Medical Biophysics II.

6th lecture: Transport processes II.

Diffusion, Brownian motion, Osmosis

25th March 2026.

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Diffusion?

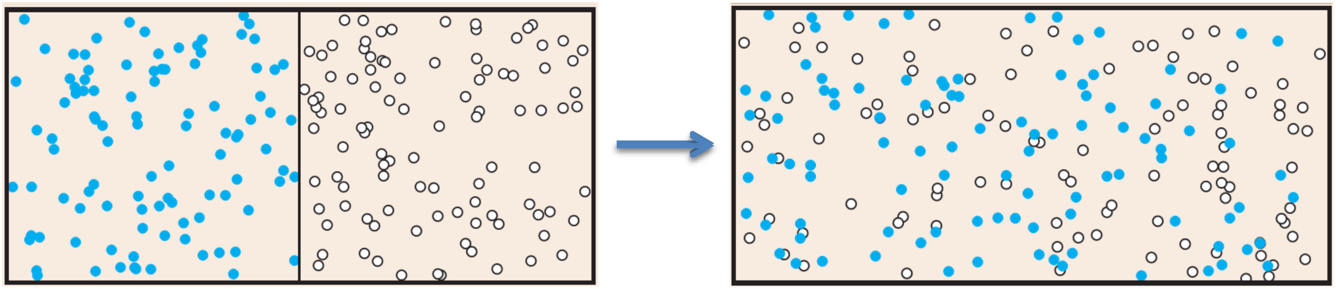
Why is it important?

- physiology: cell function – ion diffusion…
- disorders: fibrosis, oedema, vasculitis, ascites…
- diagnostics: DWI MRI…
- therapy: dialysis, physiological saline….
- drug delivery: transdermal (liposomal), inhaled…

…….

Diffusion?

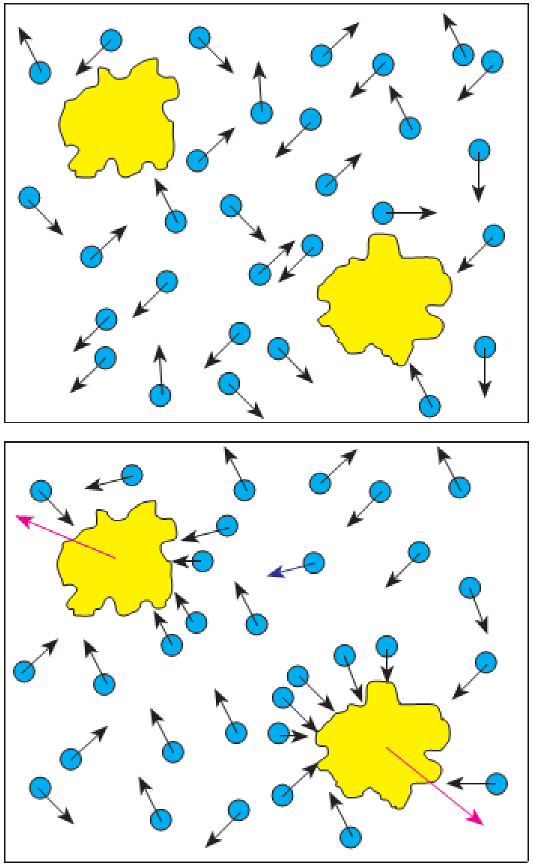
What is diffusion? A **net material flow** as the macroscopic consequence of the change in particle distribution due to the **random thermal motion** of microscopic particles.



Note: For us, the material transport of „A” in „B” is interested, so we disregard the so-called self-diffusion.

Brownian motion

The „random walk” of a larger particle is the result of random collisions with microscopic particles undergoing thermal motion.



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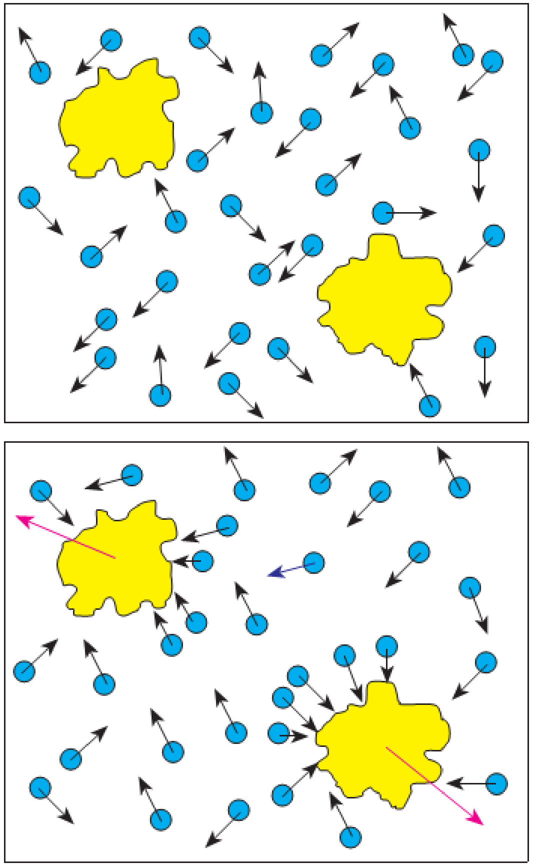

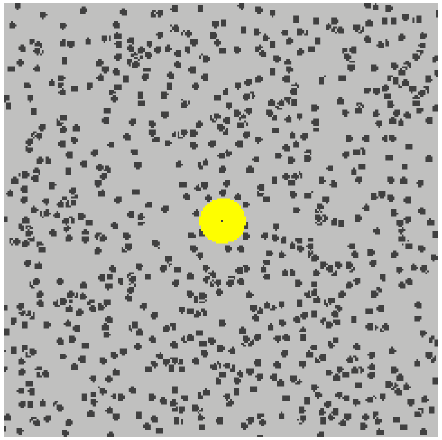

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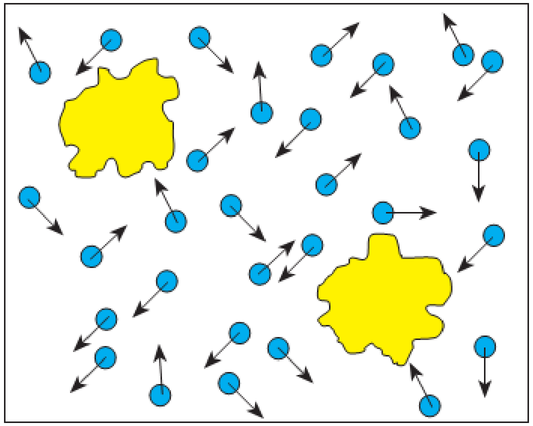
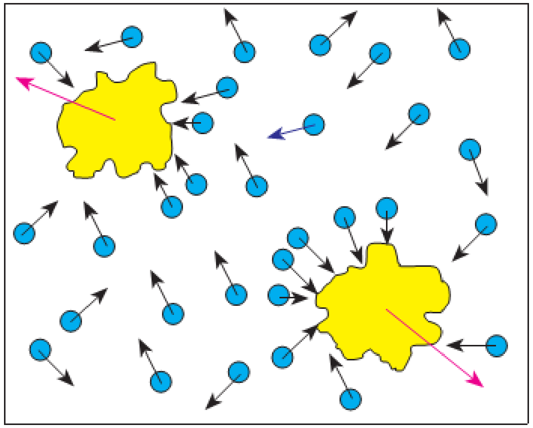
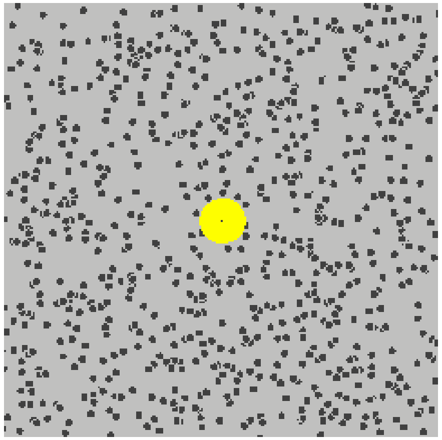
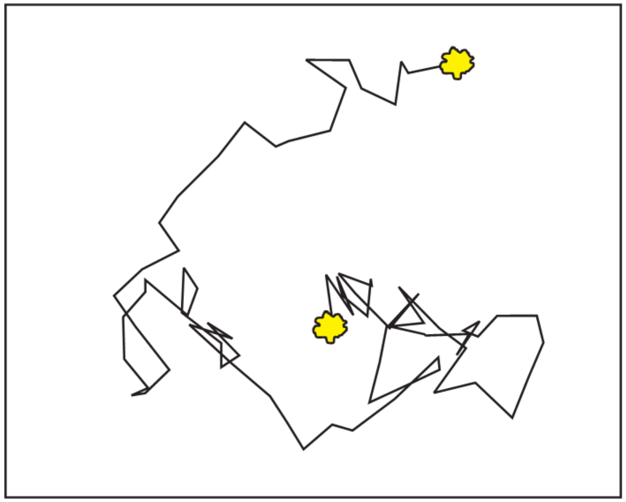


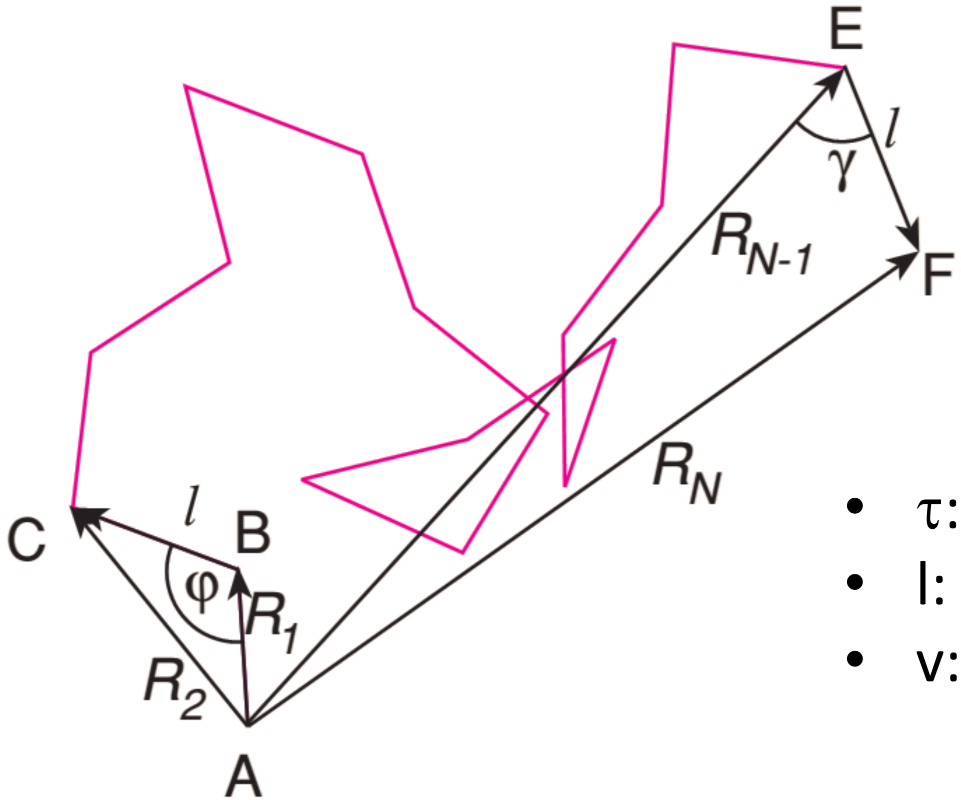






- τ: mean time between collisions
- l: mean free path
- v: mean speed of particles

How far reaches a $\overline{\text{particle}}$?



Simplification:
diffusion 1 plane

- τ: mean time between collisions
- l: mean free path
- v: mean speed of particles

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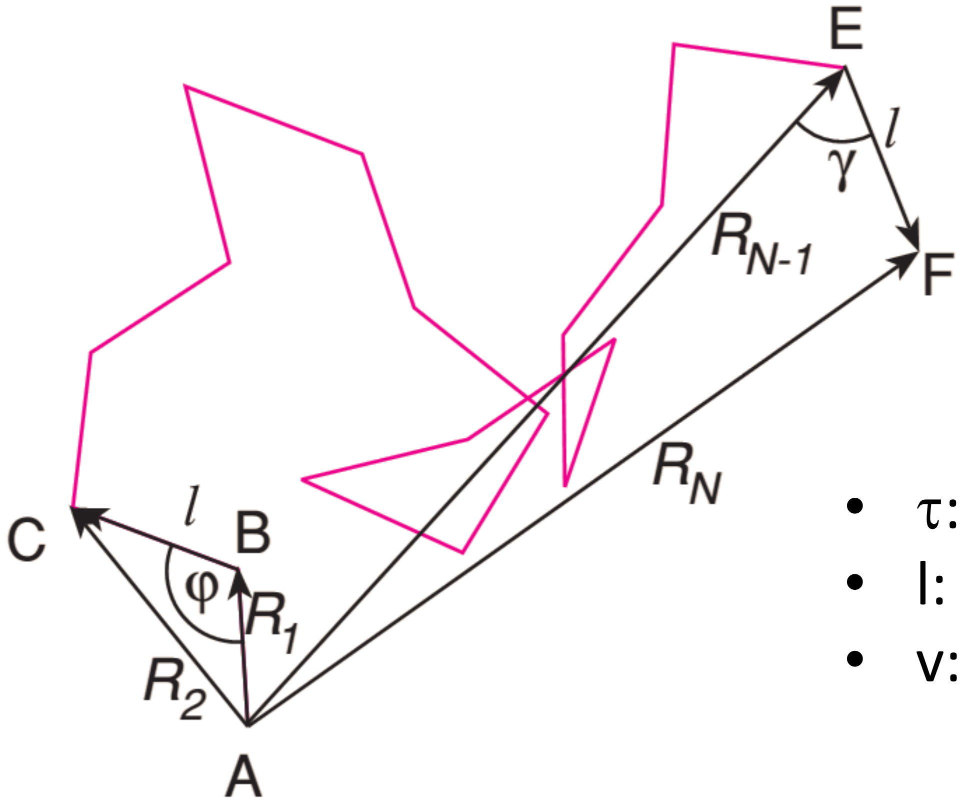

- τ: mean time between collisions
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One particle: $R_2{}^2 = R_1{}^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi$

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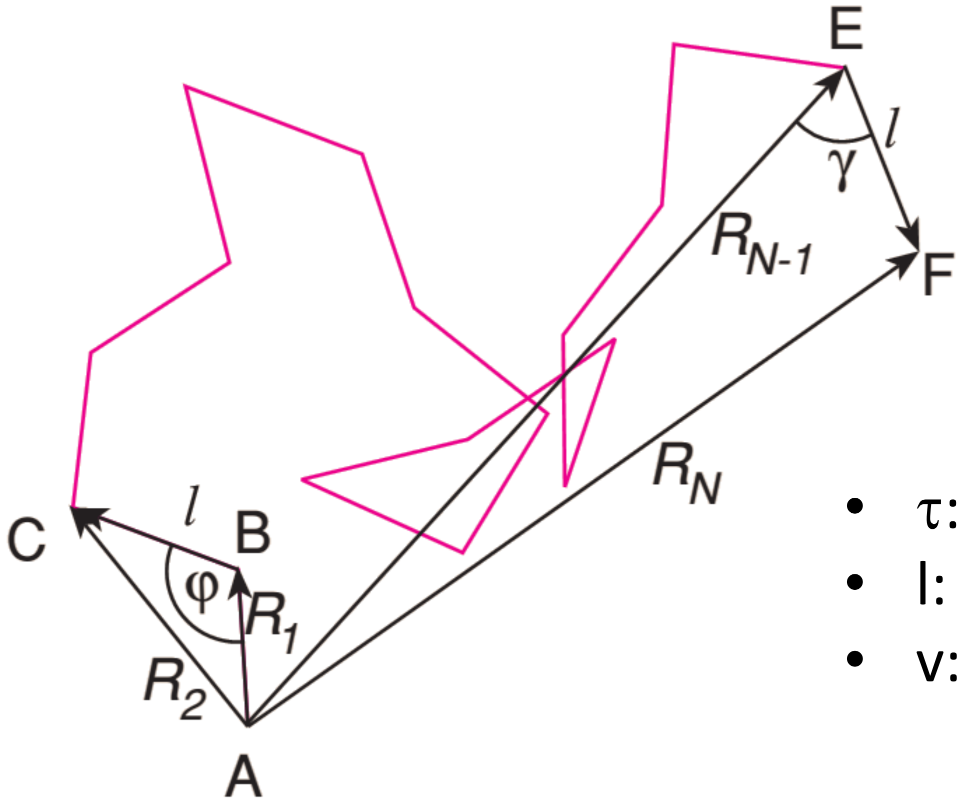

- τ: mean time between collisions
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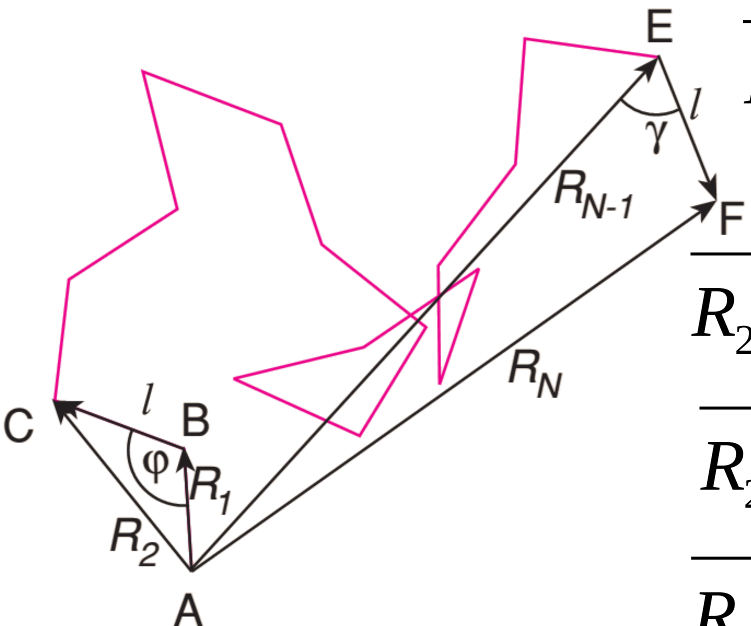
One particle: $R_2^{\,2} = R_1^{\,2} + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi$

A „mean” particle:
(mean of *n* particles): $\overline{R_2^{\,2}} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(R_1^{\,2} + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi_i \right)$

How far reaches a particle?



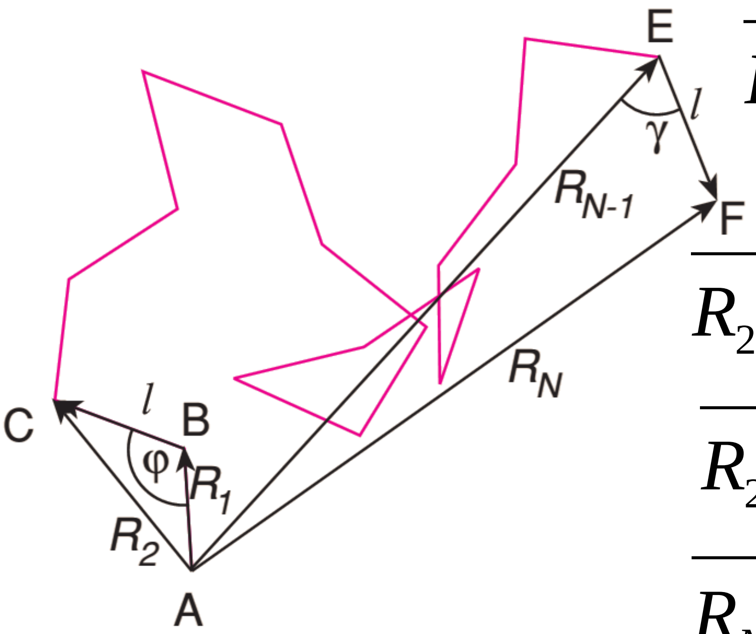
$$\overline{R_2^{\,2}} = \frac{1}{n}\cdot\sum_{i=1}^{n}\left(R_1^{\,2} + l^2 - 2\cdot R_1\cdot l\cdot\cos\varphi_i\right)$$

$$\overline{R_2^{\,2}} = \frac{1}{n}\cdot n\cdot\left(R_1^{\,2} + l^2\right) - \frac{2}{n}\cdot R_1\cdot l\cdot\sum_{i=1}^{n}\left(\cos\varphi_i\right)$$

$$\overline{R_2^{\,2}} = R_1^{\,2} + l^2 = l^2 + l^2 = 2\cdot l^2$$

$$\overline{R_N^{\,2}} = N\cdot l^2$$

How far reaches a $\overline{\text{particle}}$?



$$\overline{R_2^{\;2}} = \frac{1}{n}\cdot\sum_{i=1}^{n}\left(R_1^{\;2} + l^2 - 2\cdot R_1\cdot l\cdot\cos\varphi_i\right)$$

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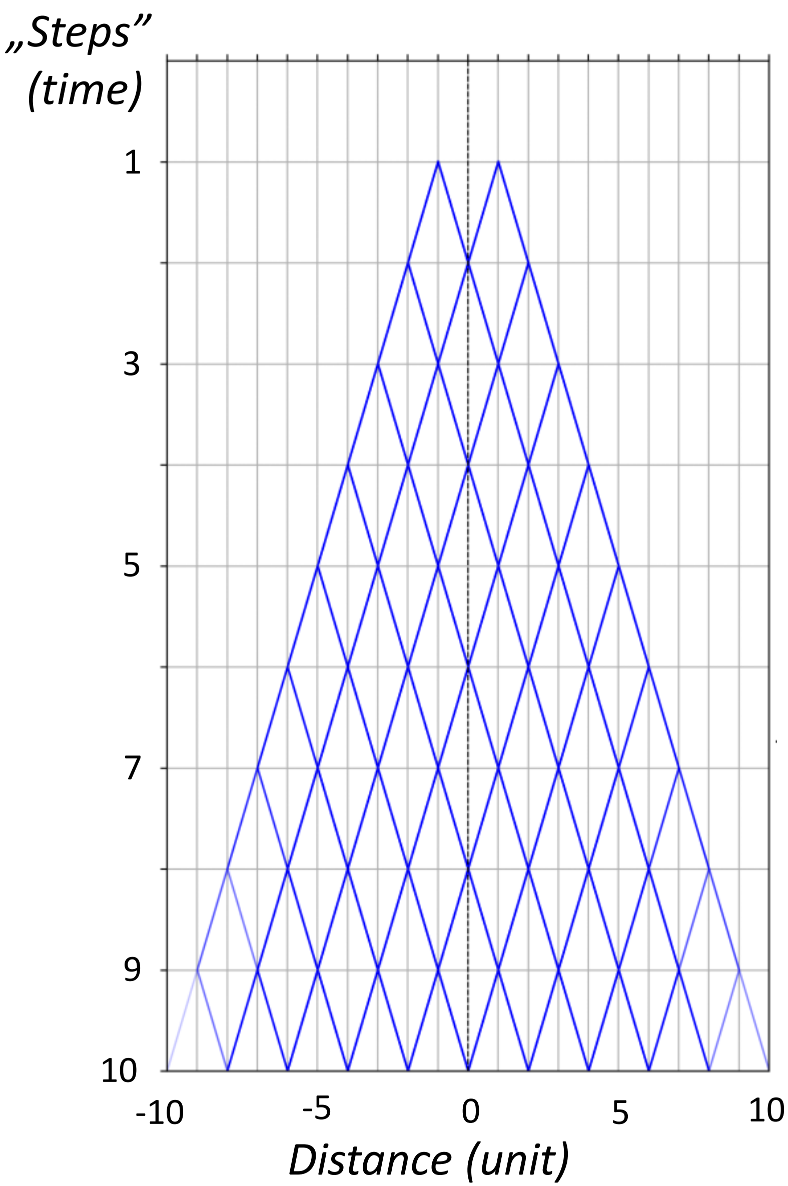
$$\overline{R_N^{\;2}} = N\cdot l^2$$

$$\overline{R_t} = \sqrt{N\cdot l^2} = \sqrt{\frac{t}{\tau}\cdot l\cdot l} = \sqrt{t\cdot v\cdot l} = \sqrt{3\cdot D\cdot t}$$

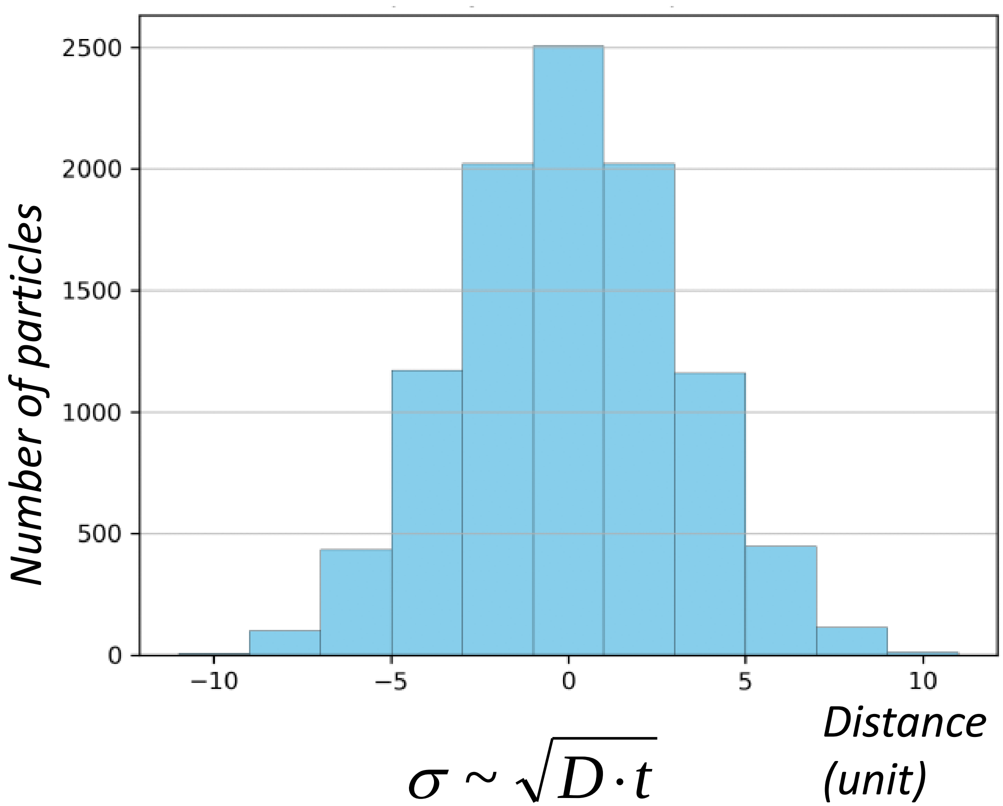
$$\frac{v\cdot l}{3} = D$$

Hova reaches the particles?



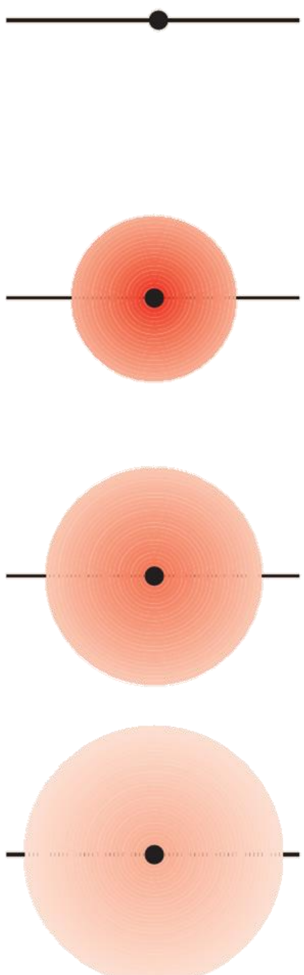
Let’s track 10,000 particles in 1D: they can move „either left or right”
follow 10 steps (time units)

Distribution after 10 steps

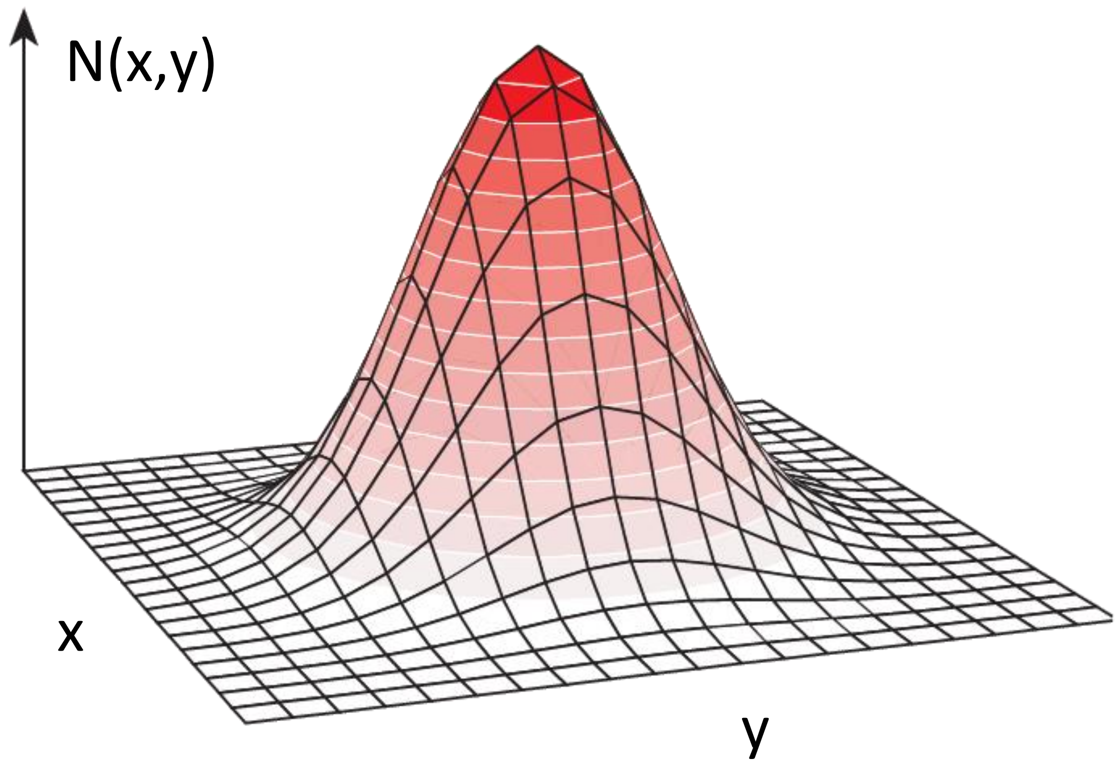


$$\sigma \sim \sqrt{D \cdot t}$$

Distribution of particles in 2D



We will do this experiment during the practice



$$\sigma \sim \sqrt{D \cdot t}$$

„Result” of thermal motion - flow

Particle flow rate: $$I_N = \frac{\Delta N}{\Delta t}; \left[\frac{1}{s}\right]$$

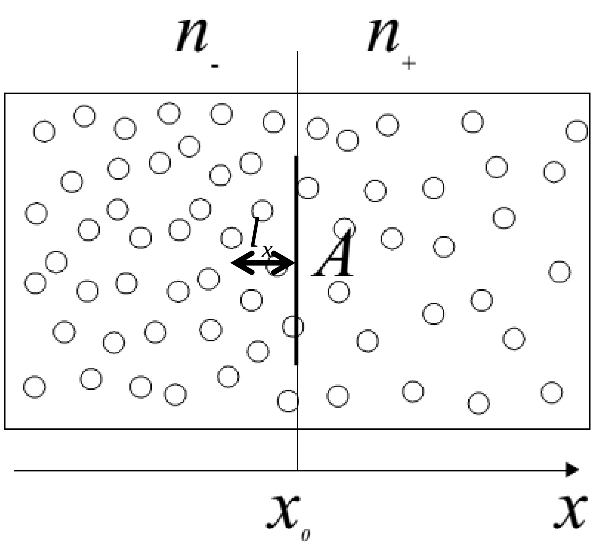
Particle flow density (flux): $$J_N = \frac{\Delta I_N}{\Delta A}; \left[\frac{1}{m^2 \cdot s}\right]$$

For a lot of particles:

Matter flow rate: $$I_\nu = \frac{\Delta \nu}{\Delta t}; \left[\frac{mol}{s}\right]$$

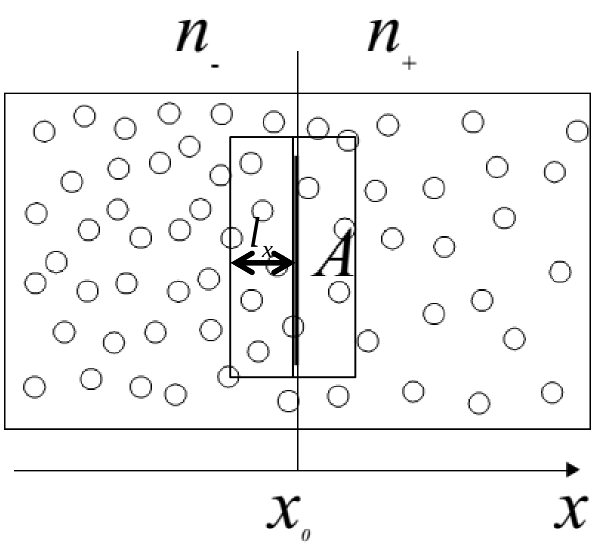
Matter flow density (flux): $$J_\nu = \frac{\Delta I_\nu}{\Delta A}; \left[\frac{mol}{m^2 \cdot s}\right]$$

Fick’s first law



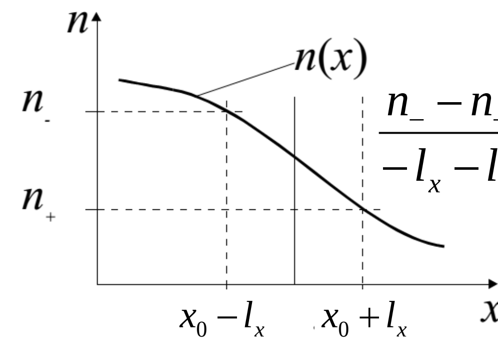
What is the net material flow, if the concentration differs?

Fick's first law



What is the net material flow, if the concentration differs?

Could be derived from thermal motion – see the textbook



$$\frac{n_- - n_+}{-l_x - l_x} = \frac{\Delta n}{\Delta x}$$

$$\Delta N = N_- - N_+ = \frac{1}{2}\cdot V_t\cdot(n_- - n_+) = \frac{1}{2}\cdot v_x\cdot\Delta t\cdot \overbrace{A\cdot(n_- - n_+)}^{l_x}$$

$$\Delta N = \frac{1}{2}\cdot v_x\cdot\Delta t\cdot A\cdot 2\cdot l_x\cdot -\frac{\Delta n}{\Delta x}$$

$$v_x\cdot l = D$$

$$J_{Nx} = \frac{1}{2}\cdot v_x\cdot 2\cdot l_x\cdot -\frac{\Delta n}{\Delta x} = -D\cdot\frac{\Delta n}{\Delta x}$$

$$J_v = -D\cdot\frac{\Delta c}{\Delta x}$$

But Δc is not the real „driving force"!
But it is thermal motion

Diffusion coefficient

D gives the amount of matter diffused across a unit area in a unit time in a case of unit concentration drop (gradient).

$$D = \frac{v \cdot l}{3}; \left[\frac{m^2}{s}\right]$$

$$D = u \cdot k \cdot T$$

Einstein-Stokes
(spheres)

$$D = \frac{k \cdot T}{6 \cdot \pi \cdot \eta \cdot r}$$

BUT!
Not directly proportional with T!

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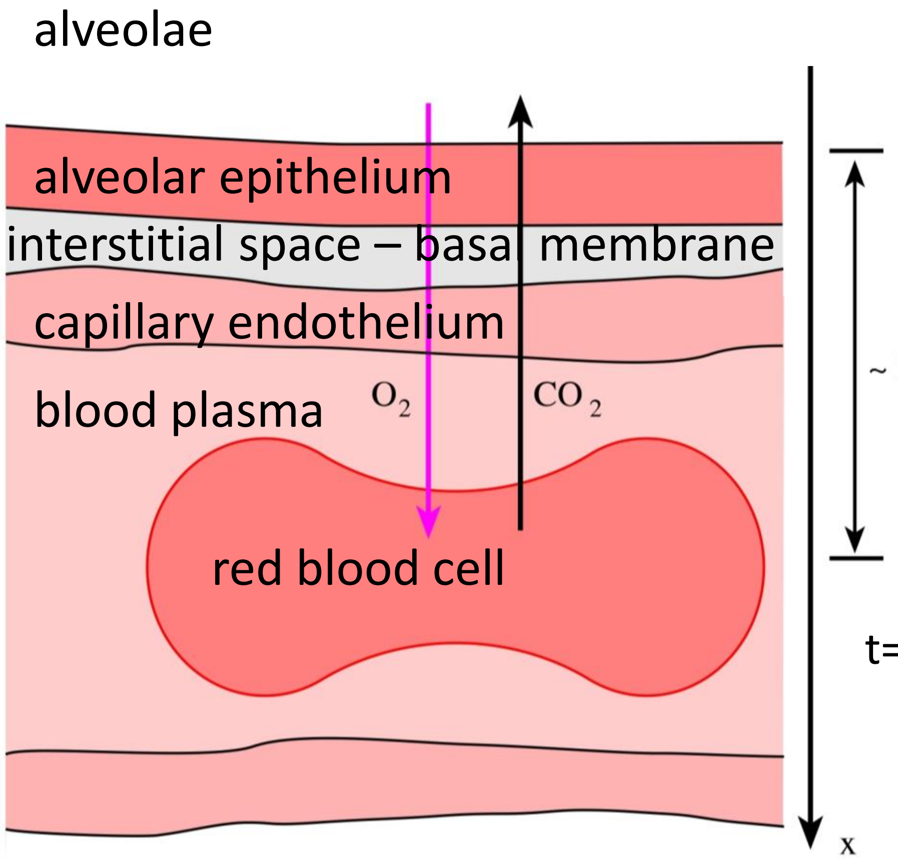


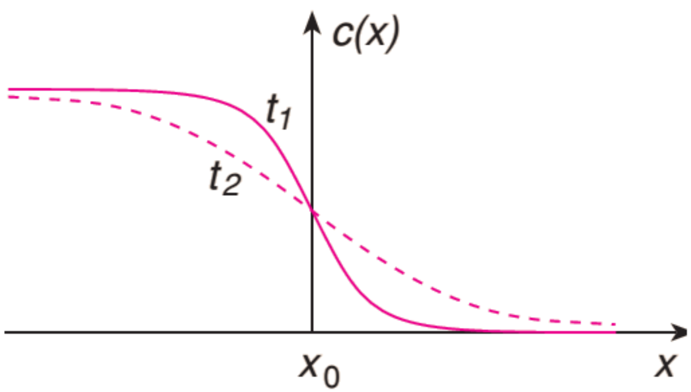
$$1{,}9 \cdot 10^{-9} \frac{m^2}{s}$$

$t \approx 500 \mu s$

t=? 1m distance

Fick’s second law

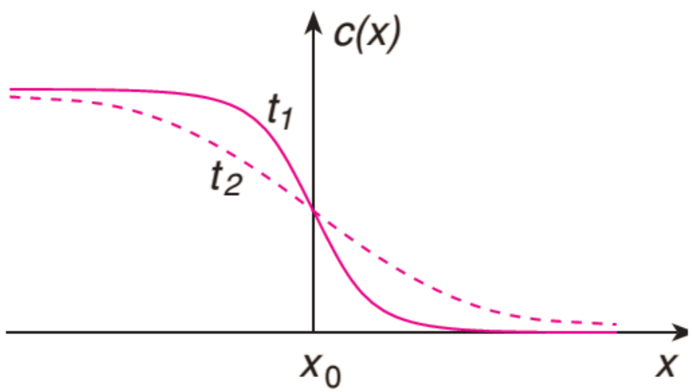
Fick II: if concentration drop is not constant
- changing in time



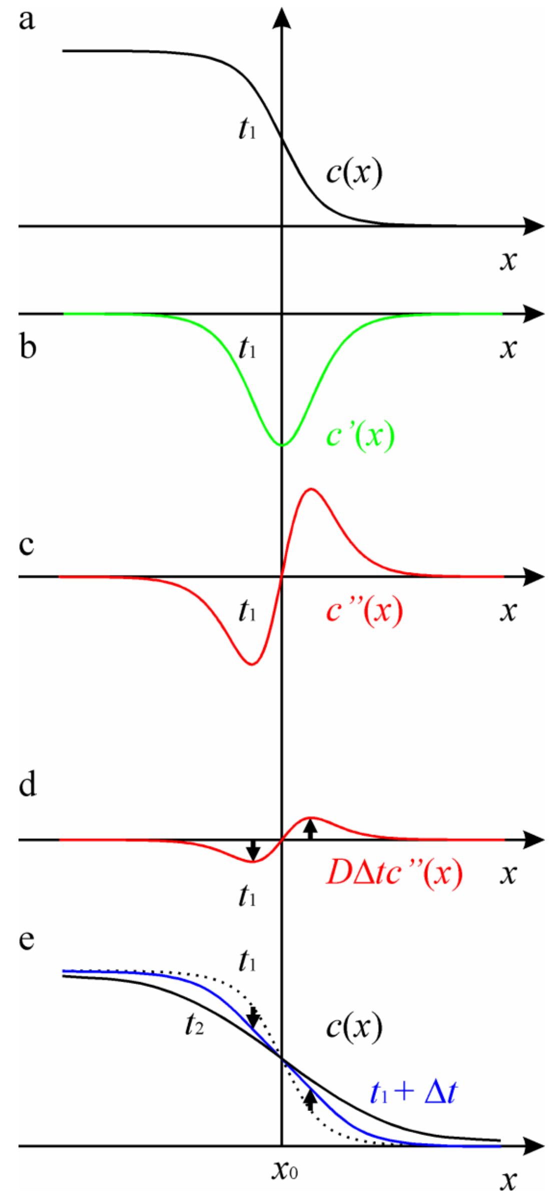
$$c(t+\Delta t)=c(t)+D\cdot\Delta t\cdot\frac{\Delta\left(\frac{\Delta c}{\Delta x}\right)}{\Delta x}$$

Fick's second law

Fick II: if concentration drop is not constant
- changing in time

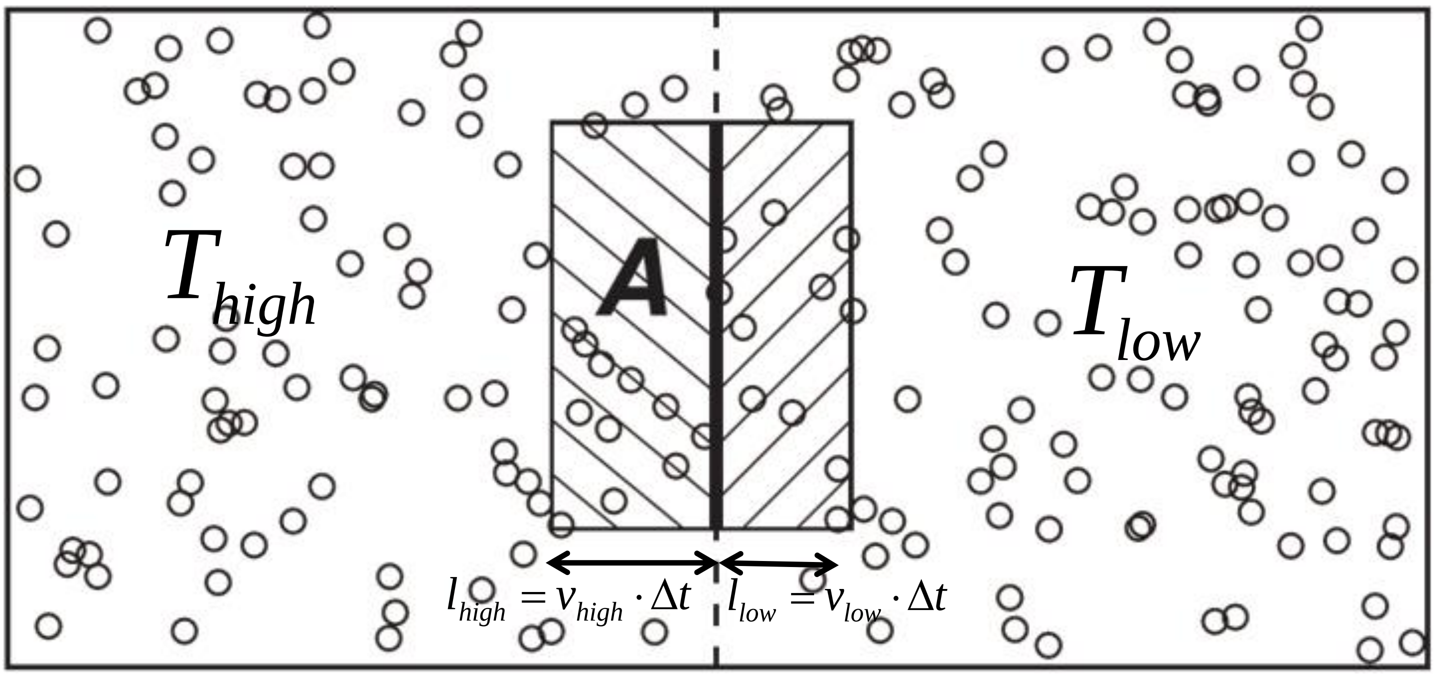


$$c(t+\Delta t) = c(t) + D \cdot \Delta t \cdot \frac{\Delta\left(\frac{\Delta c}{\Delta x}\right)}{\Delta x}$$



Thermodiffusion

What is the net material flow,
If the temperature differ (but the concentration is the same)?



$T_{high} > T_{low}$

$v = \sqrt{3 \cdot \frac{k \cdot T}{m}}$

$$J_v = -L_T \cdot \frac{\Delta T}{\Delta x}$$ (Ludwig-Soret effect)

Generalization

Onsager-relation: $J_{ext.} = L_{cond} * X_{int\_grad}$

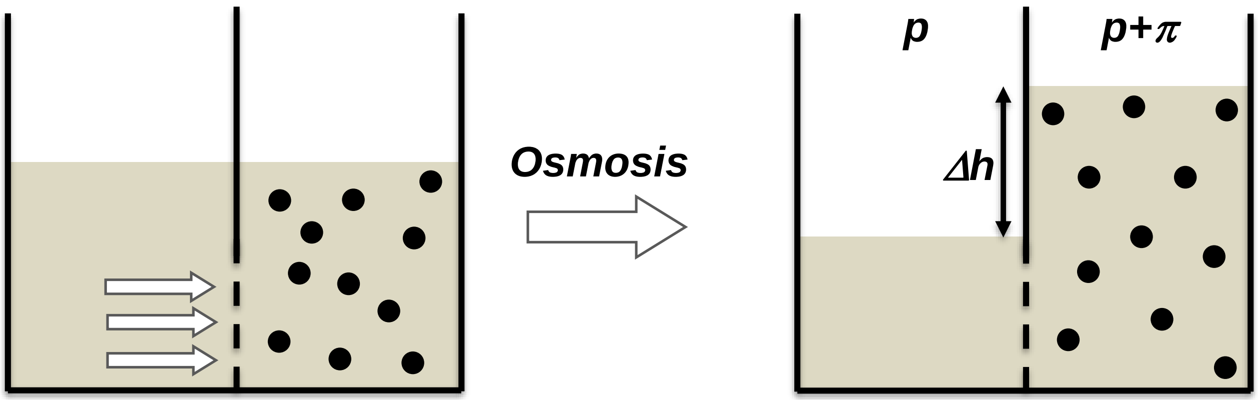
J_{ext}: flow density of an extensive quantity (eg. J_{matter})

X_{int_grad}: gradient of an intensive quantity (eg. $\frac{\Delta c}{\Delta x}$)

L_{cond}: conductivity coefficient (eg. D)

Osmosis

One-way diffusion of the SOLVENT. (permeable membrane only for *water*)



difference in the concentration of the SOLVENT

Hydrostatic pressure difference (osmotic pressure)

$$p_{osm} = \pi = c_{solute} \cdot R \cdot T$$ (Van ’t Hoff law)

Osmotic concentration (equivalent osmotic pressure, „ozmolarity”, „ozmolality”):
The concentration of a solution that keeps balance with a heterogeneous solution.
Derived units: *mOsm(/L)*, mmol/L, mmol/kg

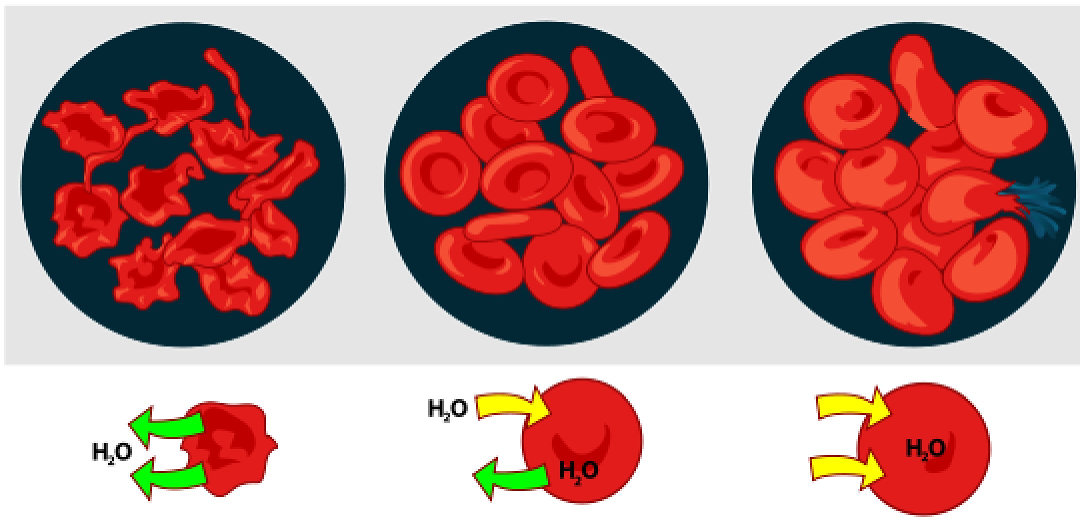
More precisely: correction needed for dissociation

Medical practice

Tonicity: „effective” osmolarity

membrane: *given cell membrane* (permeable not only for water)

non-permeable ions/molecules are important for tonicity



Hypertonic **Isotonic** **hypotonic**

„Isosmotic”, „Physiological”, „isotonic”, „normal” solutions:

Physiological/Normal/Isotonic saline: 0,9% (w/v) NaCl (isotonic)

d5W: 5% (w/v) glucose (hypotonic)

Ringer, Ringer’s lactate (isotonic)

Isosmotic not equal Isotonic!

Osmotic concentration of the blood plasma: about 300mOsm/L