 kPa)	965
water (20 ºC)	1 483
fatty tissue	1 470
muscle	1 568
bone (compact)	3 600
iron	5 950

***Problems*:**

1. Calculate the wavelength of standard pitch (440 Hz) in air and in water!

2. The frequency of a siren is 880 Hz. Calculate its wavelength in air!

3. In ultrasound therapy the mechanical waves can be used to relax muscles. In such a therapy the frequency of US so 800 kHz. Calculate the wavelength of this US in muscle!

4. Waves are propagating on the surface of water towards the shore with a velocity of 1.5 m/s. The distance between two neighboring crests is six meters. There is a piece of wood somewhere further in the water that turns up and disappears periodically as the water waves when you are looking at it from the shore. Calculate the time interval between two turn-ups.

5. The wavelength of UV light produced by the germicidal lamp used to sterilize operating rooms is 252 nm. Calculate its frequency!

6. The speed of microwaves is $3\cdot 10^8$ m/s. Calculate the wavelength of the microwave produced by a mobile phone that operates at the frequency of 1 800 MHz!

7. The wavelength of a sound wave propagating in water is 3 mm.
a) Calculate the frequency of this sound!
b) To which sound range does this sound belong?
c) What would be its wavelength and frequency in air?

8. Which statement is true?
A: In case of longitudinal waves the direction of oscillation and propagation are perpendicular to each other.
B: Mechanical waves can only be transverse.
C: Mechanical waves propagate as transverse waves only in solid materials (and partially on liquid surfaces).
D: Waves always carry material.

9. We generate plane waves in a bath with a frequency of 0.5 Hz. The speed of sounds at the deeper part of the bath is 3 m/s. When the waves reach the shallow part of the bath, they slow down to 2 m/s. The waves arrive at the deep-shallow interface with an angle of incidence $\alpha = 60^\circ$. Calculate
a) the angle of refraction,
b) the wavelength in the deep and in the shallow part of the bath!

10. A sound wave arrives from air (0ºC) at the water surface (20ºC). Angle of incidence is 10º. Calculate the angle of refraction!

11. In a bath, a plane wave propagates from the shallow part towards the suddenly deep part. As it passes the interface from shallow to deep, it refracts. The angle of incidence is 22º, the angle of refraction 31º.
a) In which part is the speed of sound greater and by how many percent?
b) Calculate the wavelength in the shallow part if it is 5.5 cm in the deeper part!

12. Determine the first four harmonic frequencies of a string fixed at both ends. The length of the string is 30 cm and the speed of transverse waves propagating in the it is180 m/s.

13. Determine the fundamental frequency of a 50 cm long string fixed at both ends, if the velocity of wave propagation is 120 m/s.

14. Which value can be the wavelength of a standing wave on a 20 cm long string fixed at both ends?
A: 30 cm **B**: 60 cm **C**: 5 cm **D**: 6 cm

15. From the four values, which one can be the wavelength of an audible sound in air?
A: 3.3 cm **B**: 1.1 cm **C**: 0.8 cm **D**: 0.6 cm

Solutions:

1. Speed of sound can be found in the table: c = 330 m/s in air, and c = 1 483 m/s in water.
$\lambda = \frac{c}{f} = \frac{330}{440} = 0.75\ \text{m}$ in air, and $\lambda = \frac{c}{f} = \frac{1\,483}{440} = 3.37\ \text{m}$ in water.

2. 37.5 cm

3. 1.96 mm

4. 4 s

5. 1 190 THz

6. 16.7 cm

7. a) 494 kHz; b) ultrasound; c) f = 494 kHz and λ = 0.668 mm

8. C

9. a) From the law of refraction: $\sin\beta = \sin\alpha \cdot \frac{c_2}{c_1} = \sin 60° \cdot \frac{2}{3} = 0.577\,35$ and $\beta = 35.3°$.
b) The frequency is the same in booths sides, but the wavelengths will be different.
In the deeper side $\lambda_1 = \frac{c_1}{f} = \frac{3}{0.5} = 6\ \text{m}$, in the shallower side $\lambda_2 = \frac{c_2}{f} = \frac{2}{0.5} = 4\ \text{m}$.

10. 51.3°

11. a) In the side where the angle is greater, so in the deeper side by 37.5%.
b) 4 cm

12. The first (longest) wavelength is: $\lambda = \frac{2l}{k} = \frac{2 \cdot 0.3}{1} = 0.6\ \text{m}$

The corresponding (fundamental) frequency is: $f = \frac{c}{\lambda} = \frac{180}{0.6} = 300\ \text{Hz}$.

We will get the other wavelength and frequency values if we substitute the values k = 2, 3 and 4 in the equation. The results are:

k	$\lambda = \frac{2l}{k}$ (m)	$f = \frac{c}{\lambda}$ (Hz)
1	0.6	300
2	0.3	600
3	0.2	900
4	0.15	1 200

13. 120 Hz

14. C

15. A

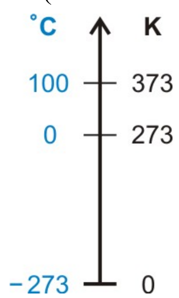
9. Thermodynamics

Changes in the temperature of our environment and the resulting phenomena such as water freezing, vapor precipitation, rain formation, etc., are essential for life on Earth. Even relatively small temperature variations of the human body (e.g., fever or hypothermia) significantly influences the course of biochemical reactions and vital functions.

Everyone has some empirical concept of the meaning of "hot" or "cold" even though these concepts are subjective. These properties of bodies are objectively described by the physical quantity called *temperature*. If we want to understand this physical quantity, we have to peer into the world of the molecules and atoms making up the bodies. The atoms and molecules (hereafter referred to as particles) are in constant motion, which can be decomposed into translation (i.e., motion from one point in space to another), rotation, vibration. These motions are collectively known as *thermal motion*. The "strength" or "power" of thermal motion may be described with an energy term. We already know the energy describing the "strength" of translational motion: kinetic energy, introduced in Chapter 5. We can assign kinetic energies to rotational and vibrational motions, too. All these kinetic energies are collectively known as *thermal energy*.

Thermal energy: the sum of the translational, rotational, and vibrational energies of particles, which describes, in general, how “powerful” thermal motion is.

Temperature (symbol: T or t): quantity that describes the state of a body related to its thermal energy. We can say that the temperature is the *gradation* or *degree* of this thermal energy. The higher the temperature the more "powerful" the thermal motion hence the greater the thermal energy. If the temperature of a body – and, simultaneously, the thermal motion of its particles – increases, then several observable consequences may occur for example, the body may expand. This phenomenon is used in classical liquid-filled thermometers (e.g., the mercury-in-glass thermometer). There are several temperature scales: the Kelvin scale is the scientific or "absolute" temperature scale. The Celsius scale is used for everyday purposes around the world, and the importance of the Fahrenheit scale is smaller. The fixed points of the **Celsius scale** are 0 °C (melting point of ice) and 100 °C (boiling point of water at normal atmospheric pressure). The fixed points of the **Fahrenheit scale** are 0 °F (the lowest air temperature he measured in Danzig [today Gdańsk, Poland] during the winter of 1708/1709) and 100 °F (his estimation of average human body temperature). The zero point of the **Kelvin scale** is the absolute zero at or below which temperature values do not exist. If we could reach this temperature, particles would stop moving altogether. The absolute zero point (i.e. 0 K) equals –273.15 °C, so this is how much the two scales are shifted relative to each other. The increments (steps), however, are similar, that is, 1 °C temperature change is equal to 1 K temperature change. Temperature measured in kelvin is usually denoted by T, while that measured in degrees Celsius is denoted by t. The conversion between the Celsius and Kelvin scales is (see also the Fig.):



$$\frac{T}{\mathrm{K}} = \frac{t}{°\mathrm{C}} + 273.15 \quad \text{and} \quad \frac{t}{°\mathrm{C}} = \frac{T}{\mathrm{K}} - 273.15$$

If two bodies of different temperatures are in interaction, then the hotter body transfers thermal energy to the colder. The process of energy transfer is called heat transfer or, simply, *heat*.

Heat (usual symbol: Q): the process or the amount of thermal energy transferred between two bodies in thermal interaction. Its SI unit is – of course – joule (J). In medical and nutritional practice the traditional calorie (cal) unit is still often used. 1 cal = 4.186 J. Attention: kilocalories (1 kcal = 1 000 cal) are sometimes, confusingly, also called calories (e.g., in "Nutrition Facts").

In order to make a body hotter, its thermal energy needs to be increased by means of energy transfer in the form of the aforementioned heat, by mechanical work, or by irradiation (e.g., sunlight). Let us consider the first case: how much heat needs to be transferred to the body in order to increase its temperature by one unit? This depends on the body's *heat capacity*.

Heat capacity (usual symbol: upper case *C*): the ratio of the heat (*Q*) transferred to a body and the resulting temperature change (ΔT):

$$C = \frac{Q}{\Delta T}$$

The unit of heat capacity is J/K (or, equivalently, J/°C). In order to make a body hotter, heat must be transferred to it; in this case both ΔT and Q are positive. In order to make a body colder, heat must be transferred away from it, then both ΔT and Q are negative. Attention: not every heat transfer leads to temperature change. E.g., we need to transfer heat in order to melt ice but this does not change the temperature, it will remain 0 °C. So the above formula may not be used in such cases. The heat capacity of a body depends on its material (quality) and "size" (e.g., its mass). Since heat capacity, in first approximation, changes simply in direct proportion to mass, we will get a physical quantity specific to the material of the body if we divide heat capacity by mass:

$$c = \frac{C}{m}$$

The name of this new physical quantity is **specific heat capacity** (usual symbol: lower case *c*). Its common unit is J/(kg·K) or, equivalently, J/(kg·°C). It gives the amount of heat that needs to be transferred to a body of 1 kg mass in order to increase its temperature by 1 K (or, equivalently, by 1 °C). Some values are shown in the following table:

The specific heat capacity of some materials

material	*c* (J/(kg·K))
silver	234
glass	840
water	4 180
body tissue (average)	3 500

The previous two formulae can be combined into a single formula which gives directly the amount of heat required to change the temperature of a body by ΔT, if its material and mass are given.

$$Q = c \cdot m \cdot \Delta T$$

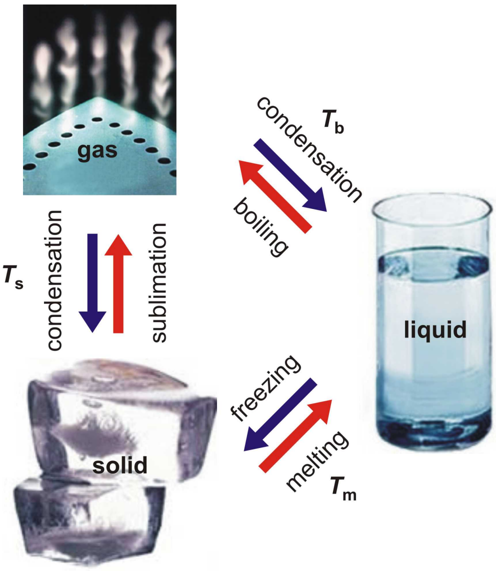
As we already mentioned, increasing (or decreasing) the temperature of a body may change many of its properties including its geometrical characteristics: length, volume. This phenomenon is called *thermal expansion*.

Thermal expansion: most bodies expand when getting hotter (both length and volume increase) and contract, shrink when getting colder (length and volume decrease). An important exception is water: when heated from 0 °C to 4 °C it shrinks (above 4 °C it will change normally). This phenomenon is one of the many *anomalous properties of water*, which has a widely-known role in the life of aquatic organisms.

Another important phenomenon related to heat transfer is the structural change of bodies occurring at certain temperatures known as *phase transition*.

States of matter: the different forms of material under different conditions (temperature, pressure, etc.). The three most important states of matter are *solid*, *liquid*, and *gas*. A body in **solid** state has a well-defined volume and shape (e.g., ice crystal). A body in **liquid** state also has a definite volume but its shape depends on the container holding it so it does not have a shape *per se* (e.g., liquid water). A body in **gas** state does not have either a definite volume or a definite shape *per se*, but it occupies the available volume and assumes the shape of the container (e.g., water vapor). States of matter are also called phases (solid phase, liquid phase, gas phase), but this term has a broader meaning which we will discuss during our university studies.

Phase transition (**phase change**): the transition of a body from one phase (or state) to another accompanied by changes in its structure and properties. The figure shows the possible transitions between the three classical states of matter. The temperatures at which the phase transitions occur are called **melting** (or **freezing**) (T_m), **boiling** (T_b), and **sublimation temperatures** (T_s). These temperatures vary with the ambient pressure. For example, the boiling temperature of water is 100 °C only at normal atmospheric pressure (101 kPa). In high mountains where air pressure is lower, water will start boiling at lower (even as low as 70 °C) temperatures. Evaporation is not limited to a definite temperature like boiling or sublimation. Liquids and solids evaporate at every temperature, although not with the same rate. Evaporation occurs from the liquid surface while during boiling vapor bubbles are formed inside the bulk liquid.



During phase transitions the bonds between particles of the body may break up or reform, and the structure of the body goes through significant changes. These changes also require the addition or withdrawal of energy.

Specific latent heat (usual symbol: L): the heat (Q) absorbed or released during the phase transition of a body divided by the mass (m) of the body:

$$L = \frac{Q}{m}$$

Unit: J/kg. It gives the amount of heat that needs to be transferred in order to transform 1 kg of a material from one phase to another one. It is called **specific heat of fusion** (L_f) in case of melting, **specific heat of vaporization** (L_v) in case of boiling and evaporation. See the table below for some examples.

Specific latent heat of some materials

material	L (kJ/kg)
gold — *heat of fusion*	67
aluminum — *heat of fusion*	396
table salt (NaCl) — *heat of fusion*	517
ice — *heat of fusion*	334.4
water — *heat of vaporization (at 30 °C and 101 kPa)*	2 400
water — *heat of vaporization (at 100 °C and 101 kPa)*	2 257

Considering the different states of matter, gases are the simplest bodies: there is no observable order among the constituent particles which are almost independent from each other. We will revise some simple quantities and principles related to gases, which, however, can also be used in a broader sense as well. First, we will define the quantity related to particle count, which is known as the *amount of substance*.

Amount of substance (usual symbol: ν [Greek lower case letter "nu"] or, especially in chemistry, n): it gives the number of constituent particles of a body by an arbitrary unit called mole. One mole (symbol: mol) is the amount of substance in a body if it contains 6.02×10^{23} particles (e.g., atoms or molecules). This value is known as **Avogadro's constant** (symbol: N_A): $N_A = 6.02\times10^{23}$ mol^{-1}. If the amount of substance (ν) is given, then the number of particles (N) can be calculated using the following formula:

$$N = \nu \cdot N_A$$

The description of gases can be further simplified by the introduction of the *ideal gas* model.

Ideal gas: a gas that consists of point-like (i.e., zero-volume) particles, and there are no interactions between the particles except for elastic collisions. Ideal gases do not exist in reality, but this model describes low-pressure atomic gases (e.g., noble gases) very well. The only motion that particles of an ideal gas make is translation, therefore the thermal energy comes from the translational kinetic energy of the particles. As a result, the temperature of an ideal gas is directly proportional to the average kinetic energy of its particles. It is important to say "average", because the motion of individual particles differs from one another, and even for one particle it changes over time. The particles collide elastically not only with each other but also with the wall of the container. During collision they exert a tiny force. These "pushing" forces are the source of the pressure of the gas.

There is a simple relationship between the physical quantities describing the state of an ideal gas: the *ideal gas law*.

Ideal gas law: a relationship between the pressure (p), volume (V), amount of substance (ν), and temperature (T) of an ideal gas:

$$pV = \nu RT$$

where R is the universal **gas constant** (R = 8.314 J/(mol·K)). If the amount of substance is given as the ratio of the particle count and Avogadro's constant ($\nu = N/N_A$) we get another form of the ideal gas law:

$$pV = \frac{N}{N_A}RT = N\frac{R}{N_A}T = Nk_BT$$

where $k_B = R/N_A$ is a new constant, **Boltzmann's constant** ($k_B = 1.38 \cdot 10^{-23}$ J/K).

The changes in the state of a gas can occur under different conditions. The following terms used to describe such conditions are used not only for gases, but for any kind of matter.

Simple conditions for state transitions and other processes: processes at constant temperature are called **isothermal**, processes at constant pressure are called **isobaric**, and processes at constant volume are called **isochoric**.

Problems:

1. Give the normal body temperature (37 °C) in kelvins.

2. A patient suffers from a disease that caused fever leading to a 2 °C rise in body temperature. Give the temperature change in kelvins.

3. Air condensates at around 73 K. Give this temperature in degrees Celsius.

4. Give the heat released by 2 liters of water when it cools down from 60 °C to 0 °C.

5. What heat would increase the body temperature of a 70 kg patient by 0.5 °C?
A: 123 J **B**: 123 kJ **C**: 123 mojo **D**: 123 cal

6. How many kg 0 °C ice should be thrown into 2 liters of 60 °C water in order to cool it to 0 °C? Suppose that the container has negligible mass and it is thermally isolated from its environment.

7. We throw a 20 g 0 °C ice cube into a glass (2 dl) of warm (30 °C) water. What will be the final temperature after the ice melts? Conditions are same as in problem #6.

8. The average daily oxygen consumption of a human is approximately 16 moles. How many oxygen molecules does it amount to?

9. A container of $V = 1.5\ \mathrm{m}^3$ volume contains oxygen gas. The pressure of the gas is 5 mbar, its temperature is 25 °C. How many moles and how many pieces of O_2 molecules are there in the container? Assume that the gas is ideal.

10. A 6 L bottle contains $16\cdot10^{23}$ N_2 molecules. Find the
a) amount of substance and
b) the pressure of the nitrogen gas if its temperature is 0 °C and it is considered ideal.

11. The air in a pump is compressed to half of its volume in a way that its temperature remains the same. How does its pressure change? Suppose the gas is ideal.

12. A metal gas container is left lying under the shining Sun. The initial pressure of the ideal gas inside is 50 bar. Its temperature increases as a result of the sunshine from 12 °C to 72 °C. What will be the final pressure?

13. What does it mean that a process is isochoric?
A: V = constant **B**: p = constant **C**: ν = constant **D**: T = constant

14. What does it mean that a process is isobaric?
A: ν = constant **B**: T = constant **C**: V = constant **D**: p = constant

15. What does it mean that a process is isothermal?
A: p = constant **B**: V = constant **C**: T = constant **D**: ν = constant

Solutions:

1. 310 K

2. 2 K

3. –200 °C

4. Because the density of water (see Chapter 6) is approx. 1 000 kg/m^3, the water of 2 liters = 2 dm^3 = 0.002 m^3 volume has a mass of: $m = \rho V = 1\,000 \cdot 0.002 = 2$ kg.
The specific heat capacity of water can be found in the table of this Chapter. Using this, the released heat corresponding to the ΔT = –60 K temperature change is:

$Q = cm\Delta T = 4180 \cdot 2 \cdot (-60) = -502$ kJ.

The negative sign indicates that heat is released and not absorbed.)

5. B

6. From problem #4 we already know that the cooling requires 502 J of heat to be removed. Because of the isolated nature of the system, this heat can only be absorbed by the ice thrown into the water, which as a result will melt. The heat absorbed when m mass of ice is melted can be calculated from the latent heat of fusion of water (334.4 kJ, see the table in this Chapter): $mL_0 = Q = 502\,000$ J.
The mass m can be determined from the equation: $m = \frac{Q}{L_0} = \frac{502\,000}{334\,400} = 1.5$ kg.
If less ice is added, then the water will not cool down to 0 °C; if more, then not all of the ice will melt.

7. 20 °C

8. Avogadro's constant (N_A) gives the number of particles in one mole. In 16 moles the number of particles is proportionally higher: $N = \nu \cdot N_A = 16 \cdot 6.02 \cdot 10^{23} = 9.63 \cdot 10^{24}$.

9. p = 5 mbar = 0.005 bar = 500 Pa and T = 25+273 = 295 K. Plug these values into the gas law formula:

$\nu = \frac{pV}{RT} = \frac{500 \cdot 1.5}{8.31 \cdot 295} = 0.306$ mol.

The number of particles can be calculated using Avogadro's constant:

$N = \nu \cdot N_A = 0.306 \cdot 6.02 \cdot 10^{23} = 1.84 \cdot 10^{23}$.

10. a) 2.66 mol; b) 1 MPa

11. If the right side of the equation of the gas law formula is constant (i.e., neither the amount of substance, nor the temperature changes) then the pV product on the left side must also remain constant. Consequently, if the volume is halved, the pressure must double.

12. 60.5 bar

13. A

14. D

15. C

10. Electricity — Electrostatics

The relationship between physiological functions and electricity have been known ever since the famous experiments of Luigi Galvani. The function of nerves and muscles is accompanied by electric processes. The measurement of electric fields created during these processes has diagnostic relevance regarding the function of the respective organs, for example, electrocardiography (ECG) in the heart and electroencephalography (EEG) in the brain.
Medical therapy uses numerous electric methods as well. For example, the defibrillator used to treat life-threatening conditions contains a capacitor, which can store electric charge and energy. In case of emergency, the heart can be "restarted" using this charge and energy.

Electric charge is a special characteristic of bodies. Some materials may be charged electrically by rubbing. Bodies charged like this will show an interaction not seen before: they exert force on each other. This interaction is called *electric interaction*. One of the scientific fields of electricity is called *electrostatics*, which deals with electrically charged objects at rest and the interactions in between.

Electric charge (symbol: q): one of the properties of bodies is the electric interaction, which is the basis of the electric field and electric phenomena. There are two kinds of charges: **positive** and **negative**. By definition, the charge of amber is negative if it is rubbed by wool (i.e., the charge of wool is positive). Bodies with like charges repel each other, while those with unlike charges attract. The SI unit of electric charge is the **coulomb** (C). The smallest charge is the **elementary charge** (symbol: e), which equals to $1.6 \cdot 10^{-19}$ C. The electric charge of a body can only be the integer multiple of this value, that is, the electric charge is a *discrete* physical quantity that cannot assume any value. Electric charge does not exist "on its own", it is always linked to bodies, so called *charge carriers*.

Charge carriers: particles which have electric charge. The most important charge carrier is the **electron**, which has a charge of $-e$. The **proton** that can be found in atomic nuclei has the same charge but with opposite sign: $+e$. Atoms, molecules, and macroscopic bodies are electrically neutral if, as in most cases, they contain the same number of protons and electrons. If, however, we remove some electrons from the atoms, molecules, or macroscopic bodies (e.g., by rubbing or irradiation)then the charge equilibrium will be broken and the given object will have a net positive charge. A positive ion formed from a neutral atom is called cation. If, on the other hand, the body acquires extra electrons the charge equilibrium will be broken in the opposite direction and it will have a net negative charge as in the case of negatively charge ions called anions.

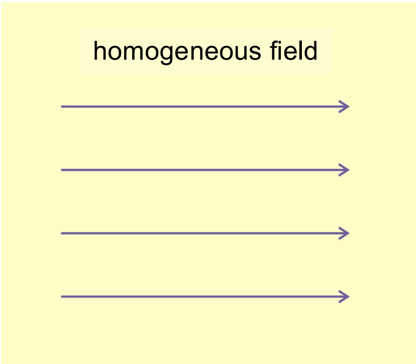
There is an interaction between electrically charged bodies (i.e., they exert force on each other) which we have not encountered before: this is called electric interaction and its force law is called *Coulomb's law*.

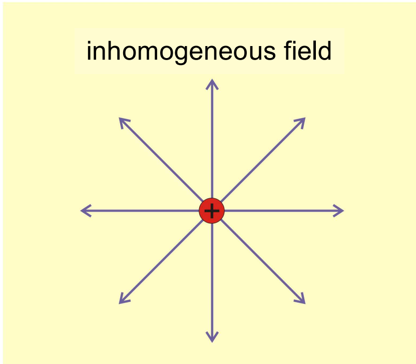
Coulomb's law: is the force law of electrostatic interaction, which gives the force acting between two electrically charged point-like bodies (their charges are q_1 and q_2), the so-called **Coulomb force,** as a function of the distance (r) between them:

$$F = k\frac{q_1 \cdot q_2}{r^2}$$

where k is a constant (Coulomb's constant) with the value: $k = 9 \cdot 10^9$ Nm^2/C^2. The Coulomb force acts along the line connecting the two bodies; in case of unlike charges it is attractive, in case of like charges it is repulsive. For example, it is the Coulomb force that keeps the negative electrons around the positive atomic nucleus. Coulomb’s law is very similar to the gravitational force law (see Chapter 4): the form of the equations is similar (both are inverse-square laws) only the masses are replaced by charges in the numerator, and a different coefficient is used. The consequence of this similarity is that the concepts, models, physical quantities used to describe the two kinds of interactions are also very similar. For example, the model of physical force field is used in case of both phenomena, which in the case of electric interaction is called the *electric field*.

Electric field: a model for the interpretation of the electric interaction which helps to imagine how two bodies act on each other even if they are far away. According to the model, charges create a force field around themselves and they exert force on each other by this field and not directly. The structure of the force field can be visualized by **field lines**. The direction of the field line – or, more precisely, the direction of the tangent of the field line – in a given point gives the direction of the force acting on the positive test charge. The density of field lines on the other hand gives the magnitude of the force. In case of a **homogeneous** (or uniform) **electric field** the force acting on the positive test charge has the same strength and the same direction at each point, therefore the field lines are parallel and their density is uniform. On the other hand, the forces exerted by an **inhomogeneous** field may have different strength and direction in different points. The electric field between the plates of a capacitor (see later) is homogeneous. The electric field of a point-like charge is inhomogeneous, or more precisely, radially symmetric (see Figures below).



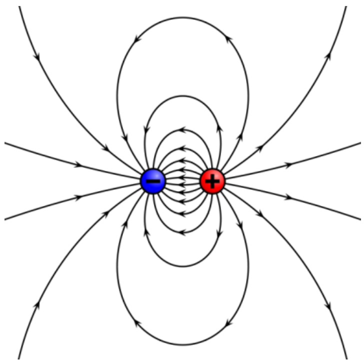


Electric dipole: two charges of equal magnitude but opposite sign ($+q$ and $-q$) at a given distance (d). The electric field of this arrangement of charges is also inhomogeneous (see Figure) and called a **dipole field**. The strength of a dipole is given by the **electric dipole moment** (p):

$$p = q \cdot d$$

Its unit is **coulomb·meter** (C·m). Another common unit is the **debye** (D). The water molecule has a strong electric dipole moment: electrons are not uniformly distributed over the molecule due to the higher electronegativity ("electron-attracting ability") of oxygen, hence they are closer to the oxygen atom. As a result, this part of the molecule will have a negative charge surplus, and the hydrogens will be relatively positive. The electric field of the heart can also be approximated with a dipole field.

Electric dipole and its field

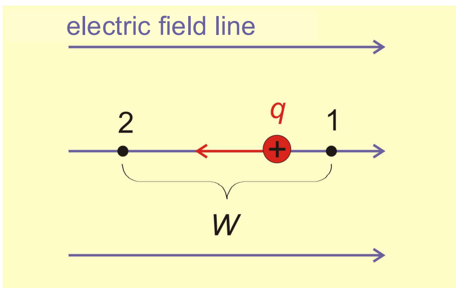


Although the field lines are useful to visualize the electric field , its exact description requires physical quantities. These physical quantities are the *electric field strength*, the *electric tension* commonly known as *voltage*, and the related concept of *electric potential*. The first one characterizes the electric field from the aspect of force while the other two from the aspect of work and energy, respectively.

Electric field strength (usual symbol: E): force (F) acting on a positive test charge (at a given point of the electric field) divided by the charge (q):

$$E = \frac{F}{q}$$

In other words, the electric field strength is the force acting upon a unit of positive charge (i.e., +1 C). The SI unit is newton/coulomb (N/C) or volt/meter (V/m) (for volt, see below). Electric field strength is a vector quantity. In a homogeneous electric field the electric field strength is uniform with regards to both magnitude and direction.



Voltage or electric tension (usual symbol: U): The work (W) done on a positive test charge when moved against the electrical field from position #1 to position #2 divided by its charge (q) (see Figure).

$$U_{21} = \frac{W}{q}$$

In other words, voltage is the work needed to move a unit charge. Its SI unit is the **volt** (V; 1 V = 1 J/C). Attention: voltage characterizes not a single point in space but a pair of points. The index numbers used in the formula (U_{21}) are often neglected, and we simply write U. In the case shown in the Figure we proceed against the electric field, hence we exert the force against it. Therefore, the work, as well as U_{21}, will be positive. In such cases we say that point #2 is at higher *potential* than point #1. If we swapped the two points, that is, we moved in the opposite direction, then we would get the same work but with negative sign. Voltage characterizes the field

strength, too: if the field strength is greater, more work must be done, and the voltage between the two points will be greater as well.
The voltage between the interior and the exterior of a resting eukaryotic cell's membrane is around –100 mV; this is the resting voltage or, less precisely, resting potential, which is related to the asymmetrical distribution of anions and cations. The reason for the negative sign is that the electric field points in the direction the interior of the cell, and by convention, the test charge is moved from the exterior to the interior.

Electric potential (usual symbol: φ): Let us fix a reference point (0) with an arbitrarily set value of zero potential ($\varphi(0) = 0$). The $\varphi(i)$ potential of any other i point is equal to the U_{i0} voltage between the 0 and i points.

$$\varphi(i) = U_{i0}$$

In other words: the voltage between two points is equal to the difference between their electric potentials:

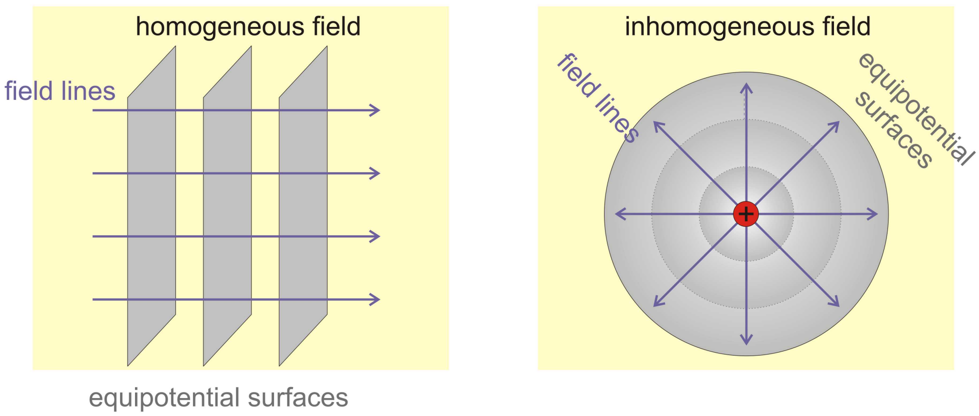
$$U_{i0} = \varphi(i) - \varphi(0)$$

as well as

$$U_{21} = \varphi(2) - \varphi(1)$$

So the meaning and, consequently, the unit of potential difference and voltage is the same. The precise name of the voltage measured between the two sides of a quiescent cell's membrane is "resting voltage" or "resting potential difference" but the term "resting potential" most often used in life sciences is also unambiguous since the same meaning is implied.

Equipotential surface: the set of all points which have the same electric potential value. Electric field lines cross equipotential surfaces at a right angle at every point of the electric field.



Capacitor or, formerly, condenser: an electrical component that is capable of storing electric charge and energy as well as creating homogeneous electric field. Its simplest form is the **parallel plate capacitor**, which consists of two parallel metallic plates with vacuum or some insulating material between them. If we charge the capacitor, one of the plates will receive $+q$, the other $-q$ electric charge. Between the two plates a nearly homogeneous electric field is formed. The electric field strength (E) is proportional to the potential difference (U) of the two plates:

$$U = E \cdot d$$

where d is the distance between the plates. By rearranging the formula, we get $E = U/d$ from which the unit of E is V/m, same as the N/C we saw above. On the two sides of the cell membrane there are relatively well conducting solutions while the membrane itself is a rather good insulator; this arrangement is reminiscent of a capacitor. Therefore, the electric model of the cell membrane also contains capacitive elements.
The greater the charge of the capacitor plates, the greater the electric field strength as well as the voltage between them. Ultimately, the electric charge and the voltage are proportional, the proportionality constant is called *capacitance*.

Capacitance (usual symbol: C): the electric charge (q) of a capacitor divided by its voltage (U):

$$C = \frac{q}{U}$$

The SI unit of capacitance is the **farad** (F; 1 F = 1 C/V). Capacitance is the measure of the charge-storing ability of a capacitor which gives the charge stored in the capacitor if its voltage is 1 V. The capacitance primarily depends on the dimensions of the capacitor: the greater the surface area (A) of the plates, the more charge it can store at a given voltage; the lower the distance (d) between the plates, the stronger the field and the more charge required to maintain a given voltage. Summarizing these will lead to the formula:

$$C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{d}$$

where we have a compound proportionality constant: $\varepsilon_0 \cdot \varepsilon_r$. Here, ε_0 is the absolute permittivity of vacuum ($\varepsilon_0 = 8.85 \cdot 10^{-12}$ (As)/(Vm)) and ε_r is the relative permittivity (a.k.a. dielectric constant) of the insulating medium between the plates (if any). The latter quantity is a mere number characterizing the insulating material that gives the factor by which the capacitance of the capacitor increases if the space between the capacitor plates is filled with this material instead of vacuum. The symbol of a capacitor (or capacitance) in circuit diagrams is shown in the Figure.

In order to charge a capacitor work must be done, and this work is stored in the energy of the electric field of the capacitor. Charging could be carried out theoretically in the following way: In the beginning both plates are electrically neutral. Transfer an electron from one plate to the other one. This does not require electric work. To transfer the second electron, however, we need to do electric work because this electron will be moved form an already little bit positive plate to a little bit negative plate against the repulsive force. If we want to further increase the charge, the transfer of each extra electron will require a little bit more work.

The electric energy stored in the capacitor: the work (W_e) needed to charge the capacitor which is stored as energy in the electric field of the capacitor. The more charge is transferred to the plates of the capacitor, the stronger the electric field and the greater the energy:

$$W = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CU^2$$

In order to transform the equation, we used the previously explained $q = CU$ formula as well.

Capacitors are used in the circuitry of different devices, e.g., defibrillators. In certain cases, we have to use more than one capacitor. Two capacitors may be connected either in parallel ("side by side") or in series ("one after the other").

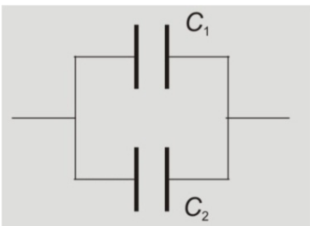
Connecting capacitors:

- if capacitors are connected **in parallel**, the individual capacitances are added. The total capacitance (C) is:

$$C = C_1 + C_2 + \cdots$$

- if capacitors are connected **in series**, the reciprocals are added:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$



parallel connection

series connection

Problems:

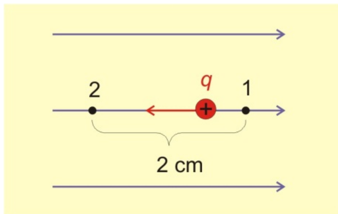
1. What is the Coulomb force between the proton and the electron of a hydrogen atom, if the distance between them is 100 pm?

2. A Cl^--ion and a Na^+-ion are exactly on the opposite sides of a cell membrane. By what force do they attract one other if the thickness of the membrane is 6 nm?

3. Two objects with the same positive charge are 1 m far away from one another. They repel one another with a $9 \cdot 10^9$ N force. What is their charge?

4. Two bodies, 'A' and 'B' have the same charge ($q_i = q_B$) so they act with a certain force on one another. We double the q_B charge. Choose the correct statement.
A: The both forces acting on each object will double.
B: Only the force acting on body 'B' will double because only the q_B charge is doubled.
C: The force acting on body 'A' won't change because the q_A charge does not change.
D: Body 'B' will repel 'A' more than 'A' will repel 'B'.

5. A body with $q = 0.1$ C charge is uniformly moved in a homogeneous electric field ($E = 1\ 200$ N/C) parallel to the field lines from position #1 to position #2.
a) What force acts on the body?
b) What work is done during the movement?
c) What is the voltage between the two positions?



6. What is the electric field strength in a point where a force of 480 N acts on a body of 0.5 C charge?

7. A body with $q = 5$ nC charge is placed into a certain point of an electric field. The field moves the body to a point which has an electric potential 2 000 V less than the first point. What work is done by the field during the movement?

8. In an X-ray tube 80 kV voltage is accelerating an electron from the cathode toward the anode of the tube.
a) What is the accelerating work done by the field?
b) This work will appear in the form of the kinetic energy of the electron. What is the speed of the electron if its mass is considered constant ($m = 9.11 \cdot 10^{-31}$ kg), i.e., we neglect the relativistic mass increase?

9. The voltage of a parallel-plate capacitor is 600 V. What is the field strength between the plates if the distance between them is 2 mm?

10. The membrane potential measured between the two sides of an excitable cell is –90 mV. Suppose that the electric field in the 10 nm thick membrane is homogeneous. Find the field strength.

11. The capacitance of the capacitor in a defibrillator is 50 μF. We charge it to a rather high voltage, 5000 V before intervention.
at) What is the charge of the capacitor?
b) What is the energy stored in the capacitor?

12. A capacitor of 50 nF capacitance has a charge of 30 μC. Determine
a) the voltage of the capacitor and
b) the energy stored in the capacitor.

13. Two capacitors (10 nF each) are connected. What is the total capacitance if they are connected
a) in parallel,
b) in series?

14. Ten capacitors (3 nF each) are connected in parallel. What is the total capacitance?

15. Two capacitors are connected in series ($C_1 = 10$ nF and $C_2 = 40$ nF). What is the total capacitance?

Solutions:

1. The distance is 100 pm = 10^{-10} m. The charge of both particles is the elementary charge but with opposite sign. The magnitude of the attractive force according to Coulomb's law is:
$F = k\frac{q_1 \cdot q_2}{r^2} = 9 \cdot 10^9 \cdot \frac{\left(1.6 \cdot 10^{-19}\right)^2}{(10^{-10})^2} = 2.3 \cdot 10^{-8}\ \mathrm{N} = 23\ \mathrm{nN}$.
This force between the two particels is ways greater than the $1.01 \cdot 10^{-47}$ N gravitational force (see Chapter 4 Problem 7)

2. $6.4 \cdot 10^{-12}$ N

3. 1 C (This could be the definition of the coulomb unit, too.)

4. A

5. a) The force can be calculated from the definition equation of the electric field strength:
$F = q \cdot E = 0.1 \cdot 1\,200 = 120\ \mathrm{N}$.
b) For the uniform moving we need to exert just as much force as exerted by the field on the charge because there is no acceleration so the total force acting on the object must be zero. The displacement and the force point in the same direction, so the work is: $W = F \cdot s = 120 \cdot 0.02 = 2.3\ \mathrm{J}$.
c) From the definition equation of voltage. $U_{21} = \frac{W}{q} = \frac{2.4}{0.1} = 24\ \mathrm{V}$ (The potential of position #2 is 24 V higher than that of position #1.)

6. 960 N/C

7. 10 μJ (The field accelerates the body and the work done by the field will "reappear" as the kinetic energy of the object.)

8. a) 12.8 fJ; b) $1.68 \cdot 10^8$ m/s

9. In the homogeneous field of the capacitor: $E = \frac{U}{d} = \frac{600}{0.002} = 300\,000\ \frac{\mathrm{V}}{\mathrm{m}}$ (The V/m unit is identical to the N/C unit.)

10. $9 \cdot 10^6$ V/m

11. a) The charge: $q = C \cdot U = 50 \cdot 10^{-6} \cdot 5\,000 = 0.25\ \mathrm{C}$.
b) The energy: $W = \frac{1}{2} \cdot C \cdot U^2 = \frac{1}{2} \cdot 50 \cdot 10^{-6} \cdot 5\,000^2 = 625\ \mathrm{J}$.

12. a) 600 V; b) 9 mJ

13. a) In case of parallel connection the capacitances are simply added: $C = C_1 + C_2 = 10 + 10 = 20\ \mathrm{nF}$.
b) In case of series connection use the reciprocal addition rule: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$. Attention, the problem is not yet done! The last step is to calculate the reciprocal of the sum of reciprocals: $C = \frac{10}{2} = 5\ \mathrm{nF}$.
(The total capacitance in series is always smaller than any of the individual capacitances.)

14. 30 nF

15. 8 nF

11. Electricity — Electric Current

Electric current can flow through the human body, for example, in muscle tissue, nerves, the heart, the brain, etc. Electric current is used by the defibrillator to restart the heart or the pacemaker. In physical therapy, direct current is used for galvanic exercise and iontophoresis, and high-frequency alternating current is applied in diathermy. Virtually every medical instrument uses electric current in numerous ways. For example, in medical imaging methods the different signals taken from the human body are always turned into electric signals, because they can be more easily and efficiently processed through signal amplification, filtering, digitization, etc.

The constituent charged particles of a body, such as ions and electrons, are in perpetual random (thermal) motion. If they are placed into an electric field, then an additional, directed collective motion arises because the electric field acts on them with a force parallel to the field lines. This collective motion is called *electric current*.

Electric current: the collective motion of particles carrying electric charge. Obviously, this requires the relatively free motion of the charge carriers. A material that contains charge carriers which can move freely is an electric **conductor**. Examples are metals due to the "free" electron cloud resulting from the metallic bond, and electrolyte solutions due to the ions moving freely in the liquid phase. If there are no freely moving charge carriers in the material, then it is an electric **insulator**. The current that is constant in time is called **direct current** (DC); in case of **alternating current** (AC) the current changes as a sine function over time. The direction of the current is defined, by convention, according to the flow direction of the positive charges (**conventional current direction)**. Due to their negative charge, electrons in metals move in the opposite direction, which can make this rule seem odd, but the electron was not yet discovered when this convention was established.

Electric current as a physical quantity (usual symbol: I): the amount of charge (q) passing through a given cross section of a conductor divided by the time elapsed (Δt):

$$I = \frac{\Delta q}{\Delta t}$$

In other words: electric current is the amount of electric charge flowing through a cross section in a unit of time. Its SI unit is **ampere** (A; 1 A = 1 C/s); *amp* is an abbreviation often used colloquially but not supported by SI. The pacemaker produces a 1 mA current in the heart, and the defibrillator is a thousand times stronger – for a very short time, though.

The electric current flowing in a conductor like a metal depends on the strength of the electric field moving the charge carriers. This strength can be characterized by voltage, for example. The electrons of a metal, however, do not move completely freely: they collide with the atoms of the metals making them lose energy, then they are accelerated again, collide again, and so on. Altogether the speed of their collective motion depends on the *resistance* exerted by the conducting material.

Ohm's law: the electric current (I) in a conductor is proportional to the voltage (U) between its two poles:

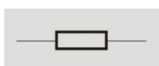
$$I = U/R \quad \text{or, equivalently,} \quad U = RI$$

where the proportionality constant is denoted by R and called *electric resistance*. So R is a constant – at least in the sense that it does not depend on voltage.

Electric resistance (usual symbol: R): ratio of the voltage (U) between two poles of a conductor and the current (I) that flows through it:

$$R = \frac{U}{I}$$

Its SI unit is the **ohm** (Ω; 1 Ω = 1 V/A). This kind of resistance is also called **ohmic resistance** to differentiate it



from other kinds of resistances. The symbol of ohmic resistance of wires and other parts in circuit diagrams (this is the IEC international standard, the U.S. standard is different):

The resistance of a conductor depends on its dimensions and material properties. Regarding dimensions: if the conductor is longer, then the same voltage corresponds to a weaker electric field, hence the motion of charges will be slower and the electric current lower; thus, the resistance is greater. If the cross-sectional area is greater, then more charge carriers can pass for a given voltage and electric field strength, therefore the current will be higher and the resistance lower. The resistance of a conductor can thus be defined as:

$$R = \rho \cdot \frac{l}{A}$$

where *l* is the length, *A* is the cross-sectional area of the conductor. ρ is a proportionality constant characteristic ("specific") of the material of the conductor, therefore it is called **specific resistance** or **resistivity**. The unit of resistivity is $\Omega\cdot$m. The resistivity of the majority of materials increases with temperature because the number of collisions will be larger due to the more intensive thermal motion of the particles in the conductor.

In certain contexts, e.g., when describing the electric properties of liquids, solutions and body tissues, the reciprocal of resistance and resistivity are preferred.

Electric conductance (usual symbol: *G*): the reciprocal of electric resistance, that is:

$$G = \frac{1}{R}$$

Its SI unit is the **siemens** (S; 1 S = 1/Ω). The conductance can also be used to give Ohm's law: $I = G \cdot U$.

Specific conductance or **conductivity** (symbol: σ): the reciprocal of resistivity, that is:

$$\sigma = \frac{1}{\rho}$$

Its SI unit is S/m. It is a physical quantity specific for the conductor's material, similarly to resistivity. For example, in case of an electrolyte solution it depends on the type and concentration of ions. Within certain limits it is directly proportional to the ion concentration. Among body tissues it is blood that has the highest conductivity, because it contains a large quantity of highly mobile ions. The conductivity of the muscle tissue is also relatively high. By contrast, the conductivity of skin and bone is rather low. There are several diagnostic methods utilizing these conductivity differences, e.g., impedance cardiography.

In electric circuits, resistors may be connected in series or in parallel.

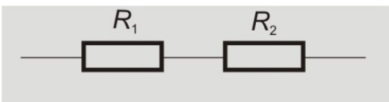
Connecting resistors:

- if resistors are connected **in series**, then the individual resistances are added. The total resistance (*R*) is:

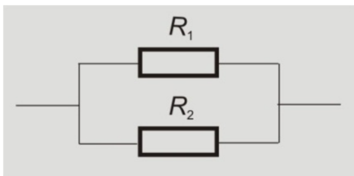
$$R = R_1 + R_2 + \cdots$$

- if resistors are connected **in parallel**, then the reciprocals of the resistances are added:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$



series connection



parallel connection

As we already mentioned, the charge carriers in the conductor lose energy due to perpetual collisions. The accelerating work of the electric field will finally increase the thermal motion of the particles, that is, it turns into thermal energy and the conductor becomes hotter; this is known as *Joule heating*.

The **work of the electric current,** a.k.a. **Joule heating** (symbol: *W*): the work done by the electric field when moving electric charge carriers which turns into another form(s) of energy; in case of ohmic resistors it turns completely into thermal energy. Joule heating is proportional to the electric current (*I*), the voltage between the poles of the conductor (*U*), and time (*t*):

$$W = U \cdot I \cdot t$$

Its unit is the **joule,** just as for all sorts of energy. Ohm's law can be used to transform the formula:

$$W = R \cdot I^2 \cdot t \quad \text{or} \quad W = \frac{U^2}{R} \cdot t$$

Electric power (symbol: P_{el}): electric work done in unit of time:

$$P = U \cdot I$$

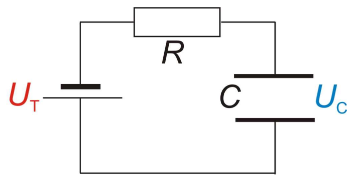
Its unit is the **watt**, just as for all sorts of power.

Electrical circuit: an interconnection of **electrical components** (e.g., capacitors, resistors, voltage source) enabling electric current flow.

Kirchhoff's circuit laws: laws regarding the distribution of electric current and voltage in electrical circuits.
– **Kirchhoff's current law** (a.k.a. **Kirchhoff's junction law**, **Kirchhoff's first law**): as a result of conservation of electric charge, the currents flowing into a junction are equal to the currents flowing out of that junction. In other words, in a branched (parallelly connected) circuit current is partitioned between the branches.
– **Kirchhoff's voltage law** (a.k.a. **Kirchhoff's loop law**, **Kirchhoff's second law**): as a result of conservation of energy, the directed sum (i.e., considering the signs) of voltages of electrical components along a loop within an electrical circuit is zero. In other words, within a loop (e.g., serially-connected circuit) voltage is partitioned between the electric components (e.g., resistances, capacitors, etc.)

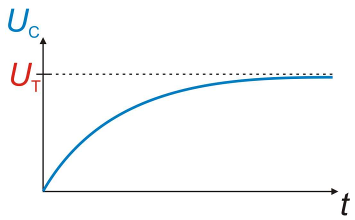
We will pay distinguished attention to a special circuit, the *RC circuit*. RC circuits can be used to model the electric properties of the cell membrane or the skin. They are also used in medical signal processing.

RC circuit: an electrical circuit consisting of an ohmic resistor and a capacitor (in series here). We will discuss two transient (short-term) phenomena: charging and discharging of the capacitor. In order to charge the capacitor we will need to connect an additional electrical component, a voltage source (battery). During discharge we remove the battery and connect the two poles of the capacitor through the resistor. Let us start with charging!



Charging the RC circuit:

- In the initial $t = 0$ point in time let the capacitor be "empty", i.e., both its charge and voltage are zero: $U_C = 0$ V.
- According to the loop law, battery voltage (U_B) is partitioned between the resistor and the capacitor. Because capacitor voltage is zero initially, resistor voltage is equal to the battery voltage.
- According to Ohm's law, a current proportional to voltage flows through the resistor.
- Because current transfers charge, it will start to "load" (charge) the capacitor. Thus, capacitor voltage (U_C) increases.
- As capacitor voltage progressively increases, resistor voltage progressively decreases because of the voltage partitioning. Capacitor voltage increases until it reaches battery voltage. Once it happens, the resistor voltage drops to zero.
- As the resistor voltage decreases, current also decreases according to Ohm's law. Accordingly, capacitor charging progressively decreases, and capacitor voltage *asymptotically* approaches battery voltage (and, in theory, will reach it in an infinite amount of time). Finally, resistor voltage and current become zero and capacitor charging complete.
- The graph of capacitor charging shown in the Figure can be described by the following function:



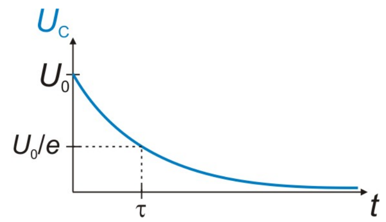
$$U_C = U_B \cdot \left(1 - e^{-\frac{t}{R \cdot C}}\right)$$

where U_C is the voltage of the capacitor at time t, U_T is battery voltage, R is resistance, and C is capacitance. The RC product in the exponent has time dimension (i.e., its unit is second; note that the expression in the exponent must be dimensionless). The RC time interval is the **time constant** of the RC circuit and it is denoted by τ. We will see its demonstrative interpretation in the next section (discharging).

It is importrant to emphasize that after the capacitor has been charged no current flows in the circuit because there is an insulator between the plates of the capacitor. That is, the capacitor presents an infinite resistance in a DC circuit.

Discharging the RC circuit:

- Let the initial voltage of the charged capacitor be U_0. If the preceding charging lasted long enough, then U_0 is equal to the battery voltage, otherwise it is less. Let us remove the battery from the circuit. Now the circuit contains only two components, the capacitor and the resistor. According to the loop law the voltage of the resistor is equal to the voltage of the capacitor, but their signs are opposite.
- Because the two plates of the capacitor are connected through the resistor, current will flow from the higher-potential ("positive plate") to the lower-potential plate ("negative plate"). Attention: this is the conventional current direction; electrons actually flow from the negative plate with electron surplus to the positive plate with electron deficiency. As the charge of the capacitor decreases, its voltage (U_C) decreases proportionally.
- In the beginning of discharging the voltages of both the capacitor and the resistor are maximal, just as the current (because of Ohm's law). As capacitor, hence resistor, voltage progressively decrease, the current decreases. As a result, the rate of capacitor discharge (i.e., the progressive decrease in voltage and current) also decreases (see Figure) until capacitor voltage reaches zero (in infinite time, theoretically), when discharge is over.



- The graph of discharging shown in the Figure can be described by the following function:

$$U_{\mathrm{C}} = U_0 \cdot e^{-\frac{t}{R \cdot C}}$$

As we already mentioned, the $\tau = RC$ product is the time constant of the RC circuit. It is the time required to discharge the capacitor to the $1/e$-th part of the initial U_0 voltage (i.e., to U_0/e).

There are several diagnostic and therapeutic methods where the physician uses alternating current. For example, the aforementioned *impedance cardiography* is used to determine the *cardiac output* (volume of blood pumped by the heart in unit time) simply and non-invasively. In this method high-frequency AC is used to avoid electric shock during examination. High-frequency AC is also used in *diathermy*. Because of their importance, we will discuss AC circuits as well.

Alternating current circuit (**AC circuit**): current and voltage changes periodically following a sine function:

$$I = I_{\max} \sin \omega t \quad \text{and} \quad U = U_{\max} \sin(\omega t + \varphi)$$

where $I_{\max}$ and $U_{\max}$ are maximal values or **peak values**, ω is the angular frequency, φ is the phase shift between current and voltage change. If the circuit contains only ohmic resistors, then there is no phase shift. If, however it also contains non-ohmic components, capacitors, for instance, then phase shift arises. Besides peak values of voltage or current, we also use so called **effective values**, which are values averaged in a certain way. Effective current or voltage of an AC circuit are equal to the direct current or voltage, respectively, of a DC circuit that produces the same Joule heating in a given conductor. The effective values in case of sinusoidal AC are also known as **root-mean-square** or **RMS** values:

$$I_{\mathrm{eff}} = I_{\mathrm{RMS}} = \frac{I_{max}}{\sqrt{2}} \quad \text{and} \quad U_{\mathrm{eff}} = U_{\mathrm{RMS}} = \frac{U_{max}}{\sqrt{2}}$$

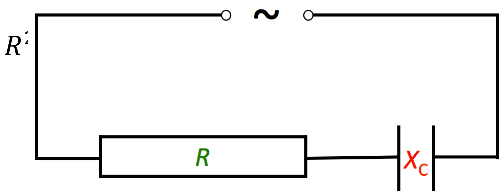
Capacitor in an AC circuit. Capacitors display an interesting behavior in AC circuits. While in a DC circuit a capacitor presents an infinite resistance (there is no conduction between the two plates due to the insulation), in an AC circuit it presents a finite "resistance" called **capacitive reactance** (symbol: X_C). In a DC circuit the current flows only for a very short time during charging or discharging (the τ time constant is usually a fraction of a second), but in an AC circuit the current is maintained: the alternating current keeps continuously charging and discharging the capacitor with alternating electric polarity. The capacitive reactance (X_C) of a capacitor with C capacitance is:

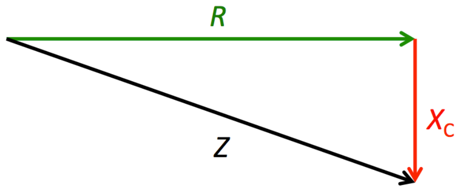
$$X_{\mathrm{C}} = \frac{1}{\omega C}$$

where ω is the angular frequency of the AC. As ω increases, reactance decreases. In case of high-frequency AC the capacitive reactance is negligible. The unit of reactance is also the ohm (Ω).

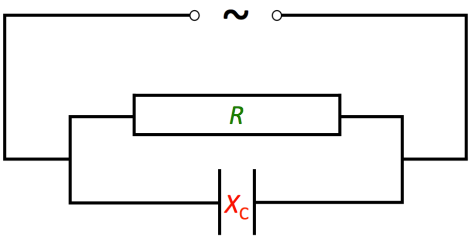
Impedance: If an AC circuit contains both resistors and capacitors, the total "resistance" of the whole circuit is called impedance (usual symbol: Z; unit: ohm). The impedance, however, cannot be calculated simply by adding the R and X_C values (in serially-connected or series RC circuit) or their reciprocals (in parallelly-connected or parallel RC circuit), because of the phase shift that arises between U and I. Instead, the two quantities have to be considered as vectors perpendicular to each other which defines a right triangle so that impedance is the hypotenuse (see figure). We can use the *Pythagorean theorem* ($a^2 + b^2 = c^2$, where c is the hypotenuse) to find Z.

– **Impedance of a series RC circuit**: the legs of the right triangle are ohmic resistance (R) and capacitive reactance (X_C), and the hypotenuse is the impedance (Z):

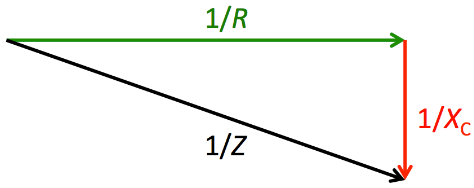




– **Impedance of a parallel RC circuit**: the legs of the right triangle are the reciprocals of the ohmic resistance ($1/R$) and the capacitive reactance ($1/X_C$), ans the hypotenuse is the reciprocal of the impedance ($1/Z$):



$$\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C}\right)^2 = \left(\frac{1}{Z}\right)^2$$



Electrodes. Most components of electronic devices are made of metals where the charge carriers are the mobile electrons. However, there are several examples of electrical circuits which contain non-metallic components such as solutions, non-metallic crystals, tissues or body parts of living organisms, vacuum, or some kind of gas or vapor. These non-metallic components must, of course, be connected to the rest of the circuit:. The metallic parts in direct connection with a non-metallic component are called **electrodes**. Some examples of devices that contain electrodes with the indication of the non-metallic component:

Some examples of devices containing electrodes

device	non-metallic component
Galvanic (or Voltaic) cell; electrolytic cell; pH meter	electrolyte solution
ECG (electrocardiograph); defibrillator	the body around the heart
EEG (electroencephalograph)	the head
microelectrodes	interior or exterior of a cell
ultrasound generator	piezoelectric crystal
gas discharge lamps	metal vapor or gas
vacuum tube (cathode ray tube; X-ray tube, photomultiplier tube)	vacuum
semiconductor devices (diode, transistor, integrated circuit etc.)	semiconducting crystal

There are two main types of electrodes in DC circuits: anodes and cathodes.

– **Anode**: the electrode that attracts anions from the non-metallic component (i.e., the electric potential of an anode is higher than that of the connected non-metallic material).

– **Cathode**: the electrode that attracts cations from the non-metallic material connected to it (i.e., the electric potential of a cathode is lower than that of the connected non-metallic material).

These definitions are much more general than those learned in electrochemistry, but they are consistent. Some electrodes are neither anodes nor cathodes because the sign of their potential relative to the non-metallic material is not characteristic and may vary over time (for example, ECG electrodes). Other electrodes may be both anodes and cathodes at a time because they are in contact with two non-metallic components with different signs of relative potential (see *dynode* in Biophysics).

***Problems**:*

1. A rheumatism patient receives iontophoretic therapy (administration of ionic pharmaceutical by means of direct current). The voltage is 40 V, the resistance of the treated body part is 12 500 Ω.
a) What is the electric current flowing through the treated area?
b) What amount of electric charge flows through the treated area in a 10-minute treatment?
c) How many drug molecules enter the body during this period if they are in the form of monovalent ions? Give the amount of molecules in moles, too.

2. During a one-minute-long X-ray trans-illumination $1.875 \cdot 10^{18}$ electrons pass through the X-ray tube.
a) How much charge passes through the tube?
b) What is the electric current flowing in the tube?

3. In the first moment of a defibrillator intervention the 6 kV voltage connected to the body creates a 0.2 A current. Find the a) resistance and the b) conductance of the involved body part.

4. The cross-section area of the copper wire of a 20-m-long extension lead is 1.5 mm^2. The resistivity of copper is $1.78 \cdot 10^{-8}$ Ωm. Find
a) the resistance of the wire,
b) the conductance of the wire, and
c) the conductivity of copper.

5. During one of the biophysics labs we fill a graduated cylinder (height: 6 cm, cross section area: 2 cm^2) with a salt solution of 12 mS/m conductivity. Determine
a) the conductance,
b) the resistivity, and
c) the resistance of the solution in the cylinder.

6. We connect two resistors (5 kΩ each). Find the total resistance in case of
a) series connection and
b) parallel connection.

7. We connect fifty resistors (10 kΩ each) in a) parallel and in b) series. Find the total resistance for both cases.

8. The resistance of the tungsten filament in a traditional light bulb at the operating temperature is 529 Ω. We connect the bulb to the mains power which has an effective voltage of 230 V.
a) What heat is produced in the light bulb during a day?
b) What is the power of the bulb?

9. The power of a traditional light bulb is 15 W.
a) How much heat is produced by the bulb during one week of continuous operation?
b) What is the current flowing in the light bulb if it is connected to 230 V?

10. A simplified representation of a defibrillator would be an RC circuit. The capacitor used in the device (C = 20 μF) was charged before treatment to a rather high voltage, 5 kV. Then it is attached to the chest using two so-called paddle electrodes. The capacitor discharges through the chest as a resistor (R = 1200 Ω).
a) What is the energy stored in the charged capacitor?
b) What is the electric current flowing through the body in the very beginning of discharging?
c) What is the time constant of the RC circuit that is created during the intervention?
d) What is the voltage of the capacitor 0.1 s after beginning the intervention?
e) In what time does the voltage of the capacitor decrease to the thousandth, i.e. to 5 V?

11. a) The resistor of an RC circuit has a resistance of 10 MΩ, the time constant is 1 s. What is the capacity of the capacitor?
b) To what percent of the original value does the voltage of the capacitor decrease in 2 s?

12. The time constant of an RC circuit is 0.6 s.
a) To what voltage can the capacitor be charged in 1 s if the voltage source is 100 V?
b) In what time will the capacitor be discharged from that value to its half?

13. The time constant of an RC circuit is 40 s. The capacitor is charged with a 9 V battery. What time does it require to charge the capacitor (from zero) to 8.9 V?

14. The domestic mains power in Europe oscillates according to the function: $U = 325\ \mathrm{V} \cdot \sin(314\ \mathrm{s}^{-1} \cdot t)$. Find:
a) the peak value of the voltage,
b) the effective (RMS) value of the voltage,
c) the angular frequency of the AC, and
d) its frequency.

15. A 20 μF capacitor is connected to a voltage source. What is the capacitive reactance, if the voltage source provides
a) DC,
b) the mains voltage customary in Europe,
c) 5000 Hz AC.

16. An AC characterized by the $U = 34\ \mathrm{V} \cdot \sin(6283\ \mathrm{s}^{-1} \cdot t)$ function is connected to a 500 nF capacitor. Find
a) the peak voltage,
b) the effective voltage, and
c) the capacitive reactance of the capacitor.

Solutions:

1. a) From Ohm's law: $I = \frac{U}{R} = \frac{40}{12500} = 0.0032\ \mathrm{A} = 3.2\ \mathrm{mA}$.

b) The charge that flows during the Δt = 10 min = 600 s treatment duration can be calculated from the definition equation of electric current: $\Delta q = I \cdot \Delta t = 0.0032 \cdot 600 = 1.92\ \mathrm{C}$.

c) The charge of a monovalent ion is e. So the number of drug molecules is: $N = \frac{\Delta q}{e} = \frac{1.92}{1.6 \cdot 10^{-19}} = 1.2 \cdot 10^{19}$.
The amount of substance in moles is: $\nu = \frac{N}{N_A} = \frac{1.2 \cdot 10^{19}}{6.02 \cdot 10^{23}} = 1.99 \cdot 10^{-5} = 19.9\ \mu\mathrm{mol}$.

2. a) 0.3 C; b) 5 mA

3. a) 30 kΩ; b) 33.3 μS

4. a) The cross-sectional area is: $A = 1.5\ \mathrm{mm}^2 = 1.5 \cdot 10^{-6}\ \mathrm{m}^2$. The resistance is:
$R = \rho \cdot \frac{l}{A} = 1.78 \cdot 10^{-8} \cdot \frac{20}{1.5 \cdot 10^{-6}} = 0.237\ \Omega$
b) Conductance is the reciprocal of resistance: $G = \frac{1}{R} = \frac{1}{0.237} = 4.21\ \mathrm{S}$
c) Conductivity is the reciprocal of resistivity: $\sigma = \frac{1}{\rho} = \frac{1}{1.78 \cdot 10^{-8}} = 5.62 \cdot 10^{7}\ \frac{\mathrm{S}}{\mathrm{m}}$

5. a) 40 μS; b) 83.3 Ωm; c) 25 kΩ

6. a) In case of series connection resistances are simply added: $R = R_1 + R_2 = 5 + 5 = 10\ \mathrm{k\Omega}$
b) In case of parallel connection the reciprocals (i.e., the conductances) are added: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$
and then $R = \frac{5}{2} = 2.5\ \mathrm{k\Omega}$

7. a) 200 Ω; b) 500 kΩ

8. a) The mains AC power source has an effective voltage of 230 V which is equivalent in heating effect with a 230 V DC. The tungsten filament is an ohmic resistor, the Joule heating produced per day (=24·3600 s) is:
$Q = W = \frac{U^2}{R} \cdot t = \frac{230^2}{529} \cdot 24 \cdot 3600 = 8.64 \cdot 10^{6}\ \mathrm{J} = 8.64\ \mathrm{MJ}$

(Only a small part of this energy will appear as visible light).
b) The electric power of the light bulb: $P = U \cdot I = \frac{U^2}{R} = \frac{230^2}{529} = 100$ W.

9. a) 9.07 MJ; b) 65.2 mA

10. a) The energy stored in the capacitor is: $W = \frac{1}{2}CU^2 = \frac{1}{2} \cdot 20 \cdot 10^{-6} \cdot 5000^2 = 250$ J.
b) The current flowing through the body from Ohm's law is: $I = \frac{U}{R} = \frac{5000}{1200} = 4.12$ A (only in the very first moment, then it decreases rapidly).
c) The τ time constant is the product of resistance and capacity: $\tau = RC = 1200 \cdot 20 \cdot 10^{-6} = 24$ ms.
d) The voltage of the capacitor (U_C) decreases exponentially during discharging:
$U_C = U_0 \cdot e^{-\frac{t}{R \cdot C}} = 5000 \cdot e^{-\frac{0,1}{0.024}} = 77.5$ V.
e) Now the time (t) is the unknown, which is in the exponent so we have to solve an exponential equation – we have to use logarithm:
$5 = 5000 \cdot e^{-\frac{t}{0.024}}$
$0.001 = e^{-\frac{t}{0.024}}$
$1000 = e^{+\frac{t}{0.024}}$
$\ln 1000 = \frac{t}{0.024}$
$t = 0.024 \cdot \ln 1000 = 0.166$ s.

11. a) $C = 0.1$ μF; b) 13.5%

12. a) Charging is described with the $U_C = U_T \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$ ffunction where U_T is the charging voltage (voltage of the power source), U_C the voltage of the capacitor at a given t point in time, and τ (= RC) is the time constant of the circuit. The voltage in question is:
$U_C = U_T \cdot \left(1 - e^{-\frac{t}{\tau}}\right) = 100 \cdot \left(1 - e^{-\frac{1}{0.6}}\right) = 81.2$ V.
b) Use the same steps as in problem 10. part e):
$t = \tau \cdot \ln\frac{U_0}{U_C} = \tau \cdot \ln\frac{U_0}{\left(\frac{U_0}{2}\right)} = \tau \cdot \ln 2 = 0.415$ s.

13. 3 min

14. a) The peak value can be read easily from the function: 325 V.
b) The effective (root-mean-square) voltage: $U_{\text{eff}} = \frac{U_{max}}{\sqrt{2}} = \frac{325}{\sqrt{2}} = 230$ V. (This is the value known as mains voltage in Europe.)
c) The angular frequency can also be read from the function: 314 s^{-1}.
d) Frequency can be calculated from angular frequency from the formula learned in the chapter about circular motion: $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50$ Hz.

15. a) In case of DC ($\omega = 0$) the reactance of the capacitor is infinity (Note: $1/0 = \infty$).
b) The frequency of mains voltage in Europe is 50 Hz (see e.g. problem 14. part d)), so the capacitive reactance is:
$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 50 \cdot 20 \cdot 10^{-6}} = 159\ \Omega$.
c) If the frequency is 5000 Hz that's 100 times higher than that in part b) so the reactance is one hundredth, i.e., 1.59 Ω.

16. a) 34 V; b) 24 V; c) 318 Ω

12. Magnetism and Electromagnetic Induction

Some birds, fish and other animals are able to sense magnetic field and use the geomagnetic field (the Earth's magnetic field) for navigation. Humans have been using magnets for navigation for about 2500 years (e.g., in China small fish were used as compass). Even this very special phenomenon has many applications in medicine. The currents that flow during muscular and neural activity generate magnetic field. Measuring this – e.g. in the form of Magnetoencephalography (MEG) examination – may provide extra information about the examined organ.
Magnetic field may influence the function of cells and organs to some extent even though our knowledge about the phenomenon is still limited – making it a debated field of science. There are promising attempts for certain therapeutic uses (e.g., treatment of urinary incontinence).
Magnetic fields are used in nuclear magnetic resonance imaging (MRI or MR). MRI is a modern and high-resolution imaging method that conveys information-rich images of high diagnostic value.

In Ancient Greece it was observed that some rocks from the town of Magnesia (Μαγνησια) in Asia Minor attract or repel each other. This is the origin of the term *magnet*. The mentioned rocks show *magnetic interaction*. The description of magnetic phenomena is not simple, therefore we will give up the exact mathematical description of the topic in this chapter just as in the problems.

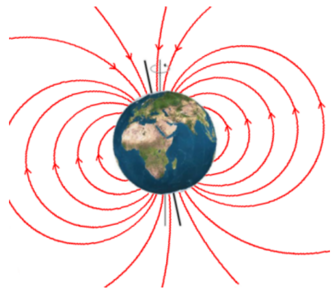
Magnet: a body with a special magnetic property. Magnets exert forces on one another that cannot be explained with other known interactions (e.g., gravitation or electric interaction). This interaction is called **magnetic interaction**. On one hand there are the rocks mentioned above, which are *permanent magnets* that typically contain large quantities of iron, nickel, or cobalt. On the other hand there are *electromagnets* (see later), which show magnetic properties as long as there is electric current passing through them. A magnet has two poles called the *north pole* and the *south pole* – the north pole being the one which points in the direction of the north pole of the Earth if the magnet can align itself freely. Like poles repel and unlike poles attract each other.

There are weaker and stronger magnets, which can be judged based on the force acting between them. The strength of a magnet is described by the *magnetic moment*.

Magnetic moment (usual symbol: m or μ (lower case Greek letter *mu*)): a physical quantity describing the strength of a magnet. Elementary particles of the atom – the electron, the proton, and the neutron – also have magnetic properties, hence magnetic moment. They can be considered miniature magnets or compasses.

As in the case of other interactions acting over long distances – gravitation and electric interaction – the concept of force field is used in this case as well.

Magnetic field: a model postulating that magnets create a magnetic field around themselves and exert forces by this field on one another even over relatively long distances. The structure, direction and strength of a magnetic field is visualized by field lines. If a needle-shaped magnet (compass) is placed in a given point of space, then the forces acting on it will rotate it until it becomes aligned with the field lines. The density of field lines indicate the strength of the field. As we already mentioned, some elementary particles also act as tiny compasses. In a magnetic field, they, or at least their magnetic moment, also align with the field lines. This phenomenon is exploited during nuclear magnetic resonance imaging (MRI): as a first step, the patient is placed in a very strong magnetic field. Atomic nuclei with magnetic moment – like hydrogen nuclei which are lone protons – align with the magnetic field lines. Note that individual magnetic moments in lieu of an external magnetic field are randomly oriented (i.e., they orient in random directions). As a result of this alignment the patient becomes "magnetized", i.e., it will attain a non-zero net magnetic moment. During the MRI measurement the changes in the net magnetic moment are investigated.



Let us briefly pause to compare the nature of electric and magnetic interactions: in what aspects are they similar or different? There are two kinds of poles (or "charges") in both cases: positive and negative electric charges as well as north and south magnetic poles. In both cases like poles repel and unlike poles attract. The difference is that in case of electricity a body is either charged positively or negatively: the opposite charges can be separated; in magnetism, by contrast, the two poles cannot be separated. A body cannot carry only one of the magnetic poles. If a bar magnet is cut in two, then both halves will contain the north and south poles. As a consequence the structure of the electric and magnetic fields are different. In an electric field the field lines emerge from a positive charge and end at a negative charge. By contrast, magnetic field lines are closed loops: they end at the same pole where they originate from (see, for example, the Earth's magnetic field).

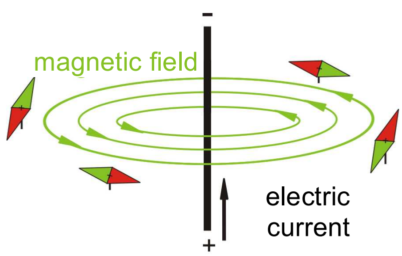
Earth is a magnet, too; its magnetic field is shown in the Figure. A magnetic compass is aligned north-south corresponding to the field lines. The geomagnetic field is not homogeneous at all: there are relatively large differences in the field strength. The strength of a field is characterised by the concept and physical quantity of *magnetic flux density* (= approx. "*magnetic field line density*").

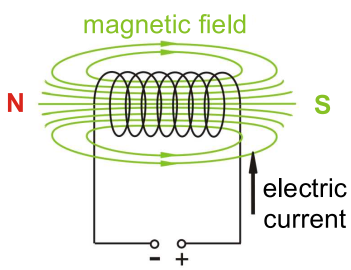
Magnetic flux density (usual symbol: B): physical quantity characterizing the strength of a magnetic field (i.e., density of field lines) and the forces acting on a magnet in an external magnetic field (i.e., the magnetic field of another magnet). Although B characterizes the strength of the magnetic field, it is not called "*magnetic field strength*" (for reasons not detailed here) but only in colloquial speech. The SI unit of magnetic flux density is the **tesla** (T). For example, the geomagnetic field has an average magnetic flux density of 50 μT; while the same value for magnetic fields used in MRI appliances is between 1–10 T, i.e., 100 000 times greater. In a homogeneous magnetic field the direction and magnitude of magnetic flux density is the same everywhere. As the Figure shows, the geomagnetic field is inhomogeneous. *Attention*: often (and confusingly) the physical quantity "magnetic flux density" is just called "magnetic field". That is, there is a *physical quantity* magnetic field, which is related to but distinct from the *phenomenon and concept* of magnetic field.

Let us *review briefly* what happens to a magnet when placed in an external magnetic field (i.e., the magnetic field of another magnet). There is an interaction between the magnet and the magnetic field. The strength of the interaction depends on two factors: the magnetic moment of the magnet (m) and the magnetic flux density (B). The forces acting during the interaction rotate the magnet to align it with the field lines. This interaction can be described energetically. The energy of the interaction between the magnet and the external field depends again on the aforementioned two factors, or, more precisely, it is proportional to their product ($m \cdot B$). When the magnetic moment is aligned with the field lines, its energy is lower, therefore this state is more advantageous. If we want to "flip" the magnet in the presence of the external magnetic field, then we need to exert force and work, that is, transfer energy to it, which will increase the energy of the magnet.

We mentioned in the beginning of this Chapter that besides permanent magnets there are electromagnets, too. Electromagnets are based on the *magnetic effect of electric current*.

The magnetic effect of electric current: a conductor generates a magnetic field around itself if electric current flows through it. This can be easily detected by small magnets, compasses (see Figure). The magnetic flux density is proportional to the electric current. The field becomes more concentrated (so the flux density increases) if the conductor is wound up into a coil (a.k.a. solenoid): the fields created by individual turns are added up creating a combined field with a flux density proportional to the number of turns. The field inside the coil (and even near the ends) is not only strong but quite homogeneous as well, especially in case of long coils.

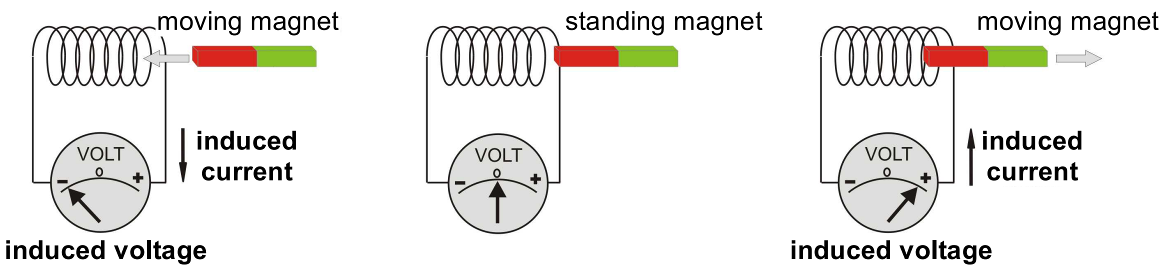




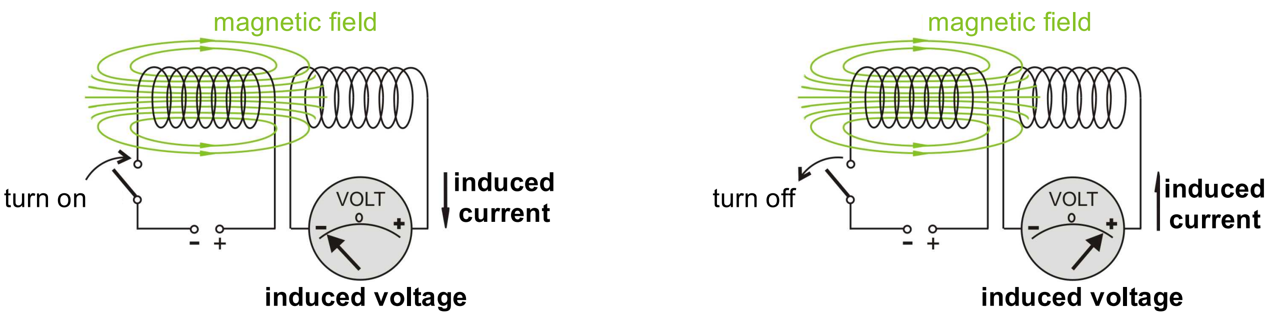
An electromagnet is a coil with electric current passing through it. If the current is switched off, its magnetism disappears. Electromagnets are used in technology and industry as well as in medicine: the very strong magnetic field of an MRI device can only be created by electromagnets. The practical advantages of electromagnets over permanent magnets are: strong and homogeneous field, the strength of field can be controlled by the current, and it can be switched off.

The phenomenon that a conductor with current passing through generates magnetic field can be re-worded: moving charges induce a magnetic field. One wonders, is symmetry at work here? That is, can the phenomenon be reversed? In other words, do moving magnets generate an electric field, too? The answer actually is yes, as we know it since the observations of Faraday. The phenomenon is called *electromagnetic induction*.

Electromagnetic induction: a magnetic field that changes over time generates an electric field.
An example of electromagnetic induction: take a coil and connect it to a voltmeter. Because the circuit contains no power source, the voltmeter indicates zero voltage, naturally. If we now take a permanent magnet and move it inside the coil, then the voltmeter indicates a voltage whenever the magnet is moving. The faster the magnet moves, the higher the voltage. The voltage created this way is called **induced voltage**. If the magnet is moved in the opposite direction (outward), then the sign of induced voltage will also be opposite. The phenomenon can also be observed if it is the coil that is moved rather than the magnet. We may conclude that the magnetic field change, caused by moving the magnet, induced an electric field, hence voltage and current, in the coil. The induced current will likewise generate a magnetic field around the coil with a polarity opposite to that of the magnet (i.e., attracts or repels the magnet departing or approaching, respectively). These forces can be sensed very well when moving the permanent magnet. That is, the induced voltage and current will act against the phenomenon creating it (in our case, the motion of the magnet), which is known as **Lenz's law**. This the simple "trick" of nature precludes the creation of electric energy without investing work.



Another example of electromagnetic induction: take two coils, connect a voltage source to one and a voltmeter to the other. Place them close to each other. When the first circuit's voltage source is switched on, the voltmeter of the second circuit indicates voltage for a short time. When the voltage source is switched off, the voltmeter indicates voltage for a short time again, but with opposite sign. Explanation: when the voltage source is on, there is a current in the first circuit creating a magnetic field in the coil as well as in the second coil placed close to the first one. In the moment of switching the voltage source on or off, the magnetic field changes: it will build up and break down, respectively. During the change of the magnetic field voltage is induced in the second coil.



It is generally true about electromagnetic induction that *the faster and greater the change in the magnetic field the greater the induced voltage*.

Regarding the second example of electromagnetic induction: voltage will be induced not only in the second coil (connected to the voltmeter) alone but also in the first coil that induced the magnetic field. This is called *self-induction*.

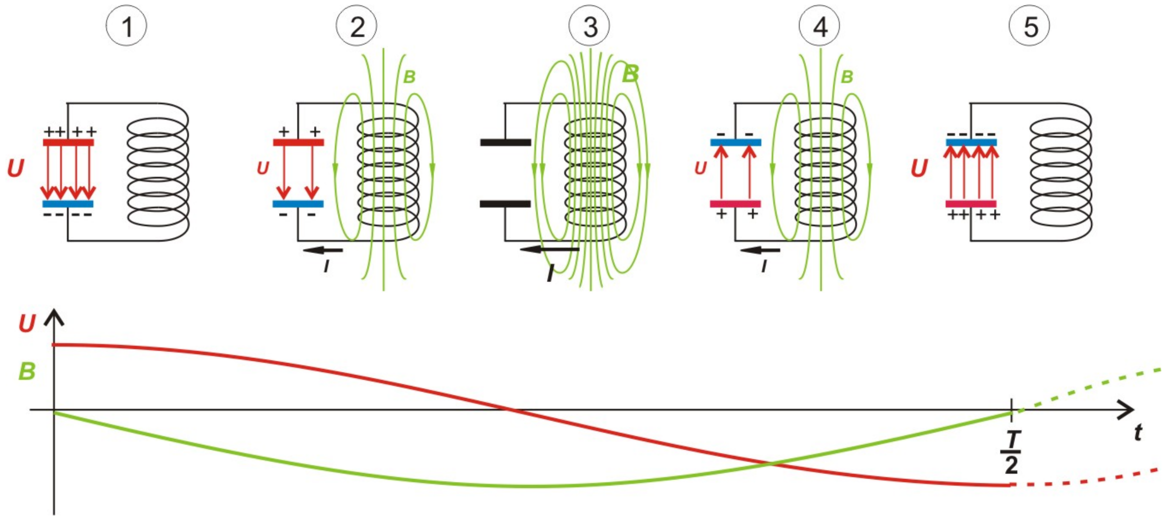
Self-induction: induction of voltage in the same coil – or, in general, any kind of conductor – in which the current change causes a change in the magnetic field. Self-induction occurs every time a circuit is switched on or off.

Electromagnetic induction has practical importance in case of current generators, transformers, etc. An example for self-induction is the LC circuit, which is used to produce electromagnetic oscillations. Since LC circuits are used in medical practice, e.g., in diathermy or ultrasound generation, we present a brief and qualitative review of this circuit.

LC circuit or **resonant circuit**: a circuit that consists of a coil and a capacitor (without ohmic resistor in the ideal case; L represents the coil). A voltage (and current) oscillation (vibration) may arise in the circuit which is initiated by charging the capacitor. Notable events of a half period of the oscillation are explained in the following (see Figure and follow the numbers):

1. Initially the charge and voltage of the capacitor are maximal, and no current flows in the circuit yet. Therefore, there is no magnetic field around the coil.
2. As capacitor discharge begins, current (I) flows from its positive plate towards the negative, and this current induces a magnetic field (B) in the coil. The "build-up" of the current and the magnetic field proceeds only gradually, because self-induction in the coil hinders the changes according to Lenz's law.
3. The capacitor is now empty, hence capacitor voltage is zero. In this instant the electric current and magnetic flux density are maximal.
4. The current, hence flux density, begin to decrease. Again, due to Lenz's law, this proceeds gradually. Current, with less and less intensity, keeps flowing in the same direction and re-charges the capacitor with opposite polarity. Current and the magnetic field decrease, but the electric field and capacitor voltage increase.
5. Current decreases to zero together with the magnetic field of the coil, but the capacitor charge and voltage reaches the original maximal value with opposite polarity.

Now the half period is over. The steps will repeat in the opposite direction until we reach the end of the whole period and the initial state.



Several quantities change periodically during the operation of an LC circuit: charge, voltage, electric field strength, and energy of the capacitor; the current flowing through the coil, the coil's magnetic field, and the energy stored in the magnetic field. Electric and magnetic oscillations are coupled; this is why we refer to them as **electromagnetic oscillations**.

There are also interconversions during the operation of the LC circuit: in steps 1–3 the energy stored in the capacitor is converted into magnetic field energy of the coil, then *vice versa*. Their sum is constant according to the law of conservation of energy. The oscillations go on undamped with a constant amplitude following a sine wave. This, of course, is only valid for "ideal" LC circuits that contain zero ohmic resistance. In real LC circuits there is always some ohmic resistance, e.g., due to the connecting wires, so there is always some energy loss, *via* Joule heating, causing the damping of the oscillation and leading to the termination of the spontaneous, free oscillation. Such a damping is undesired in many applications, e.g., medical ultrasound. The oscillation can be maintained, however, by the investment of energy in the form of a "driving force" (e.g., in a sine wave oscillator). Let this problem be addressed later, as part of the Biophysics curriculum.

Problems:

1. Which quantity characterizes the strength of a magnet?

2. Compare the electric and magnetic interactions. Which statement is correct?
A: Like electric charges attract but like magnetic poles repel.
B: While electric charges may exert attractive or repulsive force, magnets may exert only attractive.
C: Electric charges can be isolated from each other but magnetic poles cannot.
D: Magnetic poles can be isolated from each other but electric charges cannot.

3. Which quantity characterizes the strength of a magnetic field (sometimes called magnetic field by itself)?

4. What is the SI unit of magnetic flux density (B)?
A: tesla (T) **B**: volt (V) **C**: ampere (A) **D**: Siemens (S)

5. A magnet is placed in an external magnetic field. By what factor would the strength of the interaction between them increase if both the magnet's moment and the magnetic flux density of the external field is doubled?
A: 1 **B**: 2 **C**: 4 **D**: 8

6. How can a nearly homogeneous magnetic field be created?

7. What is the phenomenon "electromagnetic induction"?
A:. Creating a magnetic field with a coil.
B: The magnetization of a body.
C: Creating an electric field with a changing magnetic field.
D: The orientation of compasses using an external magnetic field.

8. Consider the second Figure in the paragraph about "electromagnetic induction". In which case is there no electric current induced in the second coil?
A: There is constant current flowing in the first coil while the second coil is moved toward the first.
B: There is constant current flowing in the first coil while it is moved toward the second coil.
C: Both coils are motionless, and an increasing current is flowing in the first.
D: Both coils are motionless, and a constant current is flowing in the first.

9. What is the name of the phenomenon when a voltage is induced in a coil because of the changing current flowing through it?

10. What are the components of an ideal resonant circuit?

Solutions:

1. magnetic moment

2. C

3. magnetic flux density (B)

4. A

5. C

6. e.g., a conductor coil with current flowing through it

7. C

8. D

9. self-induction

10. coil and capacitor (without ohmic resistor)