Medical Biophysics II.

6^{th} lecture: Transport processes II.
Diffusion, Brownian motion, Osmosis
19^{th} March 2025.

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Diffusion?

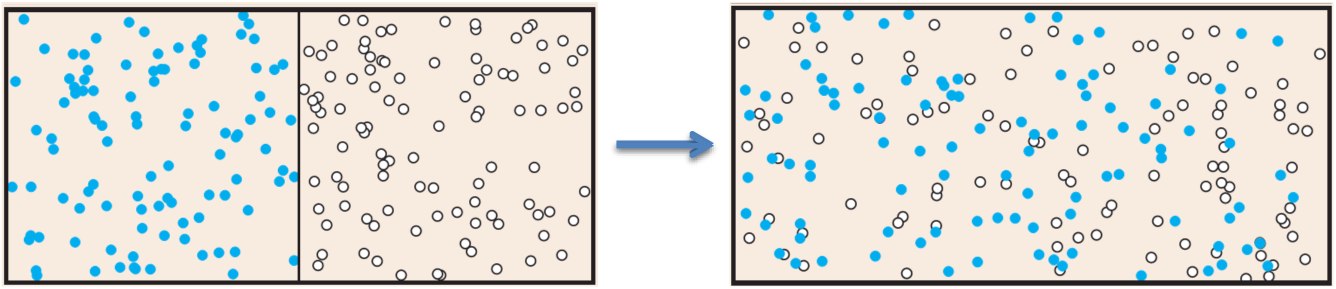
Why?

- physiology: cell function – ion diffusion…
- disorders: fibrosis, oedema, vasculitis, ascites…
- diagnostics: DWI MRI…
- therapy: dialysis, physiological saline….
- drug delivery: transdermal (liposomal), inhaled…

…….

Diffusion?

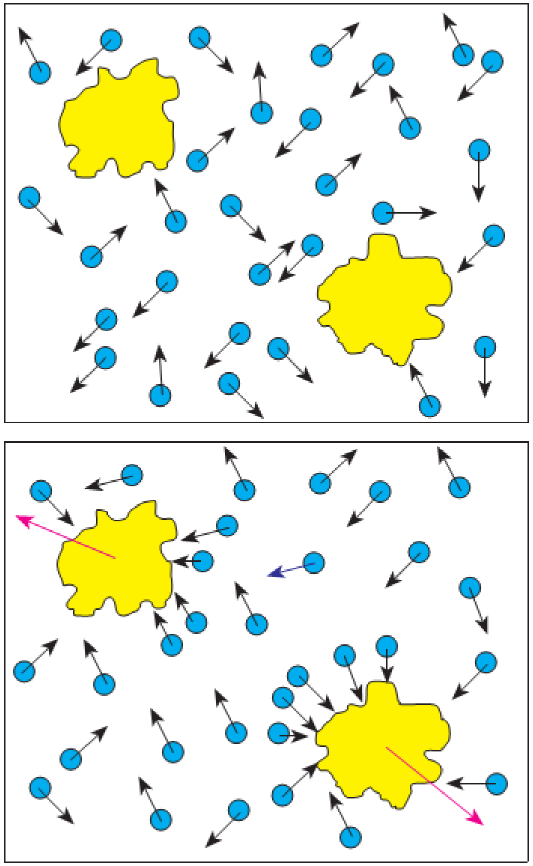
What is diffusion? A **net material flow** as the macroscopic consequence of the change in particle distribution due to the **random thermal motion** of microscopic particles.



Note: For us, the material transport of „A” in „B” is interested, so we disregard the so-called self-diffusion.

Brownian motion

The „random walk” of a larger particle is the result of random collisions with microscopic particles undergoing thermal motion.



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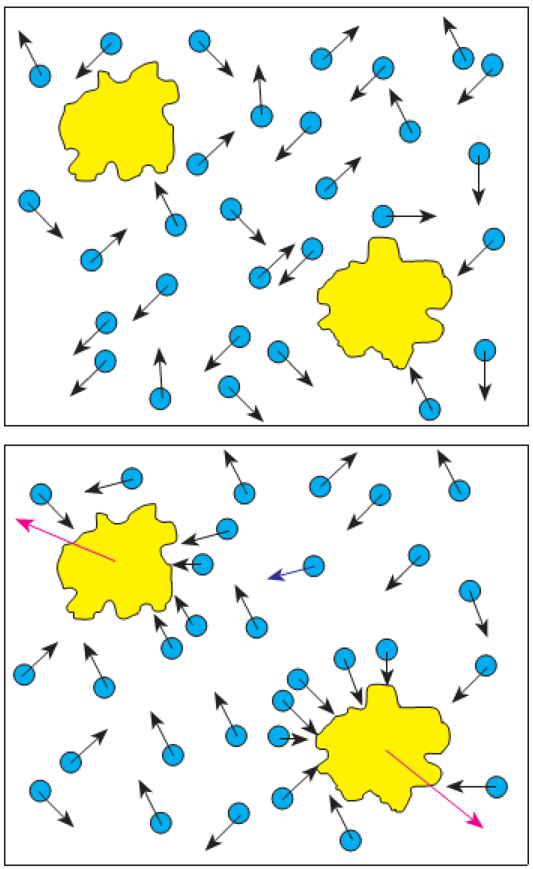

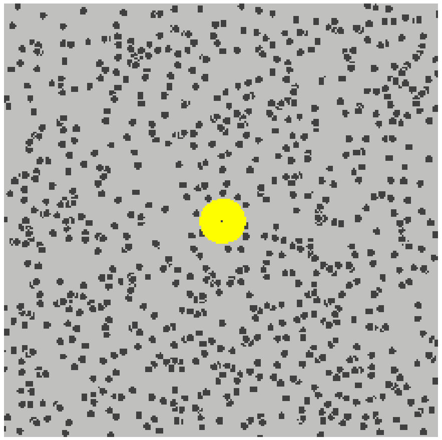

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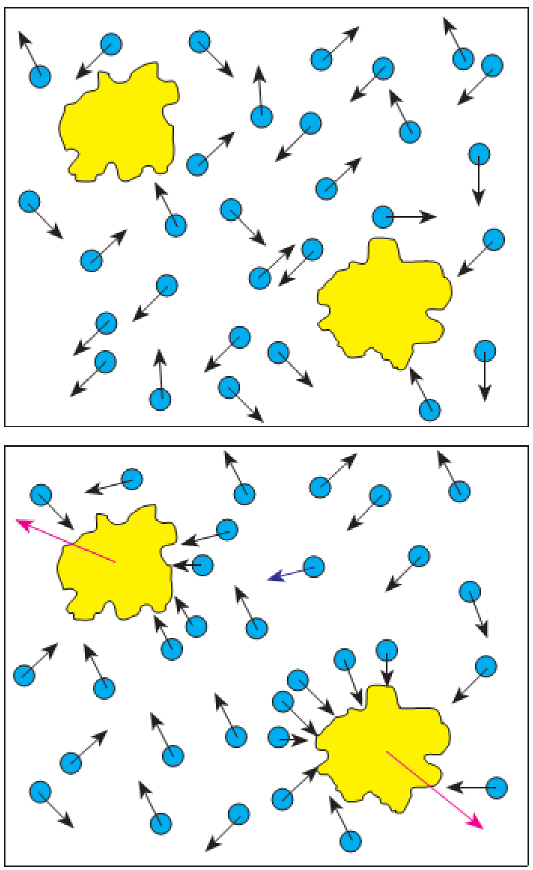
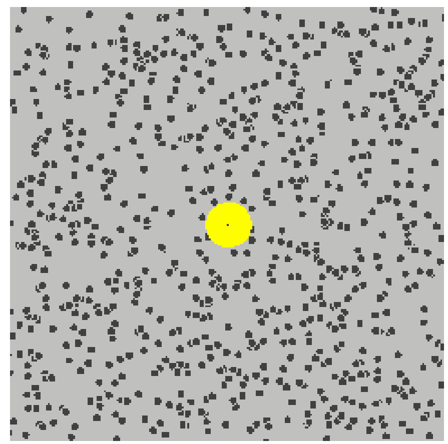
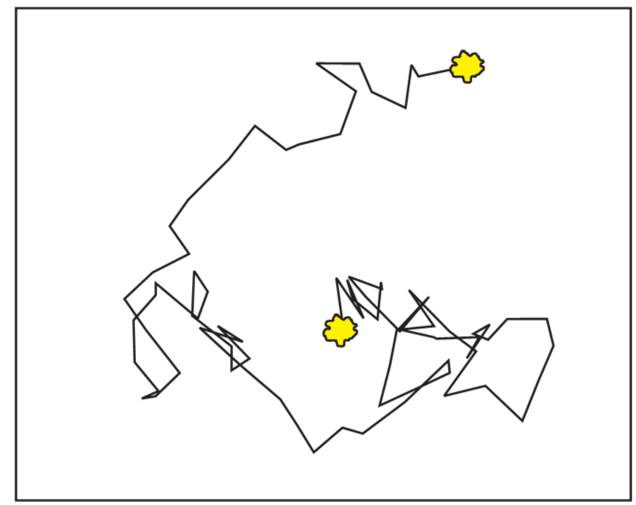
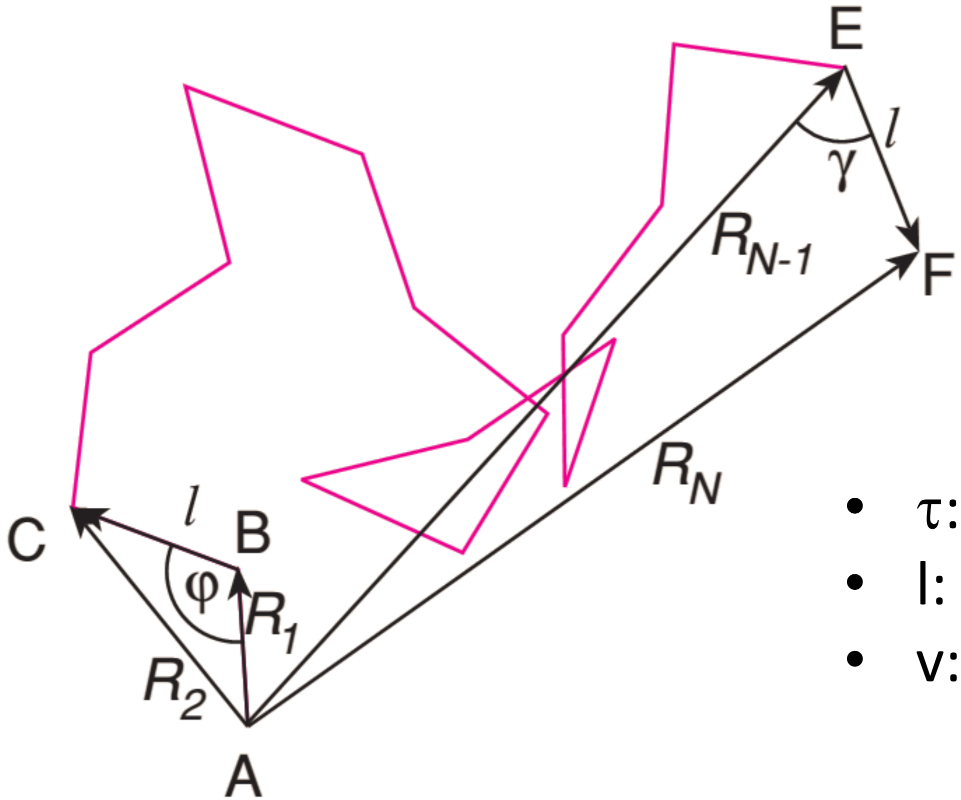






- τ: mean time between collisions
- l: mean free path
- v: mean speed of particles

How far reaches a $\overline{\text{particle}}$?



Simplification:

diffusion 1 plane

- τ: mean time between collisions
- l: mean free path
- v: mean speed of particles

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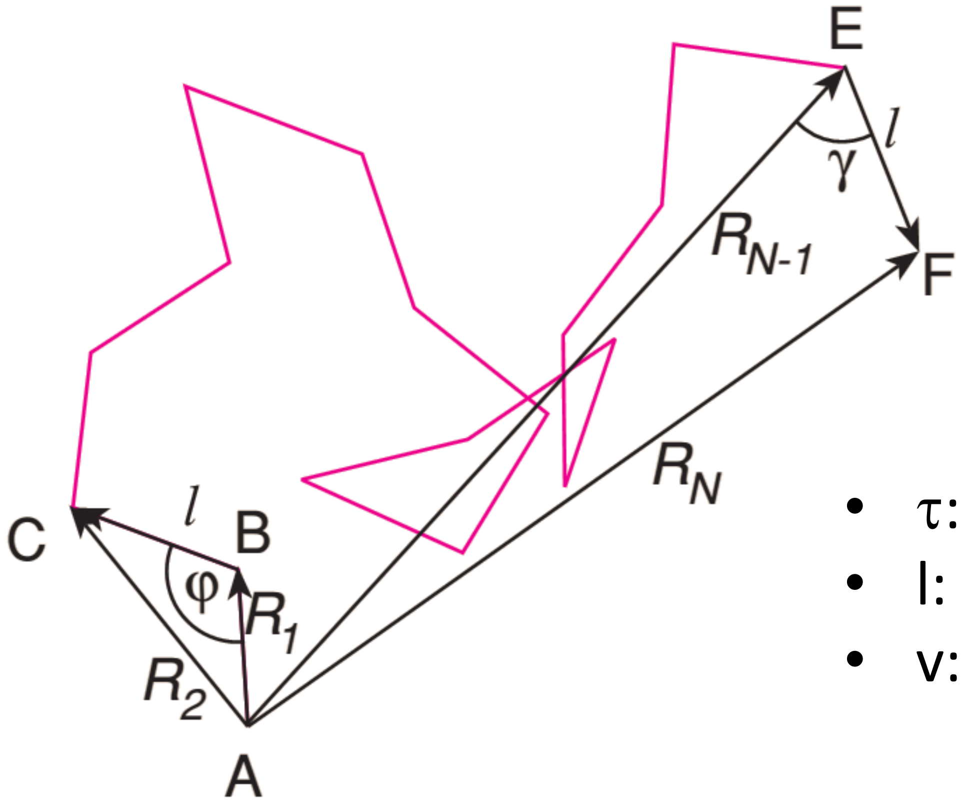

- τ: mean time between collisions
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One particle: $R_2{}^2 = R_1{}^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi$

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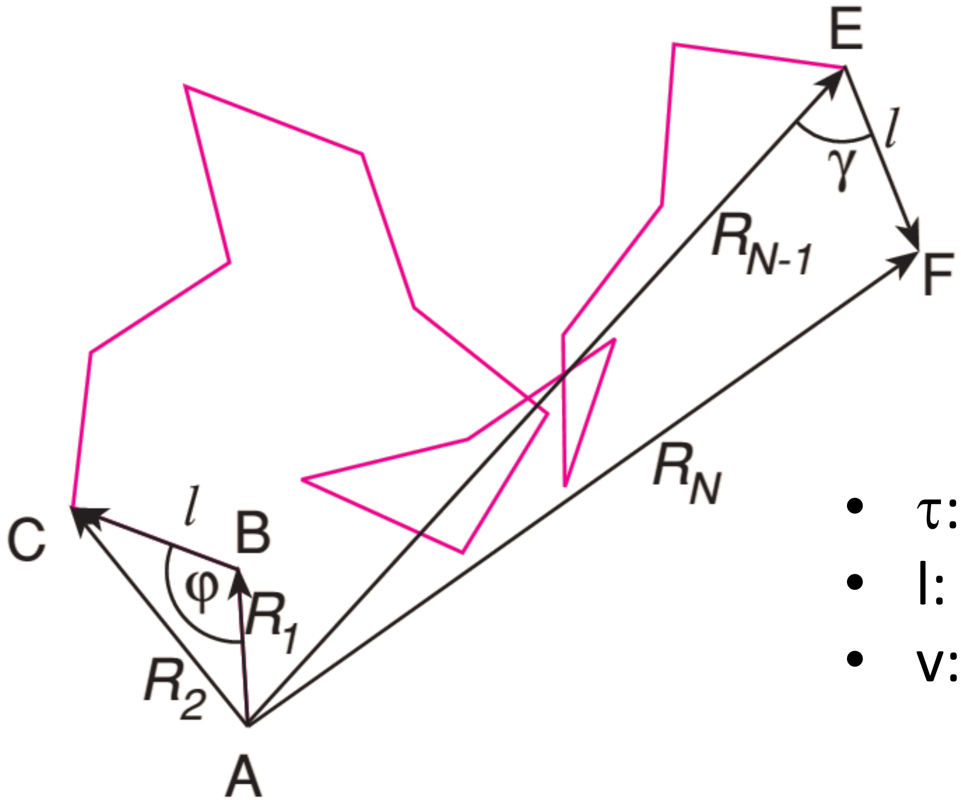

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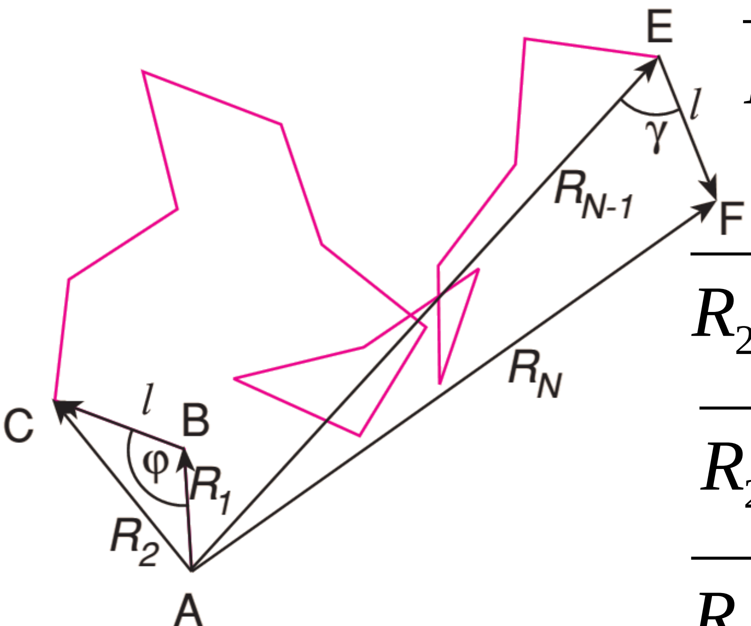
One particle: $R_2{}^2 = R_1{}^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi$

A „mean" particle:
(mean of *n* particles): $$\overline{R_2{}^2} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(R_1{}^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi_i \right)$$

How far reaches a $\overline{\text{particle}}$?



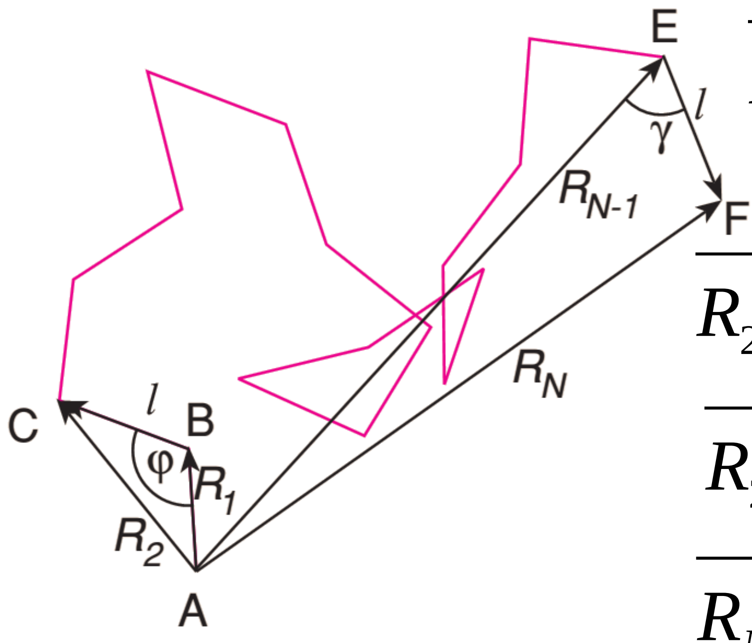
$$\overline{R_2^{\ 2}} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(R_1^{\ 2} + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi_i \right)$$

$$\overline{R_2^{\ 2}} = \frac{1}{n} \cdot n \cdot \left(R_1^{\ 2} + l^2 \right) - \frac{2}{n} \cdot R_1 \cdot l \cdot \sum_{i=1}^{n} \left(\cos\varphi_i \right)$$

$$\overline{R_2^{\ 2}} = R_1^{\ 2} + l^2 = l^2 + l^2 = 2 \cdot l^2$$

$$\overline{R_N^{\ 2}} = N \cdot l^2$$

How far reaches a $\overline{\text{particle}}$?



$$\overline{R_2^{\;2}} = \frac{1}{n}\cdot\sum_{i=1}^{n}\left(R_1^{\;2} + l^2 - 2\cdot R_1\cdot l\cdot\cos\varphi_i\right)$$

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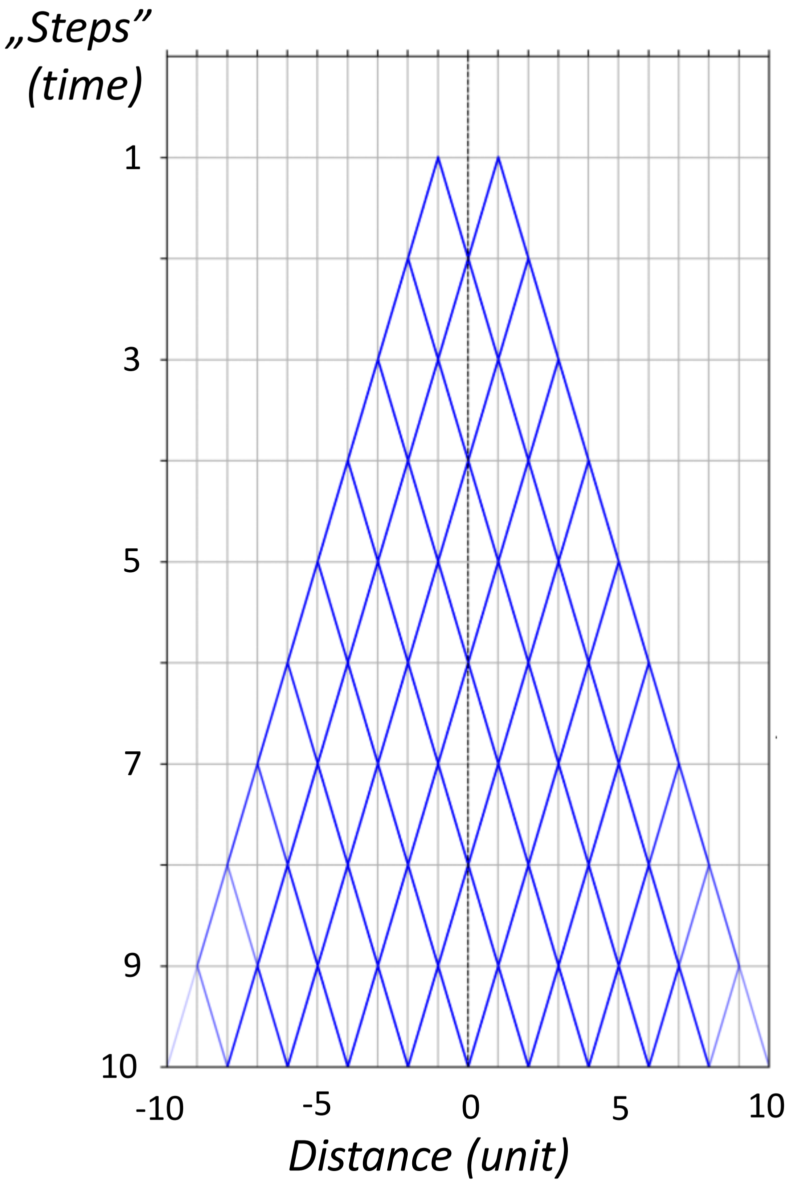
$$\overline{R_N^{\;2}} = N\cdot l^2$$

$$\overline{R_t} = \sqrt{N\cdot l^2} = \sqrt{\frac{t}{\tau}\cdot l\cdot l} = \sqrt{t\cdot v\cdot l} = \sqrt{3\cdot D\cdot t}$$

$$\frac{v\cdot l}{3} = D$$

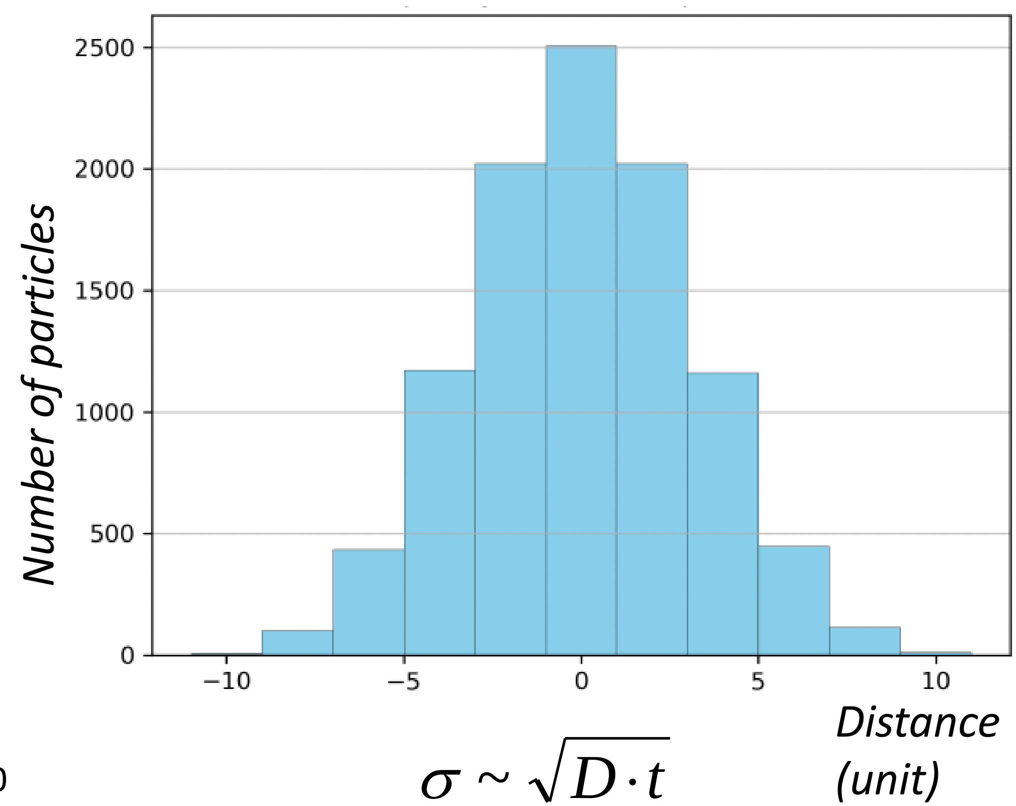
Hova reaches the particles?



Let’s track 10,000 particles in 1D: they can move „either left or right”

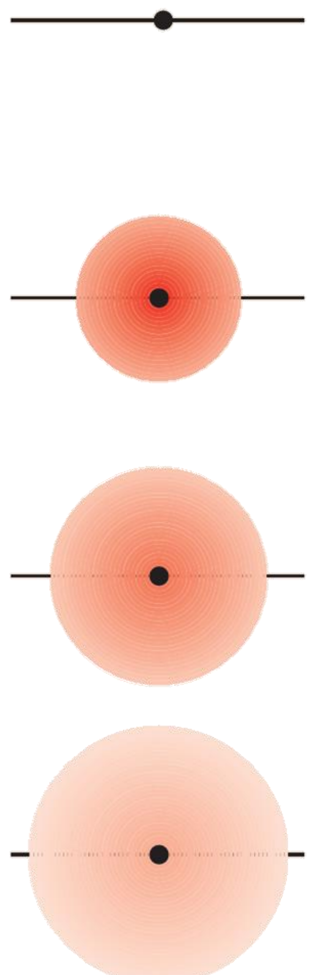
follow 10 steps (time units)

Distribution after 10 steps

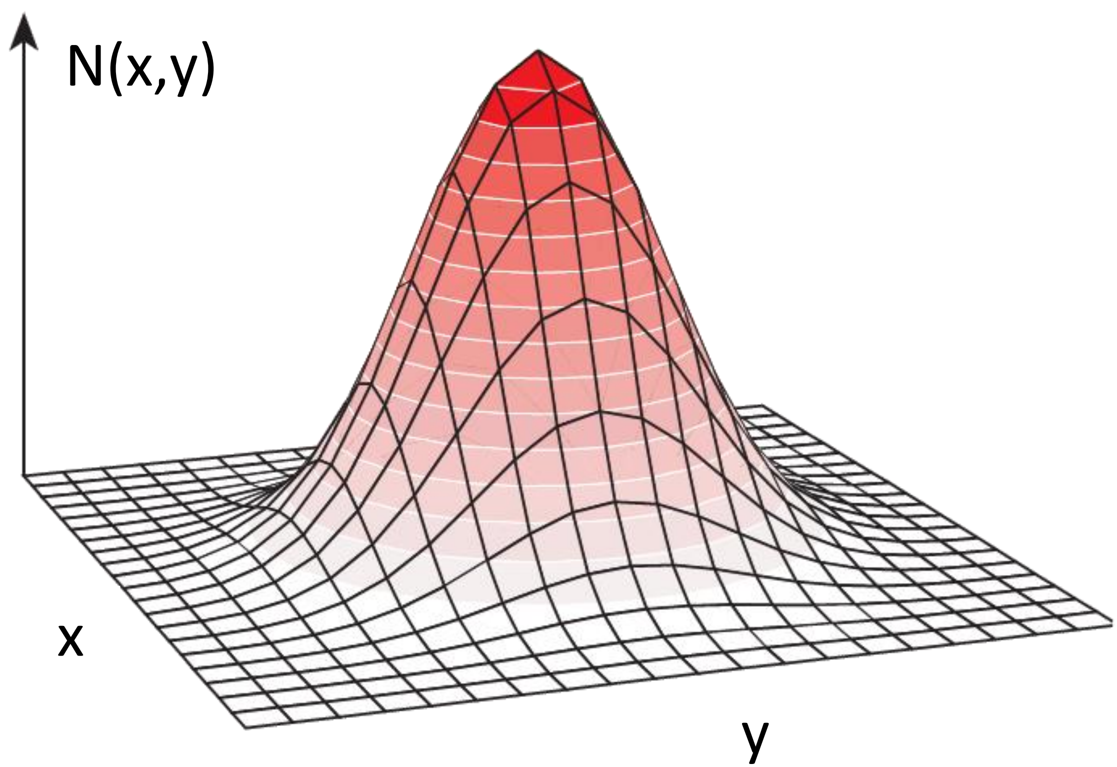


$$\sigma \sim \sqrt{D \cdot t}$$

Distribution of particles in 2D



We will do this experiment during the practice



$$\sigma \sim \sqrt{D \cdot t}$$

„Result” of thermal motion - flow

Particle flow rate: $$I_N = \frac{\Delta N}{\Delta t};\left[\frac{1}{s}\right]$$

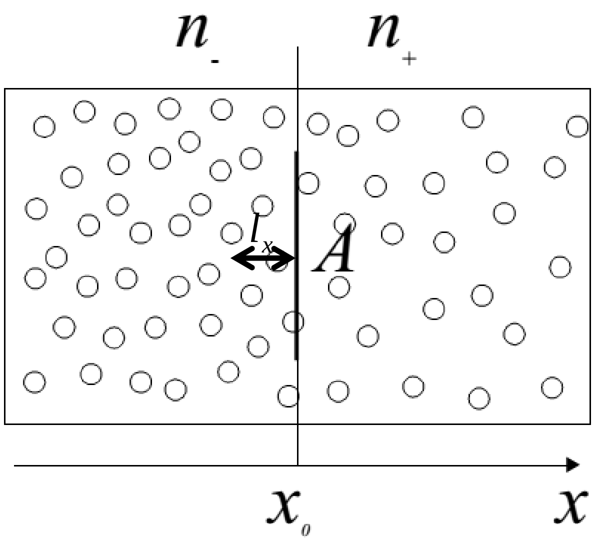
Particle flow density (flux): $$J_N = \frac{\Delta I_N}{\Delta A};\left[\frac{1}{m^2 \cdot s}\right]$$

For a lot of particles:

Matter flow rate: $$I_\nu = \frac{\Delta \nu}{\Delta t};\left[\frac{mol}{s}\right]$$

Matter flow density (flux): $$J_\nu = \frac{\Delta I_\nu}{\Delta A};\left[\frac{mol}{m^2 \cdot s}\right]$$

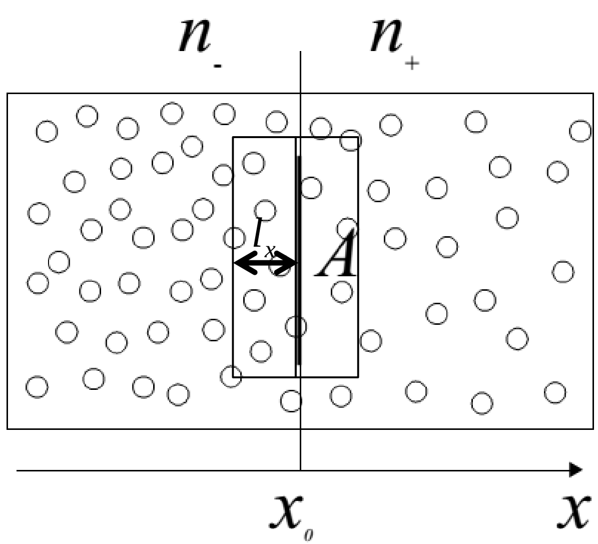
Fick's first law



What is the net material flow, if the concentration differs?

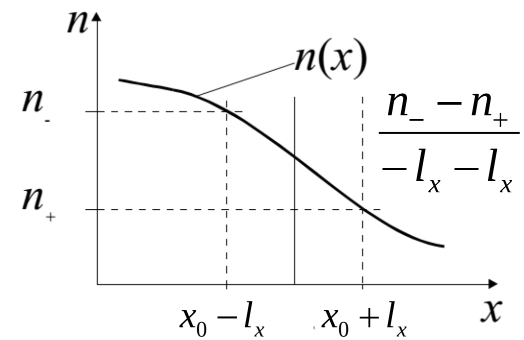
$$\boxed{J_\nu = -D \cdot \frac{\Delta c}{\Delta x}}$$

Fick's first law



What is the net material flow, if the concentration differs?

Could be derived from thermal motion – see the textbook



$$\frac{n_- - n_+}{-l_x - l_x} = \frac{\Delta n}{\Delta x}$$

$$\Delta N = N_- - N_+ = \frac{1}{2}\cdot V_t \cdot (n_- - n_+) = \frac{1}{2}\cdot v_x \cdot \Delta t \cdot \overbrace{A\cdot(n_- - n_+)}^{l_x}$$

$$\Delta N = \frac{1}{2}\cdot v_x \cdot \Delta t \cdot A \cdot 2 \cdot l_x \cdot - \frac{\Delta n}{\Delta x}$$

$$v_x \cdot l = D$$

$$J_{Nx} = \frac{1}{2}\cdot v_x \cdot 2 \cdot l_x \cdot - \frac{\Delta n}{\Delta x} = -D \cdot \frac{\Delta n}{\Delta x}$$

$$\boxed{J_v = -D \cdot \frac{\Delta c}{\Delta x}}$$

But Δc is not the real „driving force”!
But it is thermal motion

Diffusion coefficient

D gives the amount of matter diffused across a unit area in a unit time in a case of unit concentration drop (gradient).

$$D = \frac{v \cdot l}{3}; \left[\frac{m^2}{s}\right]$$

$$D = u \cdot k \cdot T$$

Einstein-Stokes
(spheres)

$$D = \frac{k \cdot T}{6 \cdot \pi \cdot \eta \cdot r}$$

BUT!
Not directly proportional with T!

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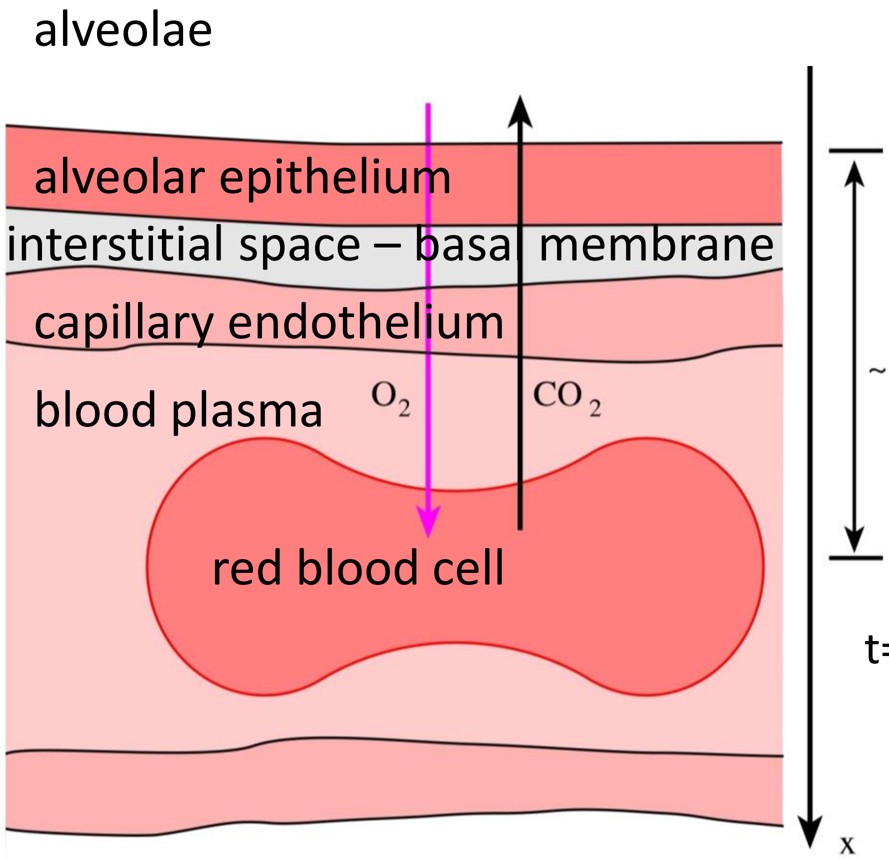


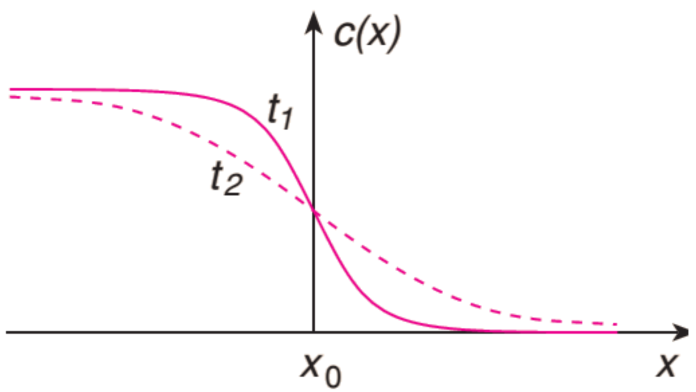
$$1{,}9 \cdot 10^{-9} \frac{m^2}{s}$$

$$t \approx 500 \mu s$$

t=? 1m distance

Fick's second law

Fick II: if concentration drop is not constant
- changing in time



$$c(t+\Delta t) = c(t) + D \cdot \Delta t \cdot \frac{\Delta\left(\frac{\Delta c}{\Delta x}\right)}{\Delta x}$$

Fick's second law

Fick II: if concentration drop is not constant
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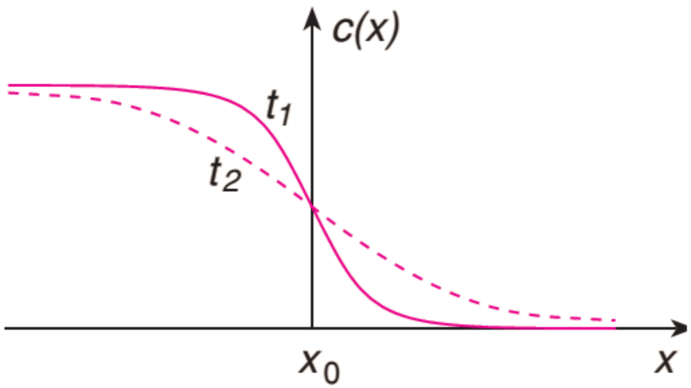

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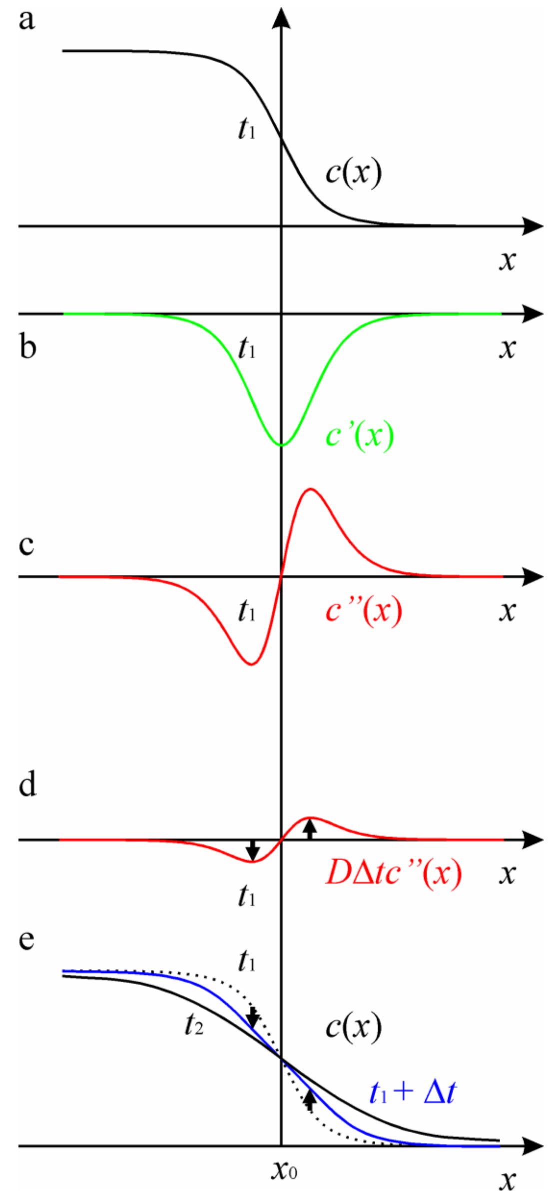
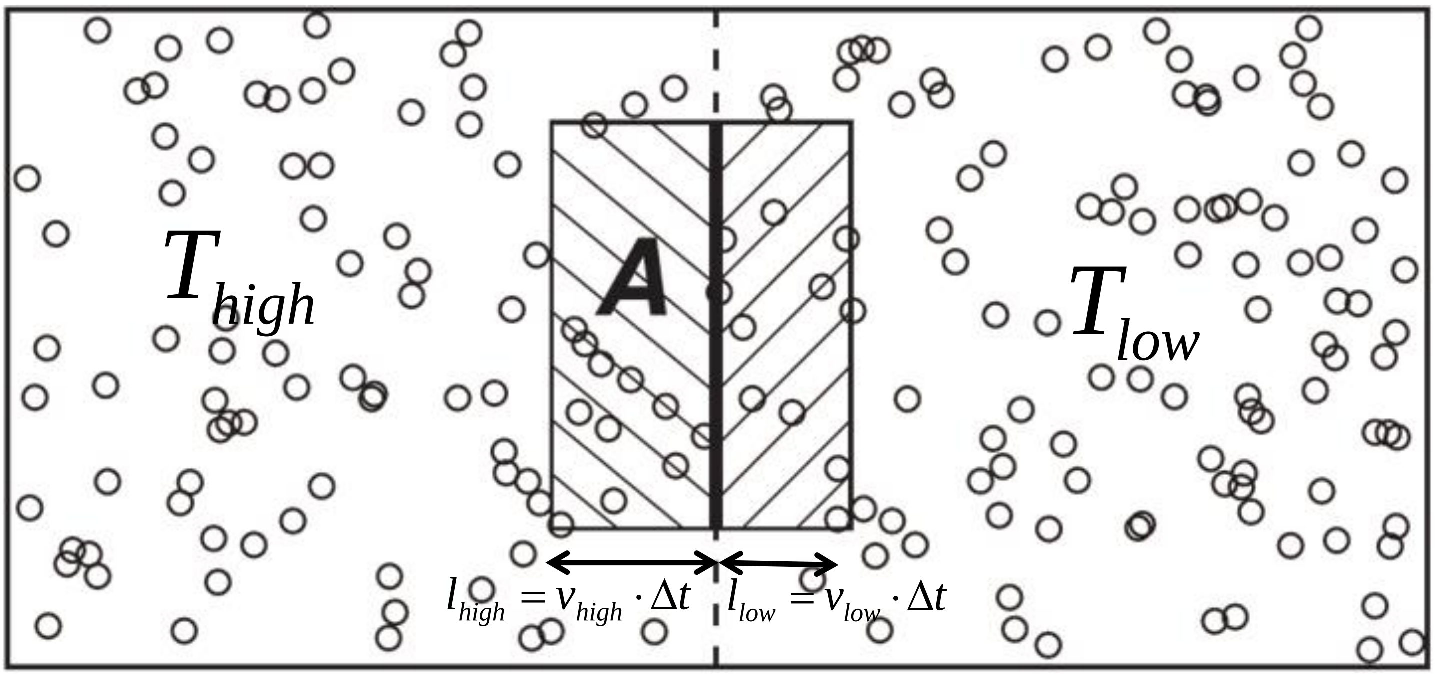


Thermodiffusion

What is the net material flow,
If the temperature differ (but the concentration is the same)?



$T_{high} > T_{low}$

$v = \sqrt{3 \cdot \frac{k \cdot T}{m}}$

$J_{\nu} = -L_T \cdot \frac{\Delta T}{\Delta x}$ (Ludwig-Soret effect)

Generalization

Onsager-relation: $J_{ext.} = L_{cond} * X_{int\_grad}$

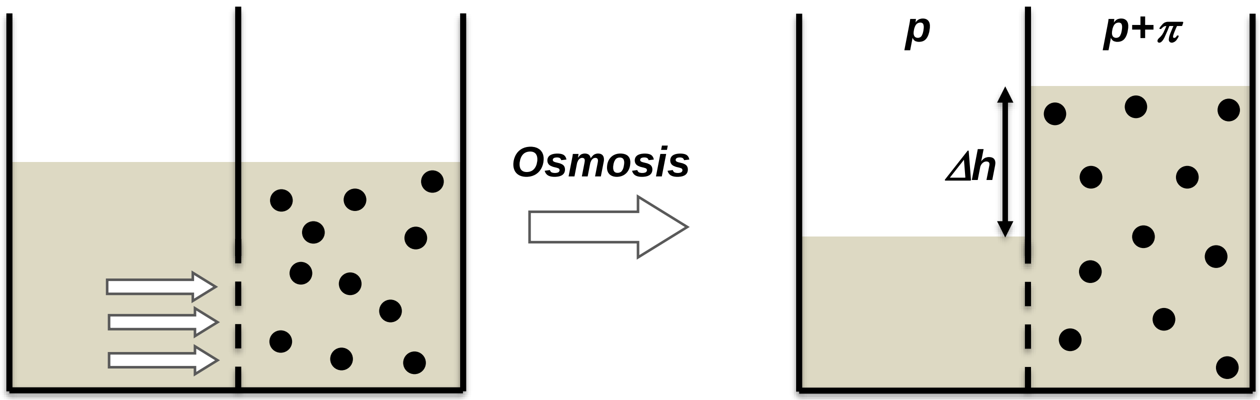
$J_{ext.}$: $flow\ density\ of\ extensive\ quantity$ (eg. J_{matter})

X_{int_grad}: $gradient\ of\ intensive\ quantity\ (eg.\frac{\Delta c}{\Delta x})$

L_{cond}: $conductivity\ coefficient\ (eg.D)$

Osmosis

One-way diffusion of the SOLVENT. (permeable membrane only for *water*)



difference in the concentration of the SOLVENT

Hydrostatic pressure difference (osmotic pressure)

$$p_{osm} = \pi = c_{solute} \cdot R \cdot T \quad \text{(Van 't Hoff law)}$$

Osmotic concentration (equivalent osmotic pressure, „ozmolarity”, „ozmolality”):
The concentration of a solution that keeps balance with a heterogeneous solution.
Derived units: *mOsm(/L)*, mmol/L, mmol/kg

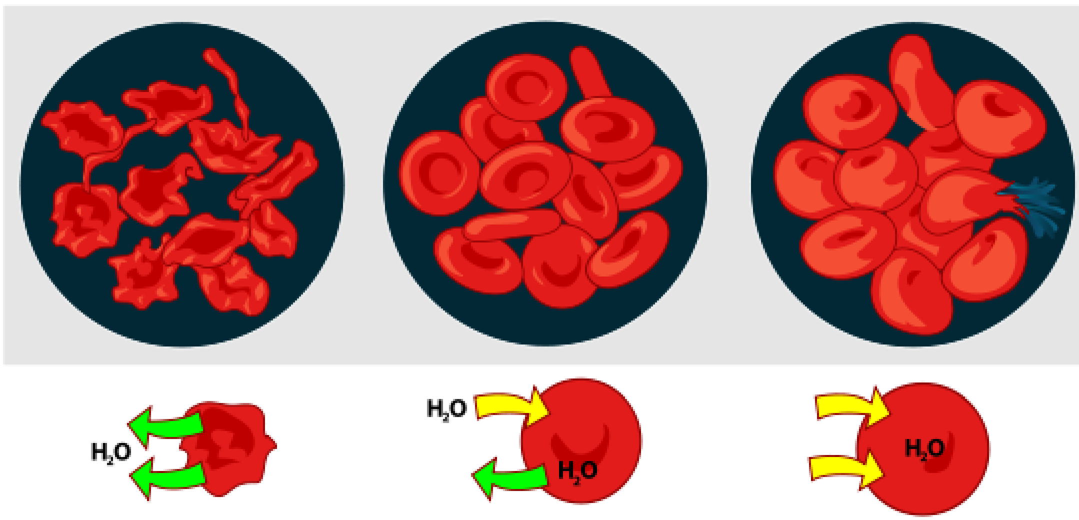
More precisely: correction needed for dissociation

Medical practice

Tonicity: „effective" osmolarity

membrane: *given cell membrane* (permeable not only for water)

non-permeable ions/molecules are important for tonicity



Hypertonic **Isotonic** **hypotonic**

„Isosmotic", „Physiological", „isotonic", „normal" solutions:

Physiological/Normal/Isotonic saline: 0,9% (w/v) NaCl (isotonic)

d5W: 5% (w/v) glucose (hypotonic)

Ringer, Ringer's lactate (isotonic)

Isosmotic not equal Isotonic!

Osmotic concentration of the blood plasma: about 300mOsm/L

OMHV