

# Flow of fluids and gases

## Blood as fluid

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MiklósKellermayer

Department of Biophysics and Radiation Biology

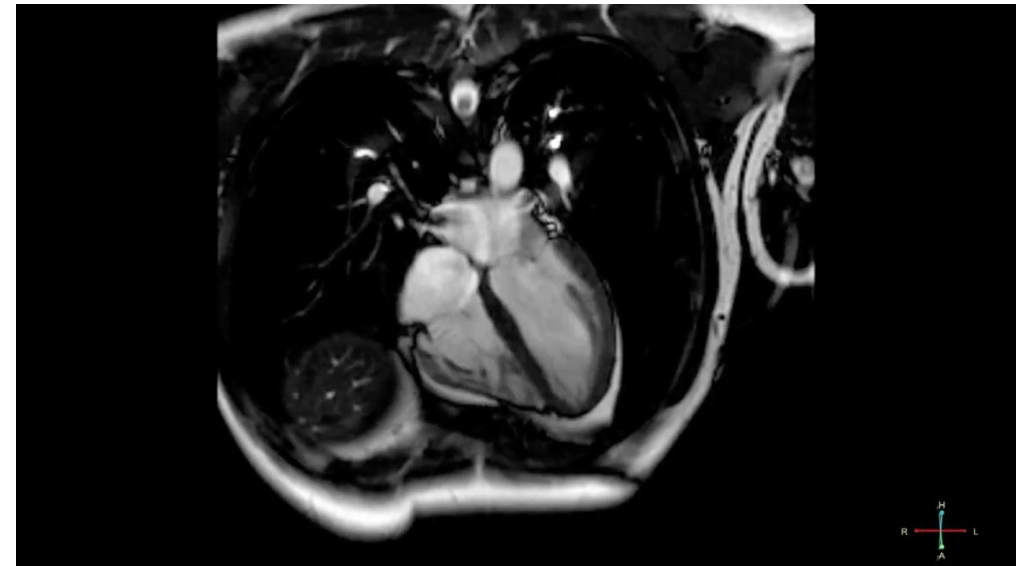


**SEMMELWEIS**  
EGYETEM 1769

# Importance of the physics of fluids

## I. Hemodynamics

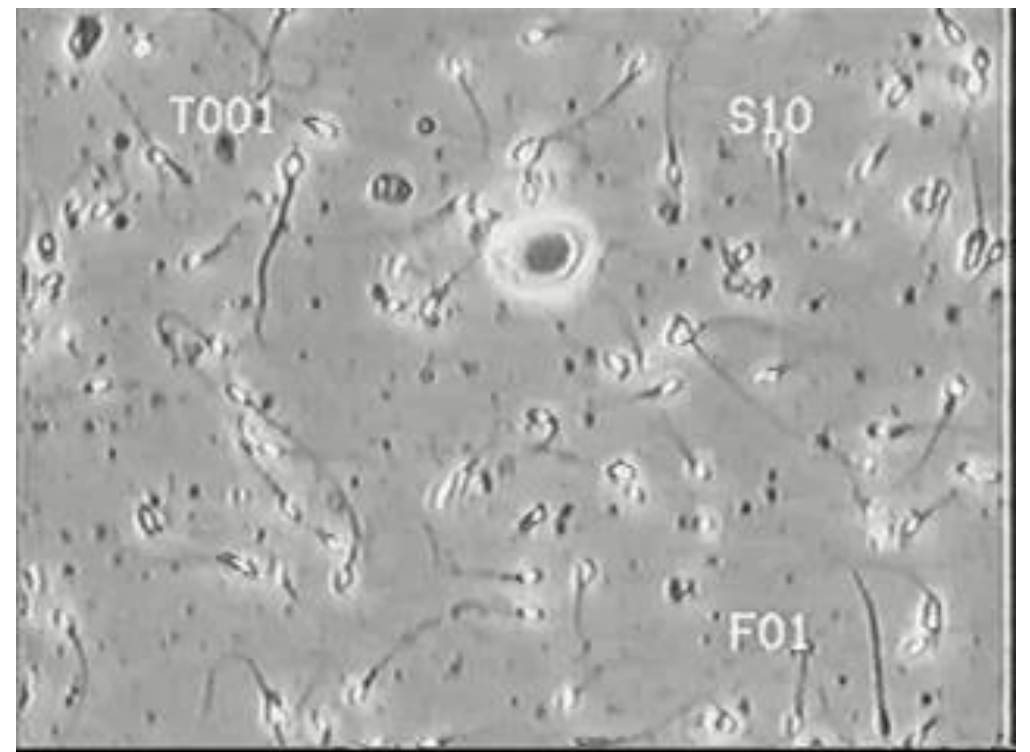
E.g.: What are the characteristics of blood flow in the circulatory system?  
(similarly: flow of gases in respiratory system)



## II. Motion in viscous fluids

E.g.: What is the force exerted by a single spermatoocyte during its motion?

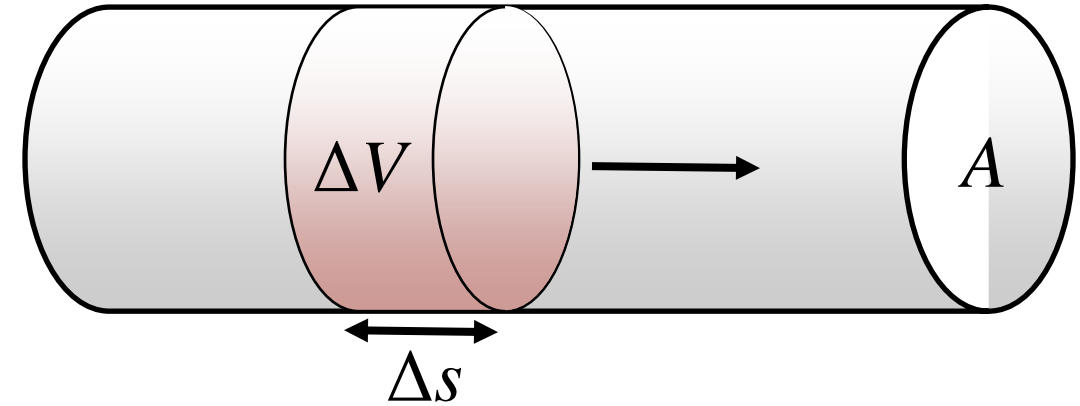
N.B.: gases - in contrast to fluids - are compressible. However, in the physiologically relevant range of pressure differences ( $\sim 100$  Pa), the changes in their volume/density are negligible.



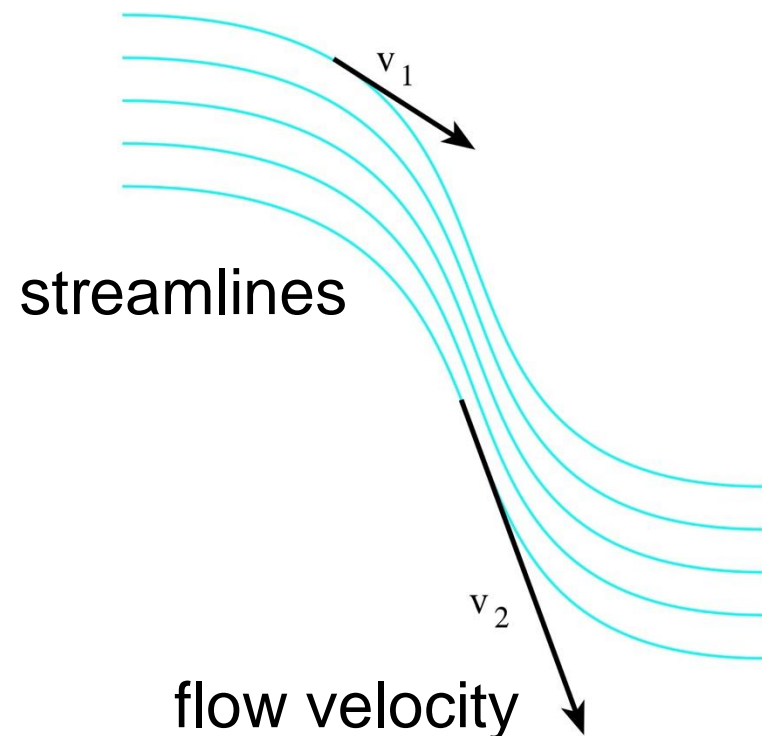
# Basic principles I.

Volumetric flow rate ( $I_V$ ):

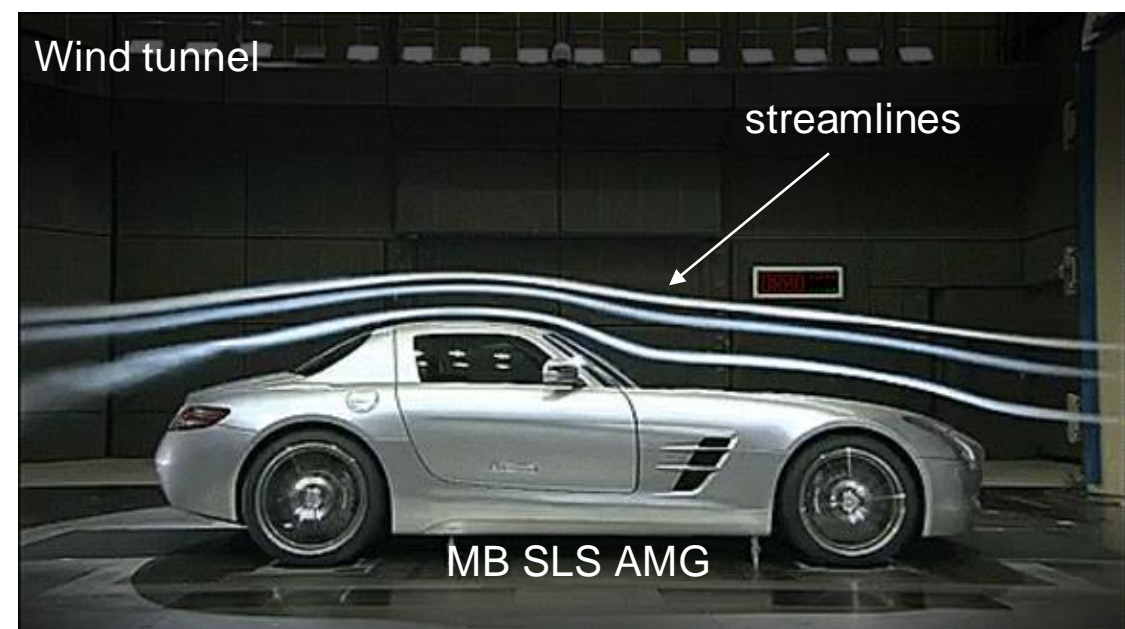
$$I_V = \frac{\Delta V}{\Delta t} = A \frac{\Delta s}{\Delta t} = A \bar{v}$$



Average velocity:  $\bar{v} = \frac{I_V}{A}$



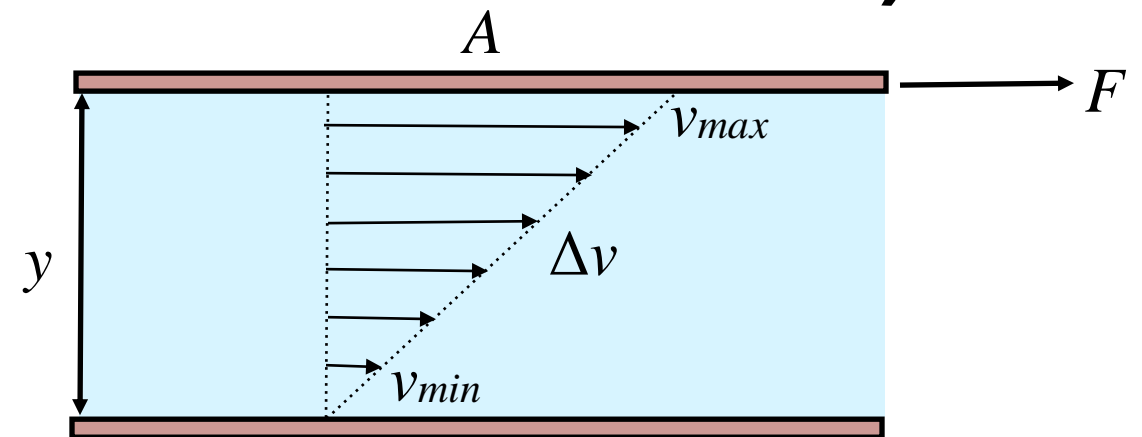
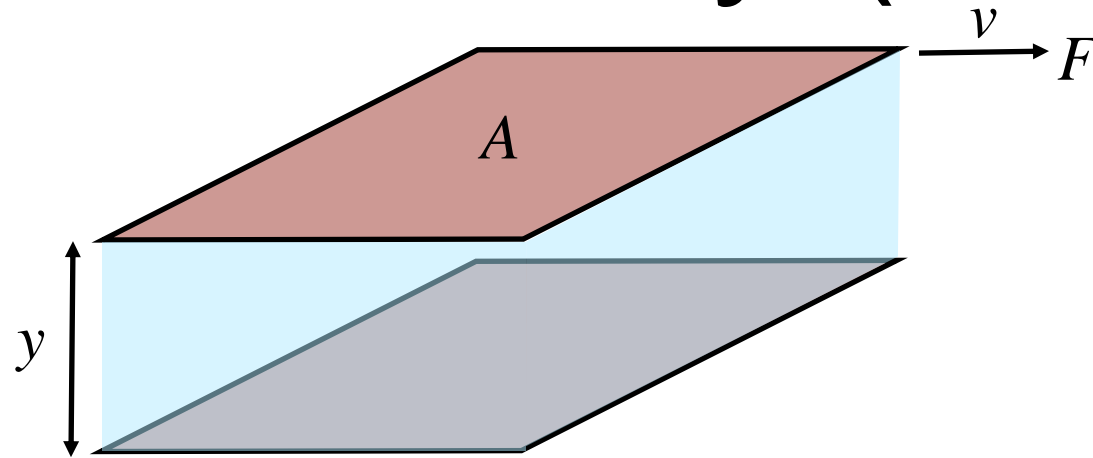
(direction of tangent = direction of velocity  
density of streamlines = magnitude of velocity)



Flow can be made visible

# Basic principles II.

## Viscosity (internal friction)



$F$  = shear force  
 $A$  = area of fluid layer  
 $\eta$  = viscosity  
 $v$  = flow velocity  
 $y$  = distance between fluid layers  
 $F/A$  = shear stress ( $\tau$ )  
 $\Delta v/\Delta y$  = velocity gradient ( $D$ )

$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y} \quad (\text{Newton's friction law})$$

$$\eta = \frac{\tau}{D}$$

Units of viscosity:  $1 \text{ Pas} = 1 \frac{\text{Ns}}{\text{m}^2} = 10 \text{ P}(\text{poise})$

Viscosity of distilled water (25 °C): 1 mPas (1 centipoise)

N.B.: in gases:  $\eta \sim T$

Momentum exchange between particles acts against the gliding of layers past one another.

fluids:  $\eta \sim e^{E/k_B T}$

Viscosity decreases with the increase in the relative concentration of vacancies.

# Types of fluids

## 1. Ideal

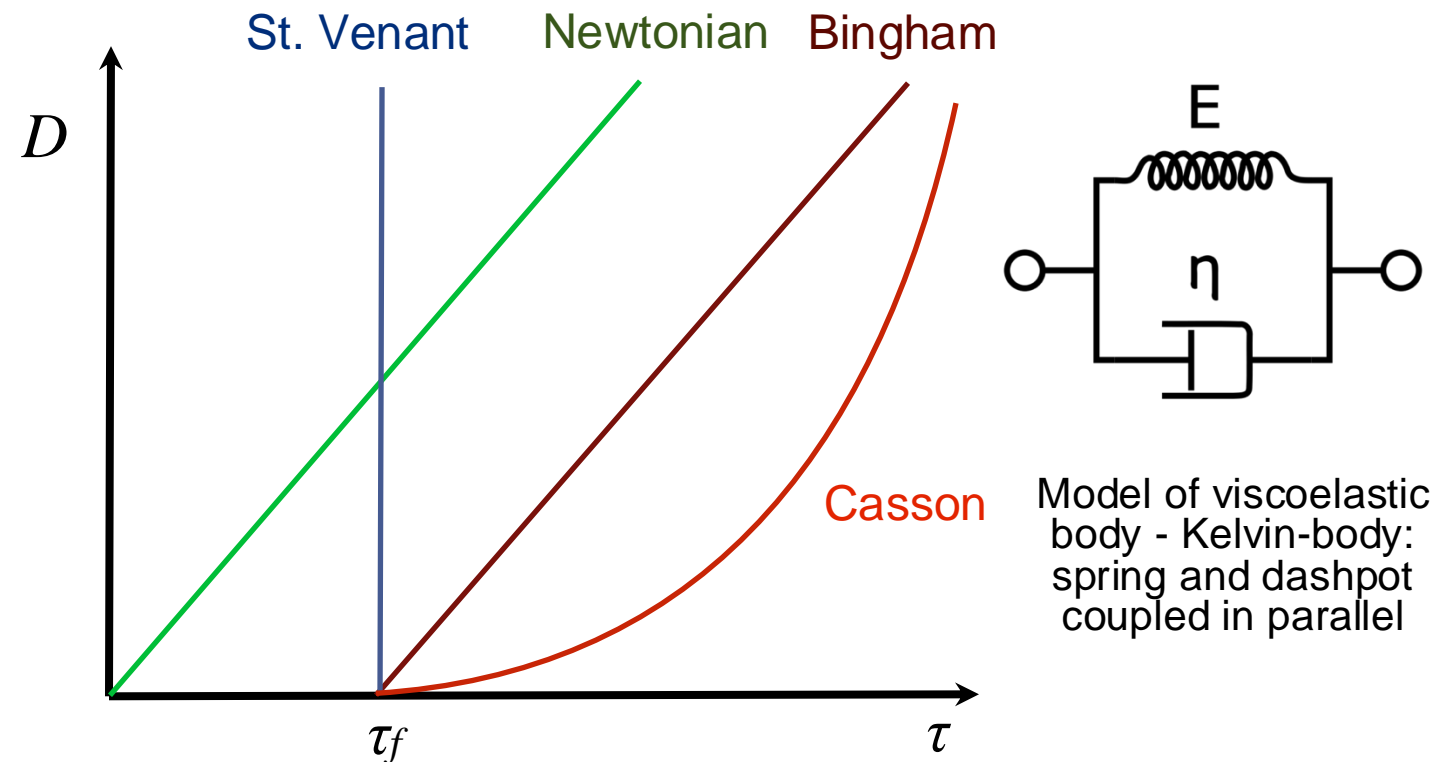
frictionless, incompressible  
 $\rho = \text{constant}$ ,  $\eta = 0$

## 2. Non-ideal (real)

a. *Newtonian (viscous)*  
 $\eta$  independent of shear stress

b. *Non-newtonian (anomalous)*  
 $\eta$  changes with shear stress

Relationship between velocity gradient and shear stress in **real** fluids



- $\tau_f$  = flow threshold
- Viscoelastic materials: combination of viscous and elastic properties (e.g., solution of polymers, macromolecules)
- Stress-relaxation: decay of stress in rapidly deformed viscoelastic body.
- Blood is a non-newtonian fluid; it displays viscoelastic properties.



# Types of fluid flow

## 1. Stationary

Volumetric flow rate stays constant (parameters characterizing flow remain unchanged).

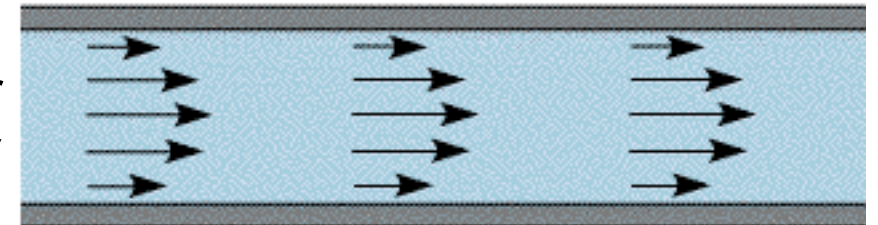
## 2. Laminar

Fluid layers do not mix.

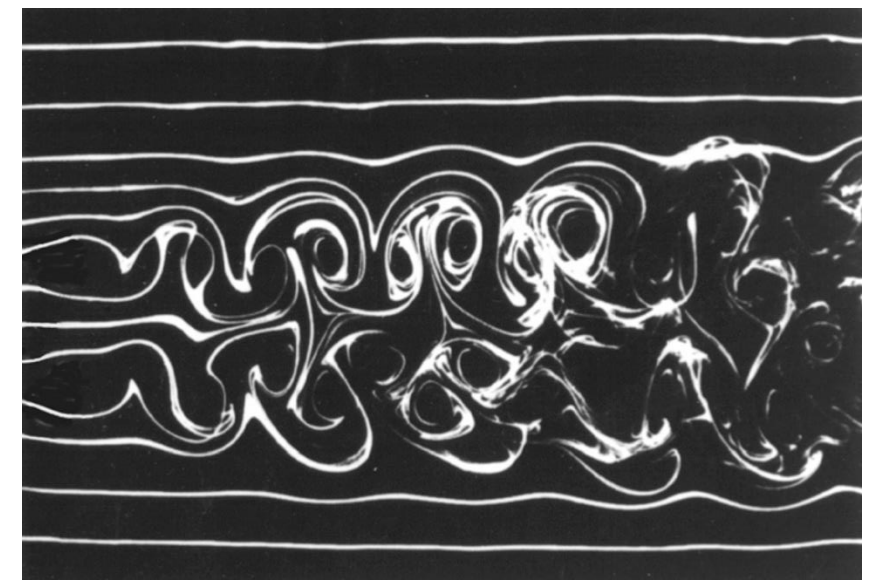
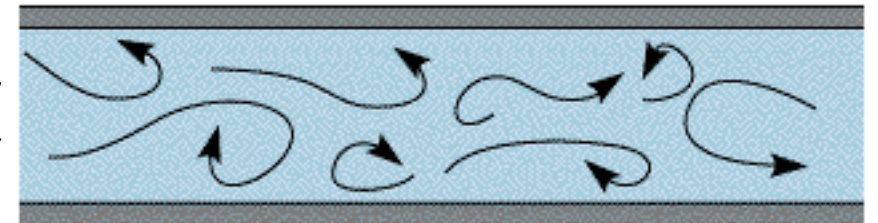
## 3. Turbulent

Fluid layers mix.

*laminar  
flow*



*turbulent  
flow*



Reynolds number (Re):

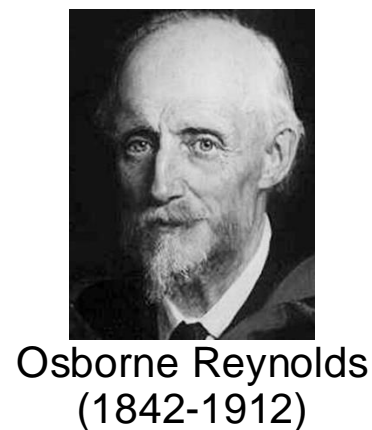
$$Re = \frac{vr\rho}{\eta}$$

$v$  = flow rate (m/s)

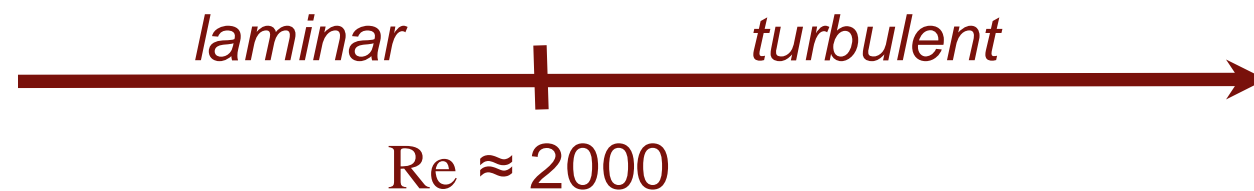
$r$  = tube radius (m)

$\rho$  = density of fluid (kg/m<sup>3</sup>)

$\eta$  = viscosity (Ns/m<sup>2</sup>)



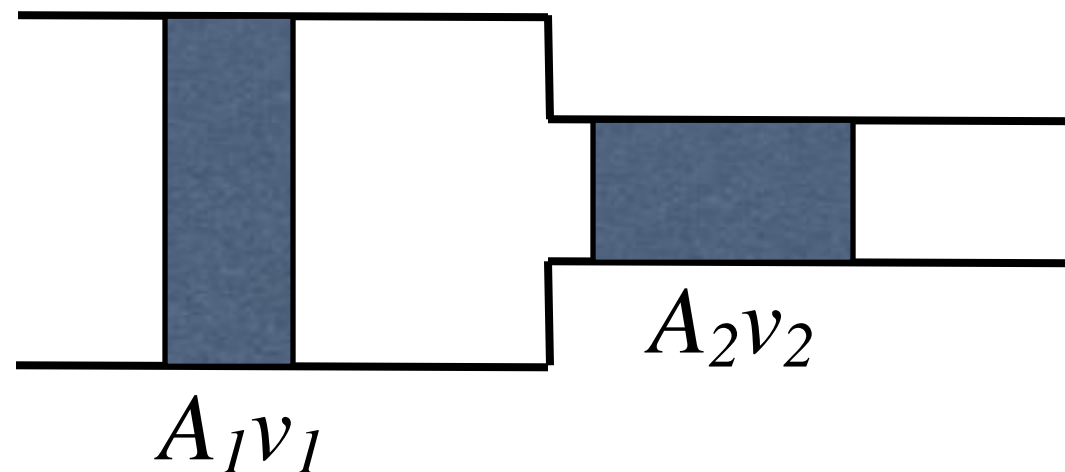
Osborne Reynolds  
(1842-1912)



"When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first." (Werner Heisenberg)

# Laws of flow in ideal fluids I.

Continuity equation:  
volumetric flow rate is constant



$$A_1 v_1 = A_2 v_2 = \text{constant}$$

$A$  = cross-sectional area  
 $v$  = flow velocity

# Laws of flow in ideal fluids II.

## Bernoulli's law - law of conservation of energy



Daniel Bernoulli  
(1700-1782)

$$p + \frac{1}{2}\rho v^2 + \rho gh = konst$$

$p$  = static pressure

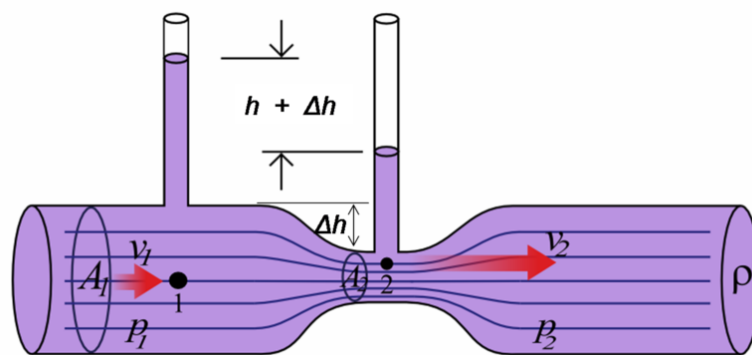
$\frac{1}{2}\rho v^2$  = dynamic pressure

$\rho gh$  = hydrostatic pressure

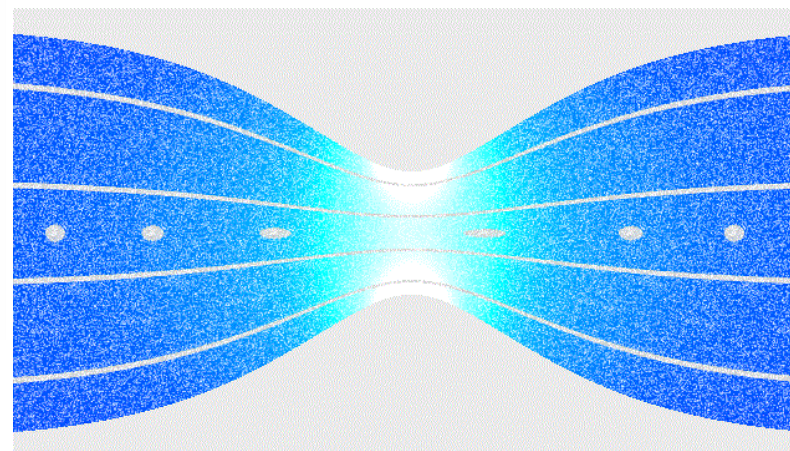
### Application: Venturi-effect



Giovanni Battista  
Venturi  
(1746-1822)



Static pressure drops at the  
tube narrowing



Venturi tube

- Biphase pulse (*pulsus bisferiens*) in aorta insufficiency
- Aspirator (vacuum pump)
- Inspirator (Bunsen-burner)
- Sensors (volumetric flow rate)
- Vacuum cleaner
- Atomizer, spray, carburetor
- Leadpipe and barrel of wind instruments
- Diffuser

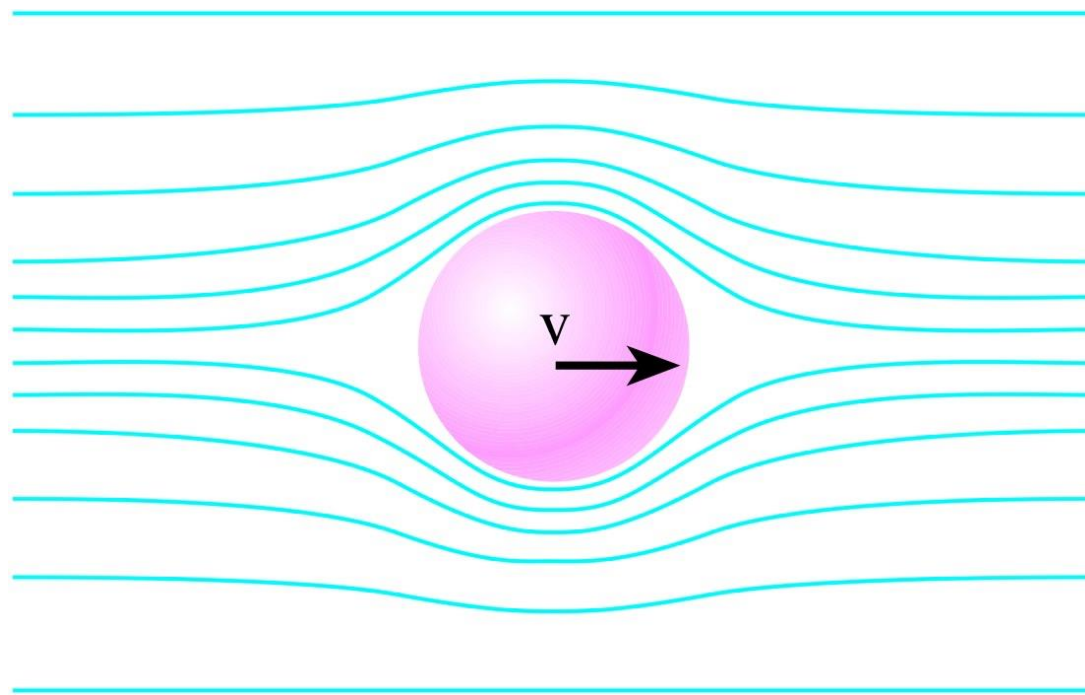


# Laws of flow in viscous fluids I.

## Stokes' law



Georg Gabriel  
Stokes (1819-1903)



$$F = \gamma v = 6r\pi\eta v$$

$F$  = force

$\gamma$  = drag coefficient (shape factor)

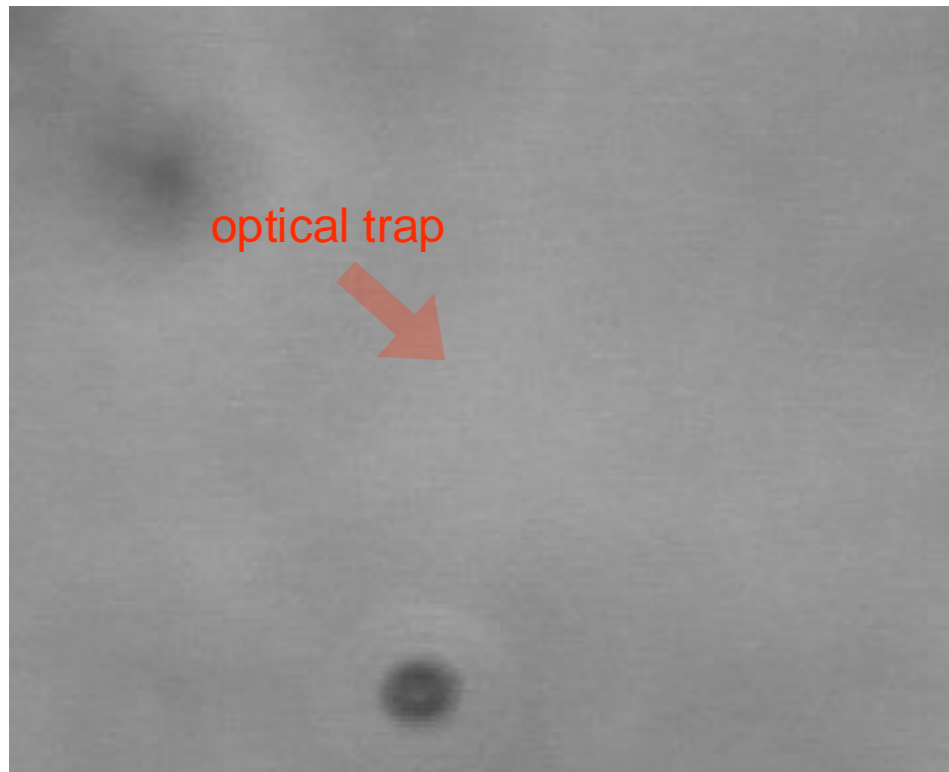
$v$  = flow rate

$r$  = radius of sphere

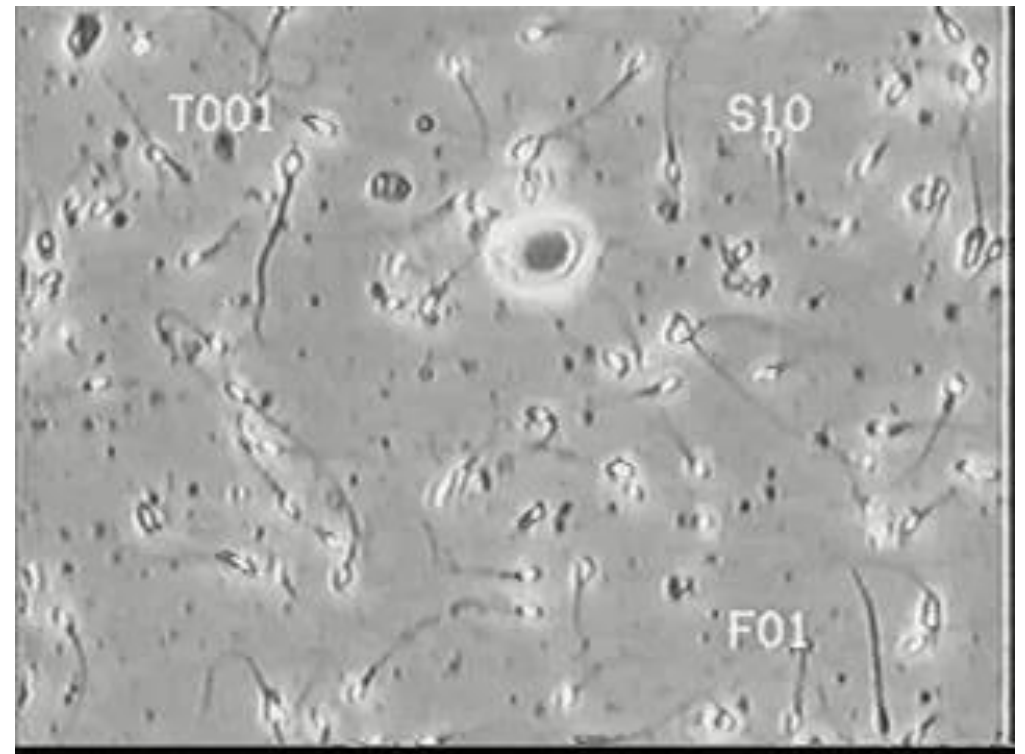
$\eta$  = viscosity

# Stokes force

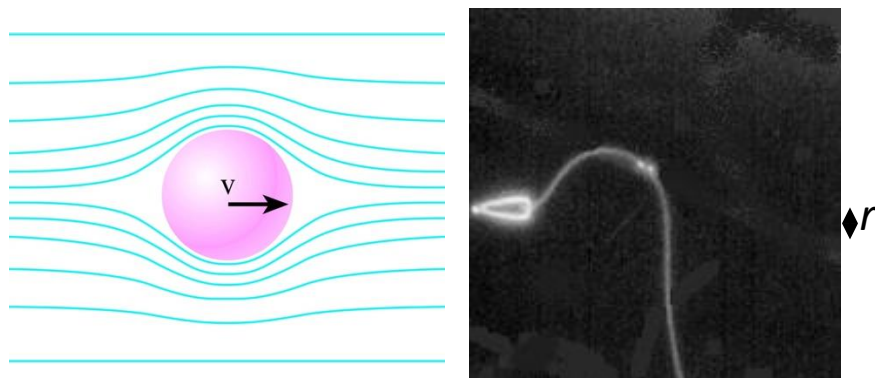
Hydrodynamic drag force (Stokes force):  $F = \gamma v = 6r\pi\eta v$



Affects stationary objects in moving fluid  
(microbead captured in optical trap)



Affects objects moving in stationary  
fluid (moving spermatozoa)



$$r = 1.6 \mu\text{m} = 1.6 \times 10^{-6} \text{ m}$$

$$v = 50 \mu\text{m/s} = 5 \times 10^{-5} \text{ m/s}$$

$$\eta = 10^{-3} \text{ Pas}$$

How much force is exerted by a single  
spermatozoa during its movement?

$$\gamma = 6r\pi\eta = 6 \cdot 1.6 \times 10^{-6} \cdot \pi \cdot 10^{-3} = 3 \times 10^{-8} \text{ N s/m}$$

$$F = \gamma v = 3 \times 10^{-8} \text{ N s/m} \cdot 5 \times 10^{-5} \text{ m/s} = 1.5 \times 10^{-12} \text{ N} = \boxed{1.5 \text{ pN}}$$

# Laws of flow in viscous fluids II.

## Hagen-Poiseuille's law

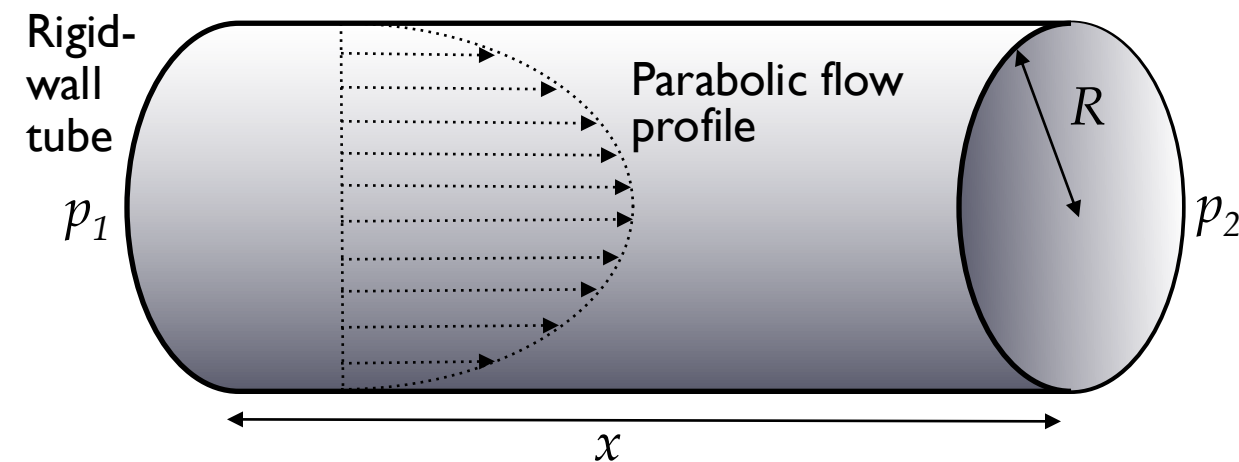


G.H.L. Hagen  
(1797-1884)



J.-L.-M. Poiseuille  
(1799-1869)

Thermodynamic current	Relevant intensive variable (its difference maintains current)	Current density	Physical law
Volumetric flow	Pressure (p)	$J_v = -\frac{R^2}{8\eta} \frac{\Delta p}{\Delta x}$	Hagen-Poiseuille



$V$  = volume  
 $t$  = time  
 $R$  = tube radius  
 $\eta$  = viscosity  
 $p$  = pressure  
 $x$  = tube length

$V/t = I_v$  = volumetric flow rate  
 $\Delta p/\Delta x$  = pressure gradient, maintained by  $p_2 - p_1$  (negative!)  
 $A$  = cross-sectional area of tube  
 $I_v$  = volumetric flow rate

$$J_v = \frac{V}{tA} = \frac{R^2}{8\eta} \frac{\Delta p}{\Delta x}$$

N.B. 1:  $A = R^2 \pi \Rightarrow I_v = \frac{V}{t} = -\frac{R^4 \pi}{8\eta} \frac{\Delta p}{\Delta x}$

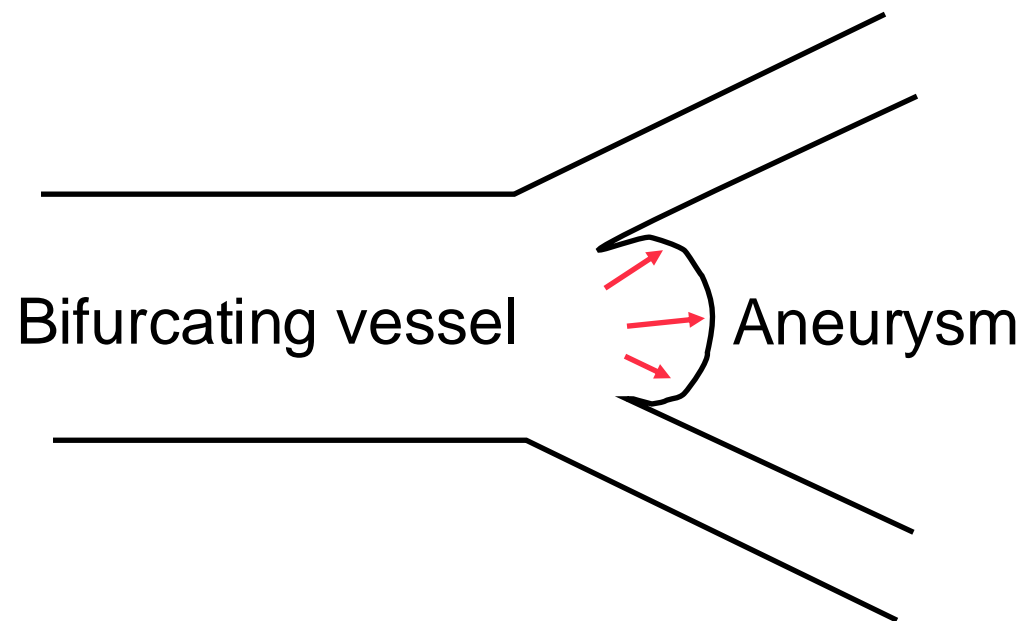
N.B. 2:  $I_v = -\frac{R^4 \pi}{8\eta \Delta x} \Delta p \Rightarrow -\Delta p = R_{tube} \cdot I_v \Rightarrow U = R \cdot I$   
 Ohm's law!

N.B. 3:  $\frac{\Delta v}{\Delta r} \sim r \Rightarrow \left( \frac{\Delta v}{\Delta r} \right)_{\max} = R \Rightarrow \tau_{\max} = R$

Shear stress is maximal at the tube wall (e.g., the von Willebrand-factor protein is stretched near the vessel wall, driven by shear stress)

# Medical significance of fluid flow

Bernoulli's law:



Formation of aneurysm (pathological expansion of blood vessel):

- Expansion of vessel: diameter increases
- Flow rate decreases, according to continuity equation
- Static pressure increases due to Bernoulli's law
- Aneurysm pregreduates - positive feedback mechanism leading to catastrophe

Hagen-Poiseuille's law:

$$\frac{V}{t} = \frac{R^4 \pi \Delta p}{8\eta \Delta x}$$

Flow intensity, hence the delivered oxygen quantity, may be **drastically**

**reduced** in certain pathological conditions:

- constriction of blood vessels (e.g., diabetes, B rger's disease)
- change in blood viscosity (e.g., fever, anaemia)
- Reduction of vessel diameter by half leads to a reduction of volumetric flow by 1/16!



# Blood as a fluid

<b>55-60% of body mass is water</b> 42 kg (70 kg body mass)		
2/3 intracellular 28 kg	1/3 extracellular 14 kg	
	1/3 plasma 4-5 kg	2/3 intersticiium 9-10 kg

**Blood:** Average volume: 5 l  
Average viscosity: 5 mPas  
Average density: 1.05 g/cm<sup>3</sup>  
Composition: 40-45 % corpuscular, 55-60 % plasma

# Determinants of blood viscosity

## **1. Hematocrit ( $htc$ , $\phi$ ):**

$$htc = \frac{V_{cells}}{V_{total}}$$

Normal range: 0.4-0.5.

Viscosity of blood as suspension  
(in the physiologically relevant  $htc$  range):

$$\lg \eta_s = A + B\phi$$

$\eta_s$ =suspension viscosity

A, B=empirical constants

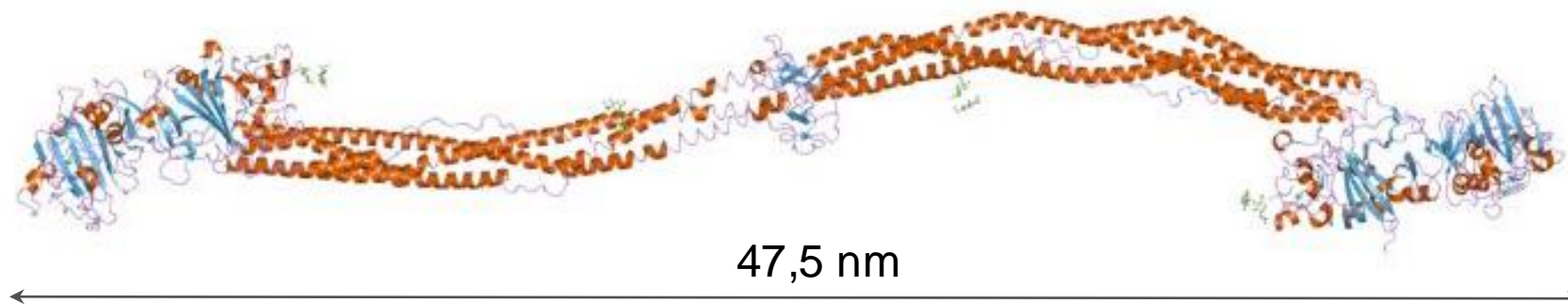
# Determinants of blood viscosity

## ***2. Plasma viscosity***

- Depends on plasma proteins.
- In *paraproteinaemias* (e.g. myeloma multiplex or plasmocytoma) the concentration of immunoglobulins is high, leading to increased viscosity.

Plasma protein	Normal concentration	% ratio	Function
Albumin	35-50 g/l	55%	maintenance of colloind osmotic pressure, transport
Globulins	20-25 g/l	38%	Part of the immune system
Fibrinogen	2-4.5 g/l	7%	Blood coagulation

# Fibrinogen, fibrin

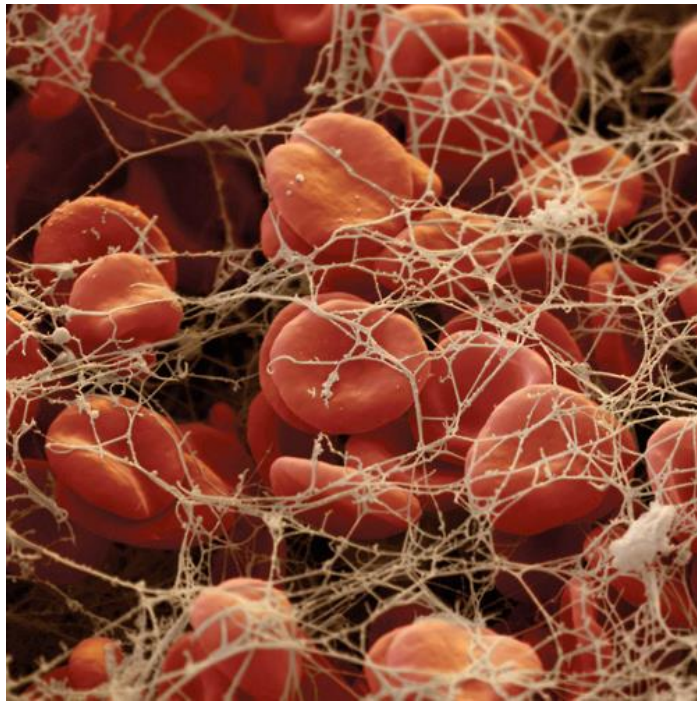


## ***Fibrinogen:***

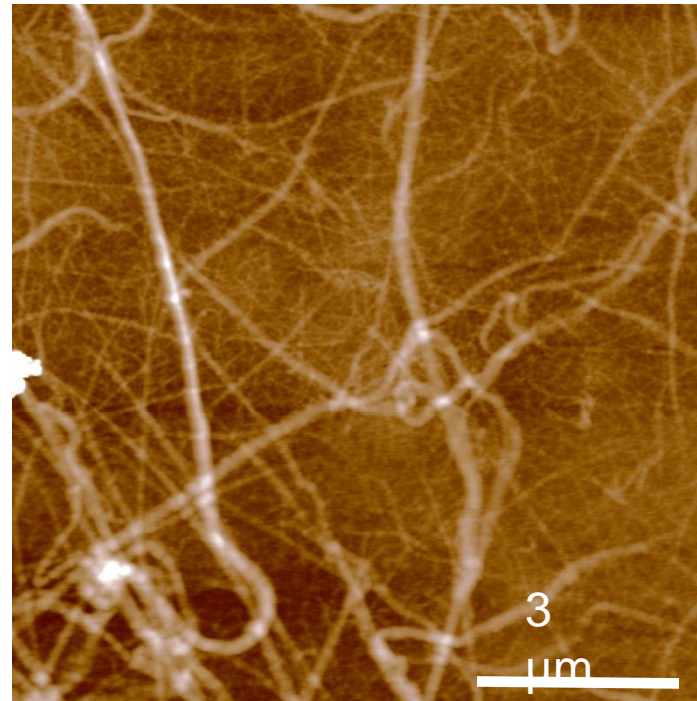
MW = 340.000 Da

In plasma: 2-4 g/l  $\approx$  10  $\mu$ M

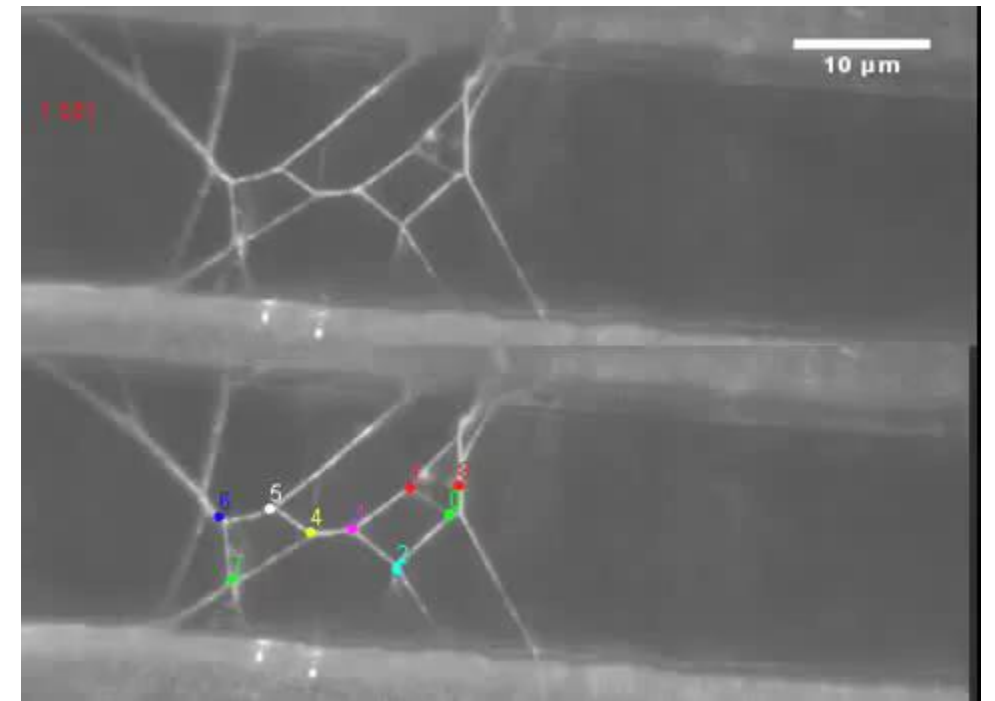
Average nearest-neighbor distance  $\approx$  55 nm



Red blood cells in  
fibrin meshwork



Fibrin polymerized in vitro  
(AFM)



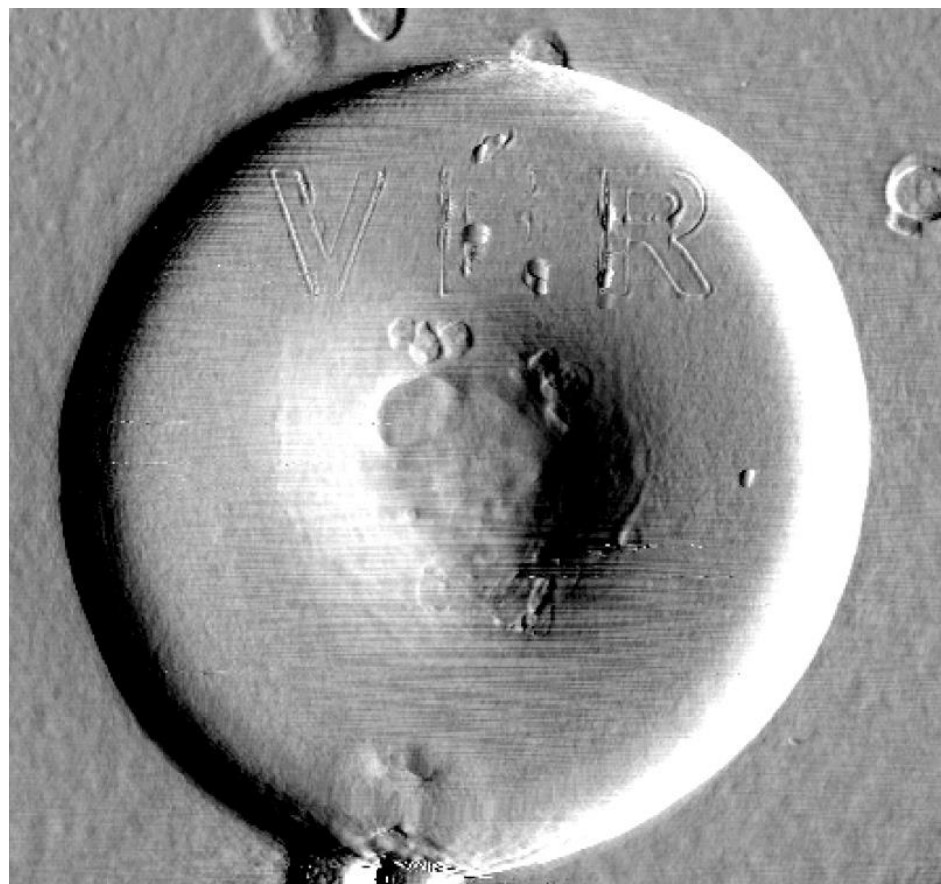
Extensibility of fluorescently  
labeled fibrin fibrils



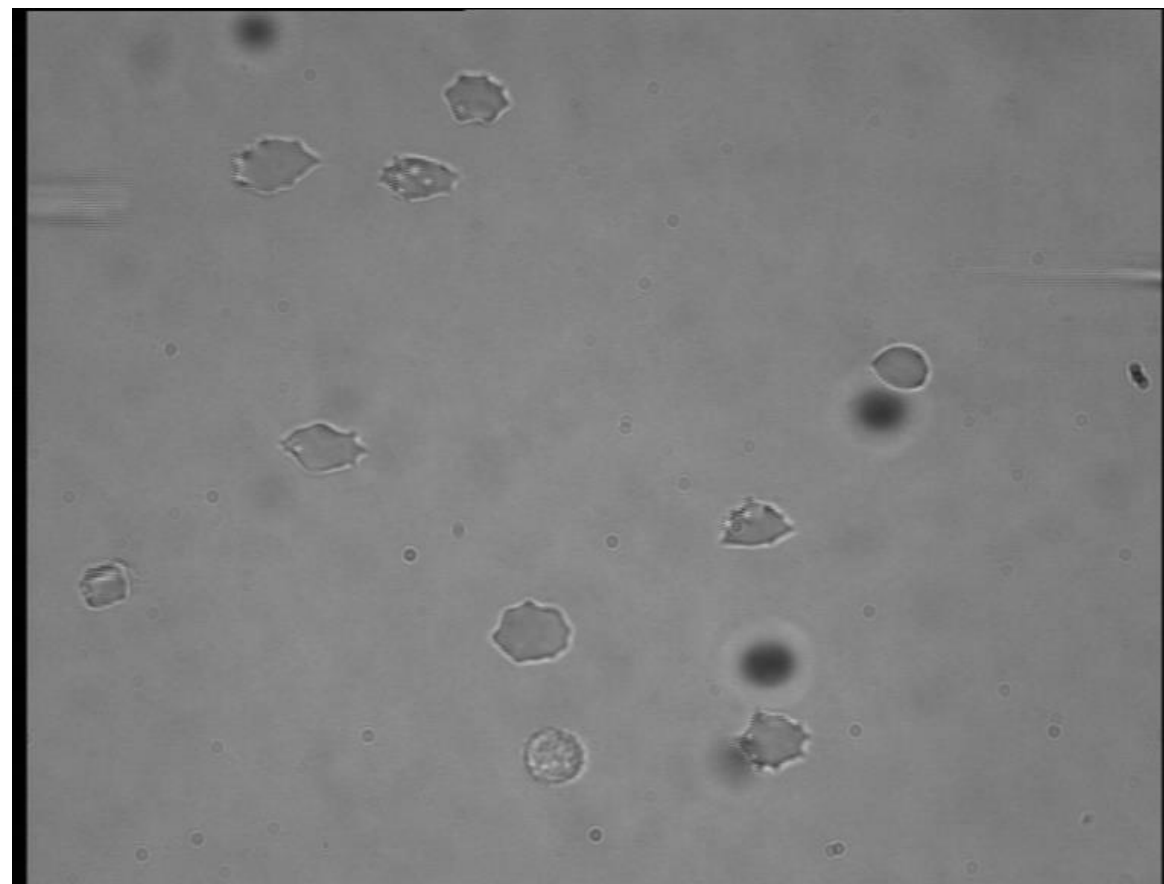
# Determinants of blood viscosity

## 3. *Plasticity of red blood cells*

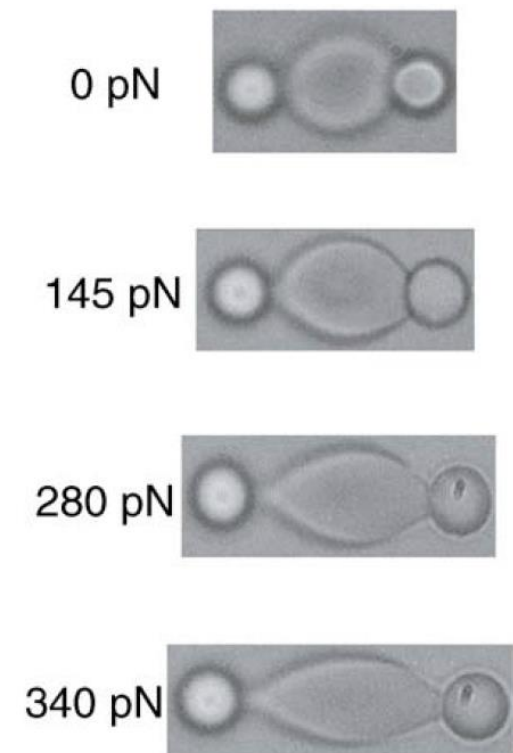
- 65% suspension of blood-cell-size particles is rock hard.
- By contrast, a 95% blood suspension is fluid, with viscosity of  $\sim 20$  mPas!
- Deformation of red blood cells: droplet, parachute, arrowhead shapes.



Fixed RBC maintaining impression (AFM).  
Disc-shaped cell with 7-11  $\mu\text{m}$  diameter.



Deformation of red blood cells  
with shear stress (fluid flow).

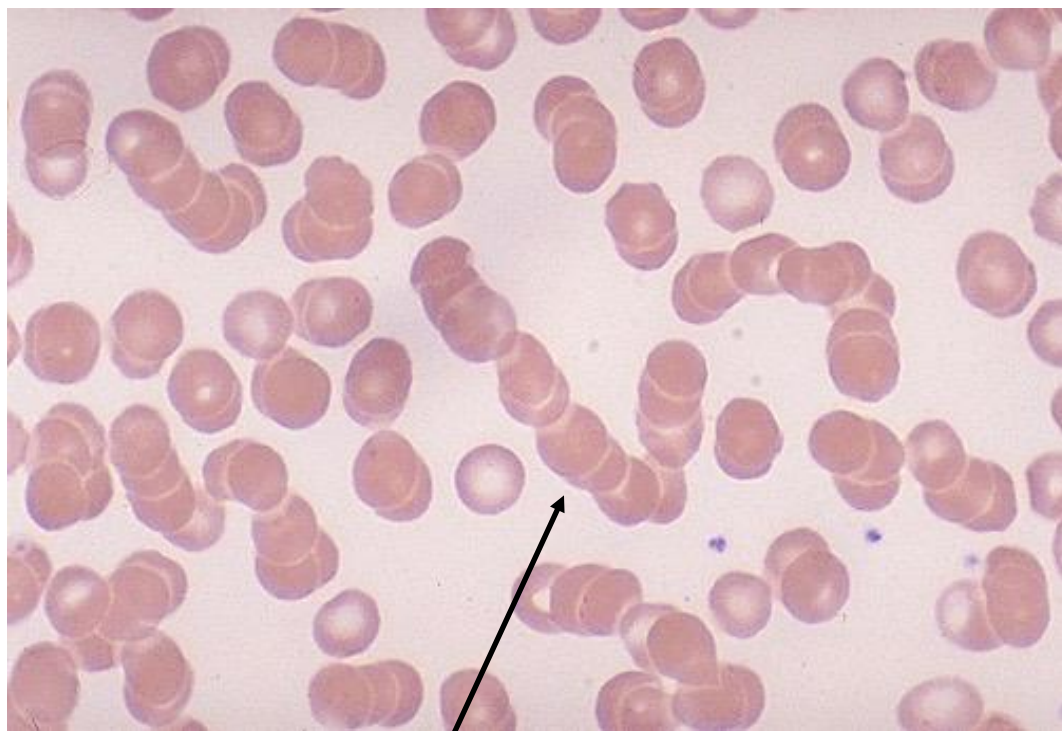


Deformation of a RBC  
with optical tweezers

# Determinants of blood viscosity

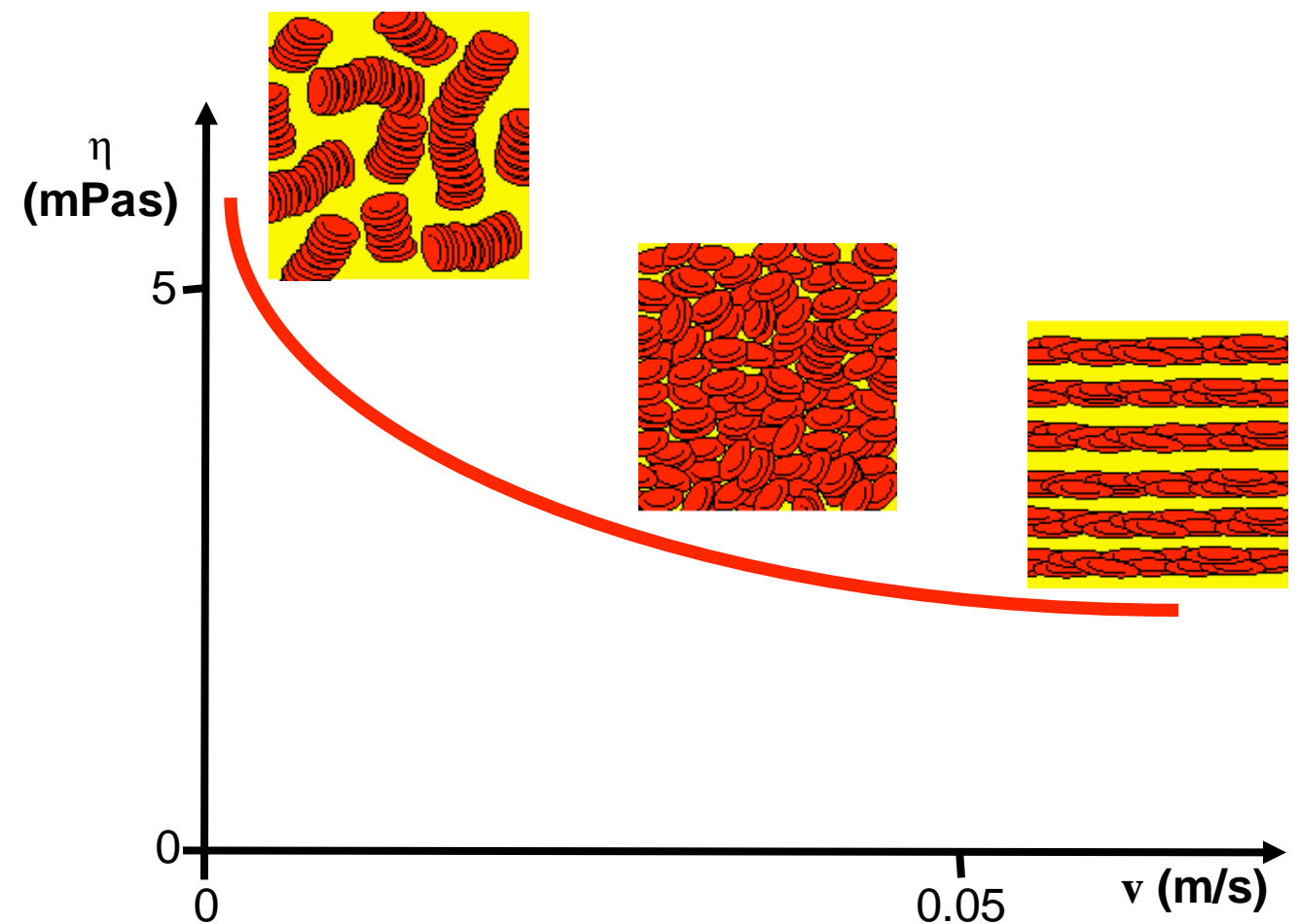
## 4. Aggregation of red blood cells

- Stack or rouleaux formation.
- More pronounced at low flow rates



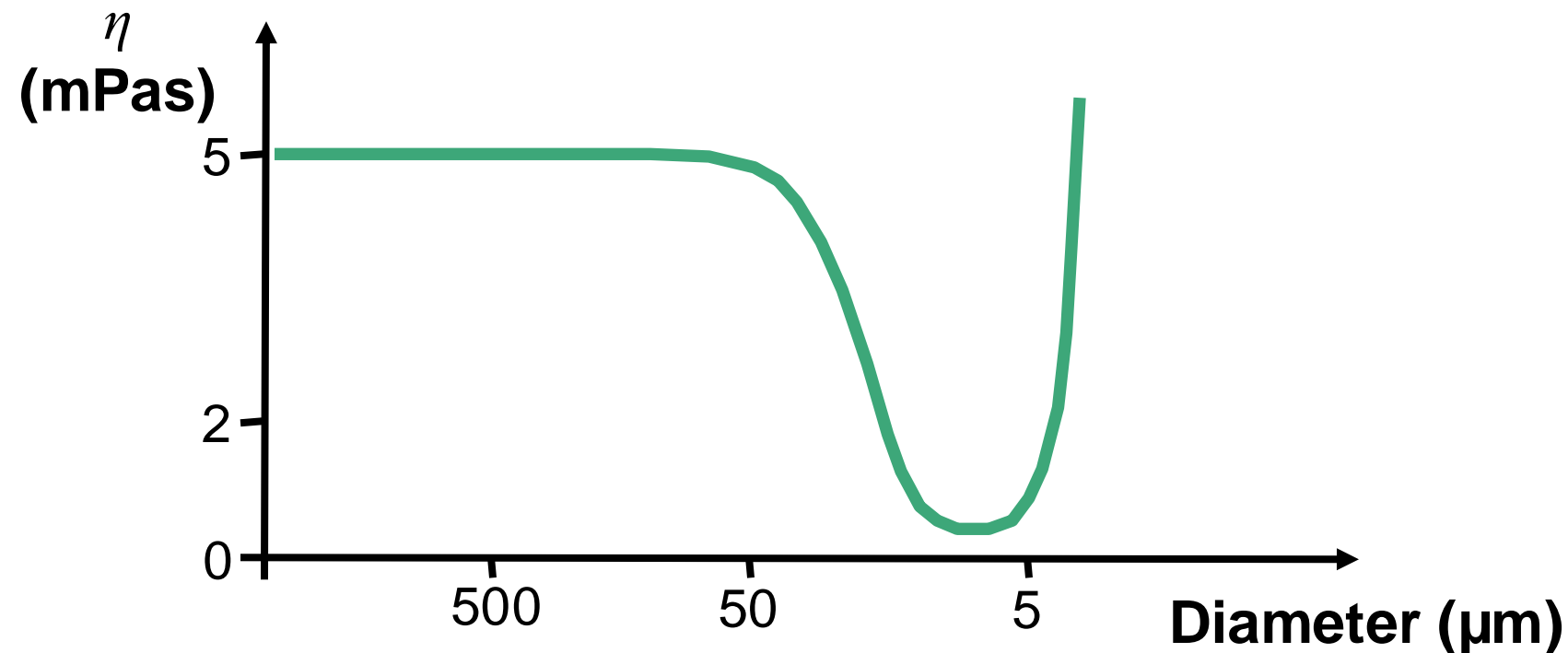
Rouleaux  
(stack)

## 5. Flow rate, velocity gradient



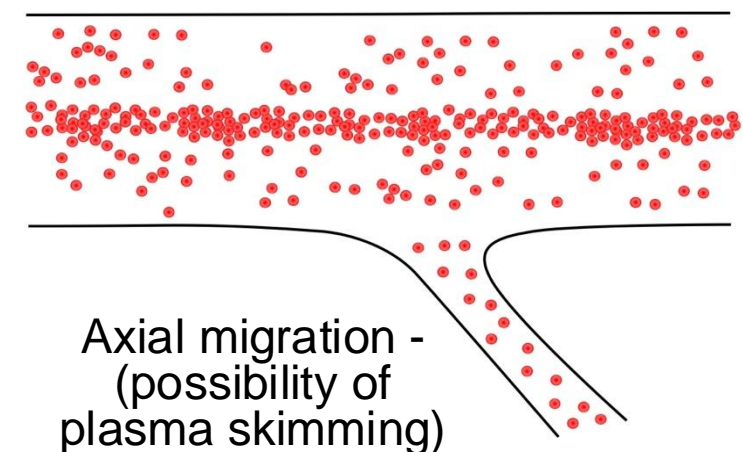
# Determinants of blood viscosity

## 6. Blood vessel diameter



### N.B.:

- With a decrease of vessel diameter, the anomalous (non-newtonian) behavior of blood becomes more pronounced.
- *Axial migration*: the red blood cells line up in the axis of the vessel (Bernoulli's law). In the axis the velocity gradient decreases, and near the vessel wall it increases. Increase in velocity gradient decreases apparent viscosity (Fåhræus-Lindquist effect).



# Feedback



<https://feedback.semmelweis.hu/feedback/pre-show-qr.php?type=feedback&qr=ABRV3IR7URHYLD6A>