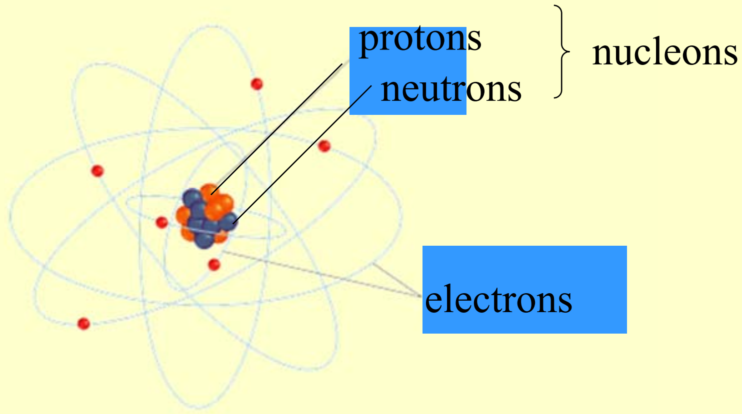
Nuclear radiation, radioactive isotopes

Atomic structure



Constituents of atoms

Particle	Symbol	Rest Energy (MeV)	Relative Charge*	Mass (kg)	Relative Mass (AMU)**
electron	e	0.51100	1-	9.11×10^{-31}	$5.4858X10^{-4}$
proton	p	938.272	0	$1.6726X10^{-27}$	1.0072765
neutron	n	939.566	1+	$1.6749X10^{-27}$	1.0086649

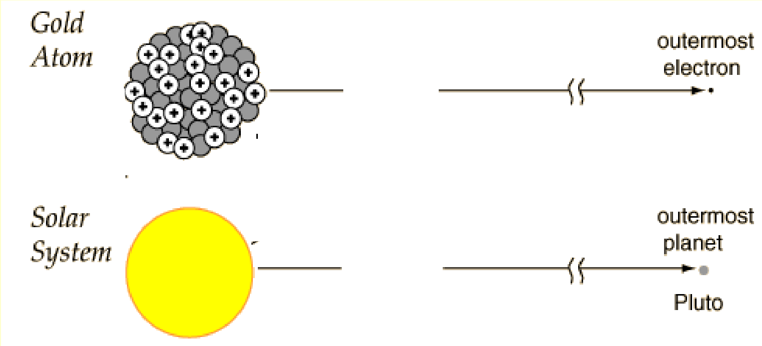
* electrons have an electric charge of -1.602×10^{-19} C
**The atomic mass unit is defined as 1/12 of the carbon (^{12}C)atom

Quick problem: protons in your body

What is the order of magnitude of the number of protons in your body? Of the number of neutrons? Of the number of electrons? Take your mass approximately equal to 70 kg.

$$N = 35kg\left(\frac{1\,proton}{1.67x10^{-27}\,kg}\right) \approx 10^{28}\,proton$$

Relative scale model of an atom and the solar system



For gold: $d_{nucleus}$ = 32 fm = 3.2×10^{-14} m,

Nuclear notation

Mass number
A = Z + N

$^{27}_{13}\mathrm{Al}$ ($^{A}_{Z}\mathrm{X}$)

Chemical symbol for the element

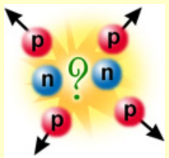
Atomic number = Number of protons

N = number of neutrons

Nuclear stability

- There are very large *repulsive electrostatic forces* between protons

  These forces should cause the nucleus to fly apart



- $\Delta M = [Zm_p + (A\text{-}Z)m_n] - M(A,Z)$ (?)

It must be that a different type of force exists within the nucleus

1911 Rutherford, Geiger and Marsden

Nuclear stability

$$\Delta M = [Zm_p + (A\text{-}Z)m_n] - M(A,Z)$$

Instead of two separate conservation laws, a single conservation law states that the sum of mass and energy is conserved. Mass does not magically appear and disappear at random. A decrease in mass will be accompanied by a corresponding increase in energy and vice versa.

$$\Delta E = \Delta Mc^2$$

Another, short-range force is present, called the *nuclear force* (Rutherford, 1911

Nuclear stability

- Another, short-range force is present, called the *nuclear force* (Rutherford, 1911)

This is an attractive force that acts between all nuclear particles

The nuclear attractive force is stronger than the Coulomb repulsive force at the short ranges within the nucleus

$$\Delta E = \Delta M c^2$$

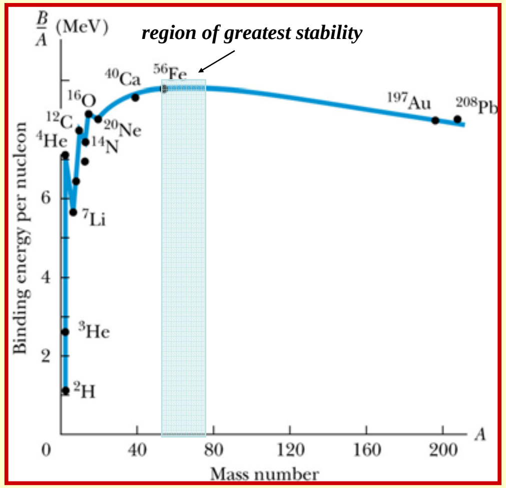
The nuclear force is

- independent of charge

- the range of action is extremely short (~fm)

Binding energy per nucleon

- The curve increases rapidly
- Sharp peaks for the even-even nuclides ^{4}He, ^{12}C, and ^{16}O
- Maximum is around A=56



Nuclear stability

The balance of proton and neutron number

is extremely important for the stability of the nucleus

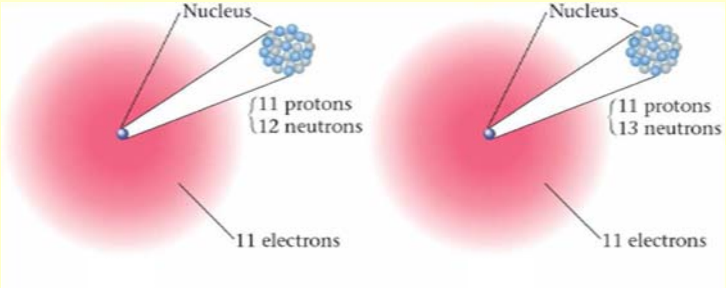
Isotopes

Greek ***isos topos = equal place***

Isotopes of an element have nuclei with

- the same number of protons
- different numbers of neutrons
- different mass number

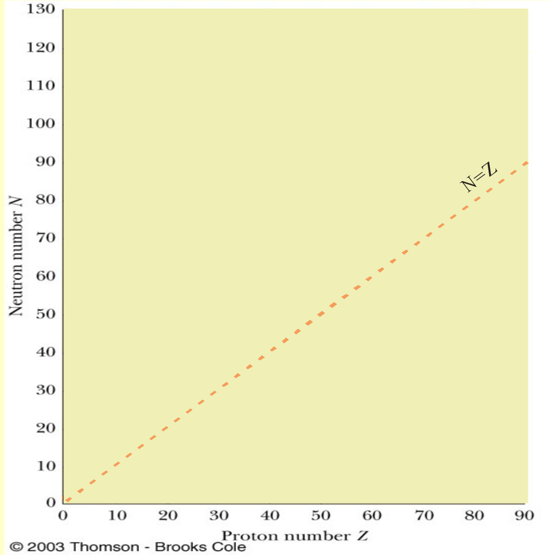
Example of isotopes



${}^{23}_{11}Na$ ${}^{24}_{11}Na$

What is/are the stable combination(s)? 1:1 ?

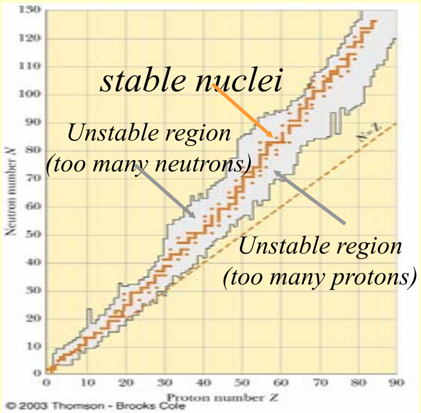
Nuclear stability chart



Nuclear stability chart

- Light nuclei are most stable if N=Z
- Heavy nuclei are most stable when N > Z

  As the number of protons increase, the Coulomb force increases and so more nucleons are needed to keep the nucleus stable



- No nucleus is stable when Z > 83

Antoine Becquerel
1903 Nobel Prize in Physics for discovering radioactivity

Image of Becquerel's photographic plate which has been fogged by exposure to radiation from a uranium salt. The shadow of a metal Maltese Cross placed between the plate and the uranium salt is clearly visible. (1896)

Radioactive decay

- *Radioactivity* is the spontaneous release of energy in the form of radioactive particles or waves
- Experiments suggested that radioactivity was the result of the decay, or disintegration, of unstable nuclei
- Three types of radiation can be emitted
  - Alpha (α) particles
  - Beta (β) particles
  - Gamma (γ) rays (Rutherford 1896, see details later)

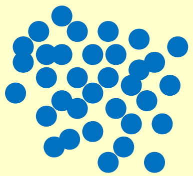
- The nuclides, as with most things in nature, want to be at their *lowest energy state* which is a stable nucleus.
- Radioactive decay occurs in nuclides where the *nucleus is unstable*.

•The nuclide reaches its stable state by undergoing radioactive decay.

Characteristics of radioactive decay

• it is *statistical process* – individual disintegrations occur ***randomly***

• it results in a decrease over time of the original amount of the radioactive nuclei



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decrease of the original amount of the radioactive nuclei over time

Activity : $$\Lambda = \left| \frac{dN}{dt} \right|$$

N: number of nuclei to be decayed

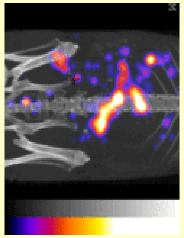
t: time

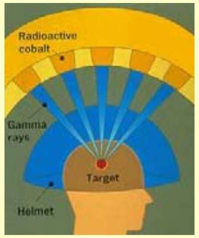
number of nuclei decayed in a unit time

measure: bequerel (Bq)
1Bq = 1 decay/sec

Typical activities in the practice

kBq,	***MBq,***	***GBq,***	***TBq***
natural background	*in vivo* diagnostics	laboratory practice	*therapy*





Radioactive decay law

Differential form $$\left|\frac{dN}{dt}\right| = -\lambda N$$

***λ* :** decay constant (measure: 1/s)

constant for a certain isotope

Activity depends both on the

- size of the population of radioactive atoms
- type of the isotope

$$\left|\frac{dN}{dt}\right| = -\lambda N$$

Solution of this equation yields

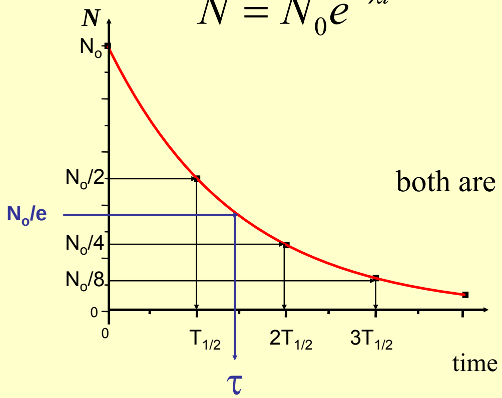
$$N = N_0 e^{-\lambda t}$$ **Integral form**

N_0 **:** number of radioactive nuclei at $t = 0$,

***N* :** the number radioactive of nuclei remaining after a period *t*

Graphical representation

$$N = N_0 e^{-\lambda t}$$

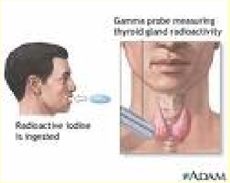


$T_{1/2}$: the half-life

τ : the mean life time

both are characteristic to the nuclei

Half-lives in Medical Uses

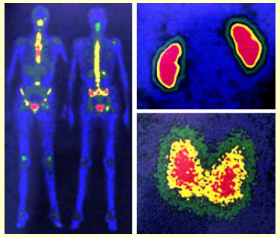


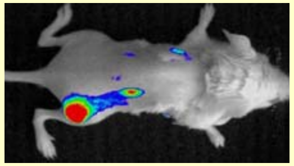
Iodine - 131 (^{131}I) ***- $T_{1/2}$ = 8 days***

Thyroid treatment

Technetium-99m (^{99m}Tc) ***– $T_{1/2}$ = 6 hours***

Isotope diagnostics





Gold-198 (^{198}Au) ***- $T_{1/2}$ = 2.7 days***

Tumor therapy

Further considerations

$$N = N_0 e^{-\lambda t}$$

If t = $T_{1/2}$ $\Longrightarrow$ $N_0 / 2 = N_0 e^{-\lambda T_{1/2}}$

$$\boxed{\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{T_{1/2}}}$$

If t = τ $\Longrightarrow$ $N_0 / e = N_0 e^{-\lambda \tau}$

$$\boxed{\lambda = \frac{1}{\tau}}$$

Definition of decay constant

Alteration of activity in time

$$N = N_0 e^{-\lambda t}$$

$$\Downarrow$$

$$\Lambda = \Lambda_0 e^{-\lambda t}$$

$$\Lambda = \lambda N$$

Specific activity : activity in a unit mass (Λ/m),

unit: Bq / kg

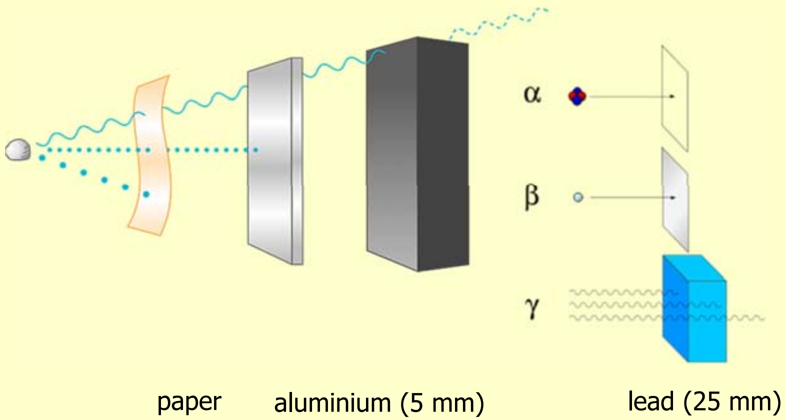
Characteristics of radioactive isotopes

Activity : depends both on the nucleus and the size of its population

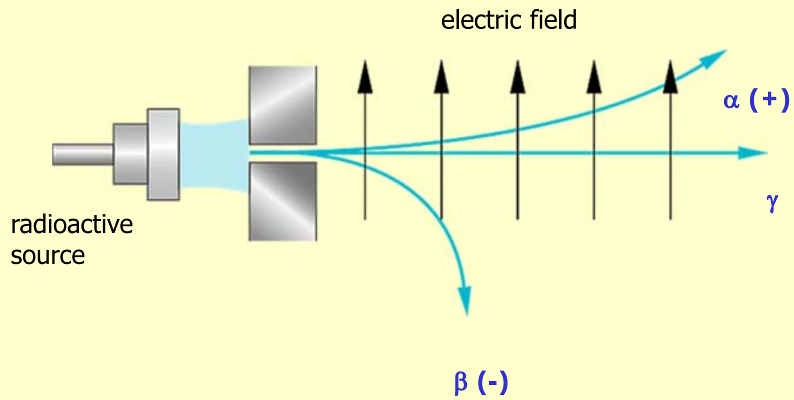
Half-life : physical parameter of each radioactive isotope

Type of radiation: physical characteristic of the nucleus

Types and nature of nuclear radiation



Deflection of radiation in electric field

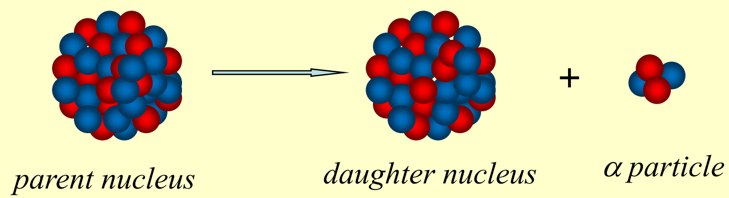


Radiation from a radium source is split by an electric field.

The radioactive decay process must obey the laws of physics

1. The conservation of mass / energy
2. The conservation of electric charge
3. The conservation of momentum
4. The conservation of nucleon number

α decay



$$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha $$

α particle is a nucleus of helium containing two neutrons and two protons

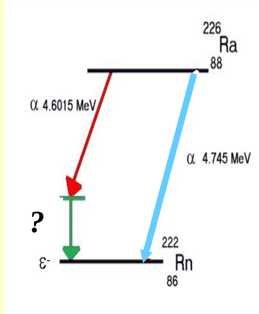
Heavy nuclei with mass numbers higher than 150 can disintegrate by emission of an α particle

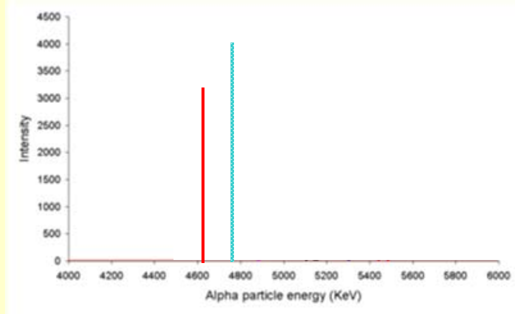
$$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha $$

example

$$ {}^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^{4}_{2}\alpha $$

Energy spectrum of α radiation

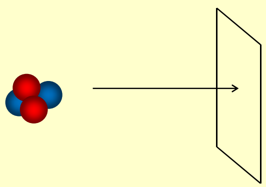




line spectrum

Energy is characteristic for the nucleus

Penetration depth of α particles



single sheet of paper

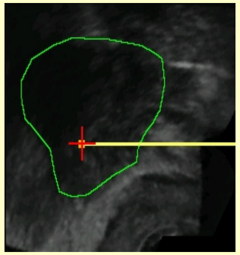
absorber	density	alpha range
air (STP)	1.2 mg/cm^3	3.7 cm
paper (20lb)	0.89 g/cm^3	53 µm
water (soft tissue)	1.0 g/cm^3	45 µm

Special precautions have to be taken to ensure that alpha emitters are not inhaled, ingested or injected by accident.

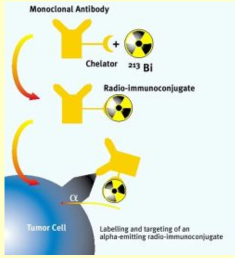
Medical application of α radiation

Diagnostics: none

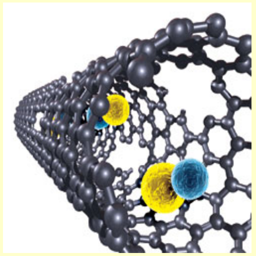
Targeted alpha **therapy** of cancer



Seed implantation by needle



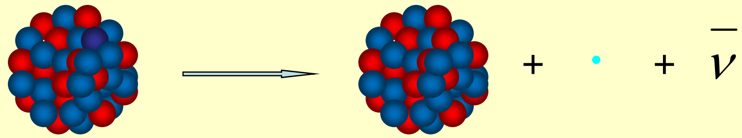
Monoclonal antibody



Carbon nano-tube

β decay

1. Neutron excess: β⁻ decay



parent nucleus *daughter nucleus*

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^{-} + \bar{\nu}$$

$$^{1}_{0}n \longrightarrow {}^{1}_{1}p^{+} + {}^{0}_{0}\beta^{-} + \bar{\nu}$$

anti-neutrino

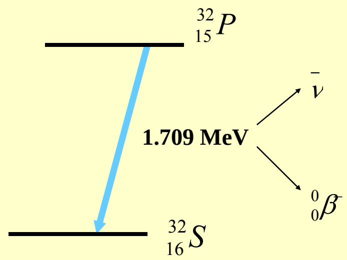
$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{0}\beta^{-} + \bar{\nu}$$

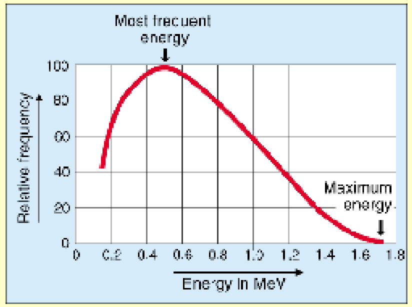
example

$$^{131}_{53}I \longrightarrow {}^{131}_{54}Xe + {}^{0}_{0}\beta^{-} + \bar{\nu}$$

$$^{32}_{15}P \longrightarrow {}^{32}_{16}S + {}^{0}_{0}\beta^{-} + \bar{\nu}$$

Energy spectrum of β radiation



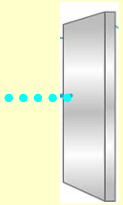


Energy distribution of the β- particles emitted during the β- decay of ^{32}P.

continuous spectrum

with maximum kinetic energy for the β particle

Penetration depth of β^- particles



5 mm aluminum

absorber	***density***	***maximum beta range*** (2.3 MeV)	(1.1 MeV)
air	1.2 mg/cm^3	8.8 m	3.8 m
water (soft tissue)	1.0 g/cm^3	11 mm	4.6 mm
aluminum	2.7 g/cm^3	4.2 mm	2.0 mm
lead	11.3 g/cm^3	1.0 mm	0.4 mm

Medical application of β^- radiation

Diagnostics: none

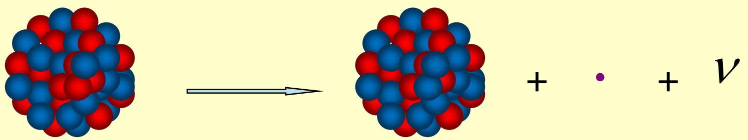
Targeted therapy: hyperthyroidism, thyroid and several other types of cancer

Brachytherapy***:*** implants into the tumours

Endovascular irradiation

β decay

2. Proton excess: β^+ decay



parent nucleus *daughter nucleus*

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \nu$$

$$^{1}_{1}p^+ \longrightarrow {}^{1}_{0}n + {}^{0}_{0}\beta^+ + \nu$$

neutrino

e^+ ***: positron – antiparticle of electron***

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \nu$$

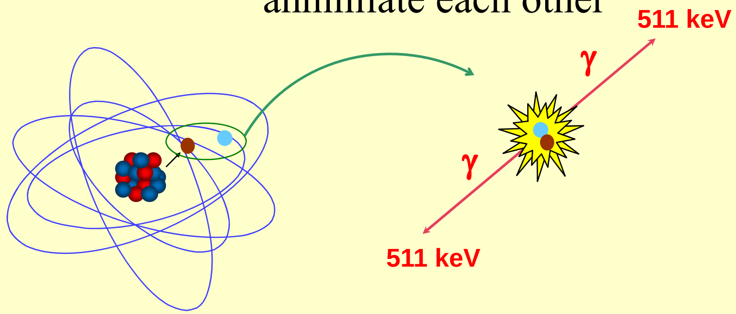
example

$$^{11}_{6}C \longrightarrow {}^{11}_{5}B + e^+ + \nu$$

¿ Fate of positron?

electron and positron are antiparticles :
particles that are identical in their significant parameters except
charge is equal but of opposite sense

Annihilation - particle-antiparticle pairs can annihilate each other



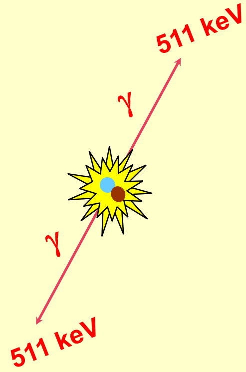
The radioisotope emits a positron that interact with an electron

The annihilation of the pair positron-electron generates two photons with 511 keV wandering in 180 degrees

1. Conservation of momentum

⇓

two photons with opposite direction are produced



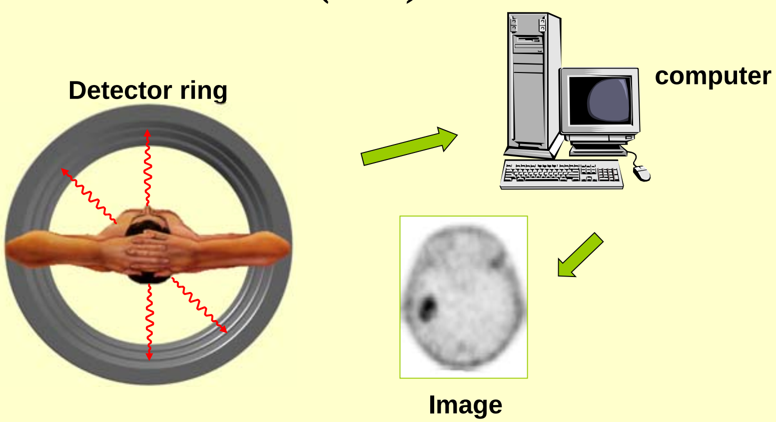
2. Energy balance:

$$m_e c^2 + m_p c^2 = 2\,hf$$

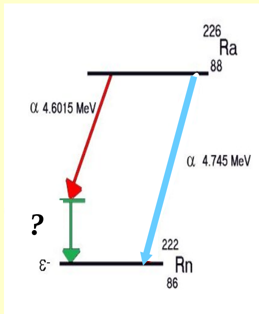
mass – energy equivalence

Medical application of of β^+ radiation

positron emission tomography (PET)

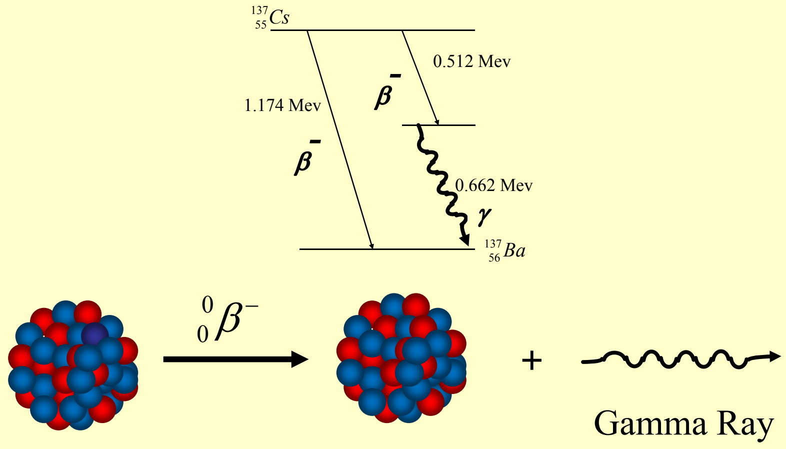


γ decay



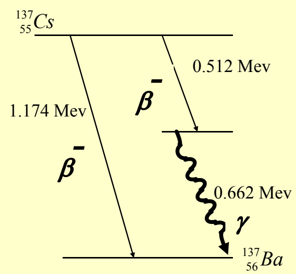
Sometimes the newly formed isotopes (after α or β decay) appear in the excited stat.

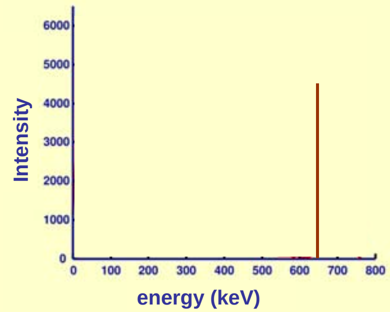
Excited nuclides have tendency to release the excess of energy by **electromagnetic radiation** - emission of gamma rays.



parent nucleus *daughter nucleus*

Energy spectrum of γ radiation

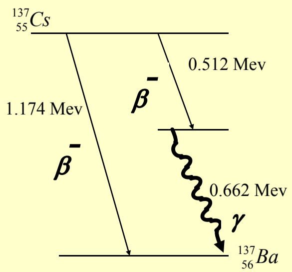


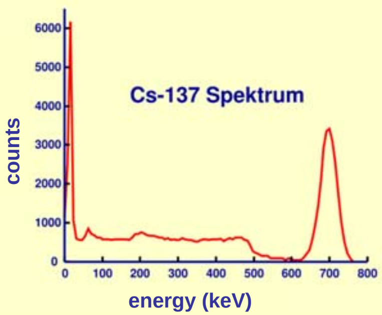


line spectrum

Energy is characteristic for the nucleus

Energy spectrum of γ radiation

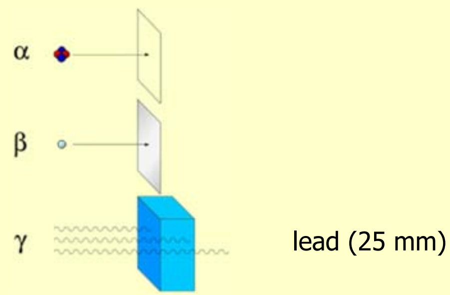




line spectrum

Energy is characteristic for the nucleus

Penetration depth of γ radiation



Penetration distance is higher than that of α or β particles, but it is highly energy dependent.

Gamma rays can travel from 1 to 100s of meters in the air and can easily go right through people (~dm).

Timing of γ emission

Lifetime of the excited nucleus:

1.Prompt γ decay: ~ $10^{-13} - 10^{-18}$ **s**

2. Isommeric transition: $\geq$ **10^{-10} s**

Isomeric transition

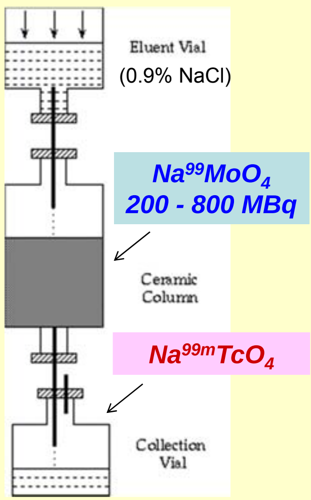
Some excited states may have a half-lifes ranging from hours up to more than 600 years

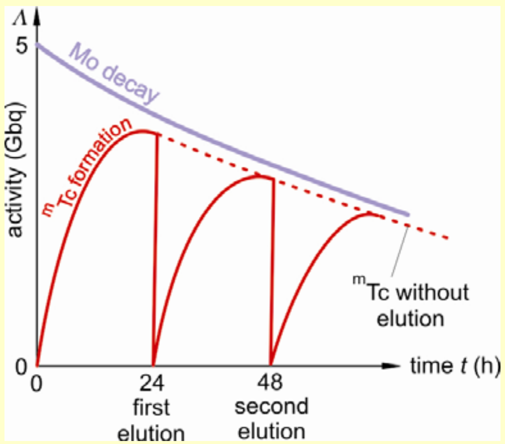
$$ {}^{Am}_{Z}X \rightarrow {}^{A}_{Z}X + \gamma $$

$$ {}^{99}_{42}Mo \rightarrow {}^{99m}_{43}Tc + \beta^{-} \rightarrow {}^{99}_{43}Tc + \gamma $$

$T_{1/2}$=67 hours $\qquad$ $T_{1/2}$=6 hours

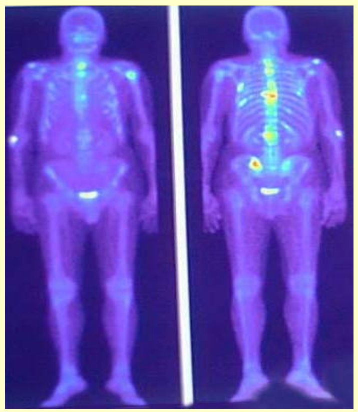
Technetium-99m generator





Medical application

Diagnostics: ideal for isotope diagnostics



Bone scan using ^{99m}Technetium labeled phosphate compound

Radionuclide	***Compound***	***Organ***	***Function***
$^{99}Tc^{m}$	***sodium pertechnetate***	***brain***	***blood flow***
$^{99}Tc^{m}$	***coagulated albumin***	***lung***	***blood flow***
$^{99}Tc^{m}$	***colloidal suspension***	***liver***	***liver function***
$^{99}Tc^{m}$	***complex phosphate***	***bone***	***bone metabolisms***
$^{99}Tc^{m}$	***red blood cells***	***heart***	***blood circulation***
^{123}I	***iodide***	***thyroid***	***metabolisms***
^{123}I	***hippuran***	***kidneys***	***renal function***
^{133}X	***gas***	***lungs***	***ventilation***

Damjanovich, Fidy, Szöllősi: Medical Biophysics

I. 1.5
- 1.5.1
- 1.5.2
- 1.5.4

II.3.2
- 3.2.1
- 3.2.2
- 3.2.3
- 3.2.4