MEDICAL STATISTICS

Physiology

Anatomy

Chemistry

…

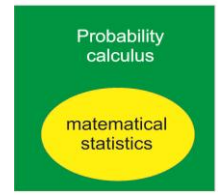
No any doubt

Statistics

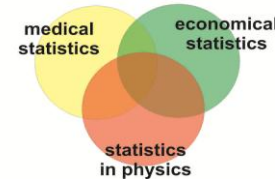
?

Medical statistics

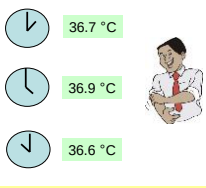
Theory: matematics

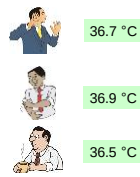


Practice: applied statistics (examples)



Example: body temperature





1. Inaccuracy of the measurement.
2. ***Daily fluctuation!!!***

3. Biological variability!!!

The measured value is not constant!

Measured value: 37.0 °C.
Is it healthy or not?

Another examples

RBC: 4.5×10^{12} 1/l ($\underline{3.9\text{-}5 \times 10^{12}\ 1/l}$) → normal range?

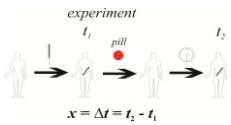
The new method in therapy is better then the old one or not?

<u>How can we prove that a medicine decreases the fewer or not?</u>

↓

Questions!

How can we answer?



1. Δt < 0

Medicine is effective.

Medicine is not effective: fluctuation, patient will be better, etc.

?

2. Δt >= 0

Consequence:

The answer is not so simply!

(Not only the medicine influences the body temperature.)

Medicine is not effective.

Medicine is effective: fluctuation, doesn't increase further, etc.

Variables

variable	range	type	variable type	
height	~50 cm … ~250 cm	real number	numerical	continuous
no. of teeth	0 .. 32	integer		discrete
blood type	A, B, AB, 0	letters	categorical	nominal
severity of cancer	1 … 4	integer		ordinal

Descriptive statistics!

Description of a variable

- Type
- Possible values
- Occurrence of the values

Numerical variables

Name	***Continuous***	***Discrete***
Definition	Infinitely large no. of values in a certain range	Only finite number of values
Example	Height, temperature, pressure …	No. of teeth, no. of children …

Categorical variables

Name	***Nominal***	***Ordinal***
Definition	No order among the values	There is a certain order
Example	Gender, blood-type …	Severity of the illness, strength of pain …

Determination of the possible values

- ***Continuous*** : giving a possible range.
  - » e.g.: height from ~50 cm - to ~ 250 cm
- ***Other*** : listing the values, if it is possible
  - » E.g.: blood type: A, B, AB, 0

Occurence

Observation: The occurrence of the values are not the same!

↓

Trial: experiment, observation, data collection.

Deal with only the case, when the trial may be repeated!

Outcome: result of one trial. (e.g.: height of a student)

Population

How many people?

↓

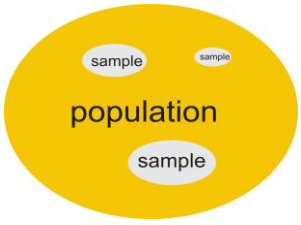
As many as possible.

↓

Ideal case: All of the people → ***population***

Sample

A smaller portion of the population.



n: no. of the elements (people) in the sample.

x: the tested variable (quantity)

x_i: i-th element from the sample

Selection of the sample

Main principle: **Random sample**

Medical statistics: if there is no any reason to exclude, must be random!

Occurrence

Frequency (*k*): no. of occurrence in the sample.

k_i: no. of occurrence of the i-th value in the sample.

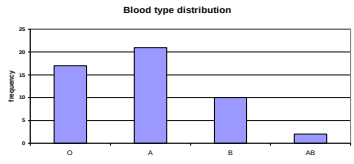
$$n = \sum_i k_i$$

Frequency distribution

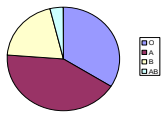
Frequency as the function of the possible values.

Blood-type	**0**	**A**	**B**	**AB**	total
frequency	17	21	10	2	50

Presentation



Bar-chart



Pie-chart

Relative frequency, proportion

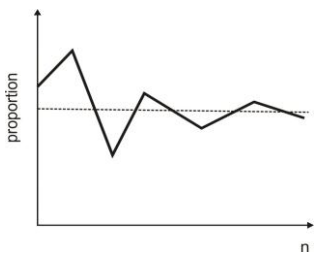
The ratio of the frequency and the total no. of the elements.

$$\sum_i \frac{k_i}{n} = \frac{1}{n}\sum_i k_i = \frac{1}{n} \times n = 1$$

Frequently it is given as percentage:

$$\frac{k_i}{n} \times 100\%$$

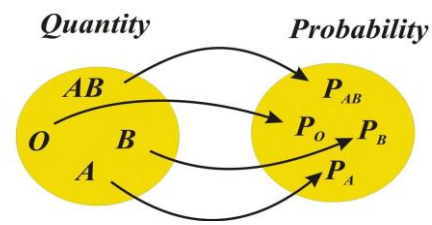
Probability (*P*)



If *n* is infinite the proportion is the probability.

Probability (P): proportion in the population.

Probability distribution



Properties of the probability

$0 \le P \le 1$ ⟶

P = 0 - never occur

P = 1 - always occur

Example: blood- type

$P_A + P_B + P_{AB} + P_0 = 1$ ⟶ $\sum_i P_i = 1$

(exclusive events)

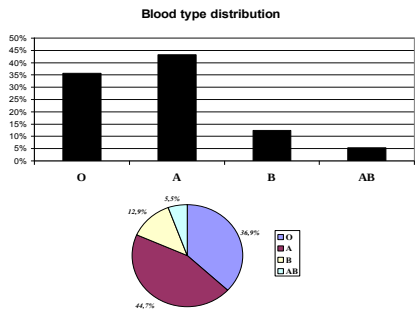
Probability and proportion

Sample	Population
n is finite	$n = \infty$
proportion	probability

Probability very frequently is unknown!

We usually use proportion instead of the probability.

Presentation



Continuous quantity

Infinite no. of possible values!!!

Class: a shorter interval in the whole range.

Class-width: the length of the class.

Frequency: no. of elements in the given class.

↓

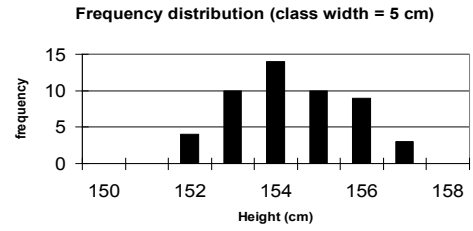
Like a discrete value!

Example

1	160 cm
2	181 cm
3	175 cm
4	163 cm
5	165 cm
6	179 cm
7	164 cm
8	185 cm
9	177 cm
10	168 cm

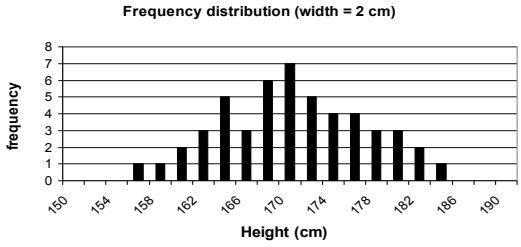
class	k_i
160-164	3
165-169	2
170-174	0
175-179	3
180-184	1
185-189	1

Presentation



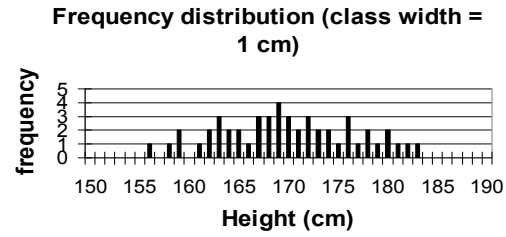
5 cm is too large!

Decrease the width!



Observation: frequency decreases!

Presentation



Reason: n is too small!

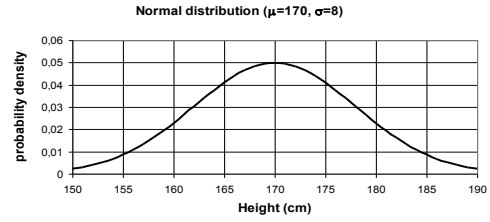
Consequence

Class-width ↓ No. of classes ↑ Frequencies ↓

We must increase the no. of the elements!

Normal distribution

If *n* and no. of classes are infinite!



Theoretical description

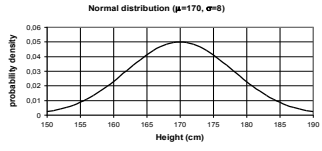
Normal or Gauss-distribution

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Parameters:

μ – expected value or mean

σ – theoretical standard deviation



Meaning of the parameters

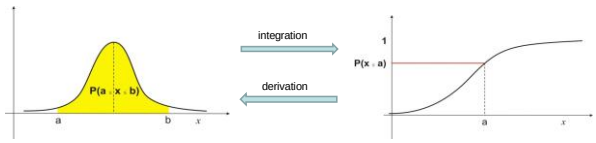
μ **(mean):**
the value belonging to the maximum of the curve.

σ (theoretical standard deviation):
the average deviation of the data from the μ.

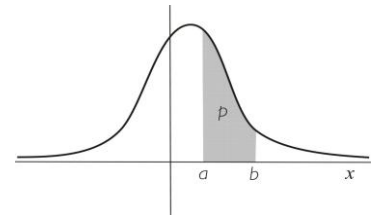
Probability density and distribution function

Probability density function

Probability distribution function



Probability



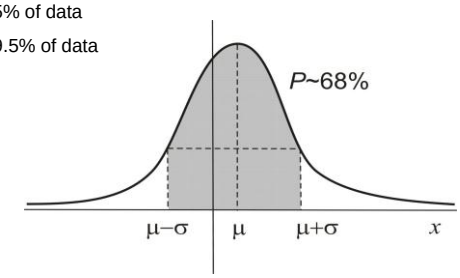
***P* is the probability that x is in the (a,b) interval.**

Standard deviation

(μ±σ) ~ 68% of data

(μ±2σ) ~ 95% of data

(μ±3σ) ~ 99.5% of data



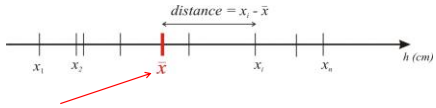
Normal distribution

Theoretical distribution that describes the population. In practice usually we don't know the parameters of this.

We usually have a ***random sample*** from the population.

We must estimate the parameters!

Estimation of the μ



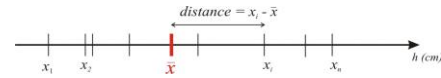
average: must be in the center of the data range.

$$\sum_i (x_i - \bar{x}) = 0 \longrightarrow \bar{x} = \frac{\sum_i x_i}{n}$$

Estimation of the σ

σ = average deviation of the data from the μ.

s (standard deviation) = average deviation of the elements from the average.



$$Q_x = \sum_i (x_i - \bar{x})^2 \geq 0$$

Standard deviation

$$s = \sqrt{\frac{Q_x}{n-1}}$$

s: the average deviation of the elements from the average.

($\bar{x} \pm s$) ~ 68%

($\bar{x} \pm 2s$) ~ 95%

($\bar{x} \pm 3s$) ~ 99.5%

Relation of parameters

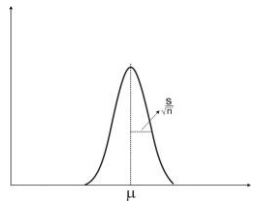
Sample	n → ∞	Population
average	⟶	μ
s	⟶	σ

μ and the average

Averages fluctuate, deviate around the μ.

Repeat the experiment!

sample	average
1	170
2	168
3	166
4	173



Standard error

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Average deviation of the averages around the μ!

Confidence interval for μ.

$$(\bar{x} \pm s_{\bar{x}}) \sim 68\%$$

~68% is the probability that μ is in this range.

(~32% that isn't)

Estimation of the μ

Average

Confidence interval

Point estimation

A simple value.

Interval estimation

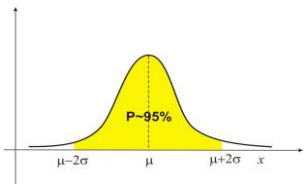
A range and the probability that the mean is in this range.

Information

	interval	probability	information content
$(\bar{x} \pm s_{\bar{x}}) \sim 68\%$	↓	↓	! ↑
$(\bar{x} \pm 2s_{\bar{x}}) \sim 95\%$			
$(\bar{x} \pm 3s_{\bar{x}}) \sim 99.5\%$			
$(\bar{x} \pm \infty) = 100\%$	∞	P =1	= 0

Normal (reference) range

Normal distribution



Other quantites

A range, in which case the probability is 95% that a value is inside the range.

But: 5% is the chance being out!!!