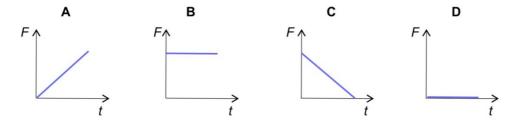
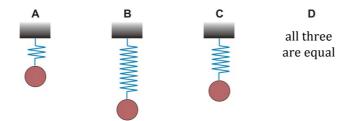
## Resonance

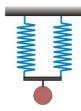
- 11.1. We hang an object of 500 g mass on a coil spring. Upon reaching equilibrium the elongation of the spring is 2,3 cm. Calculate the spring constant.
- 11.2. We hang an object of 250 g mass on a 10 cm long coil spring. Upon reaching equilibrium the spring is 7.5% longer. Calculate the spring constant.
- 11.3. The Achilles tendon may be considered a spring with a spring constant of  $3 \cdot 10^5$  N/m. What force is required to elongate the tendon by 2 mm?
- 11.4. A spring is compressed slowly and at a uniform rate. Which figure describes the restoring force correctly?



11.5. All three springs depicted in the figure below expanded by 10% when they were loaded with the same weight. Which of them has the greatest spring constant?

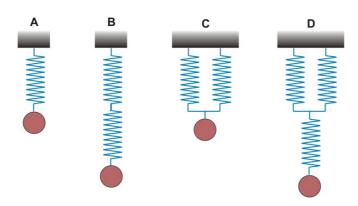


- 11.6. Two equally stiff springs of a spring constant of 450 N/m apiece are connected in parallel, then an object of 3 kg was hung on this system.
  - a) Calculate the elongation of the springs.
  - b) We would like to replace this system with a single spring so that the elongation of the replacement spring is the same as that of the original system. What should be the spring constant of the replacement spring?



- 11.7. Two equally stiff springs of a spring constant of 450 N/m apiece are connected in series, then an object of 3 kg was hung on this system.
  - a) Calculate the elongation of the springs.
  - b) We would like to replace this system with a single spring so that the elongation of the replacement spring is the same as that of the original system. What should be the spring constant of the replacement spring?

11.8. The figure below shows the schematic of four spring systems. The springs and loads of the system are similar: the springs have equal lengths and spring constants, the loads have equal masses. Which system will elongate the most due to the load?



- 11.9. Which of the following statements on harmonic oscillation is correct?
  - A: Amplitude increases with time.
  - B: Amplitude changes in a sine wave manner with time.
  - C: The restoring force is proportional to the displacement.
  - D: The distance covered increases in direct proportion with time.
- 11.10. A pendulum makes exactly 15 swinging periods in one minute. Calculate
  - a) the time period in seconds,
  - b) the frequency in hertz,
  - c) the angular frequency in 1/s.
- 11.11. The displacement of a harmonic oscillation is given by the following function:

$$x = 3 \text{ cm} \cdot \sin\left(0.6 \frac{1}{s} \cdot t\right)$$
. Give

- a) the amplitude,
- b) the angular frequency,
- c) the frequency, and
- d) the time period of the oscillation.
- 11.12. A coil spring of 60 N/m spring constant hangs vertically. We hang a ball of 0.4 kg on the spring and release it. Calculate
  - a) the eigenfrequency and
  - b) the time period of the oscillation of this spring-mass-system.
- 11.13. The time period of a spring–mass-system doubles if the mass of the load hanging on the spring is increased by 30 g. Find the original mass of the load.
- 11.14. A spring–mass-system oscillates with a time period of 3 s. If the load is reduced by  $500~\rm g$  the time period becomes 2 s. Find
  - a) the original mass of the load and
  - b) the spring constant.
- 11.15. How does the eigenfrequency of a spring–mass-system change if the mass of the load is doubled?

- 11.16. We are observing the vibration of a cantilever. The eigenfrequency of the cantilever is 20 Hz. The amplitude during the first two half-periods of the observation was 10 mm and 8 mm, respectively. What will be the amplitude of the cantilever
  - a) during the third and
  - b) during the tenth half-period?
  - c) In which half-period will the amplitude of the cantilever be less than 1 mm for the first time?
  - d) Find the value of the damping coefficient.
- 11.17. The eigenfrequency of a cantilever is 40 s<sup>-1</sup> and its damping coefficient is 0,5 s<sup>-1</sup>. The cantilever was bent from its equilibrium and released, and we are observing the arising vibration. The maximum displacement observed during the first half-period was 8 mm.
  - a) How many periods does the cantilever cover in 4 s?
  - b) To what value does its amplitude decrease from the initial 8 mm in 4 s?
  - c) By how many percent does the amplitude of the vibration decrease in one half-period and
  - d) in one whole period?
  - e) In how many seconds does the amplitude decrease from the original 8 mm to 1 mm?
  - f) How many periods does the cantilever cover during this time?

## **Solutions**

- 11.1. **213 N/m**
- 11.2. **327 N/m**
- 11.3. **600 N**
- 11.4. **A**
- 11.5. **A**
- 11.6. a) **3.27 cm** 
  - b) 900 N/m
- 11.7. a) **6.54 cm** 
  - b) **225 N/m**
- 11.8. **B**
- 11.9. **C**
- 11.10. a) **4 s** 
  - b) **0.25 Hz**
  - c) **1.57** s<sup>-1</sup>
- 11.11. a) **3 cm** 
  - b) **0.6** s<sup>-1</sup>
  - c) **0.0955 Hz**
  - d) **10.5** s
- 11.12. a) **1.95 Hz** 
  - b) **0.513 s**
- 11.13. **10** g
- 11.14. a) **0.9 kg** 
  - b) 3.95 N/m
- 11.15. decreases by 29.3%
- 11.16. a) **6.4 mm** 
  - b) **1.342 mm**
  - c) during the 12th half-period
  - d) **8.9257** s<sup>-1</sup>
- 11.17. a) **160 periods** 
  - b) decreases to 1.0827 mm
  - c) decreases by 0.623%
  - d) decreases by 1.242%
  - e) during 4.1589 s
  - f) **166.36** periods