Flow of fluids and gases
Blood as fluid

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SEMMELWEIS
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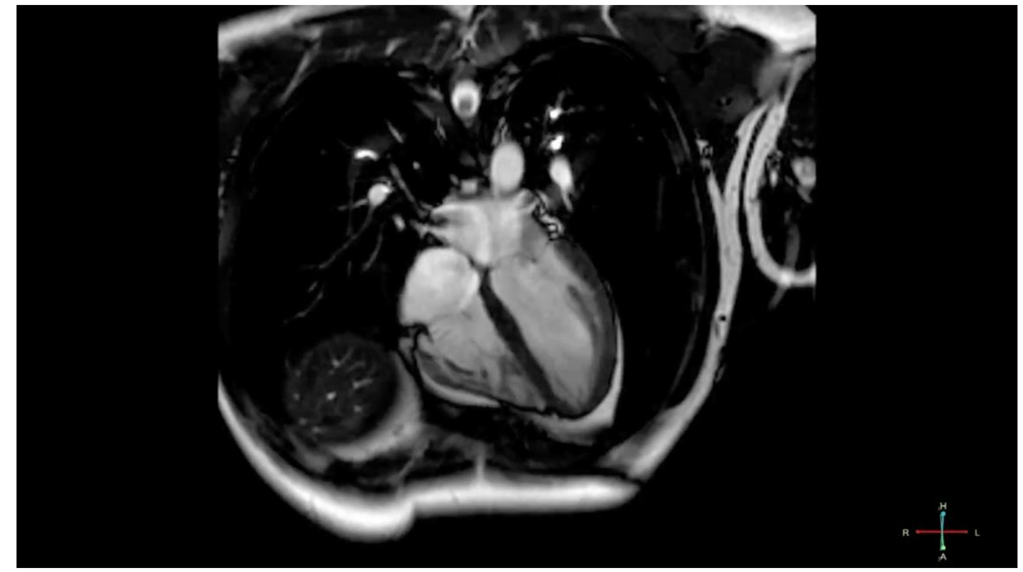
Importance of the physics of fluids

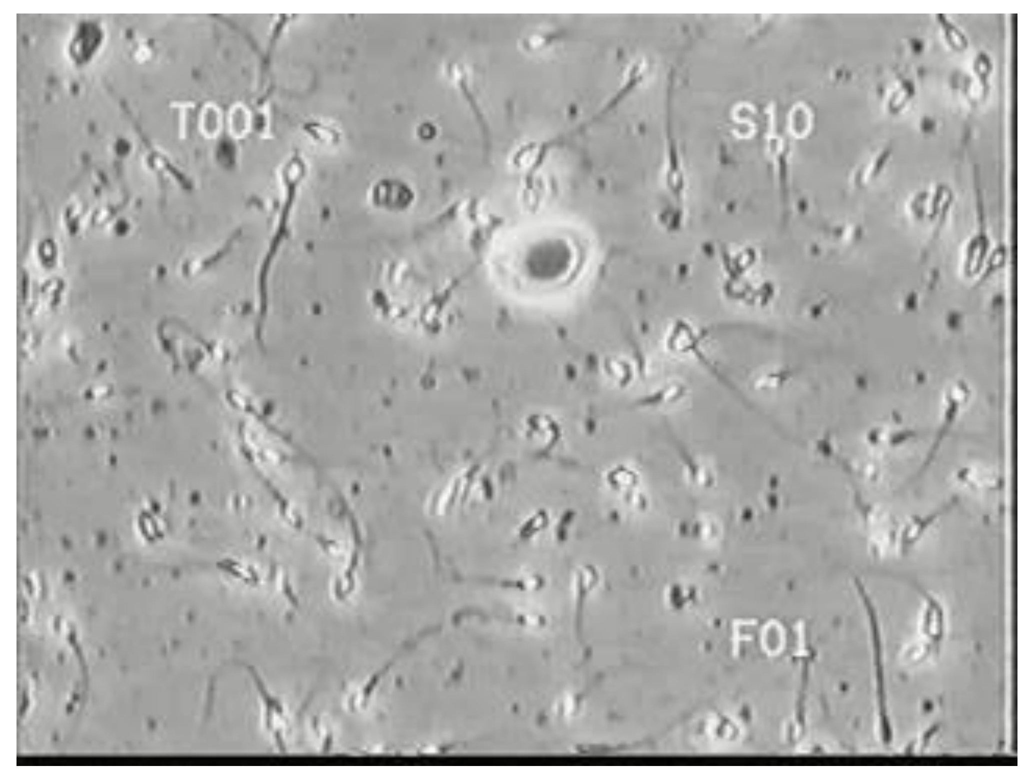
I. Hemodynamics

E.g.: What are the characteristics of blood flow in the circulatory system?
(similarly: flow of gases in respiratory system)



II. Motion in viscous fluids

E.g.: What is the force exerted by a single spermatocyte during its motion?

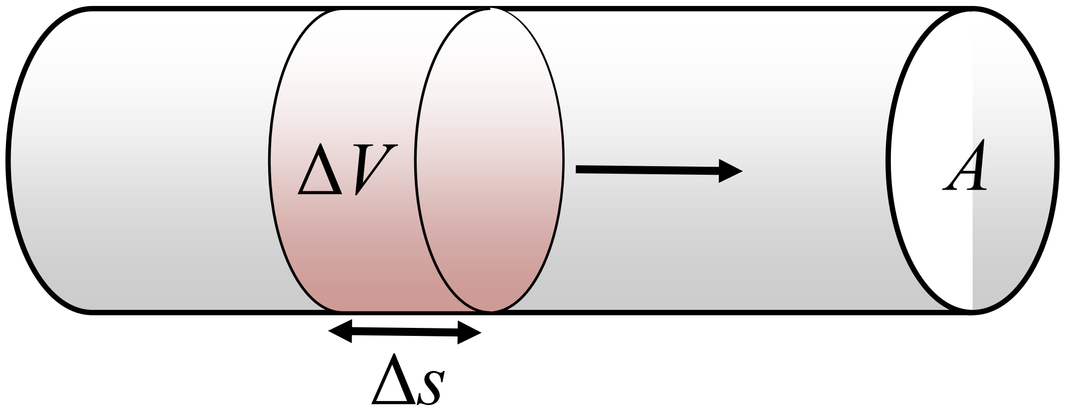


N.B.: gases - in contrast to fluids - are compressible. However, in the physiologically relevant range of pressure differences (~100 Pa), the changes in their volume/density are negligible.

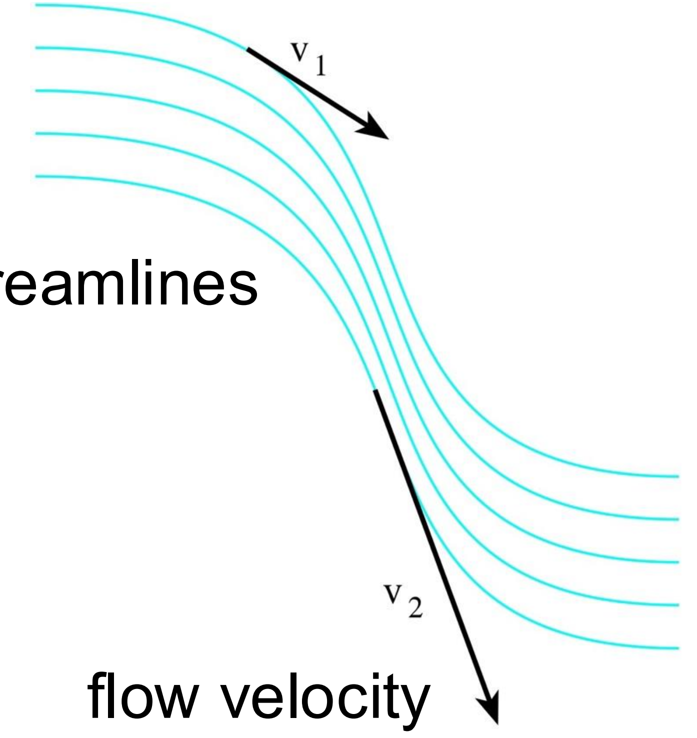
Basic principles I.

Volumetric flow rate (I_V):

$$I_V = \frac{\Delta V}{\Delta t} = A\frac{\Delta s}{\Delta t} = A\bar{v}$$

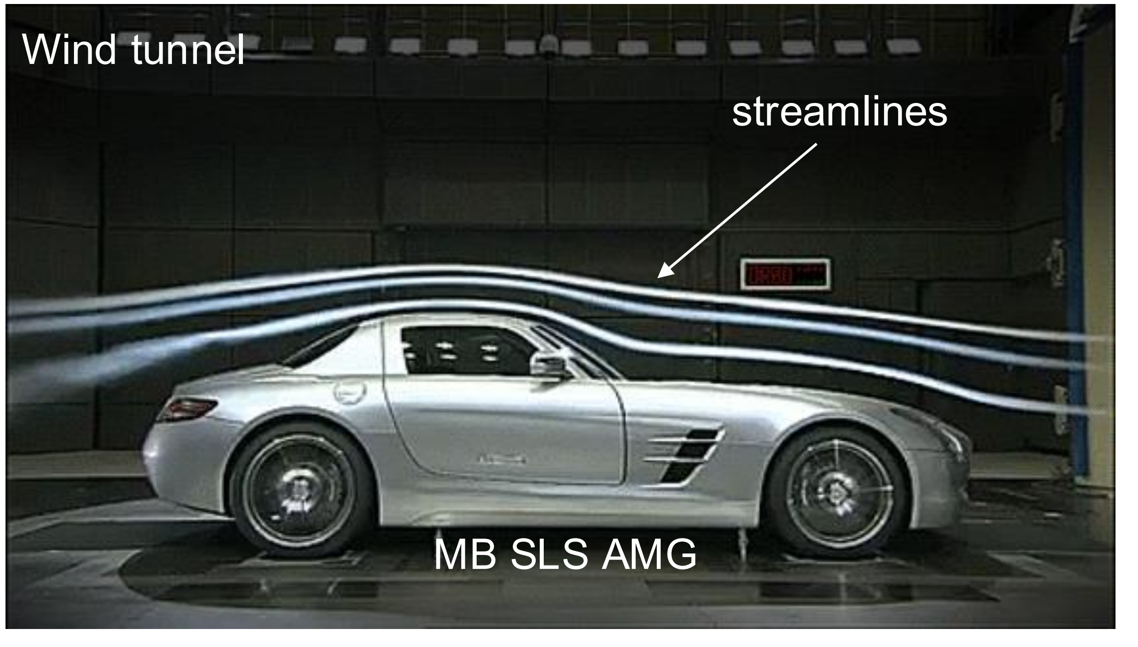


Average velocity: $\bar{v} = \frac{I_V}{A}$



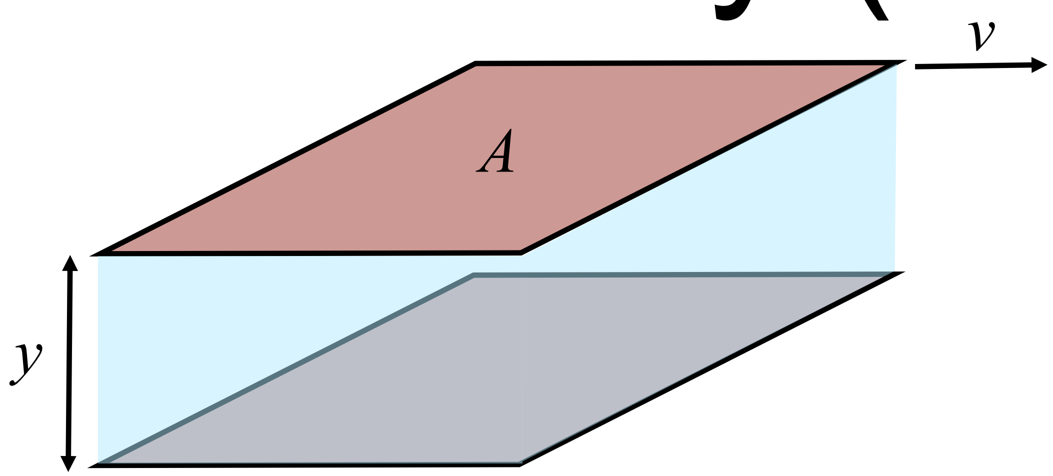
streamlines

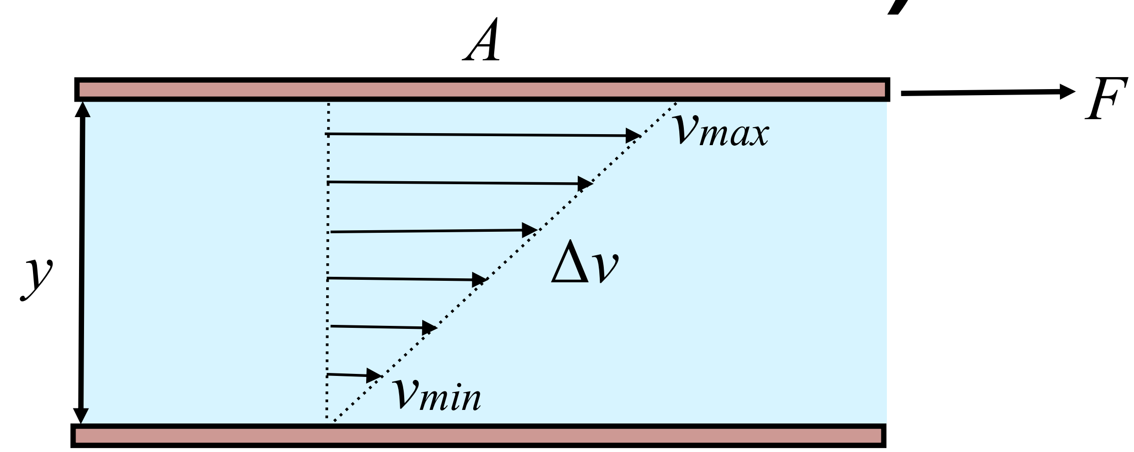
flow velocity
(direction of tangent = direction of velocity
density of streamlines = magnitude of velocity)



Flow can be made visible

Basic principles II.
Viscosity (internal friction)





F = shear force
A = area of fluid layer
η = viscosity
v = flow velocity
y = distance between fluid layers
F/A = shear stress (τ)
$\Delta v/\Delta y$ = velocity gradient (D)

$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y}$$ (Newton's friction law)

$$\eta = \frac{\tau}{D}$$

Units of viscosity: $1Pas = 1\frac{Ns}{m^2} = 10P(poise)$

Viscosity of distilled water (25 ˚C): 1 mPas (1 centipoise)

N.B.: in gases: $\eta \sim T$
Momentum exchange between particles acts against the gliding of layers past one another.

fluids: $\eta \sim e^{E/k_B T}$
Viscosity decreases with the increase in the relative concentration of vacancies.

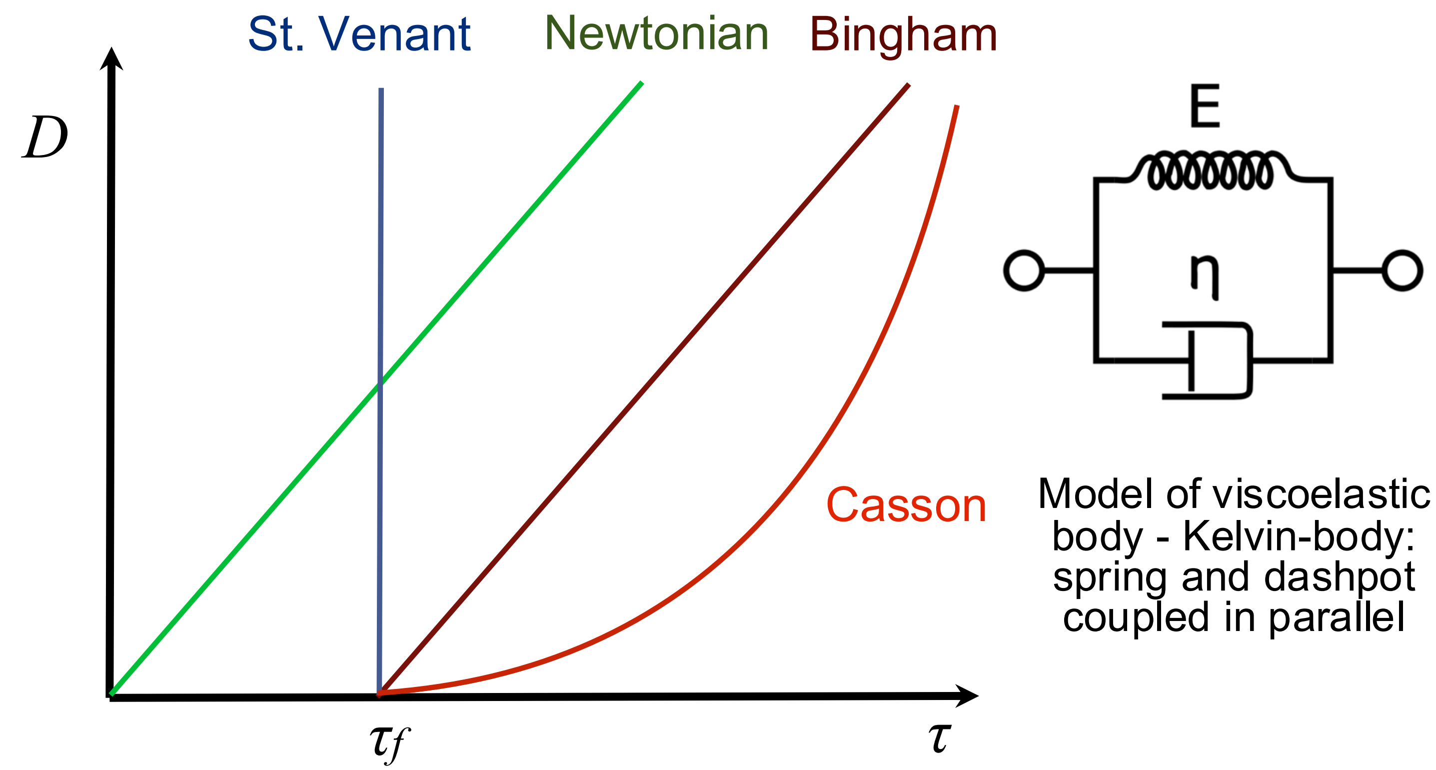
Types of fluids

1. Ideal
frictionless, incompressible
ρ = constant, $\eta = 0$

2. Non-ideal (real)
a. Newtonian (viscous)
η independent of shear stress

b. Non-newtonian (anomalous)
η changes with shear stress

Relationship between velocity gradient and shear stress in ***real*** fluids



Model of viscoelastic body - Kelvin-body: spring and dashpot coupled in parallel

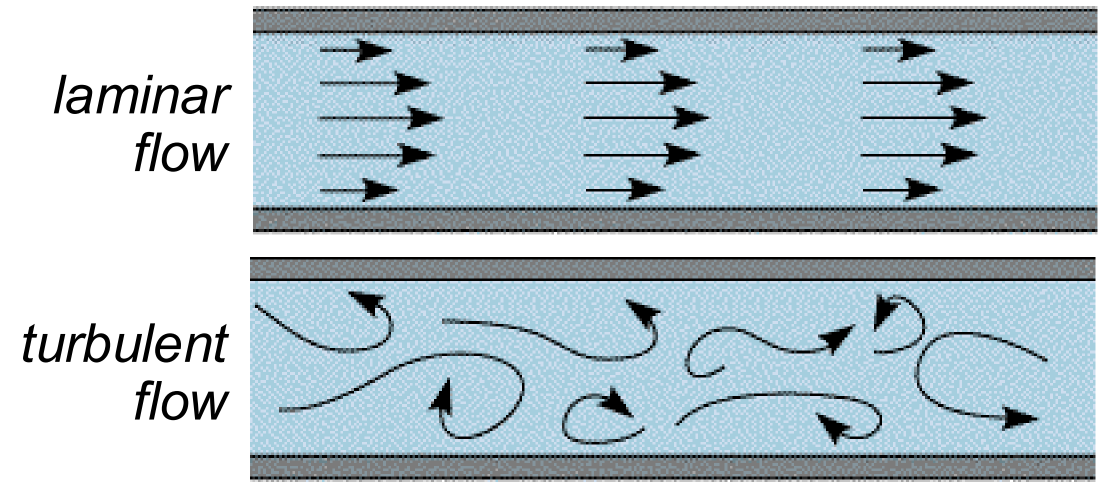
- τ_f = flow threshold
- Viscoelastic materials: combination of viscous and elastic properties (e.g., solution of polymers, macromolecules)
- Stress-relaxation: decay of stress in rapidly deformed viscoelastic body.
- Blood is a non-newtonian fluid; it displays viscoelastic properties.

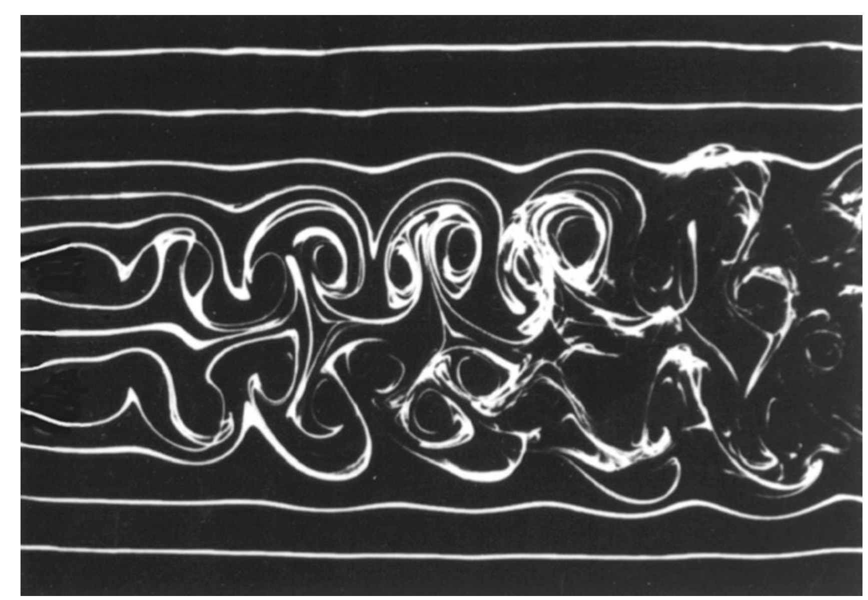
Types of fluid flow

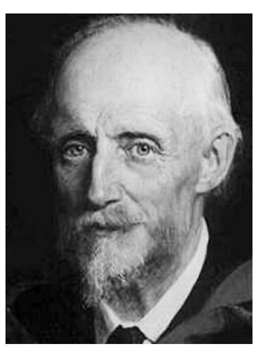
1. **Stationary**
   Volumetric flow rate stays constant (parameters characterizing flow remain unchanged).

2. **Laminar**
   Fluid layers do not mix.

3. **Turbulent**
   Fluid layers mix.







Osborne Reynolds
(1842-1912)

Reynolds number (Re):

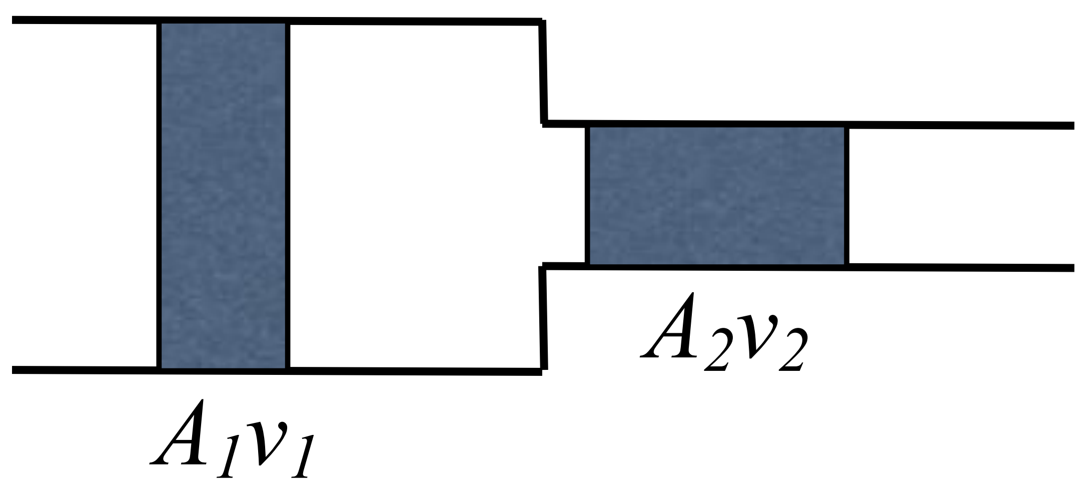
$$\mathrm{Re} = \frac{v r \rho}{\eta}$$

v = flow rate (m/s)
r = tube radius (m)
ρ = density of fluid (kg/m^3)
η = viscosity (Ns/m^2)

$\mathrm{Re} \approx 2000$

Laws of flow in ideal fluids I.

Continuity equation:
volumetric flow rate is constant



$$A_1v_1 = A_2v_2 = \text{constant}$$

A = cross-sectional area
v = flow velocity

Laws of flow in ideal fluids II.

Bernoulli's law - law of conservation of energy

Daniel Bernoulli
(1700-1782)

$$p + \frac{1}{2}\rho v^2 + \rho g h = konst$$

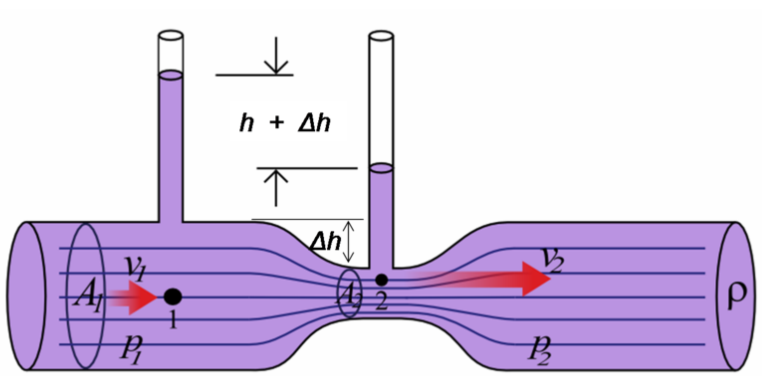
p = static pressure

$\frac{1}{2}\rho v^2$ = dynamic pressure

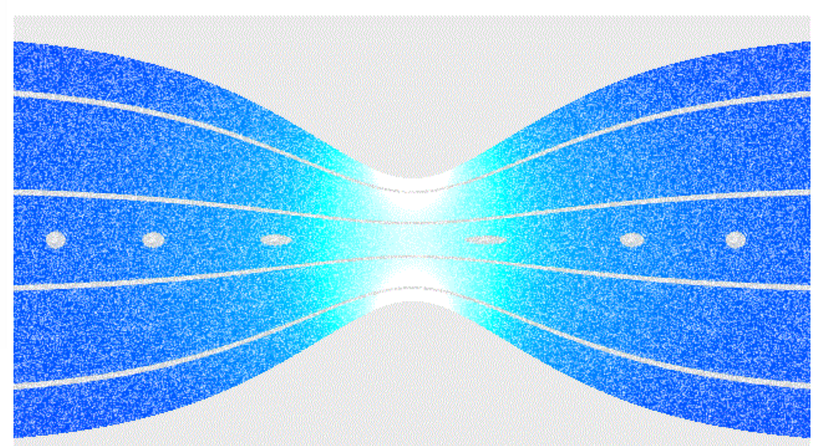
$\rho g h$ = hydrostatic pressure

Application: Venturi-effect

Giovanni Battista
Venturi
(1746-1822)



Static pressure drops at the tube narrowing



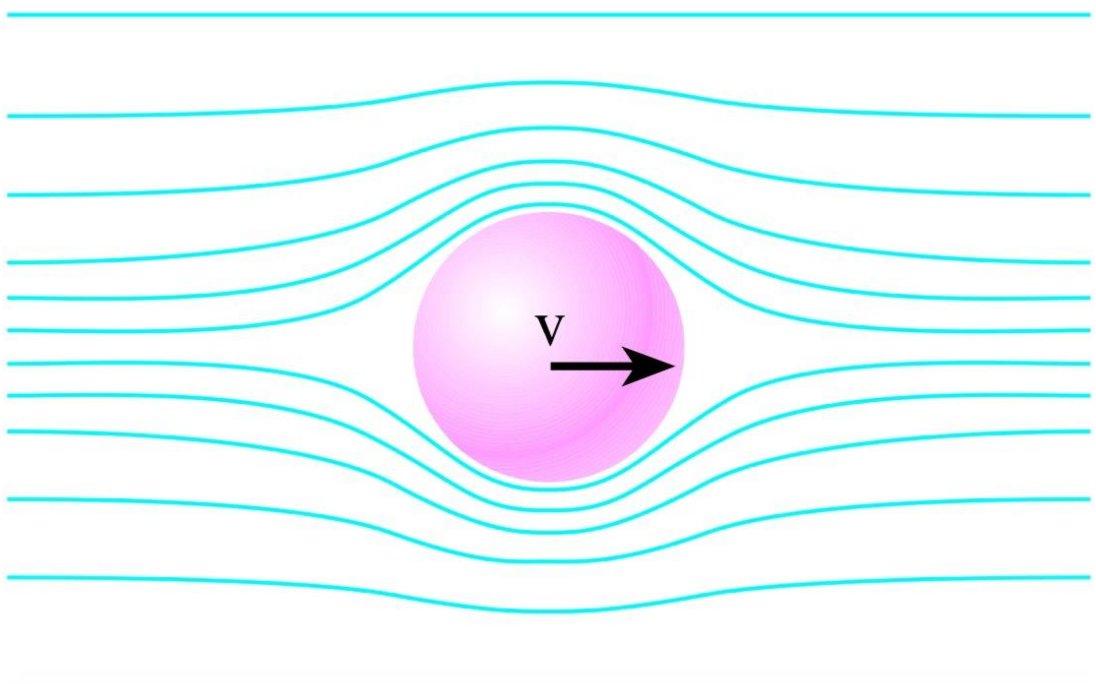
Venturi tube

- Biphasic pulse (*pulsus bisferiens*) in aorta insufficiency
- Aspirator (vacuum pump)
- Inspirator (Bunsen-burner)
- Sensors (volumetric flow rate)
- Vacuum cleaner
- Atomizer, spray, carburetor
- Leadpipe and barrel of wind instruments
- Diffuser

Laws of flow in viscous fluids I.

Stokes' law

Georg Gabriel Stokes (1819-1903)

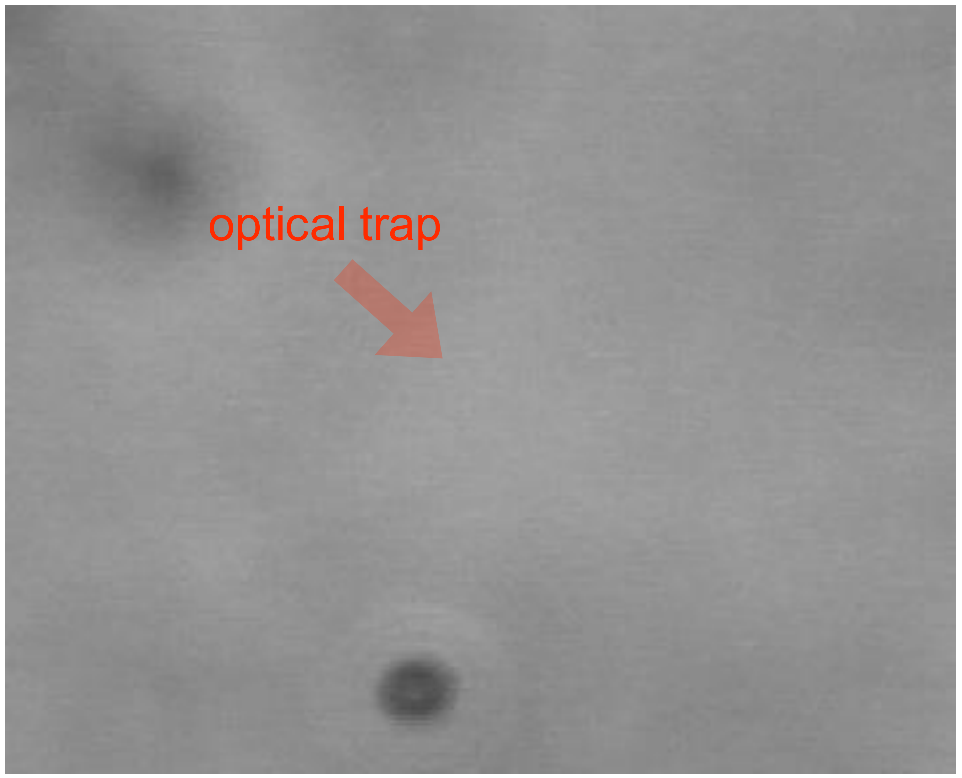


$$F = \gamma v = 6 r \pi \eta v$$

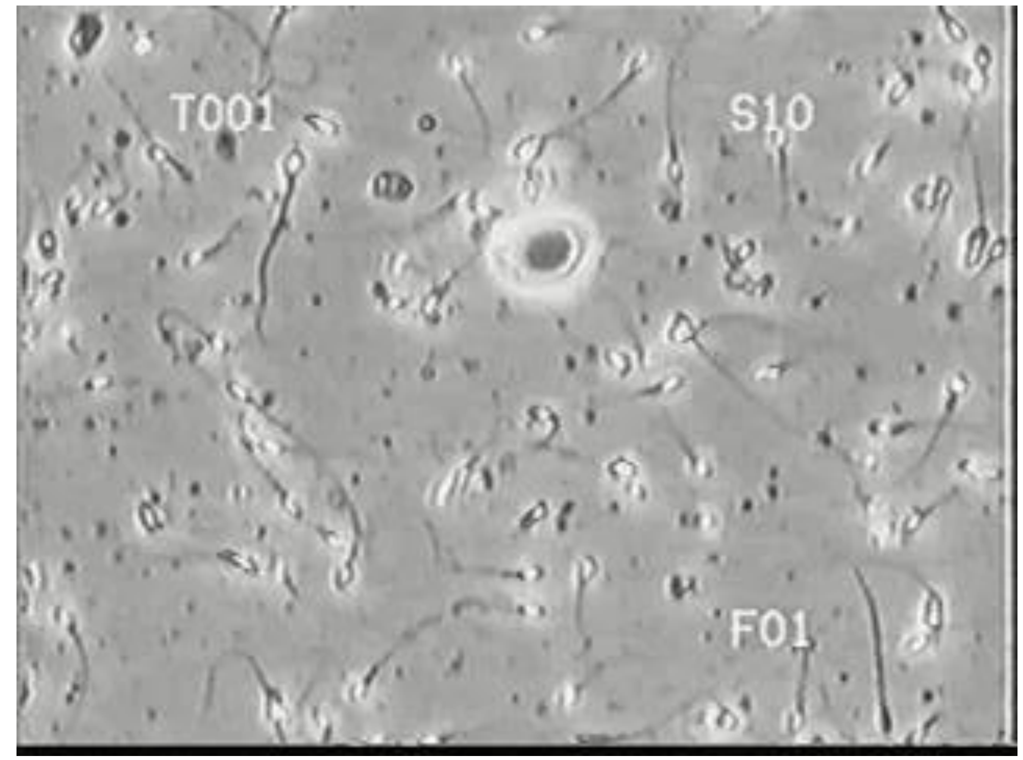
F = force
γ = drag coefficient (shape factor)
v = flow rate
r = radius of sphere
η = viscosity

Stokes force

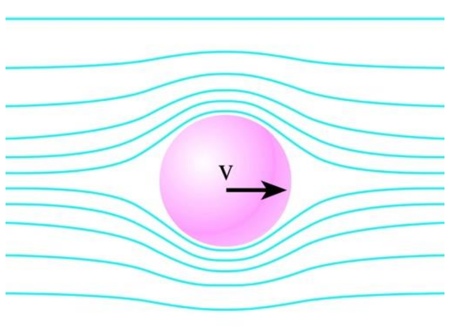
Hydrodynamic drag force (Stokes force): $F = \gamma v = 6 r \pi \eta v$

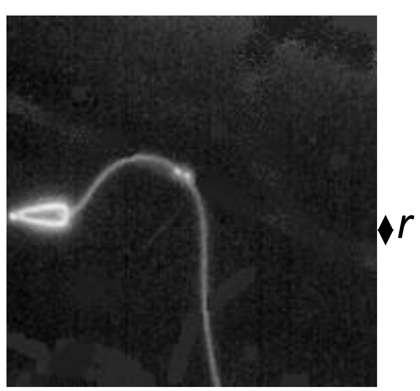


Affects stationary objects in moving fluid (microbead captured in optical trap)



Affects objects moving in stationary fluid (moving spermatocytes)





r = 1.6 μm = 1.6 x 10^{-6} m
v = 50 μm/s = 5 x 10^{-5} m/s
η = 10^{-3} Pas

How much force is exerted by a single spermatotyce during its movement?

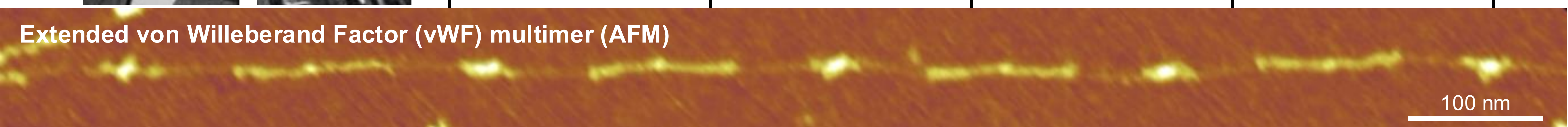
$$\gamma = 6 r \pi \eta = 6 \cdot 1.6 \times 10^{-6} \cdot \pi \cdot 10^{-3} = 3 \times 10^{-8} Ns/m$$

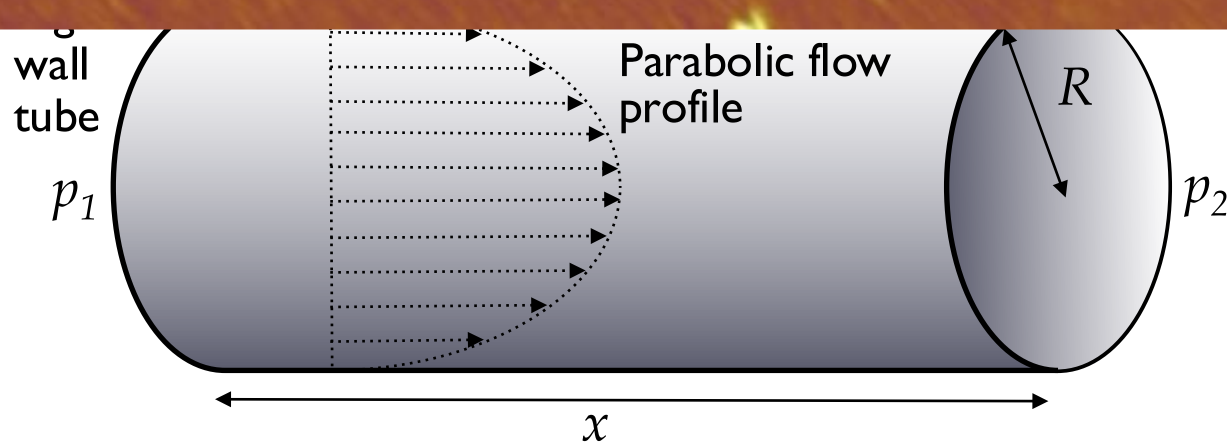
$$F = \gamma v = 3 \times 10^{-8} Ns/m \cdot 5 \times 10^{-5} m/s = 1.5 \times 10^{-12} N = \boxed{1.5\, pN}$$

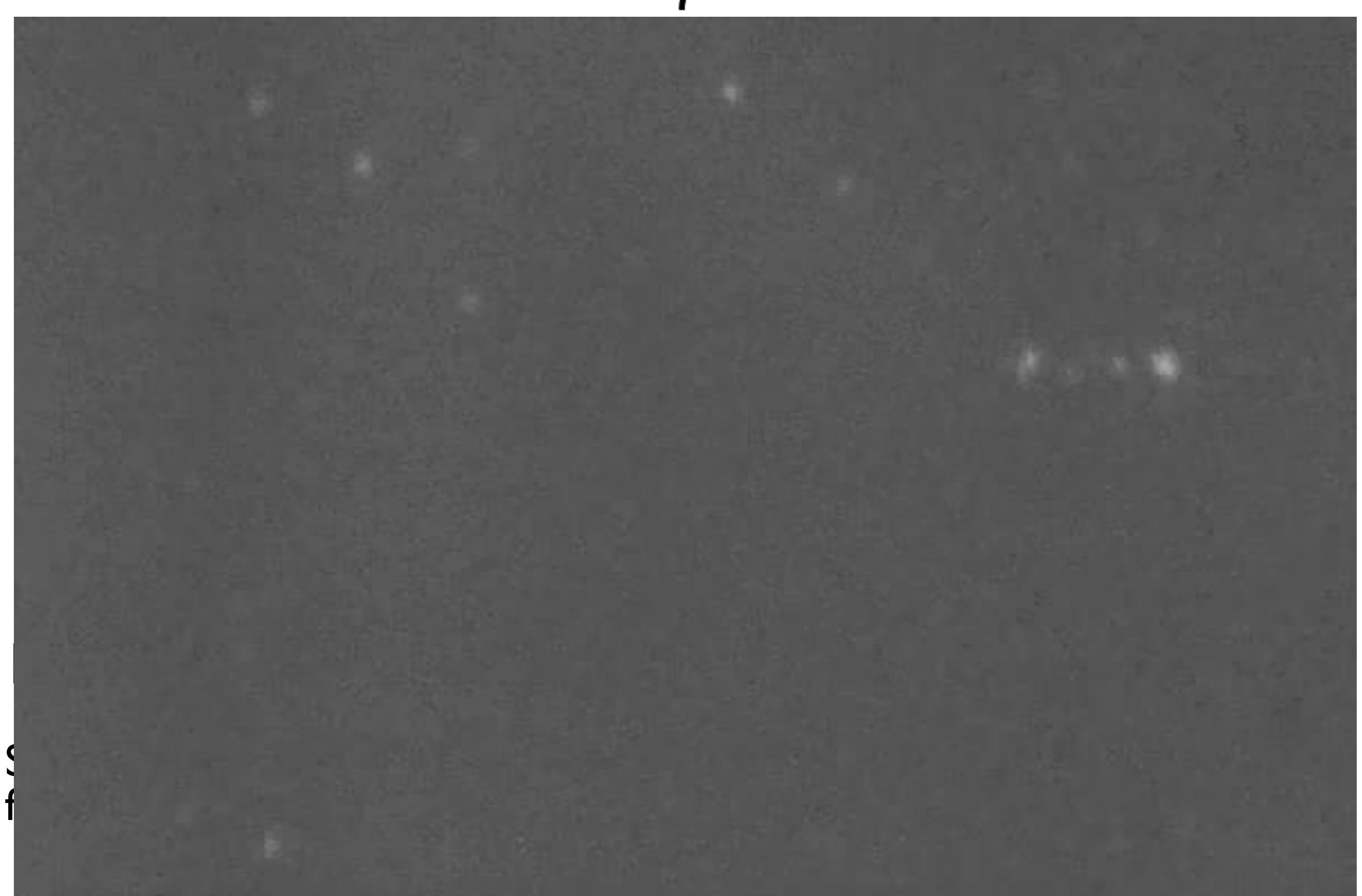
Laws of flow in viscous fluids II.

Hagen-Poiseuille's law


Thermodynamic current	Relevant intensive variable (its difference maintains current)	Current density	Physical law
Volumetric flow	Pressure (p)	$J_V = -\frac{R^2}{8\eta}\frac{\Delta p}{\Delta x}$	Hagen-Poiseuille





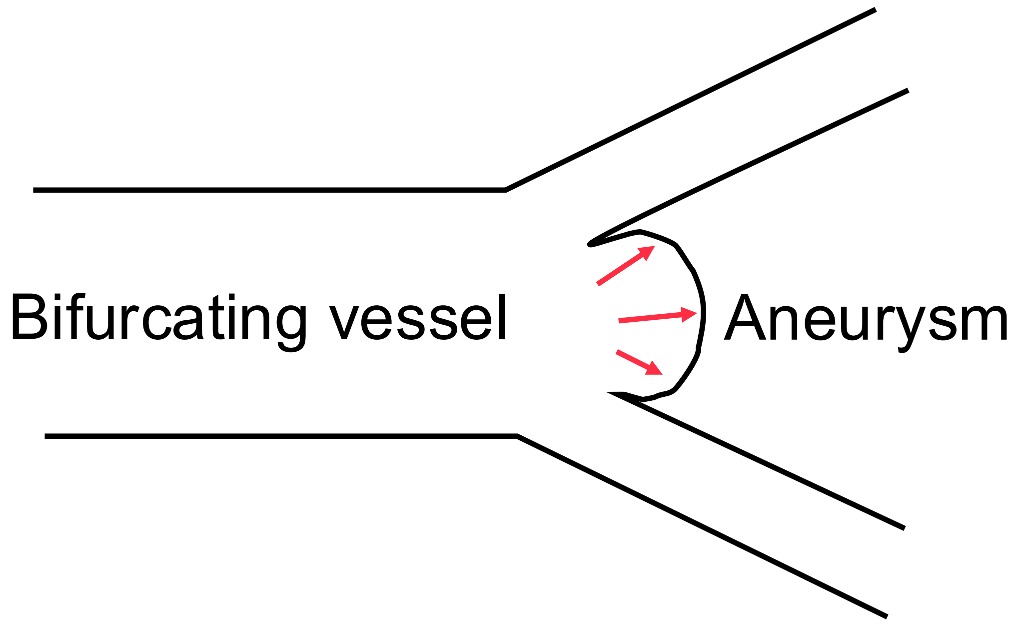


V = volume
t = time
R = tube radius
η = viscosity
p = pressure
x = tube length

$V/t = I_V$ = volumetric vlow rate
$\Delta p/\Delta x$ = pressure gradient, maintained by p_2-p_1 *(negative!)*
A = cross-sectional area of tube
I_V = volumetric flow rate

Medical significance of fluid flow

Bernoulli's law:



Hagen-Poiseuille's law:

$$\frac{V}{t} = \frac{R^{4}\pi}{8\eta}\frac{\Delta p}{\Delta x}$$

Formation of aneurysm (pathological expansion of blood vessel):

- Expansion of vessel: diameter increases
- Flow rate decreases, accoring to continuity equation
- Static pressure increases due to Bernulli's law
- Aneurysm pregrediates - positive fedback mechanism leading to catastrophy

Flow intensity, hence the delivered oxygen quantity, may be **drastically reduced** in certain pathological conditions:

- constriction of blood vessels (e.g., diabetes, Bürger's disease)
- change in blood viscosity (e.g., fever, anaemia)
- Reduction of vessel diameter by half leads to a reduction of volumetric flow by 1/16!

Blood as a fluid

<table>
<tr><td colspan="3">55-60% of body mass is water
42 kg (70 kg body mass)</td></tr>
<tr><td>2/3 intracellular
28 kg</td><td colspan="2">1/3 extracellular
14 kg</td></tr>
<tr><td></td><td>1/3 plasma
4-5 kg</td><td>2/3 interstitium
9-10 kg</td></tr>
</table>

Blood:

Average volume: 5 l
Average viscosity: 5 mPas
Average density: 1.05 g/cm^3
Composition: 40-45 % corpuscular, 55-60 % plasma

Determinants of blood viscosity

1. Hematocrit (htc, φ):

$$htc = \frac{V_{cells}}{V_{total}}$$

Normal range: 0.4-0.5.

Viscosity of blood as suspension
(in the physiologically relevant htc range):

$$\lg \eta_s = A + B\phi$$

η_s=suspension viscosity
A, B=empirical constants

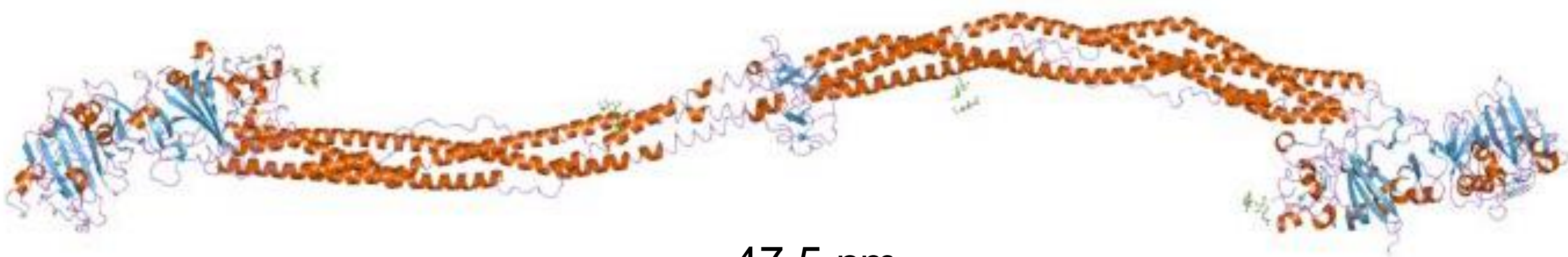
Determinants of blood viscosity

2. Plasma viscosity

- Depends on plasma proteins.
- In *paraproteinaemias* (e.g., myeloma multiplex or plasmocytoma) the concentration of immunoglobulins is high, leading to increased viscosity.

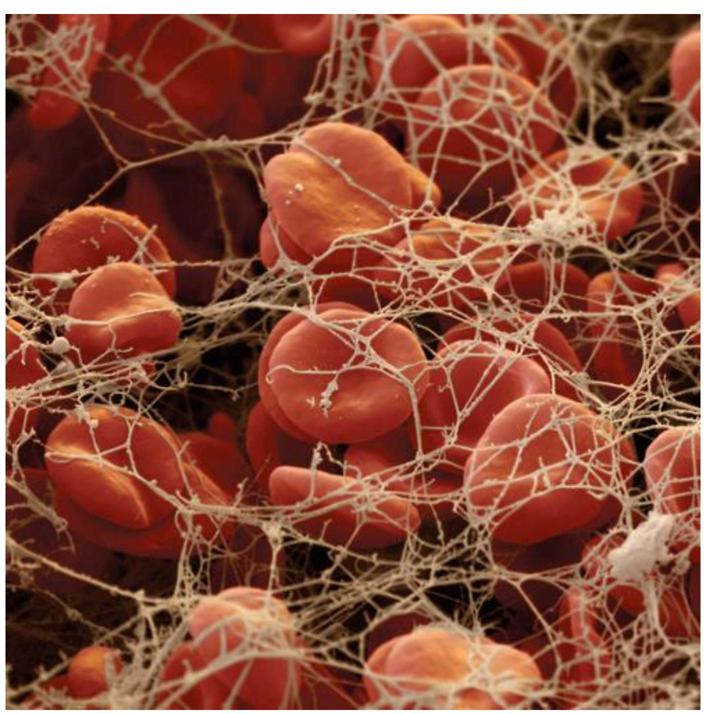
Plasma protein	**Normal concentration**	**% ratio**	**Function**
Albumin	35-50 g/l	55%	maintenance of colloind osmotic pressure, transport
Globulins	20-25 g/l	38%	Part of the immune system
Fibrinogen	2-4.5 g/l	7%	Blood coagulation

Fibrinogen, fibrin

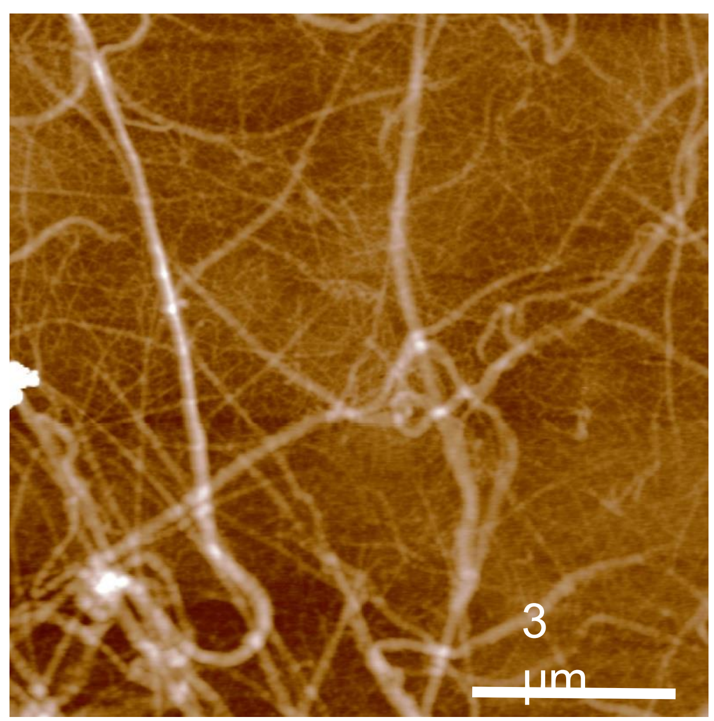


47,5 nm

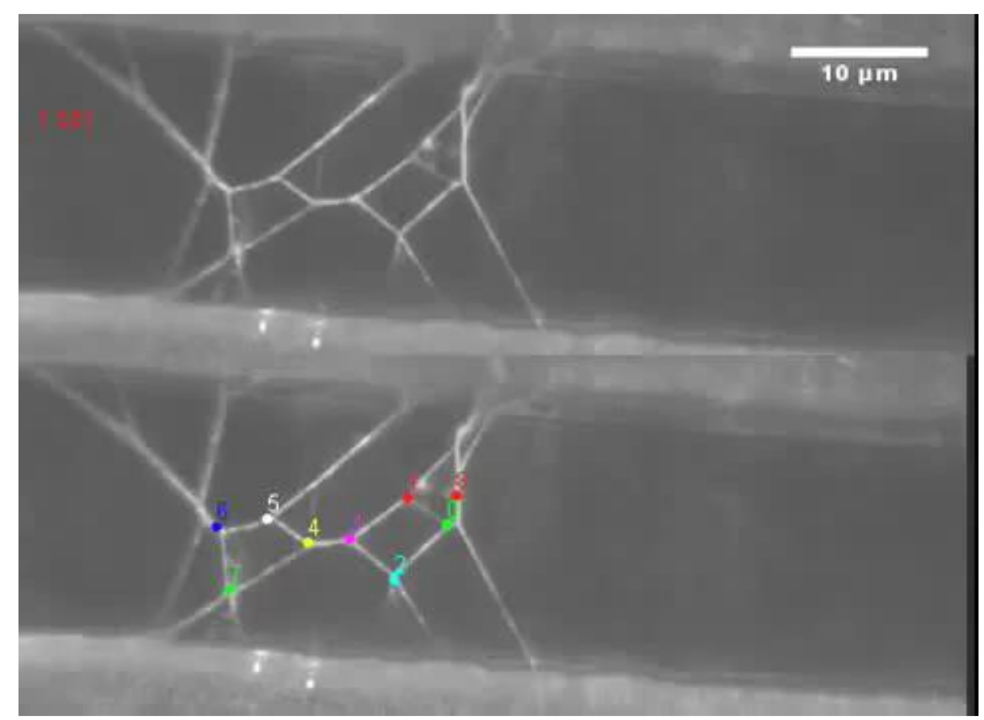
Fibrinogen:
MW = 340.000 Da
In plasma: 2-4 g/l ≈ 10 µM
Average nearest-neighbor distance ≈ 55 nm



Red blood cells in fibrin meshwork



Fibrin polymerized *in vitro* (AFM)

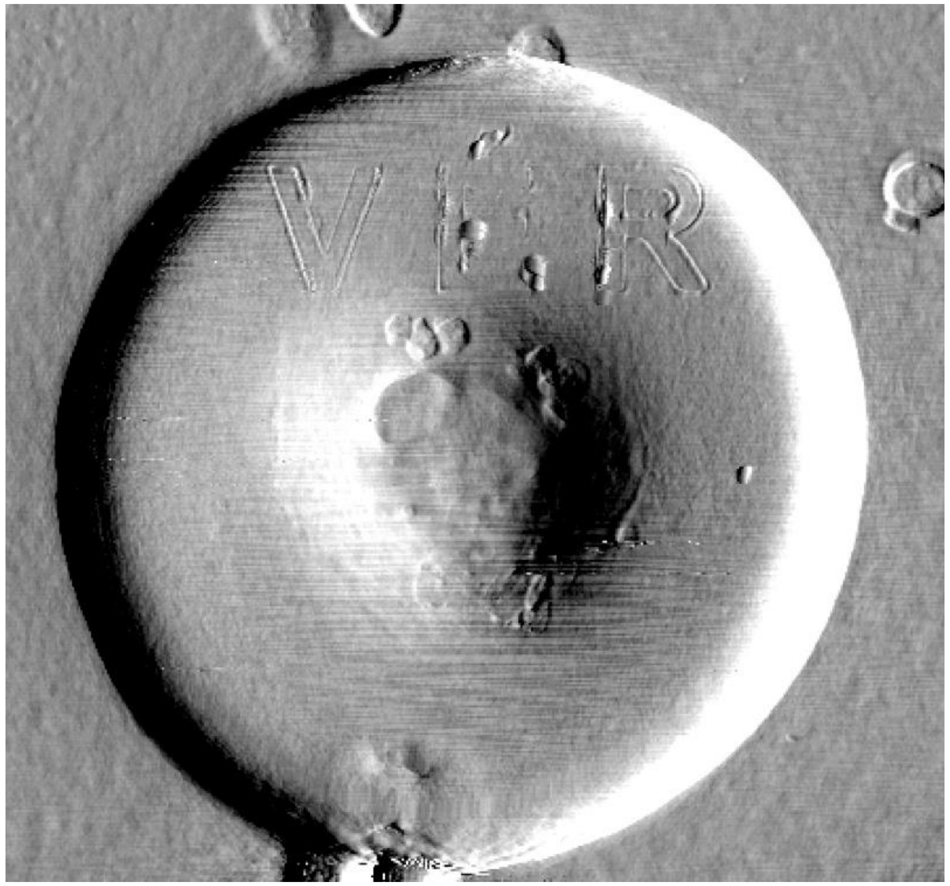


Extensibility of fluorescently labeled fibrin fibrils

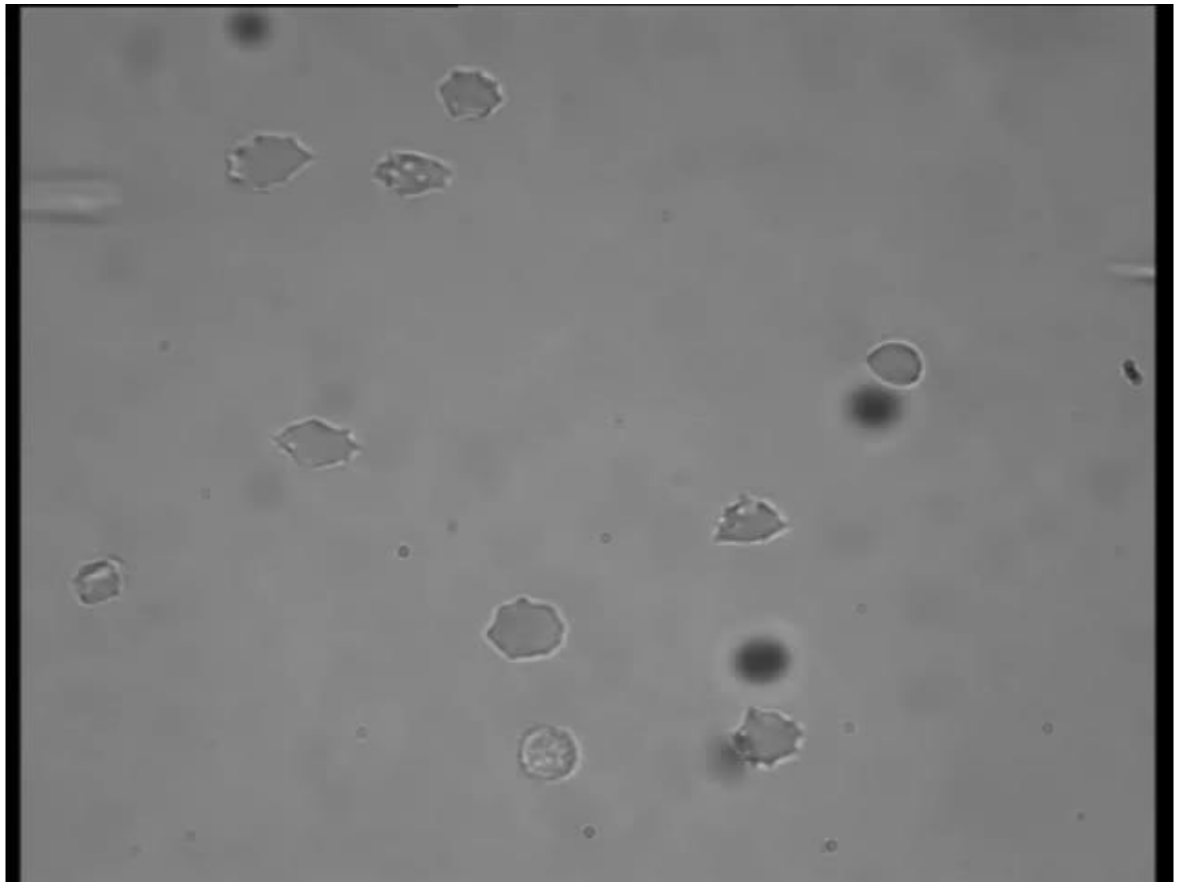
Determinants of blood viscosity

3. Plasticity of red blood cells

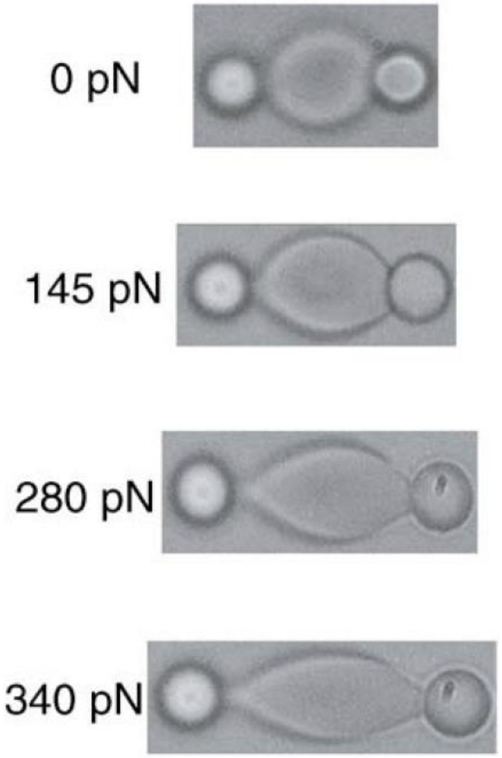
- 65% suspension of blood-cell-size particles is rock hard.
- By contrast, a 95% blood suspension if fluid, with viscosity of ~20 mPas!
- Deformation of red blood cells: droplet, parachute, arrowhead shapes.



Fixed RBC maintaining impression (AFM).
Disc-shaped cell with 7-11 μm diameter.



Deformation of red blood cells
with shear stress (fluid flow).

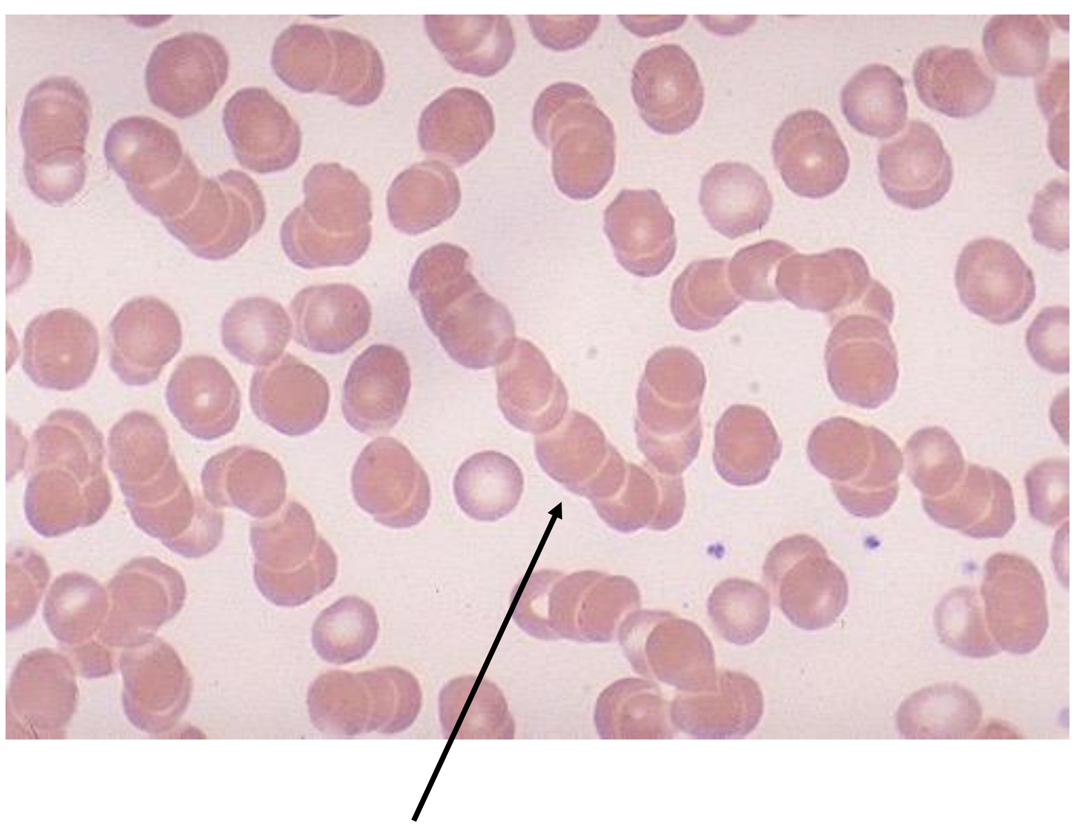


Deformation of a RBC
with optical tweezers

Determinants of blood viscosity

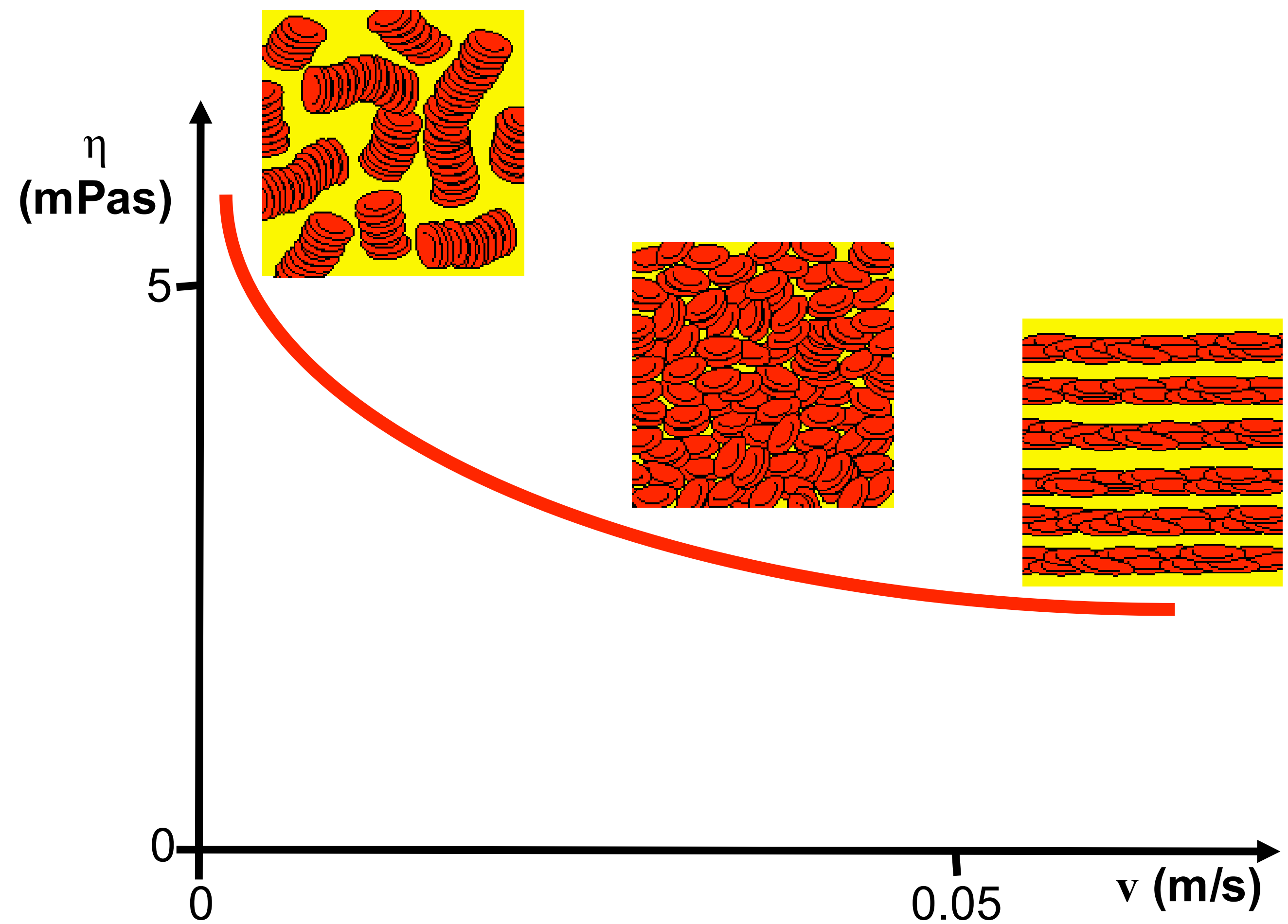
4. Aggregation of red blood cells

- Stack or roleaux formation.
- More pronounced at low flow rates



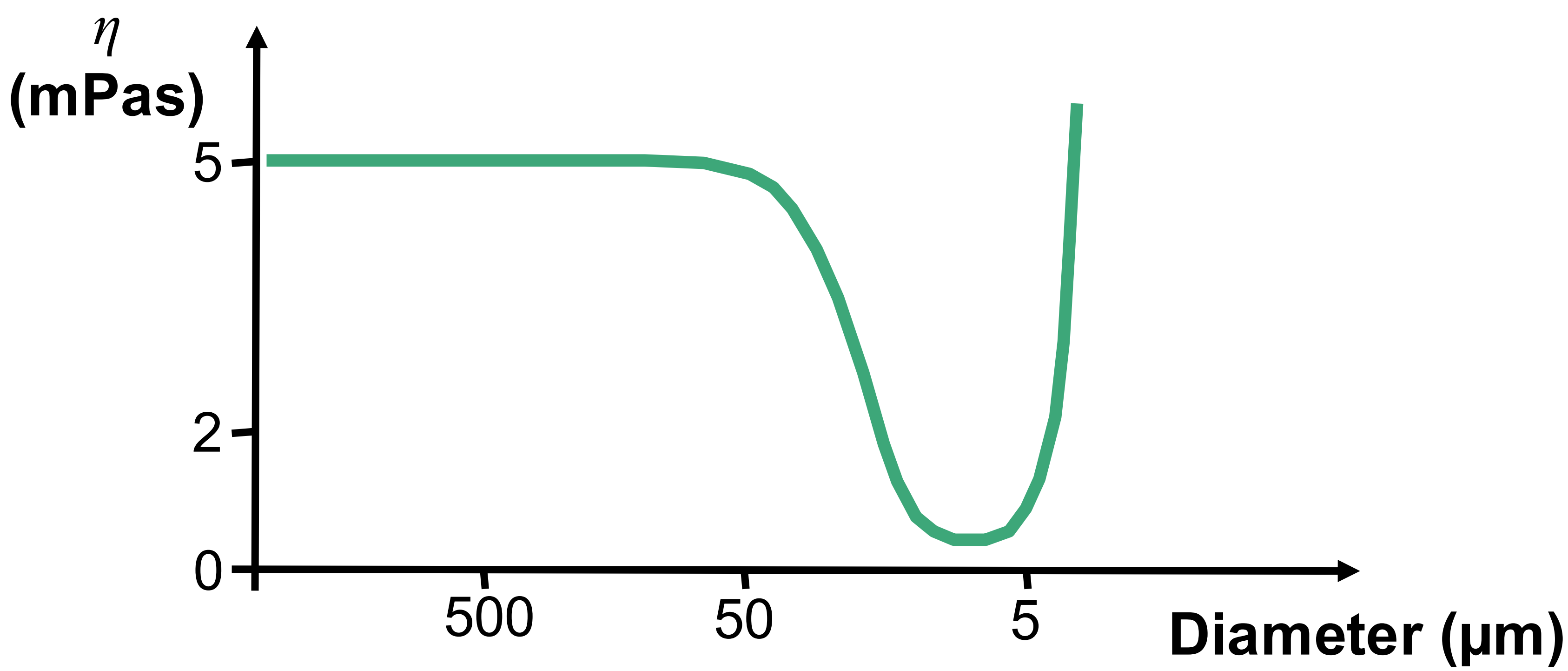
Roleaux
(stack)

5. Flow rate, velocity gradient



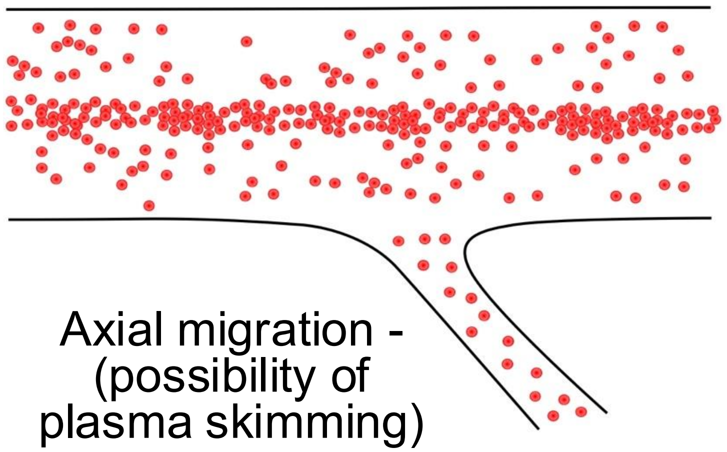
Determinants of blood viscosity

6. Blood vessel diameter



N.B.:

- With a decrease of vessel diameter, the anomalous (non-newtonian) behavior of blood becomes more pronounced.
- *Axial migration*: the red blood cells line up in the axis of the vessel (Bernoulli's law). In the axis the velocity gradient decreases, and near the vessel wall it increases. Increase in velocity gradient decreases apparent viscosity (Fåhraeus-Lindquist effect).



Axial migration - (possibility of plasma skimming)

Feedback

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