# Geometric Transformations

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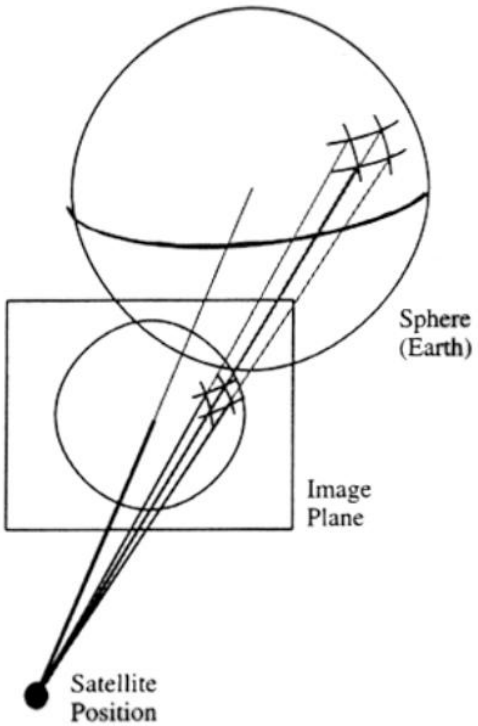
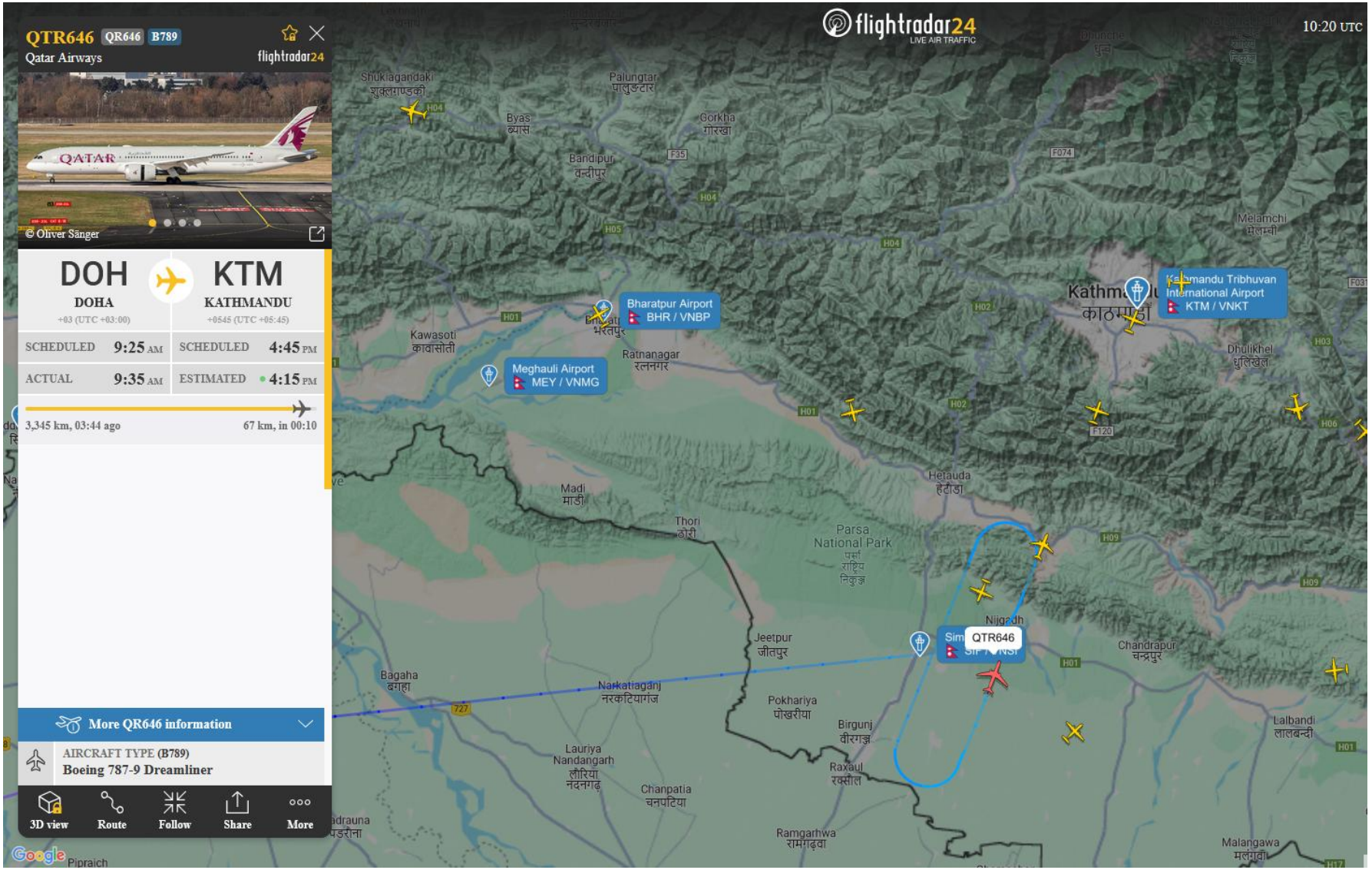
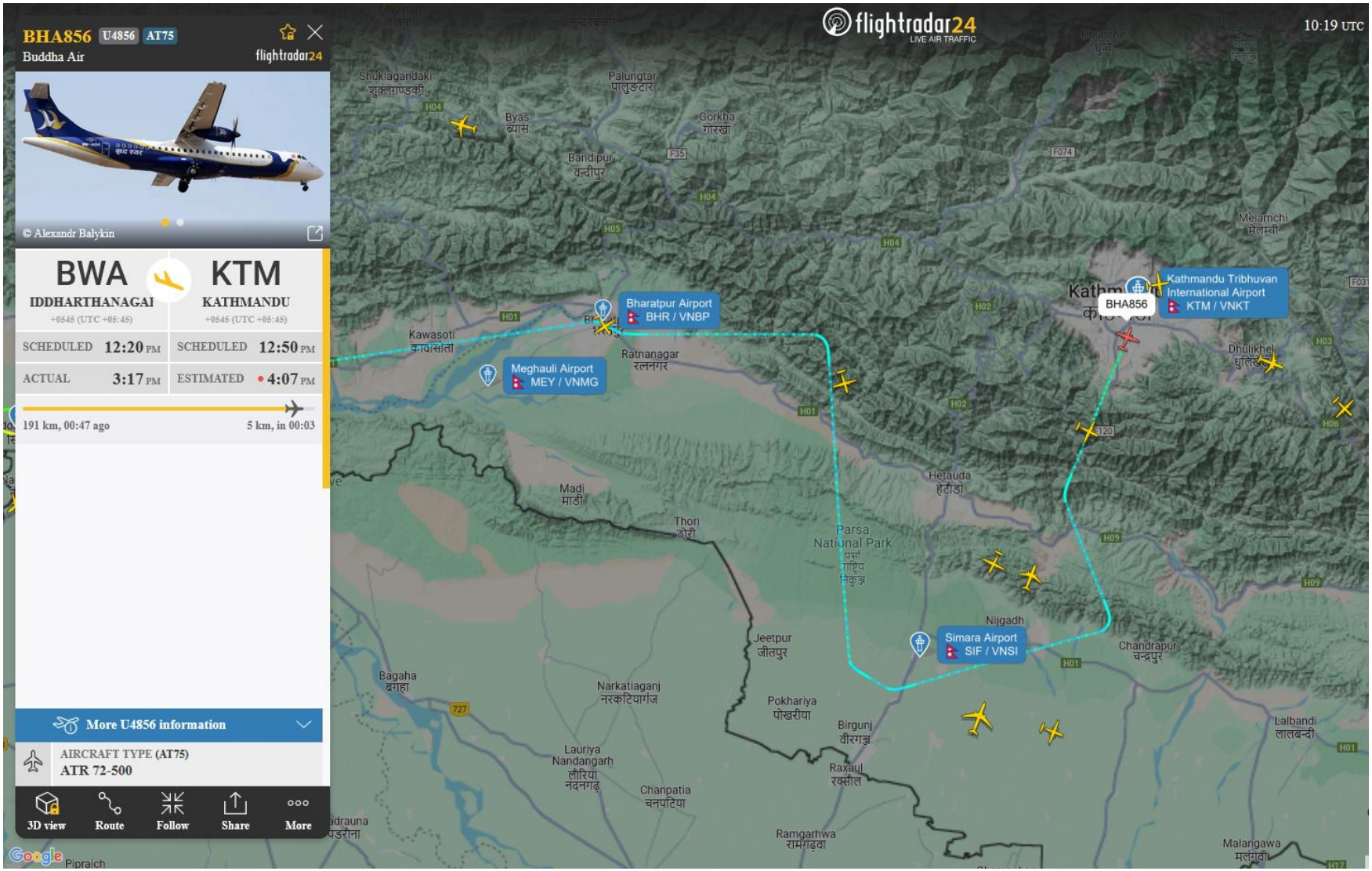
Let us consider that a traditional digital image is two-dimensional, even if it is made of a three-dimensional object. In this case, we only see the projection of the object, which can cause difficulties in evaluating the image. A typical case is a satellite image taken of the Earth's surface. In fact, digital image analysis was started in the 1970s with the beginning of the space program, at the Jet Propulsion Laboratory in California, to solve such problems.




A two-dimensional image of a spherical body (e.g. Earth) taken from a distant point (e.g. satellite) is merely a projection.


On 4 March 2015, the crew of a Turkish Airlines A330 continued an automatic **non precision RNAV (GPS) approach** below the prescribed minimum descent altitude without having obtained any element of visual reference and when this was acquired a few seconds before the attempted landing, **the aircraft was not aligned with the runway centerline** and during a 2.7g low-pitch landing, the left main gear touched down on the grass. The aircraft then left the runway completely before stopping with a collapsed nose gear and sufficient damage to be assessed a hull loss. None of 235 occupants sustained serious injury.


QTR646 QR646 B789
Qatar Airways
flightradar24
© Oliver Sänger
DOH
DOHA
+03 (UTC +03:00)
KTM
KATHMANDU
+0545 (UTC +05:45)
SCHEDULED 9:25 AM
SCHEDULED 4:45 PM
ACTUAL 9:35 AM
ESTIMATED 4:15 PM
3,345 km, 03:44 ago
67 km, in 00:10
More QR646 information
AIRCRAFT TYPE (B789)
Boeing 787-9 Dreamliner
3D view
Route
Follow
Share
More
flightradar24
LIVE AIR TRAFFIC
10:20 UTC
Bharatpur Airport
BHR / VNBP
Meghauli Airport
MEY / VNMG
Kathmandu Tribhuvan International Airport
KTM / VNKT
QTR646
Shuklagandaki
Palungtar
Byas
Gorkha
Bandipur
Melamchi
Kawasoti
Ratnanagar
Dhulikhel
Hetauda
Madi
Thori
Parsa National Park
Jeetpur
Chandrapur
Bagaha
Narkatiaganj
Pokhariya
Birgunj
Lalbandi
Lauriya Nandangarh
Raxaul
Chanpatia
Ramgarhwa
Malangawa
Pipraich


BHA856 U4856 AT75
Buddha Air
flightradar24
© Alexandr Balykin
BWA
IDDHARTHANAGAI
+0545 (UTC +05:45)
KTM
KATHMANDU
+0545 (UTC +05:45)
SCHEDULED 12:20 PM
SCHEDULED 12:50 PM
ACTUAL 3:17 PM
ESTIMATED 4:07 PM
191 km, 00:47 ago
5 km, in 00:03
More U4856 information
AIRCRAFT TYPE (AT75)
ATR 72-500
3D view
Route
Follow
Share
More
flightradar24
LIVE AIR TRAFFIC
10:19 UTC
Bharatpur Airport
BHR / VNBP
Meghauli Airport
MEY / VNMG
Kathmandu Tribhuvan
International Airport
KTM / VNKT
BHA856
Simara Airport
SIF / VNSI
Shuklagandaki
Palungtar
Byas
Gorkha
Bandipur
Melamchi
Kawasoti
Ratnanagar
Dhulikhel
Madi
Thori
Parsa
National Park
Hetauda
Nijgadh
Jeetpur
Chandrapur
Bagaha
Narkatiaganj
Pokhariya
Birgunj
Lauriya
Nandangarh
Raxaul
Chanpatia
Ramgarhwa
Lalbandi
Malangawa
Pipraich
Google

# VNKT/KTM
TRIBHUVAN INTL

JEPPESEN

6 FEB 15 (13-1)

# KATHMANDU, NEPAL
MISSED APCH CLIMB GRAD MIN 6.0% VOR DME Rwy 02

| *ATIS | *KATHMANDU Approach | *KATHMANDU Tower | *Ground |
|---|---|---|---|
| 127.0 | 120.6 125.1 | 118.1 | 121.9 |

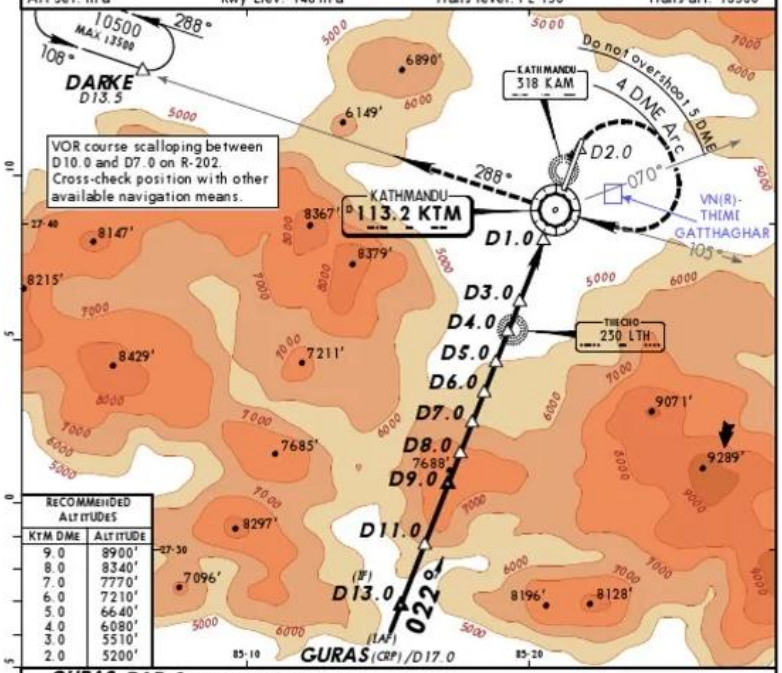
| VOR | Final Apch Crs | Procedure Alt | MDA(H) | Apt Elev 4395' |
|---|---|---|---|---|
| KTM 113.2 | 022° | D9.0 8900' (4582') | 4950' (632') | RWY 4318' |

MISSED APCH: Climb STRAIGHT AHEAD on R-022 to D2.0, turn RIGHT (MAX 185 KT) onto 4 DME Arc (do not exceed 5 DME). Crossing R-070 turn RIGHT onto R-105 inbound VOR at or above 7500'. Follow R-288 to DARKE at or above 10500'.

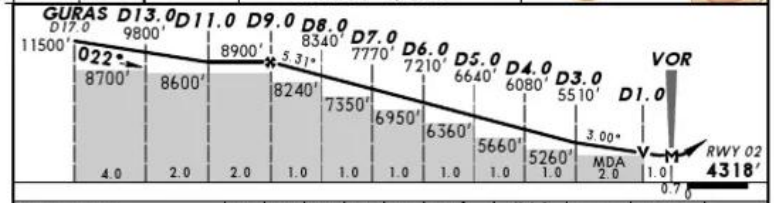
MSA KTM VOR
135° 21,100' 255° 10,500' 050° 11,600'

BRIEFING STRIP

Alt Set: hPa | Rwy Elev: 148 hPa | Trans level: FL 150 | Trans alt: 13500'




| RECOMMENDED ALTITUDES | |
|---|---|
| KTM DME | ALTITUDE |
| 9.0 | 8900' |
| 8.0 | 8340' |
| 7.0 | 7770' |
| 6.0 | 7210' |
| 5.0 | 6640' |
| 4.0 | 6080' |
| 3.0 | 5510' |
| 2.0 | 5200' |




| Gnd speed-Kts | | 70 | 90 | 100 | 120 | 140 | 160 |
|---|---|---|---|---|---|---|---|
| Descent Angle D9.0-D3.0 | 5.31° | 659 | 847 | 941 | 1129 | 1318 | 1506 |
| Descent Angle D3.0-THR | 3.00° | 372 | 478 | 531 | 637 | 743 | 849 |
| MAP at VOR | | | | | | | |

HIALS PAPI D2.0 MAX 185 KT RT 4 DME Arc

STRAIGHT-IN LANDING RWY 02
Missed apch climb gradient min 6.0%
MDA(H) 4950' (632')

| | ALS out | CIRCLE-TO-LAND 1 Max Kts | DAY MDA(H) | DAY VIS | NIGHT |
|---|---|---|---|---|---|
| A | 1600m | 100 | 4950' (632') | 2400m | NOT AUTHORIZED |
| B | 2400m | 135 | | | |
| C | 2800m | 180 | 5780' (1462') | 3700m | |
| D | 3200m | D | NOT AUTHORIZED | | |

PANS OPS

1 Circling height based on rwy 02 thresh elev of 4318'.

CHANGES: None.



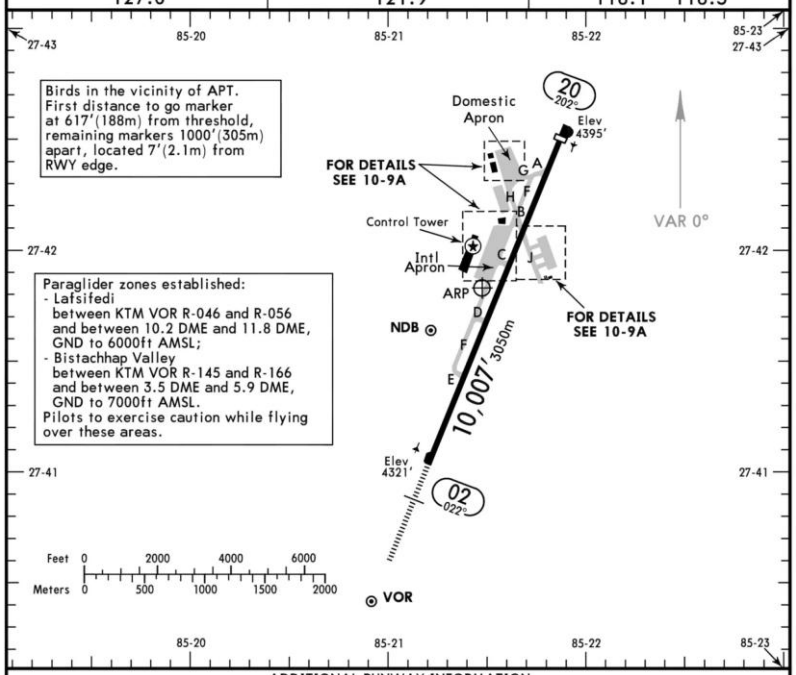
# VNKT/KTM
Apt Elev 4395'
N27 41.8 E085 21.5

JEPPESEN

17 JAN 20 (10-9) Eff 30 Jan

# KATHMANDU, NEPAL
TRIBHUVAN INTL

| *ATIS | *KATHMANDU Ground | *Tower |
|---|---|---|
| 127.0 | 121.9 | 118.1 118.5 |




ADDITIONAL RUNWAY INFORMATION

| RWY | | USABLE LENGTHS LANDING BEYOND Threshold | Glide Slope | TAKE-OFF | WIDTH |
|---|---|---|---|---|---|
| 02 | HIRL HIALS PAPI-L (angle 3.0°) REIL | | | | 150' |
| 20 | HIRL ❶PAPI-L (angle 3.0°) REIL | 9603' 2927m | | | 46m |

❶ PAPI restricted to be usable within 3 NM from rwy thresh with clearly visible.

TAKE-OFF

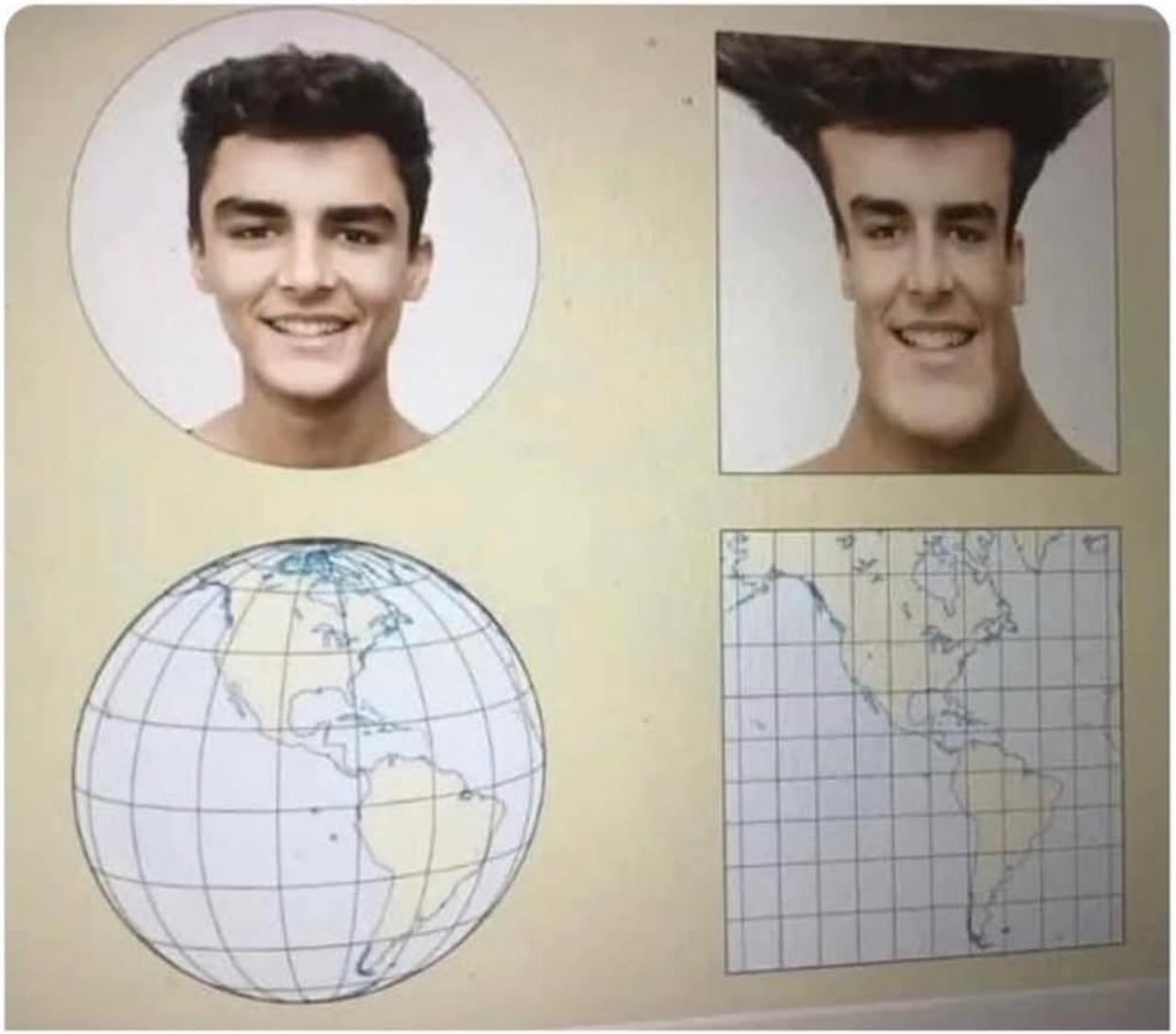
| | All Rwys JET | TURBOPROP |
|---|---|---|
| 2 Eng | 800m | 2 Eng 1600m |
| 3 & 4 Eng | | 3 & 4 Eng 800m |

CHANGES: Note.


Tinkune Park
तीनकुने पार्क
Ring Rd
Subidha Marg
Kusum Marg
Sambridhi Marg
Siddha Marg
Ishwor Kripa Marg
Mahadevsthan Marg
Ratna Marg
Himalaya College
हिमालय कलेज
Hotel A One
Cafe and Venue
होटल ए-वान
क्याफे एण्ड भेन्यु
Bajaj Service Cent
Majestic Events and
Banquet Pvt Ltd
म्याजेष्टिक इभेन्ट्स
एण्ड ब्याङ्केट प्रा. लि
INVICTUS
Maharani store
BG Brothers Futsal
Reload fashion wear
Shree Saraswati
Secondary School
Royal Jadibuti Banque
रोयल जडिबुटी
ब्याङ्केट
Google
Képek © Airbus, CNES / Airbus, Maxar Technologies, 2025., Térképadatok © Google, 2025.
Magyarország
Feltétel


RUNWAY

# Mercator projection: a simple analogy



# Resolution

The reciprocal of the limit of resolution is called the resolving power („resolution”): $f = 1 / d$.

- PPI, pixels per inch (display)
- DPI, dots per inch (printer)
- line pairs / cm
- …

Reducing the resolution: **loss of information**, additional increase of the resolution: **no additional information.**

# Zoom

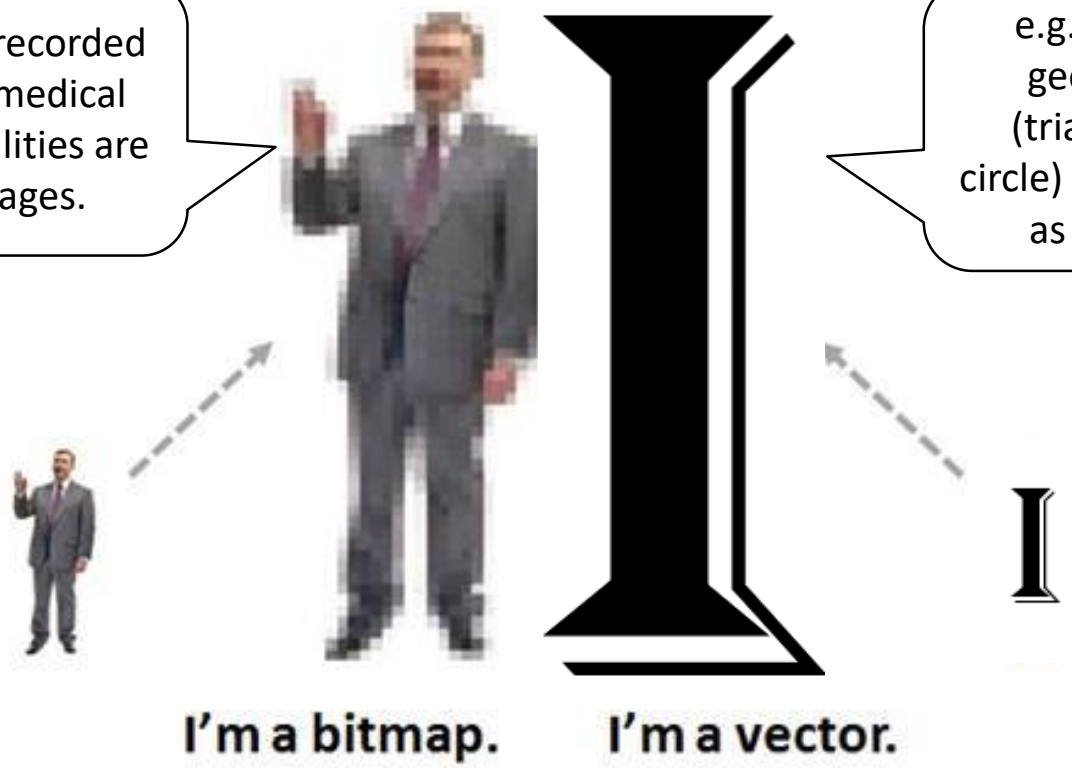
Pixel size is changed

Bitmap: zoom enables convenient assessment/analysis, but no additional information

Vector: unlimited zoom, but the shapes on the image should be defined with curves

Most images recorded by different medical imaging modalities are bitmap images.

e.g. fonts or simple geometric shapes (triangle, rectangle, circle) are usually defined as vector objects.



# File formats

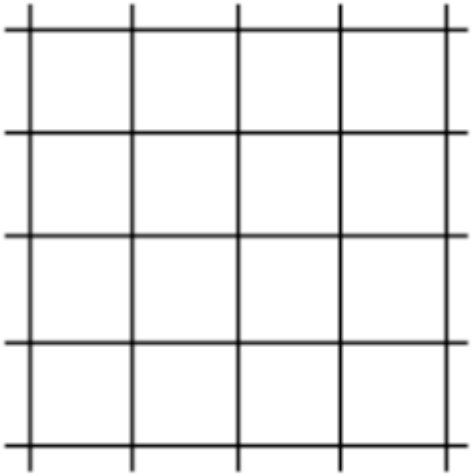
- **uncompressed**, e.g. BMP (bitmap)
- **compressed**
  - **lossless**, e.g. TIFF, GIF
  - **lossy**, pl. JPG
- **DICOM**
  - image information (including image series or stacks)
  - patient data
  - data regarding imaging mode / instrument settings, windowing
  - in the case of a CT scan, the radiation dose, etc.

small storage requirement, but some image information is lost

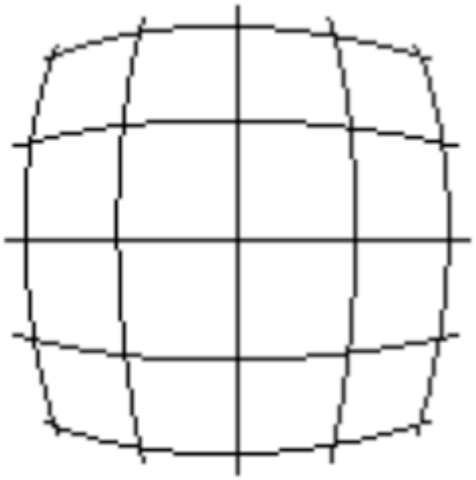
„Digital Imaging Communications in Medicine”
(standard medical image file format + network communication protocol)

# Distortion

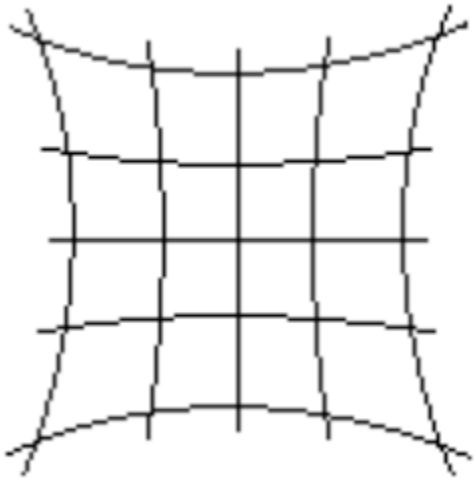
- if 3D object is imaged on a 2D display
- barrel / cushion / keystone



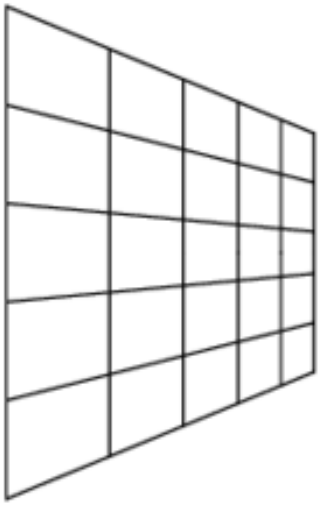
undistorted



barrel

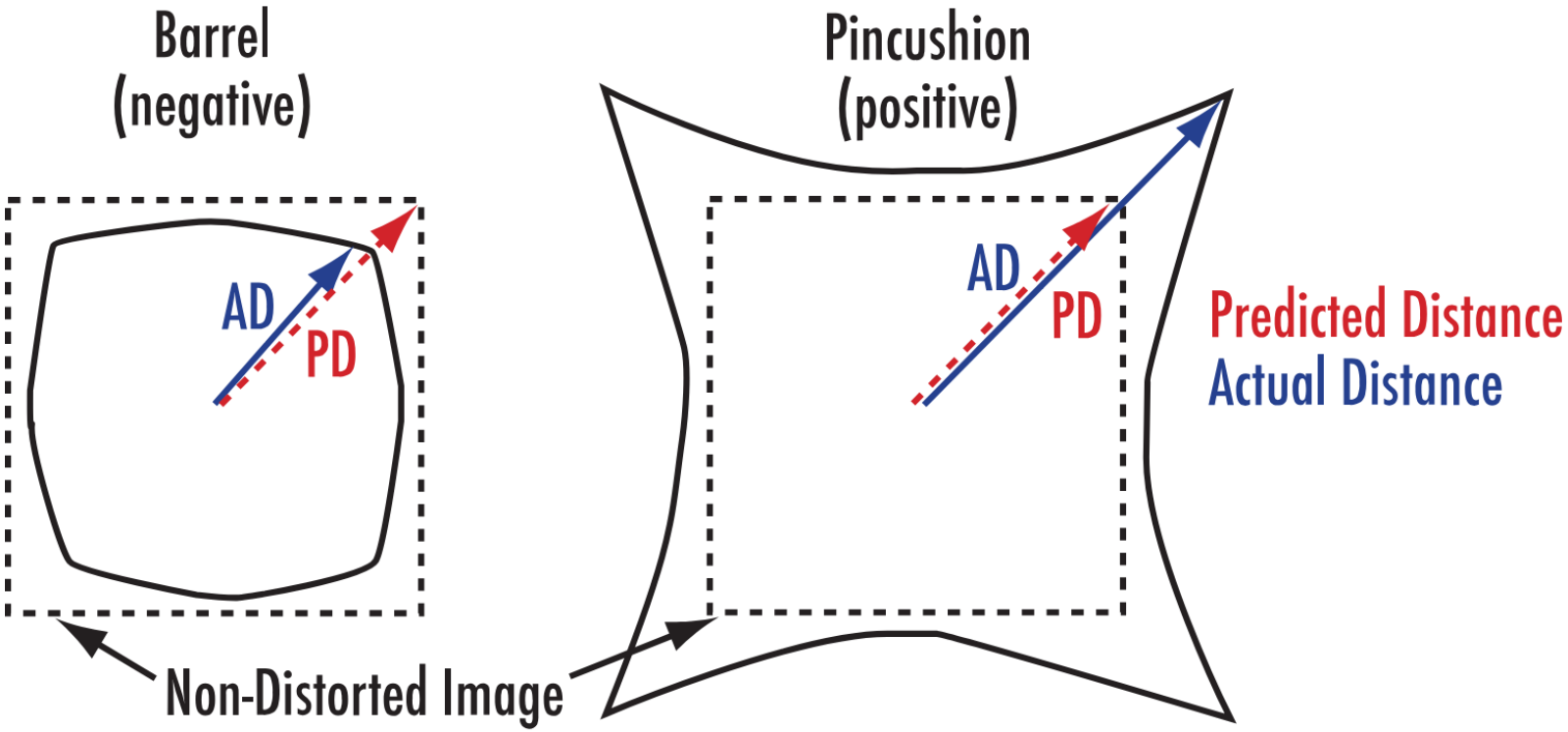


cushion



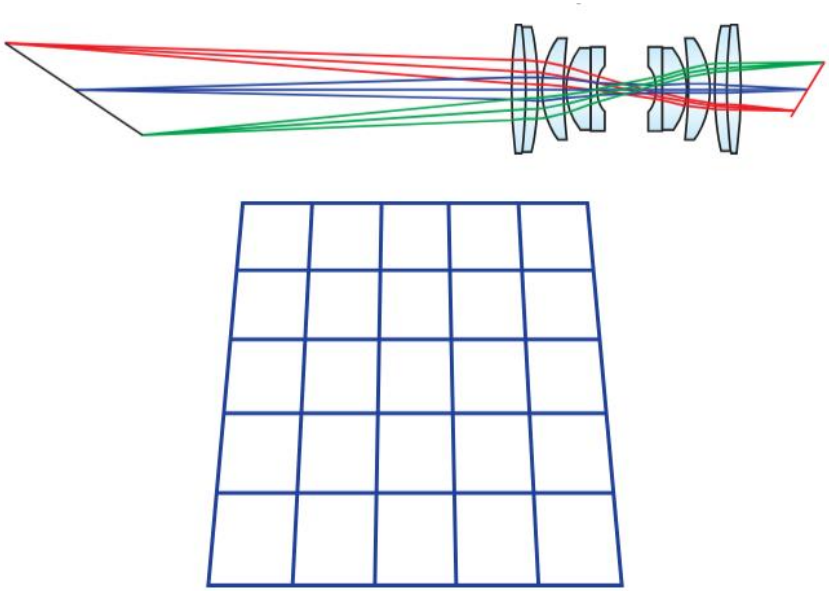
keystone

# Measure of distortion

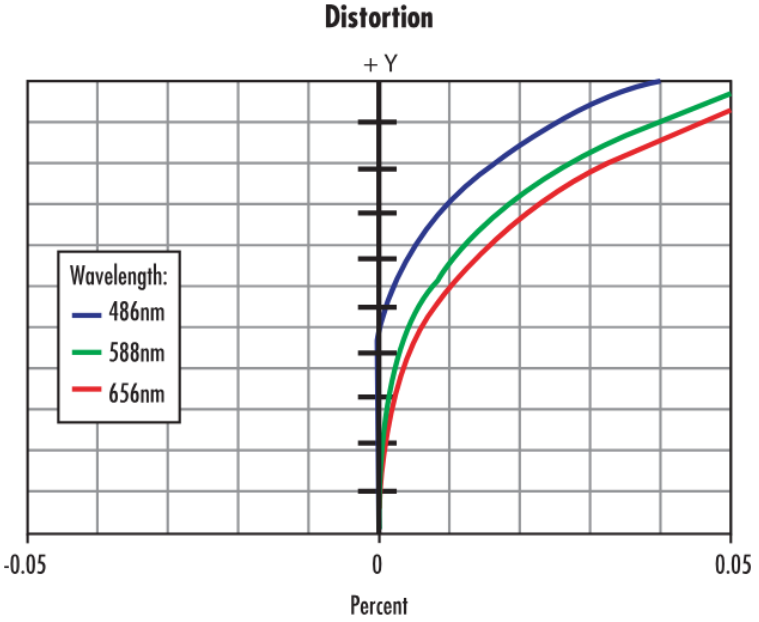



$$D\,(\%) = \frac{\mathrm{AD} - \mathrm{PD}}{\mathrm{PD}} \times 100\%$$

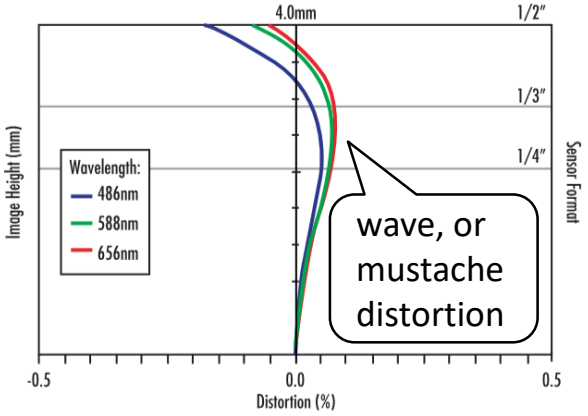
# Distortion of imaging lenses

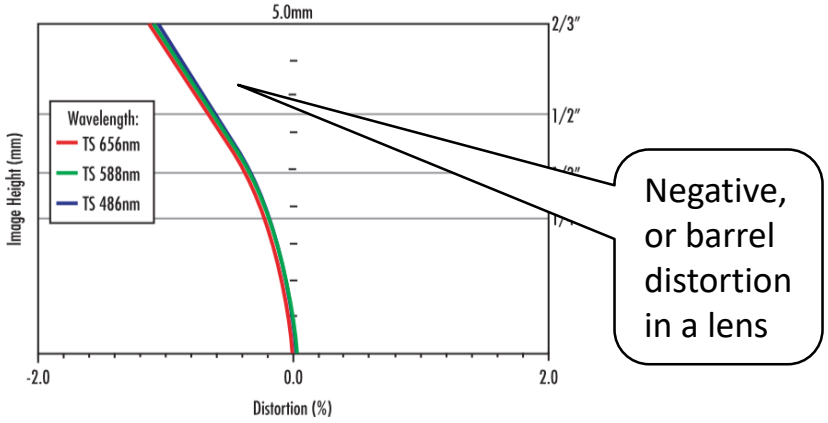


An example of keystone distortion in a lens layout and as it manifests in the image plane.



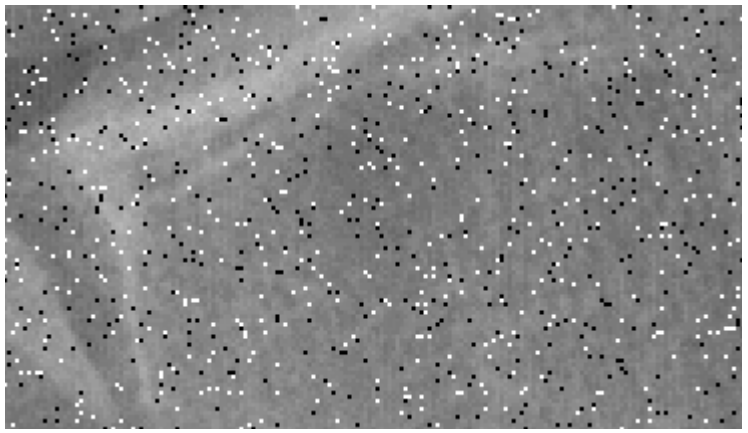

A distortion plot showing the variance of distortion with respect to wavelength.



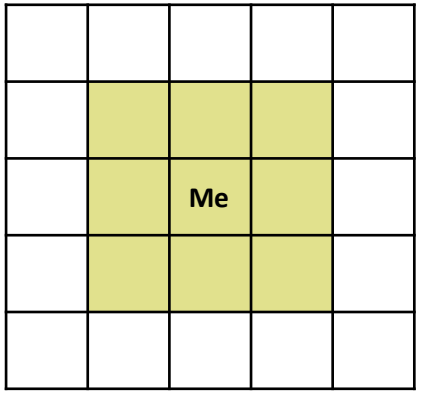
# Noise:

semiconductor detectors are temperature sensitive: **cooling** reduces the imaging noise

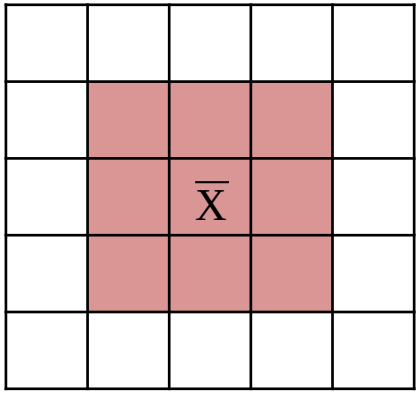


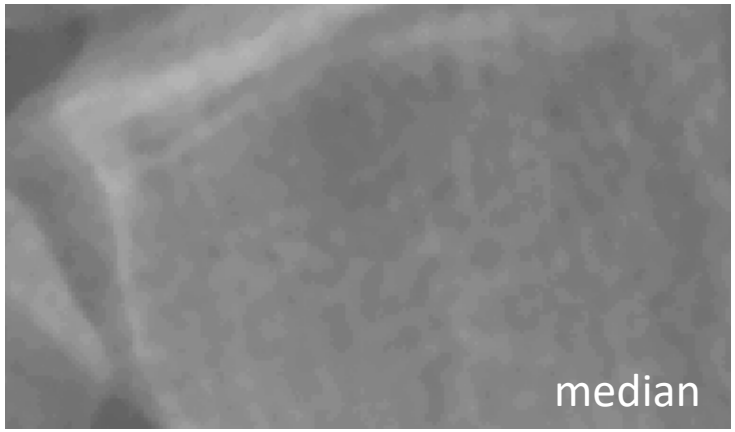
- most common source: detector
- „salt and pepper” (randomly alternating black and white pixels)

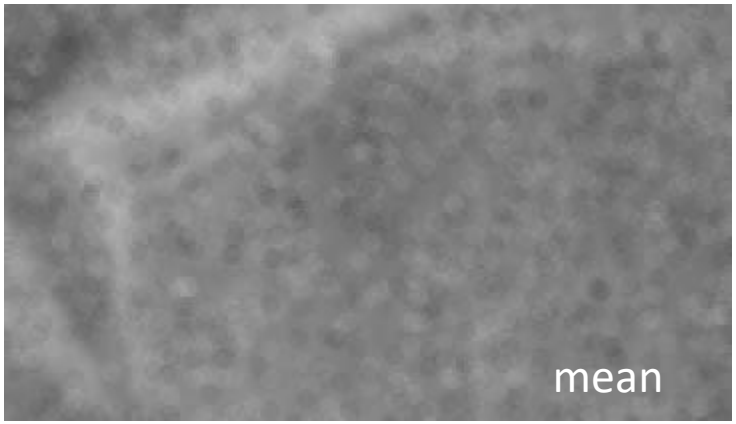
# Filtering:




highly effective against salt-and-pepper noise



# Convolution

transformation between two functions (digital image and kernel); kernel operation

**„smoothing" kernel**

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

smoothing

**„smoothing" convolution**

$$P^*_{x,y} = \frac{\sum_{i,j=-m}^{+m} W_{i,j} \cdot P_{x+i,y+j}}{\sum_{i,j=-m}^{+m} W_{i,j}}$$

P = original greyscale intensity value
x,y = coordinates of the current pixel
P* = modified greyscale intensity value
±m = size of the kernel (distance measured from the x,y coordinates)
W = weighing factor of the kernel at a given pixel determined by its i,j coordinates
i,j = coordinates across the kernel (integer numbers between -m and +m)

# Kernel-operations (example)

**„smoothing” kernel**

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$$P^*_{x,y} = \frac{\sum_{i,j=-m}^{+m} W_{i,j} \cdot P_{x+i,y+j}}{\sum_{i,j=-m}^{+m} W_{i,j}}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

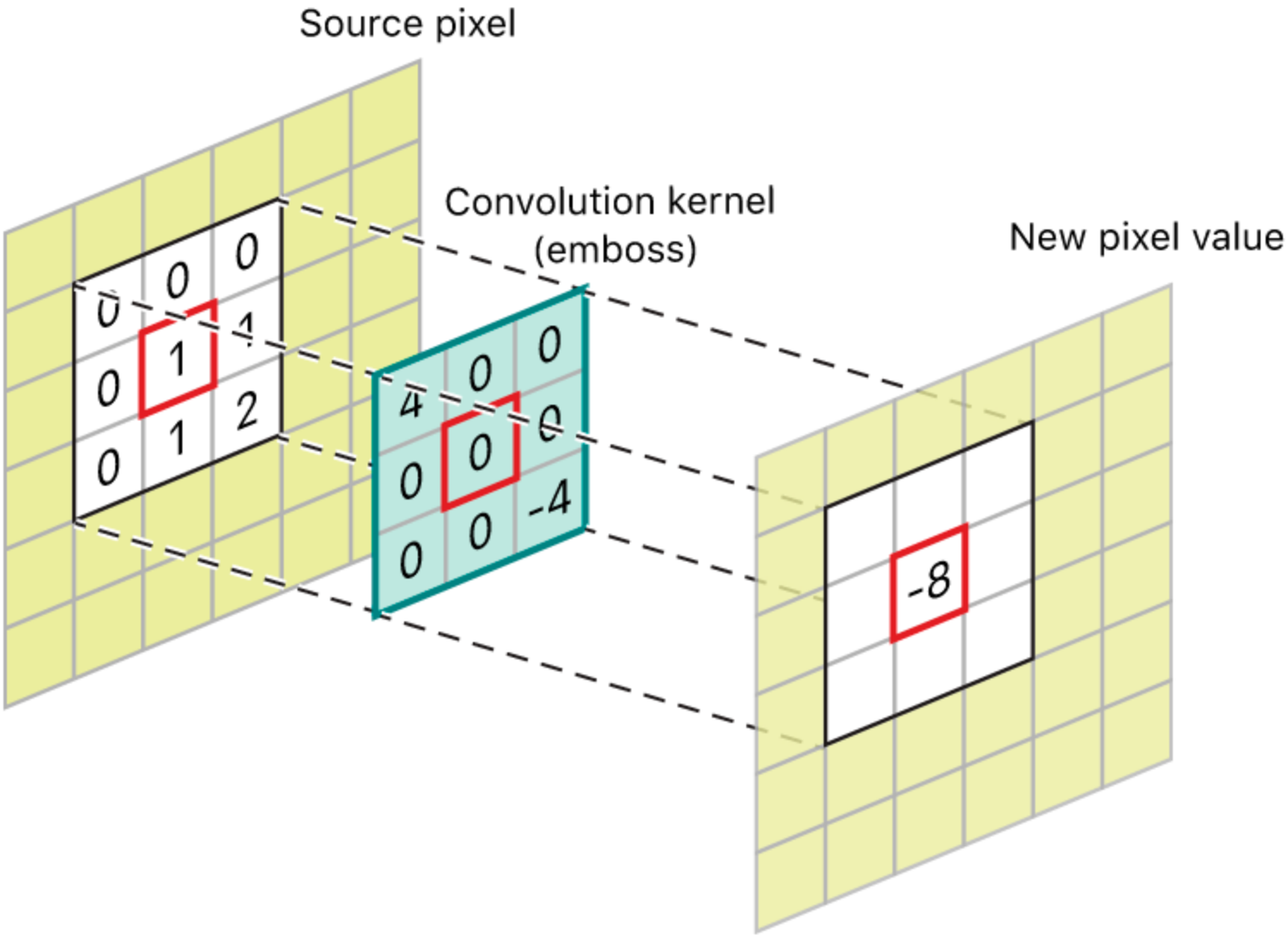
| | | |
|---|---|---|
| 15 | 25 | 38 |
| 20 | 40 | 25 |
| 10 | 36 | 20 |

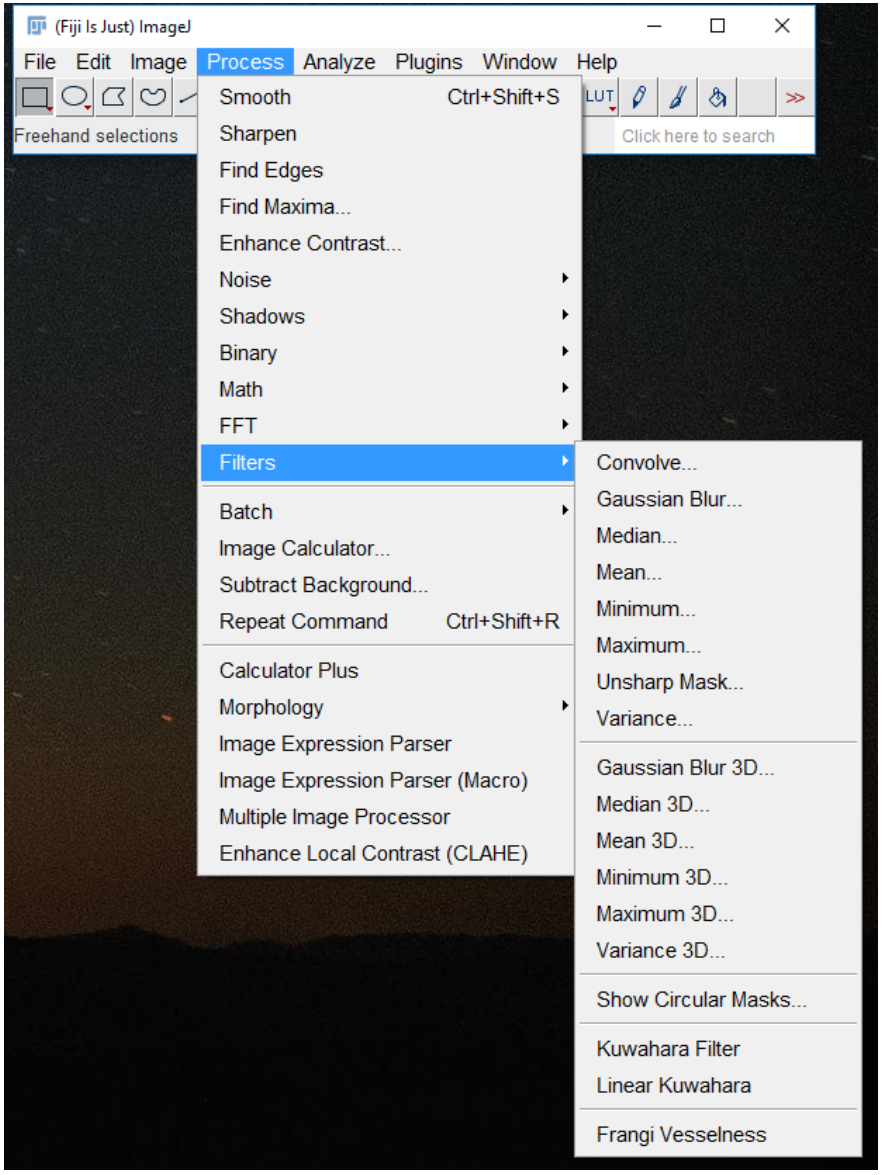
| | | |
|---|---|---|
| 15 | 25 | 38 |
| 20 | 25.44 | 25 |
| 10 | 36 | 20 |

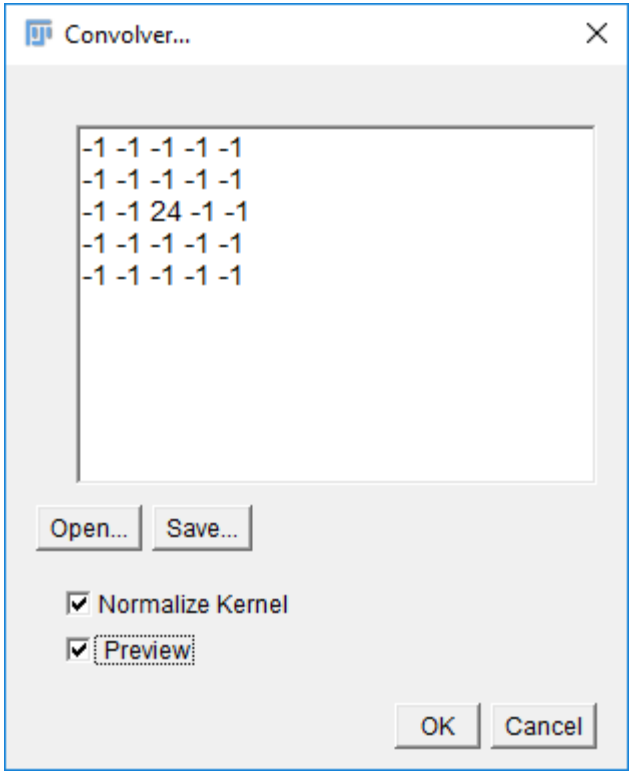
| | | | | |
|---|---|---|---|---|
| | | | | |
| | 15 | 25 | 38 | |
| | 20 | 40 | 25 | |
| | 10 | 36 | 20 | |
| | | | | |

**The following calculation is performed:**

1. The value (40) of the currently selected pixel (which falls on the central pixel of the kernel) is multiplied by 1.
2. The neighboring pixel values are multiplied by the kernel's corresponding pixel values (also 1).
3. The calculated intensity values are added and divided by the total value of the kernel (9).
4. We assign the calculated value (25.44) to the selected pixel (central pixel).


Source pixel
Convolution kernel
(emboss)
New pixel value
0 0 0
0 1 1
0 1 2
4 0 0
0 0 0
0 0 -4
-8


(Fiji Is Just) ImageJ
File Edit Image Process Analyze Plugins Window Help
Freehand selections
Click here to search
Smooth Ctrl+Shift+S
Sharpen
Find Edges
Find Maxima...
Enhance Contrast...
Noise
Shadows
Binary
Math
FFT
Filters
Batch
Image Calculator...
Subtract Background...
Repeat Command Ctrl+Shift+R
Calculator Plus
Morphology
Image Expression Parser
Image Expression Parser (Macro)
Multiple Image Processor
Enhance Local Contrast (CLAHE)
Convolve...
Gaussian Blur...
Median...
Mean...
Minimum...
Maximum...
Unsharp Mask...
Variance...
Gaussian Blur 3D...
Median 3D...
Mean 3D...
Minimum 3D...
Maximum 3D...
Variance 3D...
Show Circular Masks...
Kuwahara Filter
Linear Kuwahara
Frangi Vesselness


Convolver...
-1 -1 -1 -1 -1
-1 -1 -1 -1 -1
-1 -1 24 -1 -1
-1 -1 -1 -1 -1
-1 -1 -1 -1 -1
Open...
Save...
Normalize Kernel
Preview
OK
Cancel


L
DBM

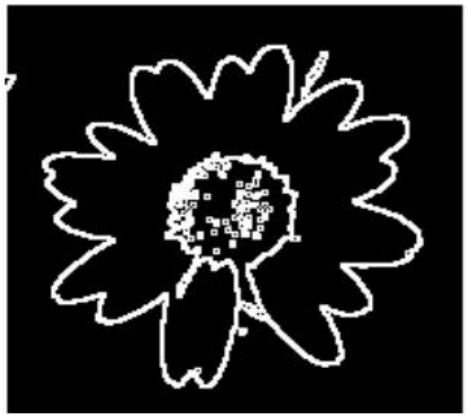
| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |

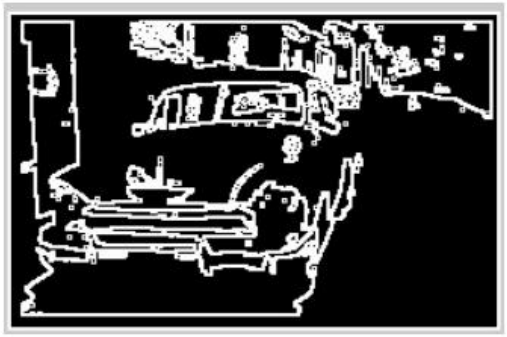
*

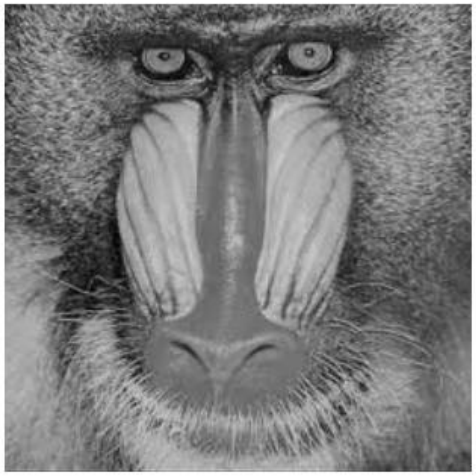
| | | |
|---|---|---|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

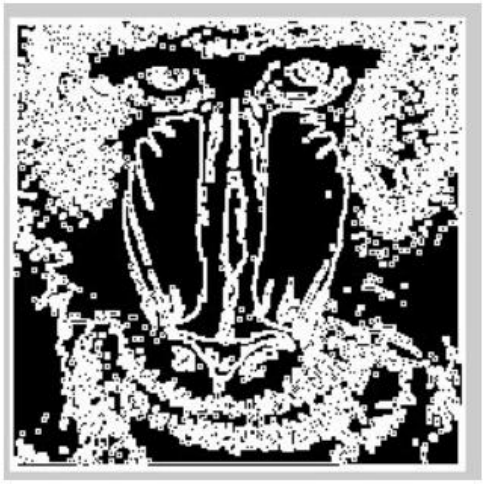
=

| | | | |
|---|---|---|---|
| 0 | -30 | -30 | 0 |
| 0 | -30 | -30 | 0 |
| 0 | -30 | -30 | 0 |
| 0 | -30 | -30 | 0 |







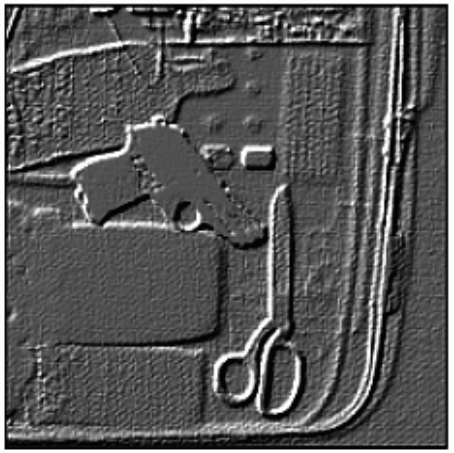


a. Delta function

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

b. Shift and subtract

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | -1 |



c. Edge detection

| | | |
|---|---|---|
| -1/8 | -1/8 | -1/8 |
| -1/8 | 1 | -1/8 |
| -1/8 | -1/8 | -1/8 |

d. Edge enhancement

| | | |
|---|---|---|
| -k/8 | -k/8 | -k/8 |
| -k/8 | k+1 | -k/8 |
| -k/8 | -k/8 | -k/8 |

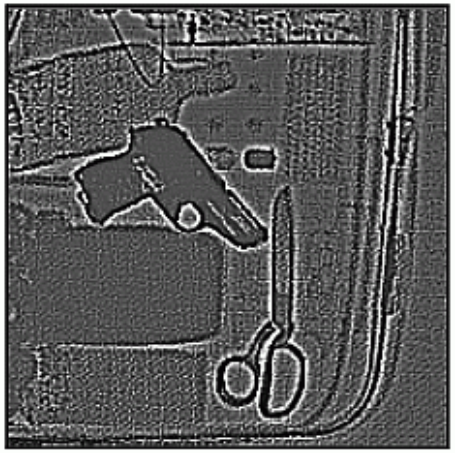


FIGURE 24-4
3×3 edge modification. The original image, (a), was acquired on an airport x-ray baggage scanner. The shift and subtract operation, shown in (b), results in a pseudo three-dimensional effect. The edge detection operator in (c) removes all contrast, leaving only the edge information. The edge enhancement filter, (d), adds various ratios of images (a) and (c), determined by the parameter, $k$. A value of $k = 2$ was used to create this image.