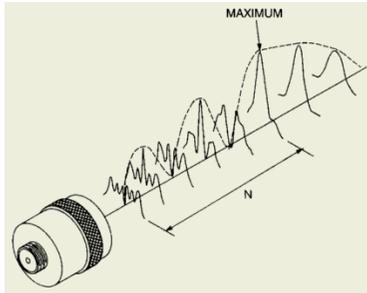
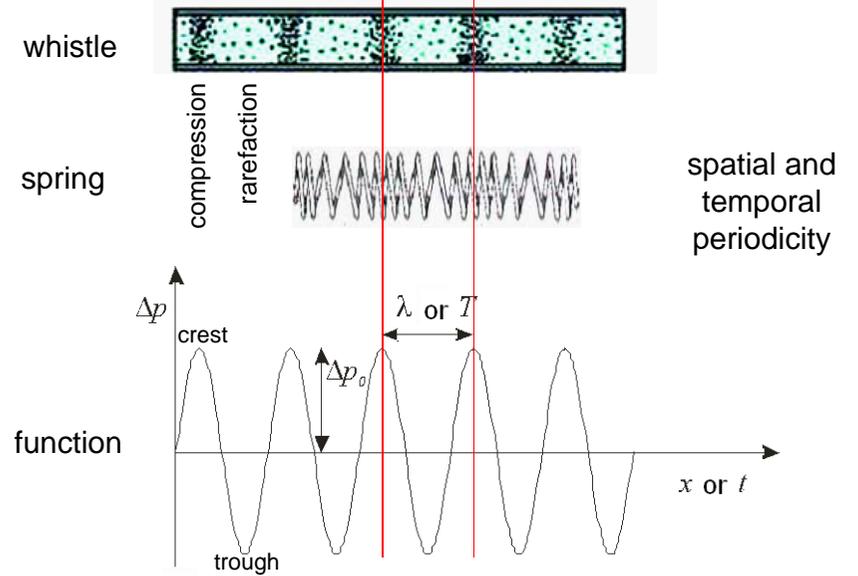


# Physics of ultrasonography



KAD 2013.02.06

# Sound: mechanical wave (model)



**longitudinal wave**  
(in the interior of liquids and gases only this type)



**transverse wave**

hydrostatic pressure      pressure change, sound pressure

$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

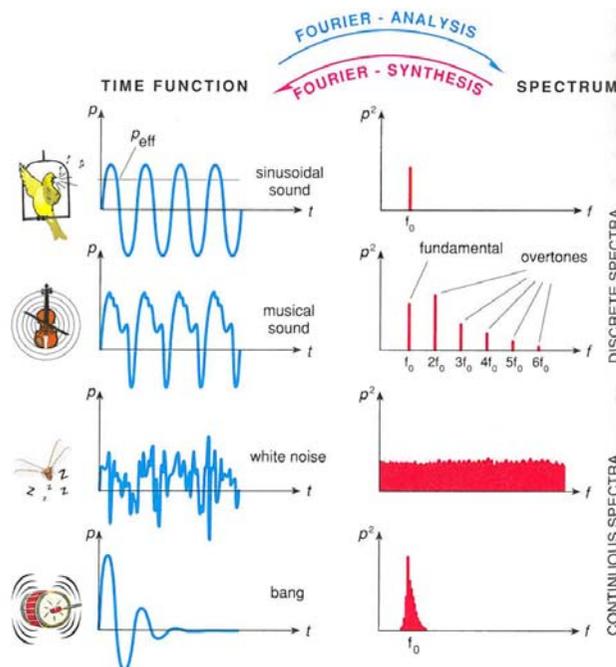
pressure DC      + AC      amplitude      phase

$$\Delta p(t, x) = \Delta p_{\text{max}} \sin \left[ 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right) \right]$$



$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$

Biophysics textbook, Fig. II.46.

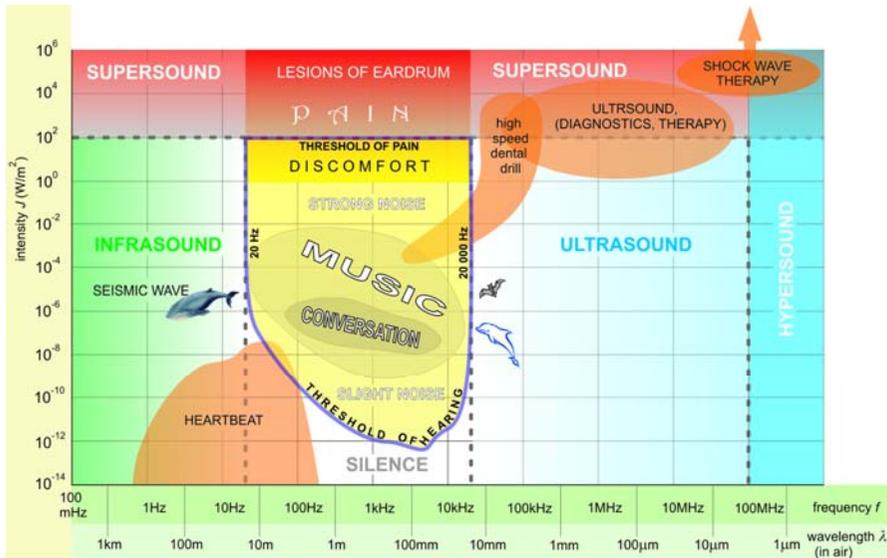


**pitch:**  
frequency of the fundamental

**timbre (tone colour):**  
relative strengths of overtones/harmonics (spectrum)

Biophysics textbook, Fig. IV.23.

# Frequency and intensity regions of sounds



Laboratory manual, Audiometry

# The role of elastic medium



$$\kappa = -\frac{\Delta V}{V \Delta p}$$

**compressibility**  
relative volume decrease over pressure

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

**speed of sound**

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

**acoustic impedance**  
(definition)

$$Z_{el} = \frac{U}{I}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

**acoustic impedance**  
(useful form)

## Supplementary material

### Acoustic impedance (way to the useful form)

$$y = y_{\max} \sin\left[\omega\left(t - \frac{x}{c}\right)\right]$$

$$p = p_{\max} \sin\left[\omega\left(t - \frac{x}{c}\right)\right]$$

$$\frac{\Delta y}{\Delta t} = v = y_{\max} \omega \cos\left[\omega\left(t - \frac{x}{c}\right)\right]$$

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

$$\frac{\Delta v}{\Delta t} = a = -y_{\max} \omega^2 \sin\left[\omega\left(t - \frac{x}{c}\right)\right]$$

$$\rho y_{\max} \omega^2 \sin\left[\omega\left(t - \frac{x}{c}\right)\right] = \frac{\Delta p}{\Delta x}$$

$$-\rho y_{\max} \omega c \cos\left[\omega\left(t - \frac{x}{c}\right)\right] = p$$

$$p_{\max} = \rho y_{\max} \omega c = \rho v_{\max} c$$

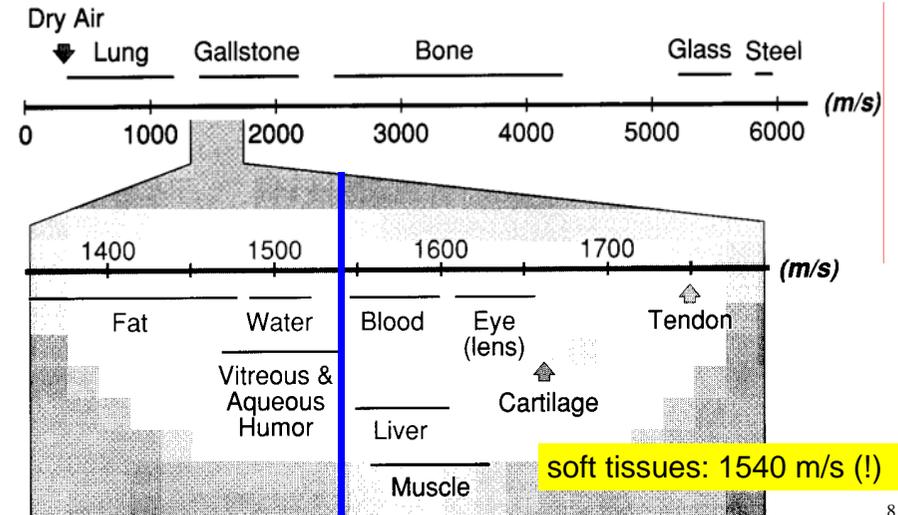
$$ma = F$$

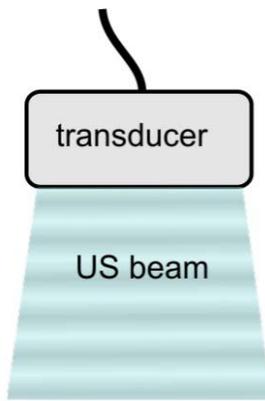
$$\frac{1}{V} ma = \frac{1}{A \Delta x} F$$

$$\rho a = -\frac{\Delta p}{\Delta x}$$

$$\frac{p_{\max}}{v_{\max}} = \rho c = Z$$

# Speed of sound/US in different media

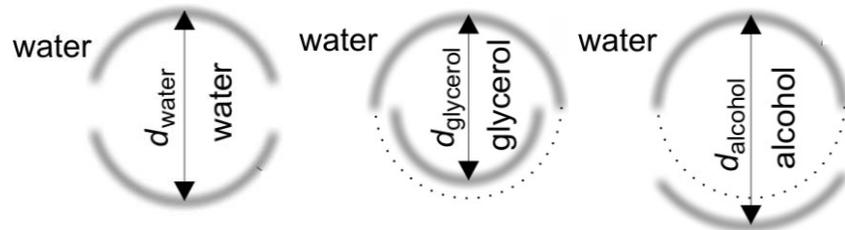




Assuming constant speed of US  
 → Artefacts

The image of the back-wall reflection appears in different distances, depending on the material in the finger of the rubber gloves

$C_{\text{water}} = 1540 \text{ m/s}$ ,  $C_{\text{glycerol}} = 1900 \text{ m/s}$ ,  $C_{\text{alcohol}} = 1200 \text{ m/s}$   
 contours of the rubber glove finger on the screen



Laboratory manual, US, Figure 15

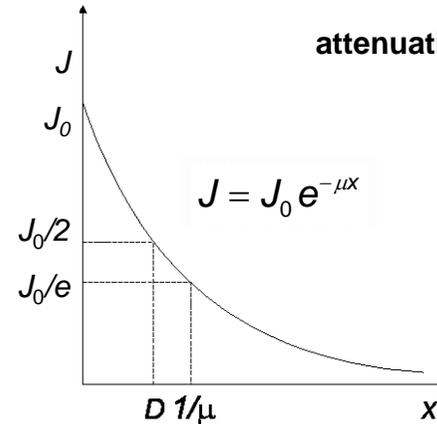
**Intensity of US**

$$J = \frac{1}{Z} \Delta p_{\text{eff}}^2$$

$$P_{\text{el}} = \frac{1}{Z_{\text{el}}} U_{\text{eff}}^2$$

intensity = energy-current density    electric analogy

**Loss of energy during propagation (absorption)**



attenuation:  $\alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

$\mu$  is proportional to frequency in the diagnostic range

specific attenuation:  $\frac{\alpha}{f \cdot x}$

$\mu$  is proportional to frequency in the diagnostic range

$$\mu \sim f^k, \quad k \sim 1(?)$$

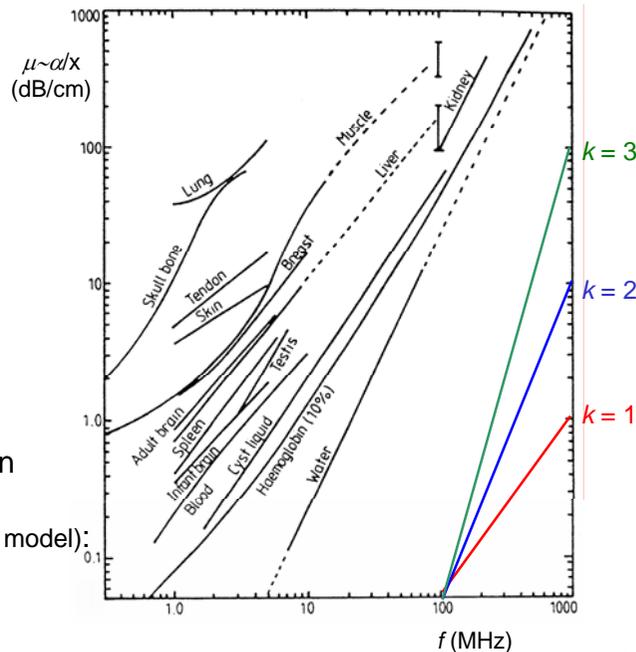
$$\log \mu \sim k \log f$$

if the graph is a linear, the power function approximation is valid

specific attenuation for soft tissues

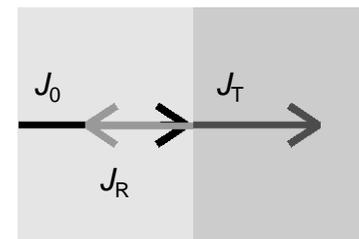
(homogeneous tissue model):

$$\frac{\alpha}{f \cdot x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$



**Phenomena at the boundary of different media**

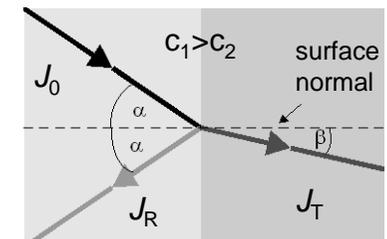
normal/perpendicular incidence



$$J_0 = J_R + J_T$$

reflection and transmission (penetration)

skew incidence



$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

## Reflection (normal incidence)

reflectivity:

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left( \frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

boundary surface	R
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

“full” reflection:

$$Z_1 \ll Z_2, \quad R \approx 1$$

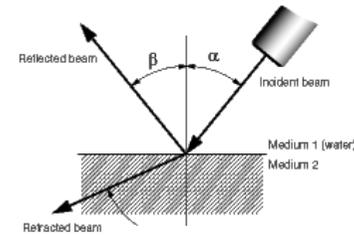
optimal coupling:

$$Z_{\text{connecting}} \approx \sqrt{Z_{\text{source}} Z_{\text{skin}}}$$



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Supplementary material



Skew incidence

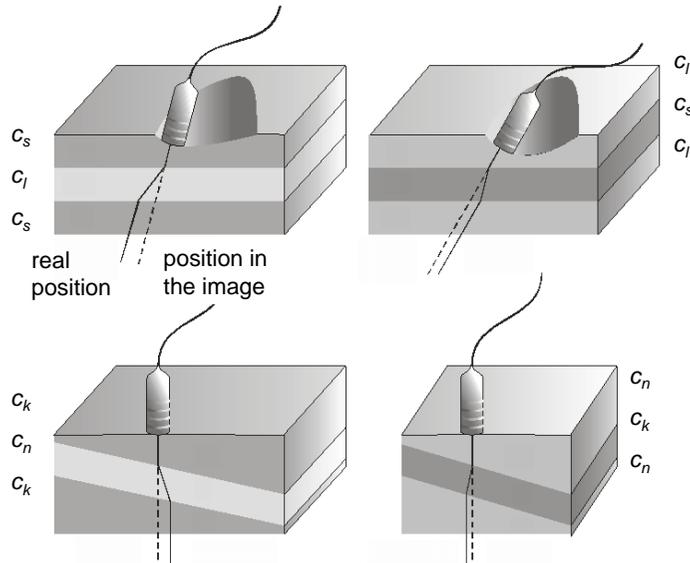
$$\text{Reflexion coefficient} = R = \left[ \frac{Z_2 \cos \alpha - Z_1 \cos \gamma}{Z_2 \cos \alpha + Z_1 \cos \gamma} \right]^2$$

$$\text{Refraction coefficient} = D = \frac{4 Z_1 Z_2 \cos^2 \alpha}{(Z_2 \cos \alpha + Z_1 \cos \gamma)^2}$$

$$\text{Refraction angle} = \gamma = \text{asin} \left( \frac{c_2 \sin \alpha}{c_1} \right)$$

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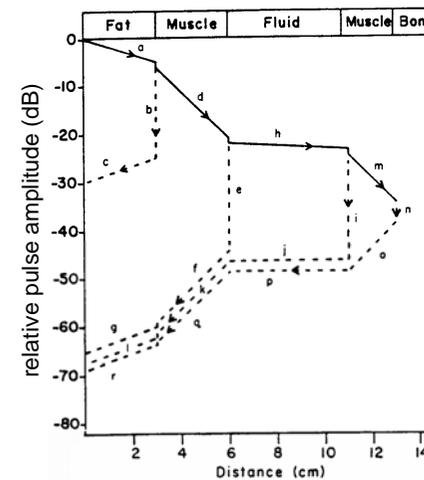
## Phenomenon of skew incidence or normal incidence and skew boundaries



Biophysics textbook, Fig. on pg. 153

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## Absorption and reflection



the later comes back the reflection, the deeper lays the reflecting surface and the weaker is the intensity

run time dependent amplification

TGC: time gain compensation

DGC: depth gain control

boundary surface	R	10lgR (dB)	T	10lgT (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

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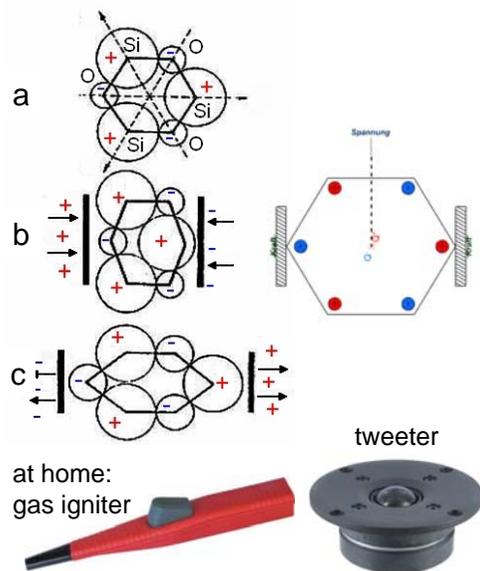
## Generation of US. Piezoelectric effect

production: inverse ~  
detection: direct ~

source of electric signal  
(sine wave oscillator)+  
transducer (piezo-crystal)

(a) Center of charge of  
positive and negative  
charges coincides.

(b) and (c) As a result of  
pressure, the charge  
centers are separated, i.e.  
a potential difference  
arises (direct ~).  
The crystal is deformed  
when voltage is applied  
(inverse ~).

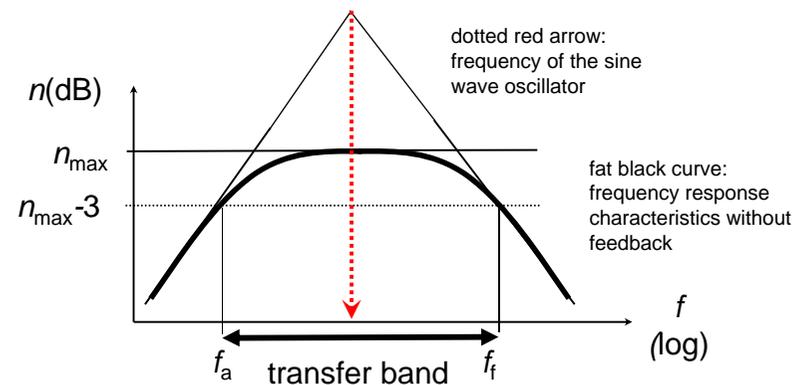


## Source of electric signal : sine wave oscillator

amplifier with positive  
feedback

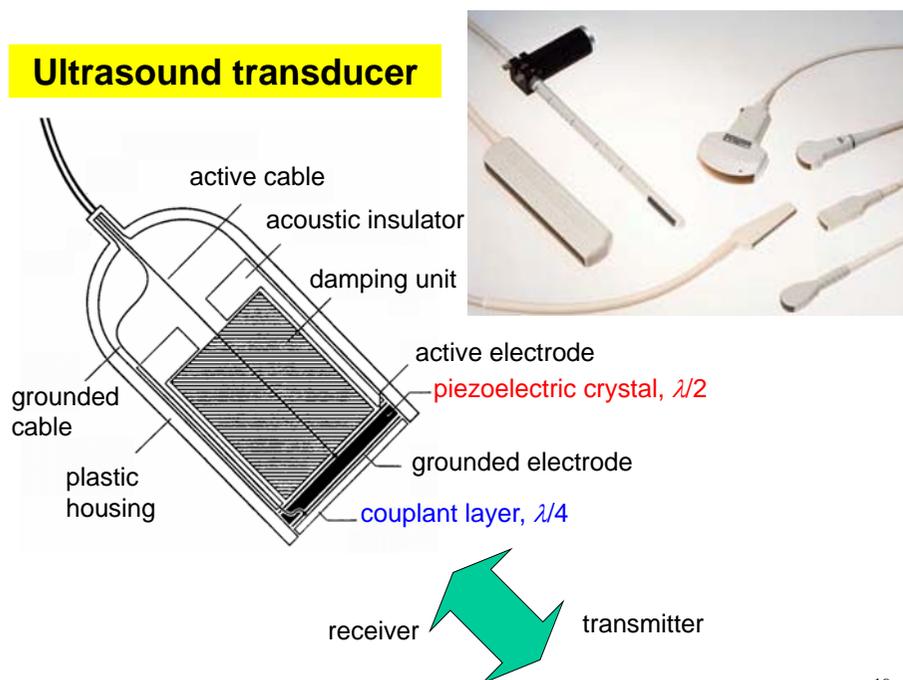
$$A_{U, \text{feedback}} = \frac{A_U}{1 - \beta A_U}$$

$\beta A_U = 1$ , amplification = „infinity“ → sine wave oscillator  
no input signal, output signal: sine voltage



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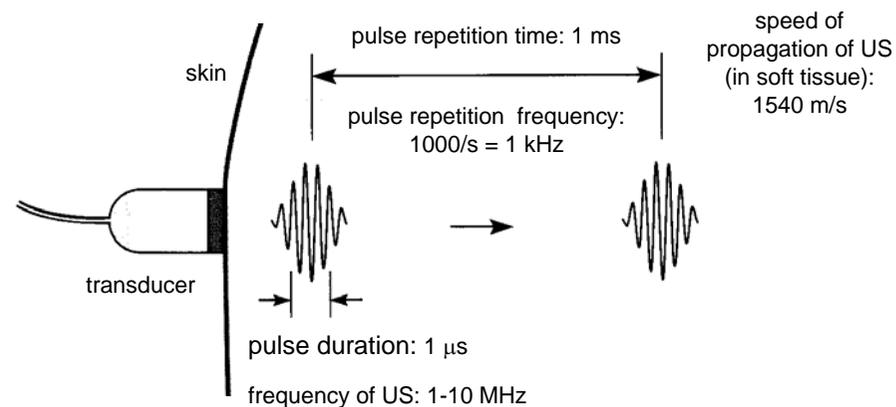
## Ultrasound transducer



19

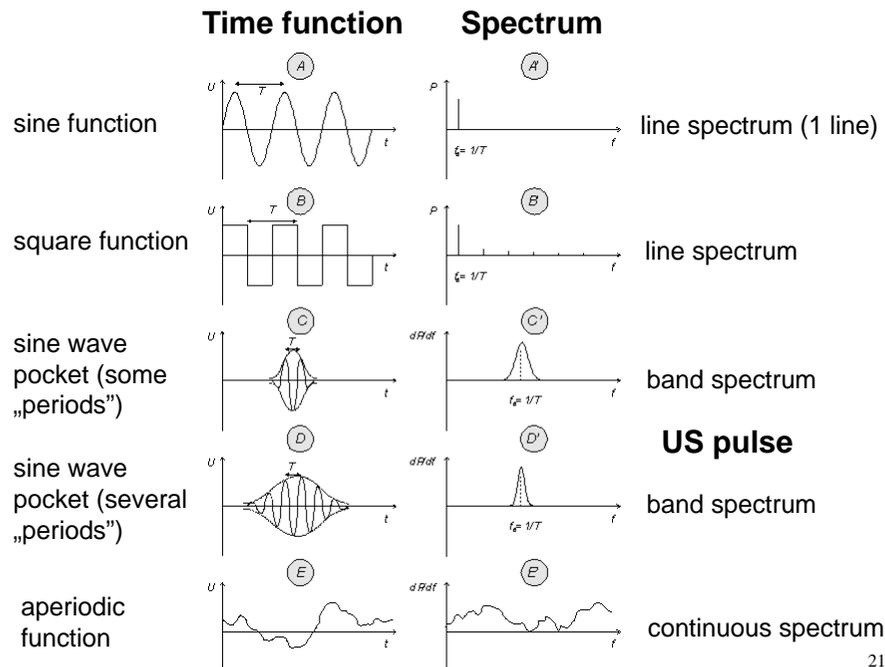
## Characteristic of US pulses

transducer: transmitter and receiver is the same unit  
**time sharing** mode: pulses instead of continuous wave US

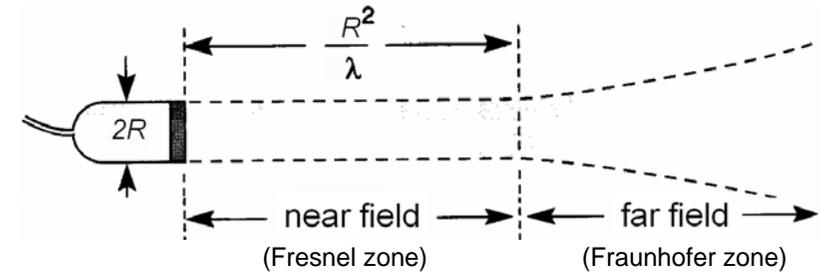


Biophysics textbook, Fig. VIII.32.

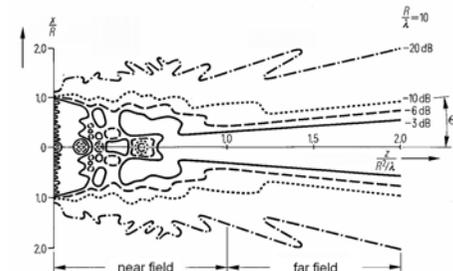
20



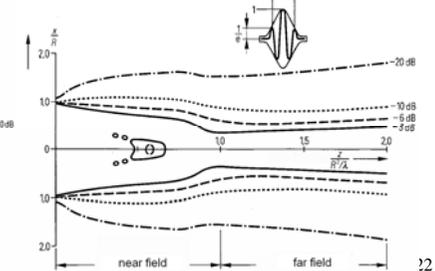
### US beam shape (simplified version)



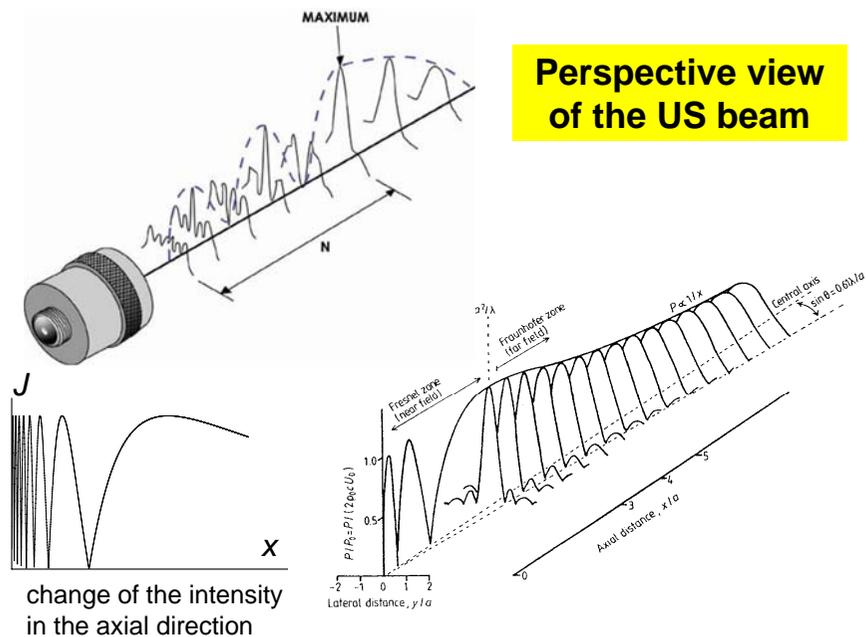
Beam shape, continuous wave US



Beam shape, pulsed wave US



### Perspective view of the US beam



### Resolving limit, resolution

**Resolving limit** is the distance between two object details which can be just resolved as distinct objects (the smaller the better).

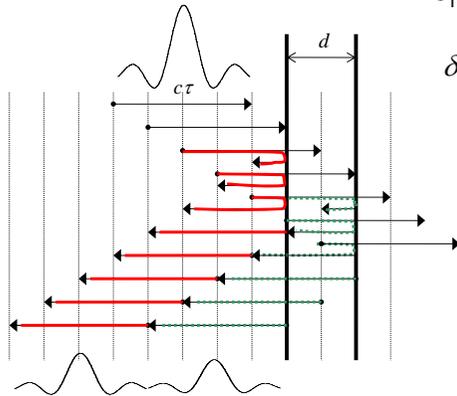
**Resolution (resolving power):** the reciprocal of the resolving limit (the greater the better)

**Axial resolving limit** depends on the pulse length. Pulse length is inversely proportional to the frequency.

**Lateral resolving limit** is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

Typical values	frequency (MHz): 2	15
wavelength (in muscle) (mm):	0.78	0.1
penetration depth (cm):	12	1.6
lateral resolving limit (mm):	3.0	0.4
axial resolving limit (mm):	0.8	0.15

## Axial resolving limit



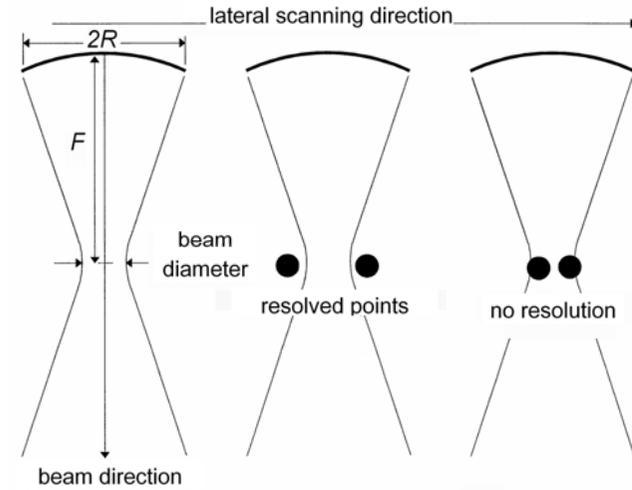
$\tau$  : pulse duration  
 $c_1\tau \cong c_2\tau = c\tau$  pulse length  
 $\delta_{ax} = d = \frac{c\tau}{2}$  resolving limit

The axial resolving limit is the half of the pulse length. The echoes from the adjacent surfaces in this case just hit another.

$$\tau \sim T = \frac{1}{f}$$

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## Lateral resolving limit

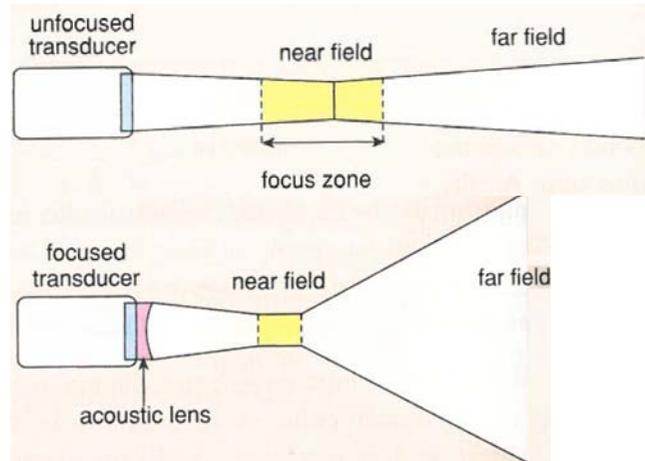


$$\left( \delta_{lat} \sim \frac{F}{2R} \cdot \lambda \right)$$

$F$ : focal length  
 $2R$ : diameter of the transducer  
 $\lambda$ : wavelength

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## Focusing of the beam

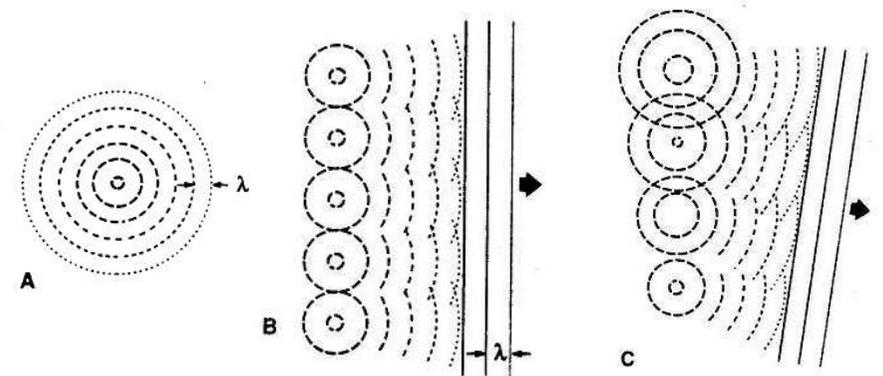


Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

cf. Textbook Fig. on p.506

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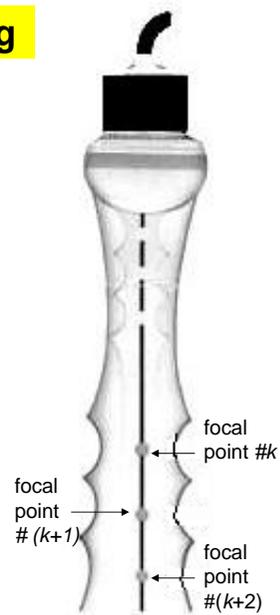
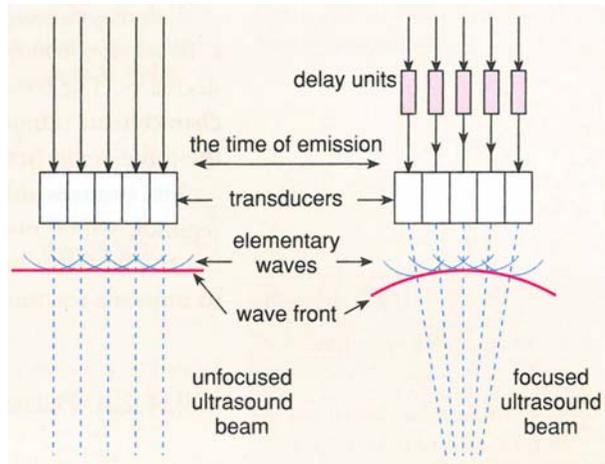
## Huygens' principle



Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.

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## Electronic focusing

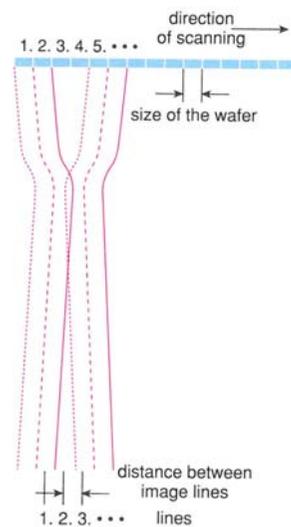


29

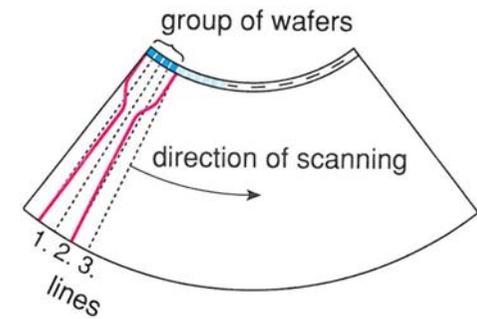
cf. Textbook Fig. on p.507

## Scanning

multi unit linear array



multi unit curved array



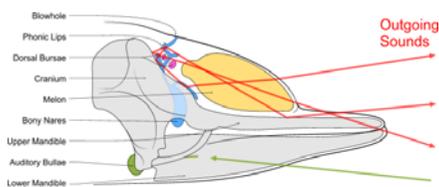
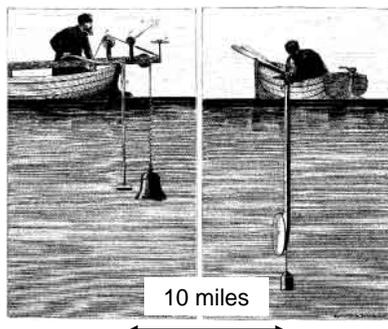
cf. Textbook Fig. VII. 36-37

30

## Echo principle

1794 Spallanzani:  
bat's navigation

1822 Colladen  
measured the speed of  
sound in water

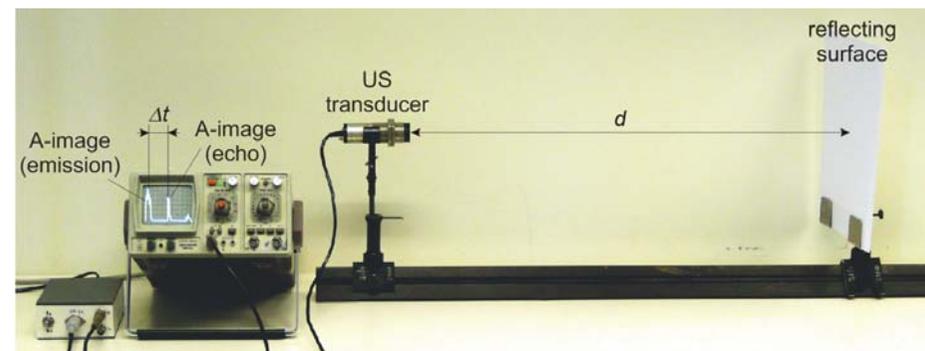


bottlenose dolphin

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## Echo principle

using a special US-head, short pulses are emitted in the air towards a reflecting surface, and the same US-head detects the echo signal

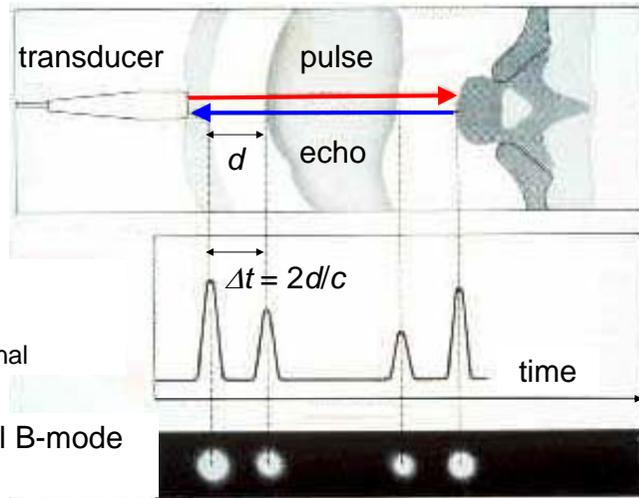


$$c\Delta t = d + d = 2d$$

32



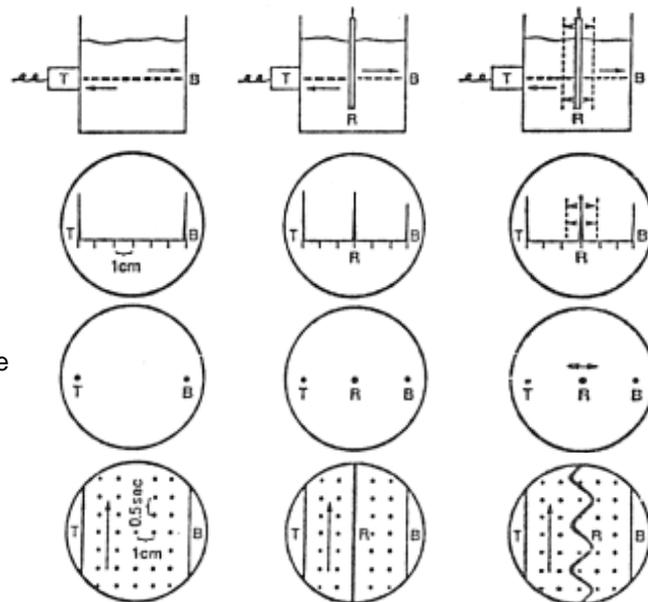
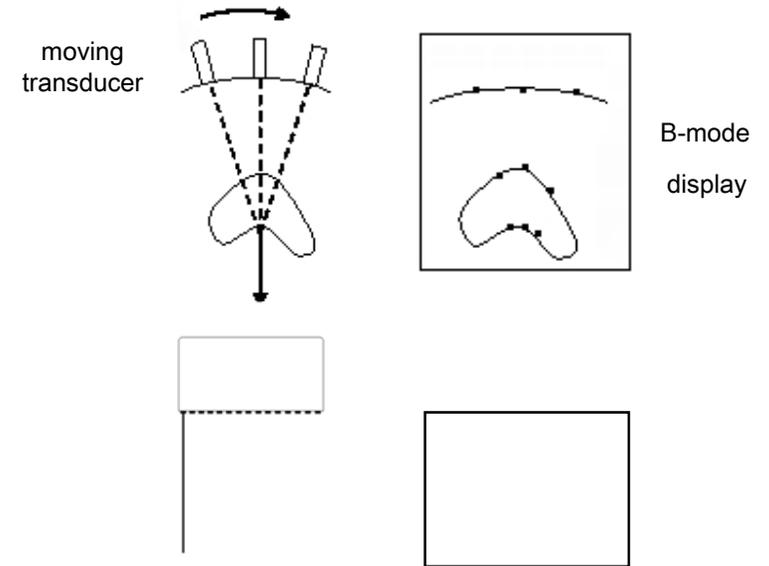
## Receiving the echos



A-mode  
(Amplitude)  
only 1-dimensional

1-dimensional B-mode  
(Brightness)

## 2-dimensional B-mode

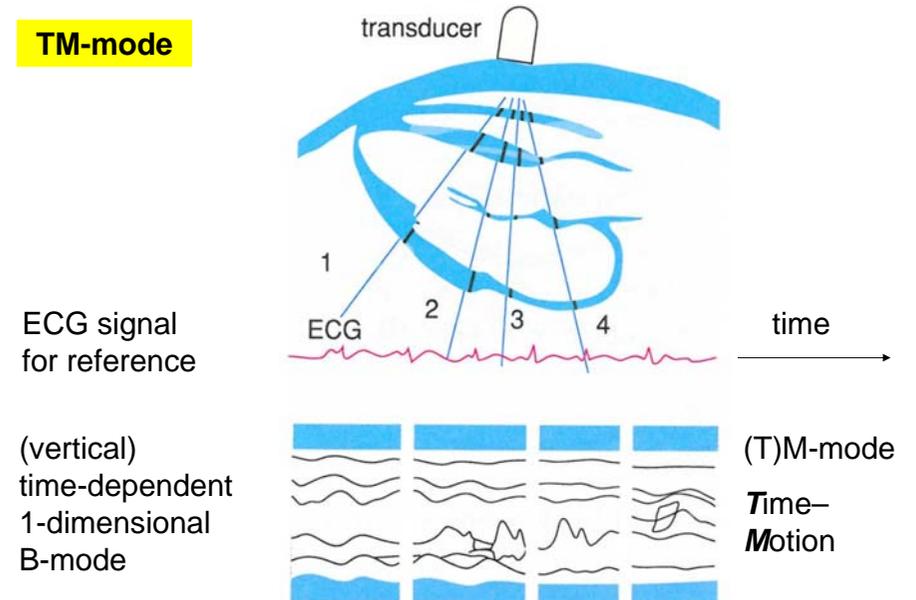


A- mode

1D B- mode

TM- mode

## TM-mode



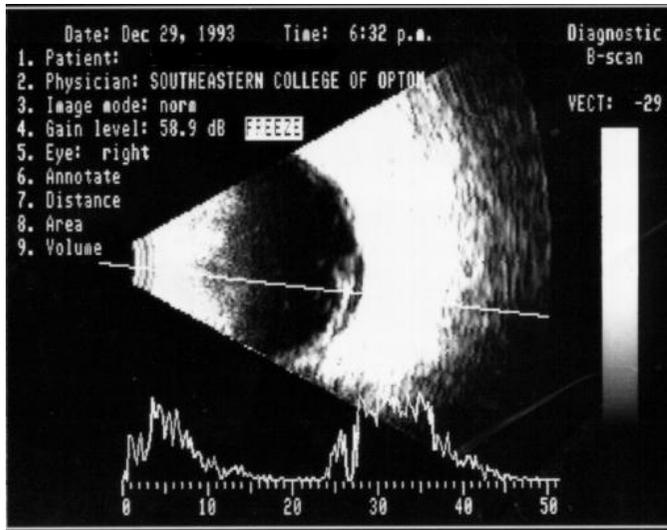
ECG signal  
for reference

(vertical)  
time-dependent  
1-dimensional  
B-mode

time

(T)M-mode  
**Time-  
Motion**

## 2-dimensional B-mode and A-mode (used in ophthalmology)



real speed of propagation for the accurate determination of distances:

cornea: 1641 m/s

aqueous humour: 1532 m/s

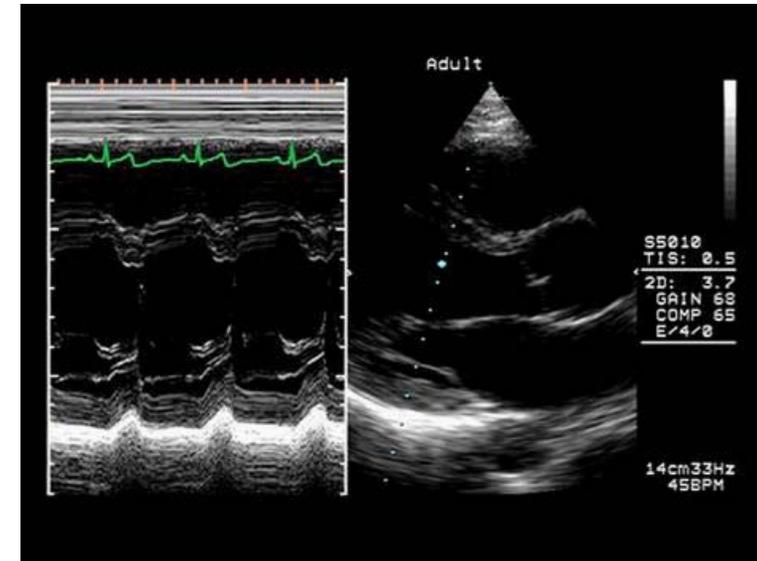
crystalline lens: 1641 m/s

vitreous body: 1532 m/s

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TM-mode

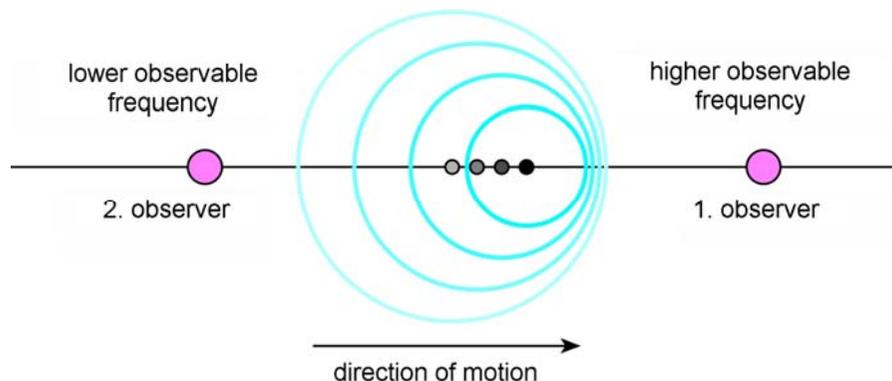
B-mode



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## Doppler phenomenon

„The pitch of a train whistle seems to get higher as it approaches, then seems to lower as the train whistle moves away.” (C. Doppler, 1842)



Teextbook Fig. VIII.39

39

$f'$ : **observed frequency**,  $f$ : original frequency

- (a) standing source and moving observer ( $v_o$ )  
 +: observer approaches the source  
 -: observer moves away from the source

$$f' = f \left( 1 \pm \frac{v_o}{c} \right)$$

- (b) moving source and standing observer  
 (if  $v_s \ll c$ , then „same” as (a))

$$f' = \frac{f}{1 \mp \frac{v_s}{c}}$$

- (c) moving source and moving observer

$$f' = f \frac{1 \pm \frac{v_o}{c}}{1 \mp \frac{v_s}{c}}$$

- (d) moving reflecting object (surface),  
 (if  $v_R \ll c$ )

$$f' = f \left( 1 \pm \frac{2v_R}{c} \right)$$

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**Doppler frequency** = frequency change = frequency shift

if  $v_i, v_R \ll c$  (i= S or O)

rearranging equation (a)  
**moving source or observer:**

$$\Delta f = f_D = \pm \frac{v_i}{c} f$$

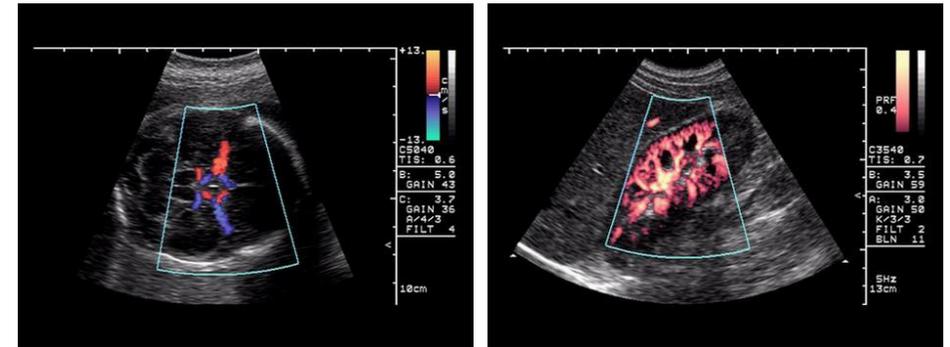
rearranging equation (d)  
**moving reflecting object or surface:**

$$\Delta f = f_D = \pm 2 \frac{v_R}{c} f$$

if  $v$  and  $c$  are not parallel, then  $v \cos \theta$  should be used instead of  $v$  (remark: if  $\theta = 90^\circ$ ,  $f_D = 0$ )

## Colour coding

towards the transducer: warm colours  
 away from the transducer: cold colours



BART: **B**lue **A**way **R**ed **T**owards

power Doppler

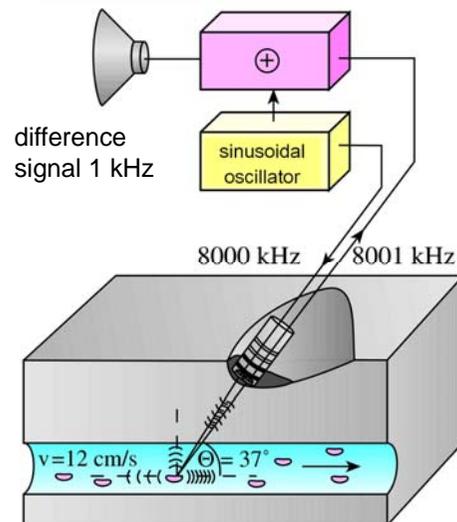
## 1-dimensional CW Doppler apparatus for measuring average flow velocity. Red blood cells as sound scatterers

CW: continuous wave  
 source and detector are separated

$$|f_D| = 2 \frac{v_R \cos \theta}{c} f$$

e.g.  $f=8000$  kHz  
 $v=12$  cm/s  
 $c=1600$  m/s  
 $\theta = 37^\circ$

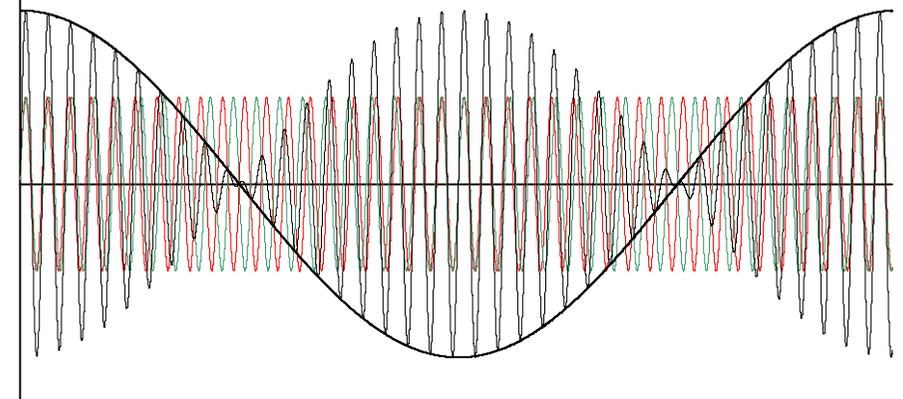
$\Rightarrow f_D=1$  kHz  
 (beating phenomenon)



Textbook Fig. VIII. 41

## Beating phenomenon

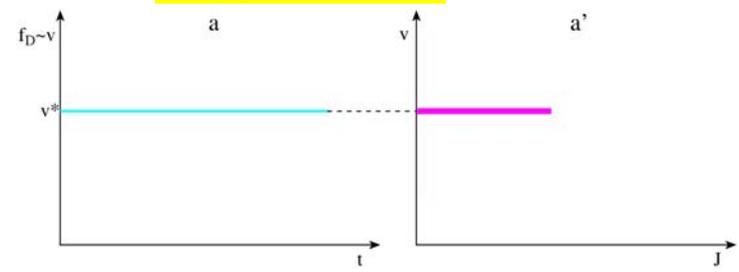
$f_{red} \geq f_{green}$  the beating frequency equals to the difference of the two interfering frequency



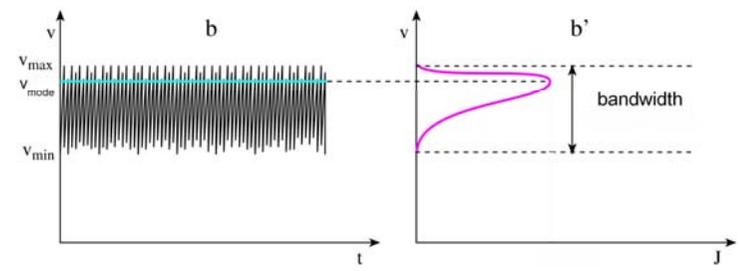
reminder:  $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$

## Doppler curves

one constant velocity ( $v^*$ )



frequency distribution (with  $v_{mode}$ )

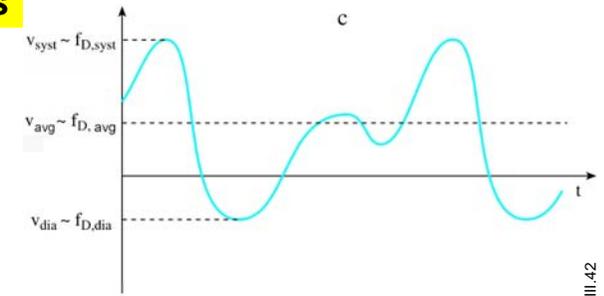


velocity distribution in TM-mode

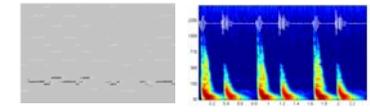
velocity distribution at a certain time

## Doppler curves

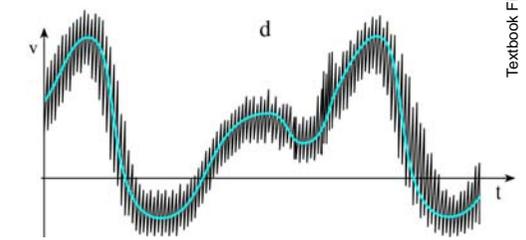
flow can be represented by one velocity in each moment



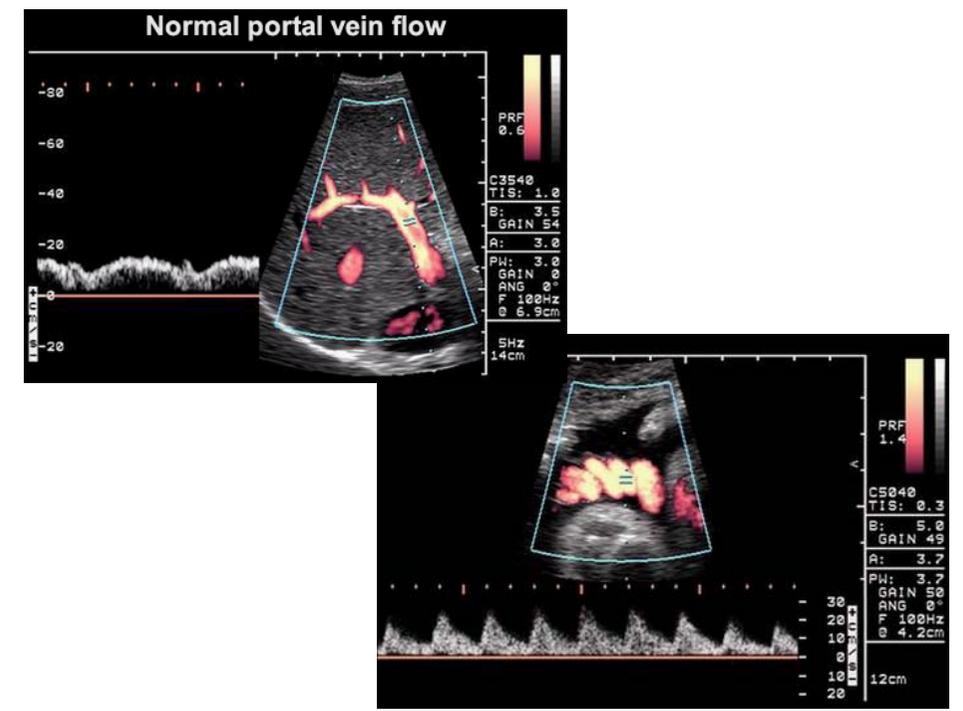
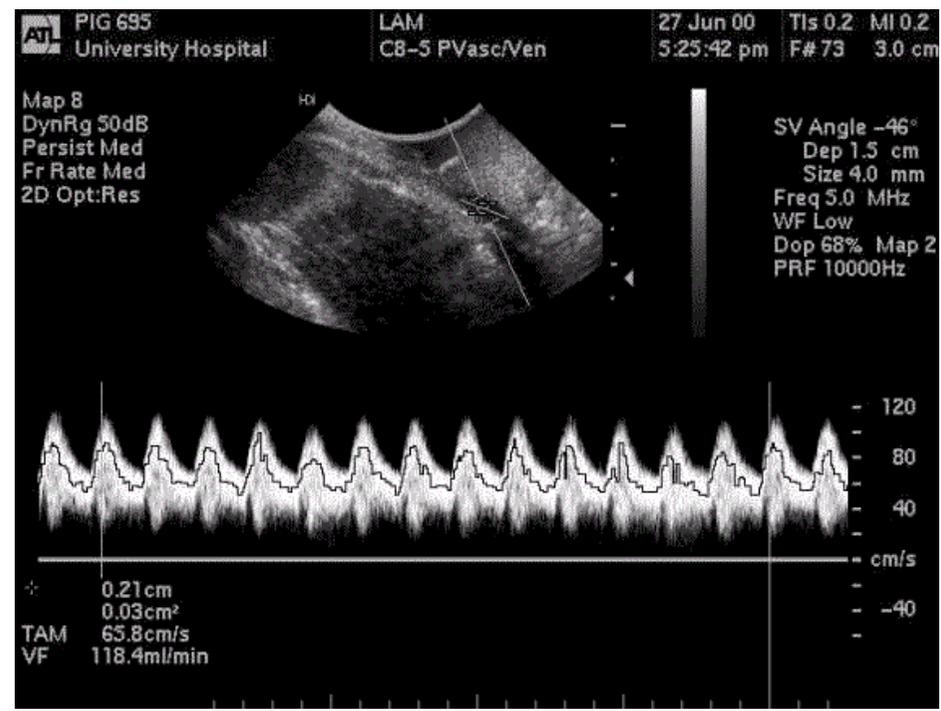
flow can be represented by a velocity distribution in each moment



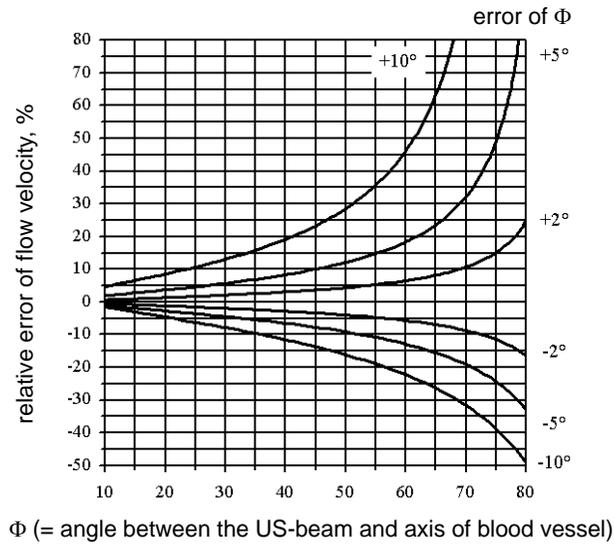
cf. voiceprint, music/heart beats in time-frequency representation



velocity distribution in TM-mode



Error of measuring the angle between the US-beam and axis of blood vessel influences the error of the flow velocity

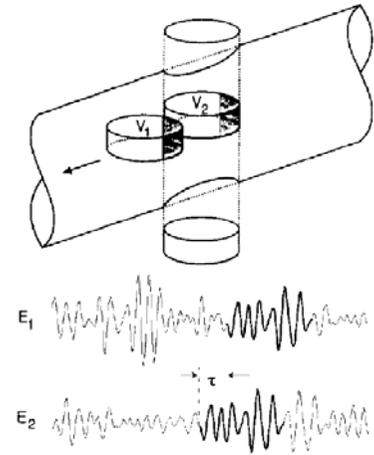


### Time domain correlation method (CVI = color velocity imaging)

If the reflecting surface and/or the scatterer are moving then the US signal at a fixed position depends on time.

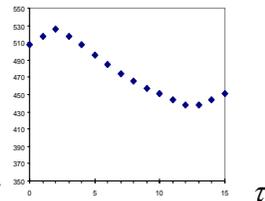
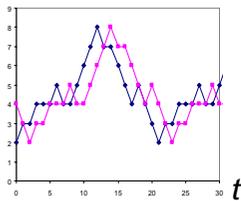
Similar US pattern can be measured at a certain distance from the earlier position (where the reflecting surface/scatterer moved).

How can be compared the similarity of different functions?



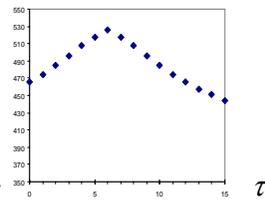
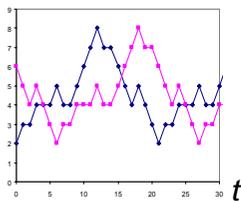
time domain

correlation function

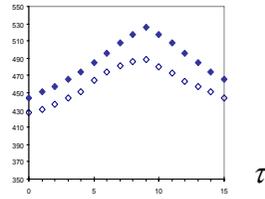
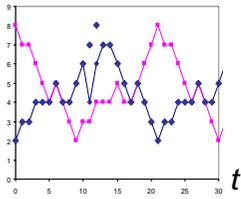


$$f_{\text{blue}}(t) = f_{\text{pink}}(t + \tau^*)$$

$$\tau^* = 2u$$

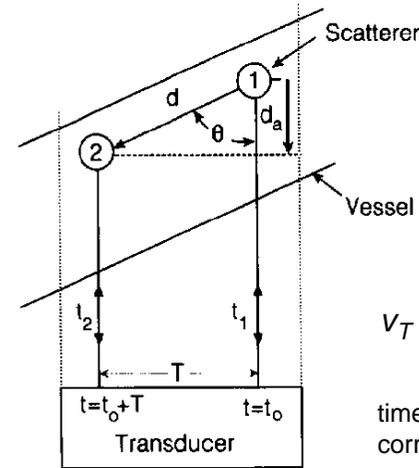


$$\tau^* = 6u$$



$$\tau^* = 9u$$

$$f_{\text{empty}}(t) \cong f_{\text{blue}}(t)$$



$$d_a = \frac{(t_1 - t_2)c}{2}$$

$$d = \frac{(t_1 - t_2)c}{2 \cos \theta}$$

$$v_T = \frac{(t_1 - t_2)c}{2T \cos \theta}$$

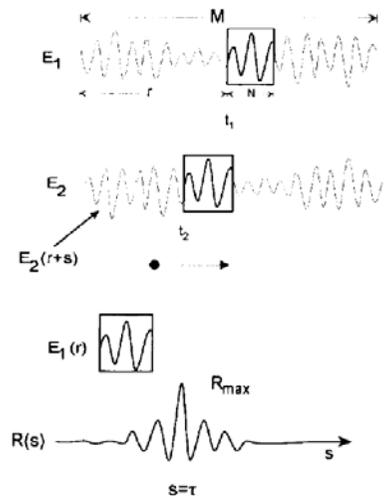
$$v_D = \frac{f_D c}{2f \cos \theta}$$

time domain correlation method

cf.: Doppler method

$T$ : pulse repetition time

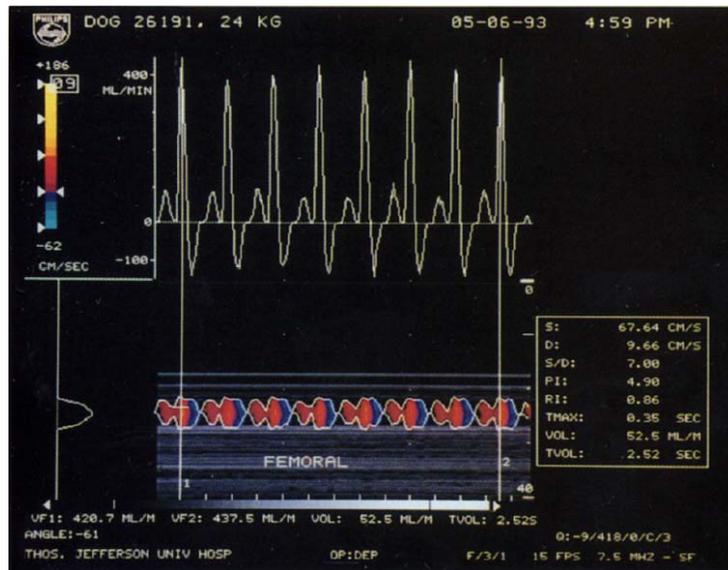
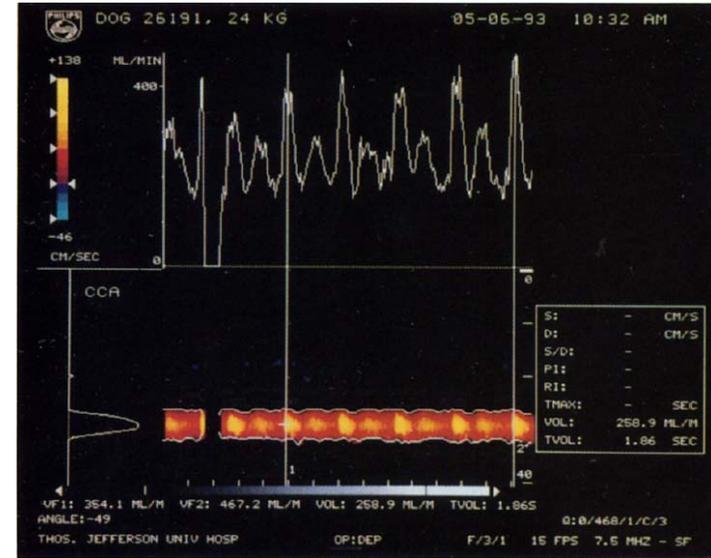
$f$ : frequency of US



correlation procedure consists of removing a window of width  $N$  at desired range from on echo signal  $E_1$

$E_1$  is correlated at different locations along another echo signal  $E_2$

the value of  $s$  producing the maximum corresponds to  $s = \tau$



## Sono-CT

image reconstruction from several multidirectional B-images

SonoCT yields better results than conventional B-mode sonography in terms of delineation, visualization of borders and artefact-free representation

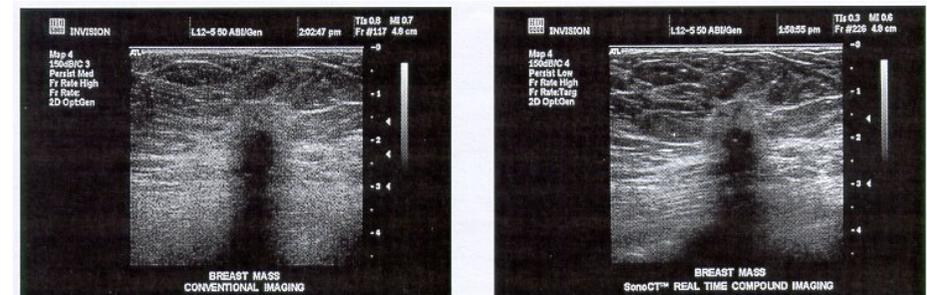
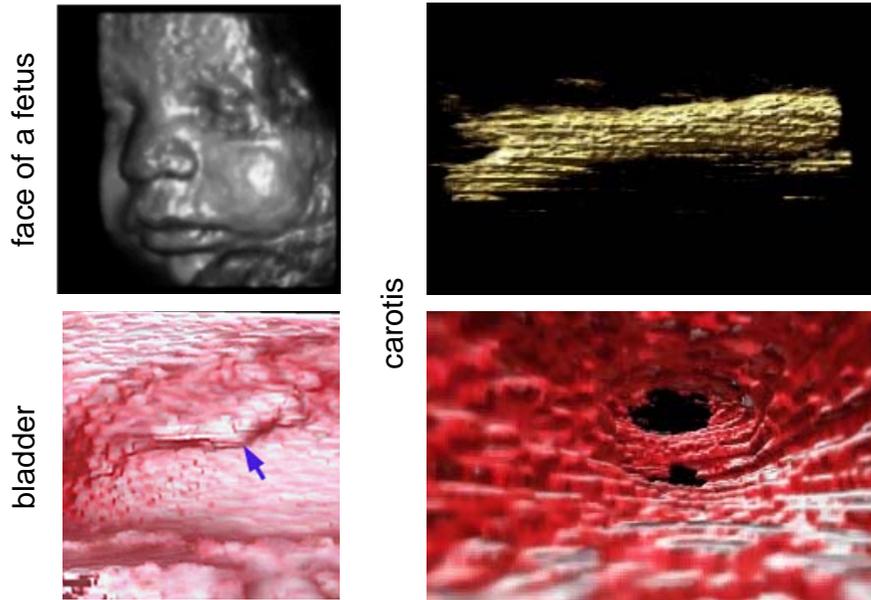


Figure 1. Breast fibroadenoma as shown on conventional ultrasound (left) and SonoCT ultrasound imaging (right).

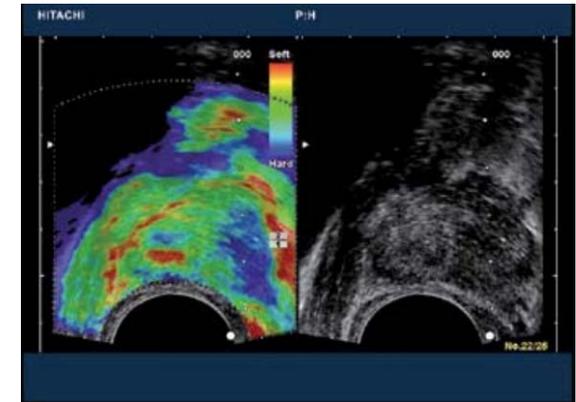
### 3D reconstruction



### Sonoelastography

**palpation:**  
one of the oldest clinical skills,  
**tissue elasticity**

**strain:**  
a region of tissue is compressed and the **degree to which it distorts** is assessed



Prostate cancer. The right panel shows the B-mode frame of a prostate ultrasound scan with the corresponding elastography frame on the left. The colour codes (here) blue for hard and red for soft.

This hard lesion, which is not apparent on the B-mode, was a carcinoma on biopsy

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#### *Cytoskeletal filaments as rigid bodies*

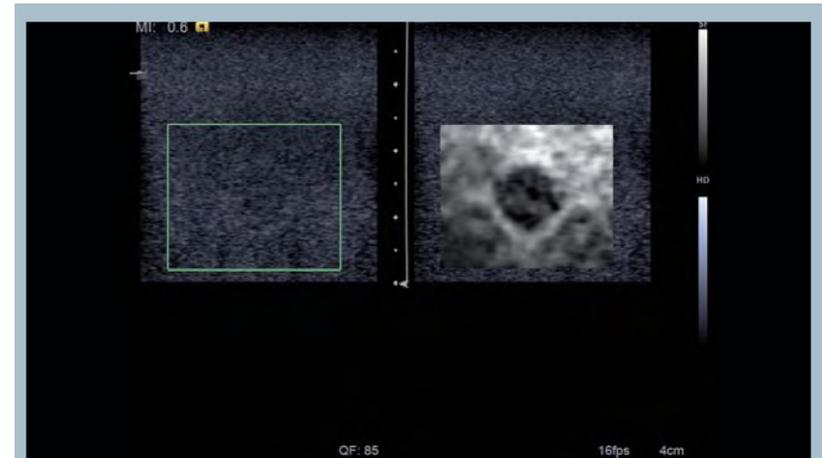
In the first approximation, cytoskeletal filaments may be described as rigid bodies, which are deformed upon the action of external force. The deformation occurs in the direction of the applied force. The elasticity of rigid bodies is described by Hooke's law:

$$\frac{F}{A} = E \frac{\Delta L}{L}, \quad (\text{V.1})$$

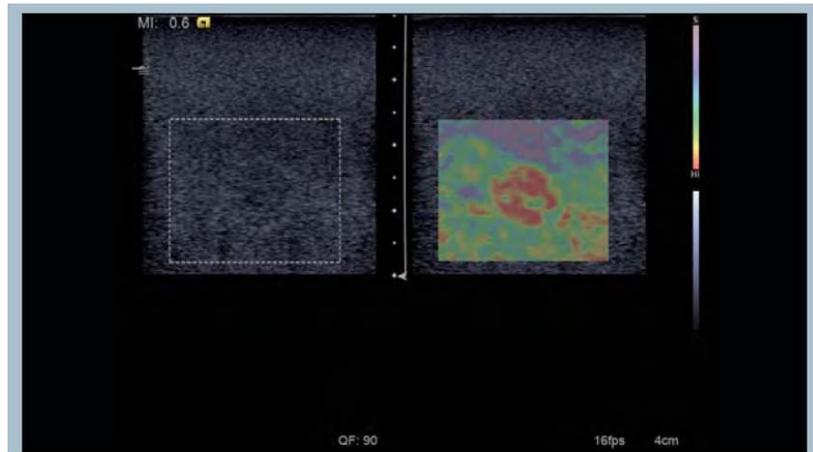
where  $F$  is force,  $A$  is cross-sectional area,  $L$  is resting length, and  $\Delta L$  is extension. The ratio  $F/A$  is stress ( $\sigma$ ),  $E$  is the Young modulus and  $\Delta L/L$  is relative extension or strain ( $\epsilon$ ). The ratio of stress and strain is known as the Young modulus (elastic modulus), the dimension of which is pressure (Pa). In the case of homogenous bodies the Young modulus is characteristic of the material properties only and is independent of shape and size.

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#### Supplementary material

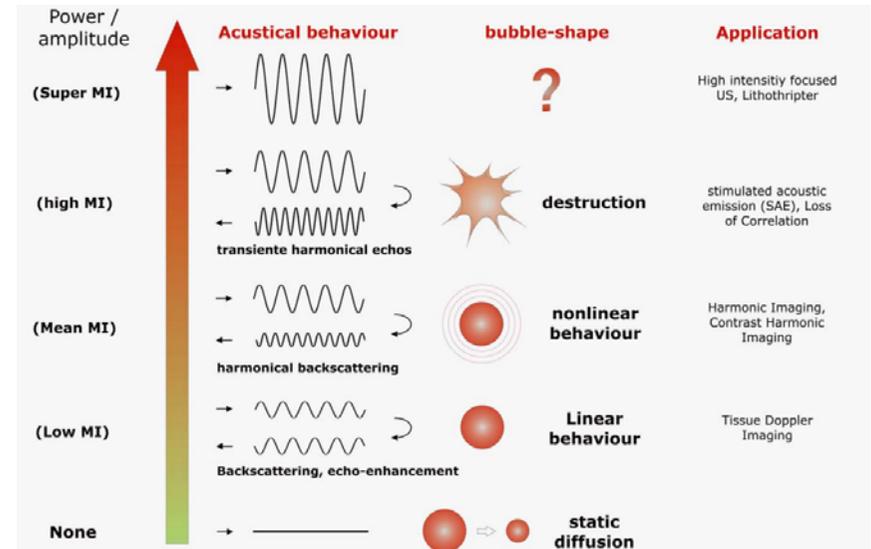


Grayscale eSie Touch Elasticity Imaging demonstrates a lesion which is more stiff (black) than the surrounding tissue. The conventional B-mode ultrasound is the same as displayed in the previous image.

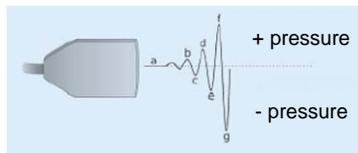


Color scale eSie Touch Elasticity Imaging demonstrates a lesion which is more stiff (red) than the surrounding tissue.

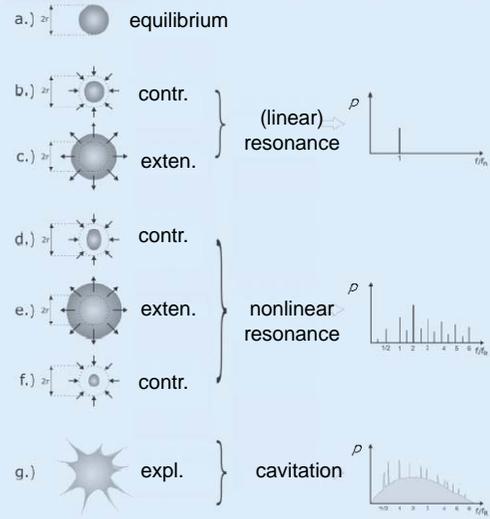
## Contrast agents



## Contrast harmonic imaging = CHI



Behaviour of microbubbles exposed to pulsed US



C. Kollmann · M. Putzer  
**Ultraschallkontrastmittel –  
 physikalische Grundlagen**  
 Radiologe 2005 · 45:503–  
 512

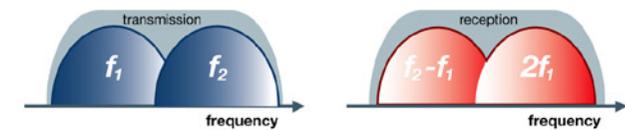
## Tissue harmonic imaging = THI

Principle:

- Simultaneous transmission of 2 pulses at different frequencies
- Reception of signals at harmonic and differential frequencies
- Cancellation of fundamental signals using Pulse Subtraction

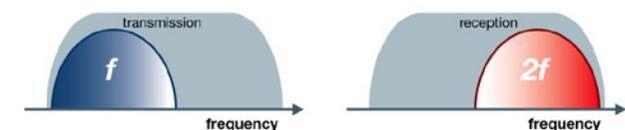
Toshiba

Differential Tissue Harmonic Imaging



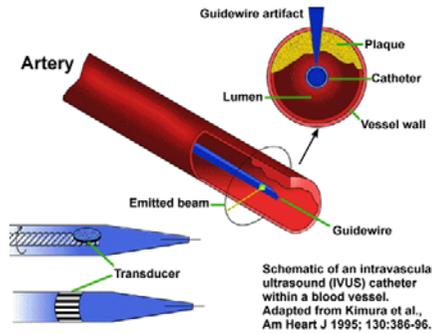
Philips

All Other Tissue Harmonic Imaging Methods



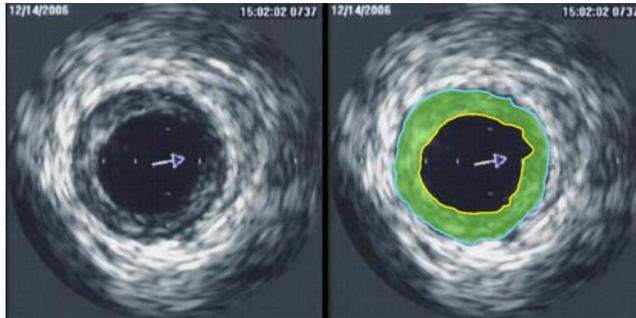
## Intravascular ultrasound (IVUS)

20-40 MHz, frame rate: 30 Hz

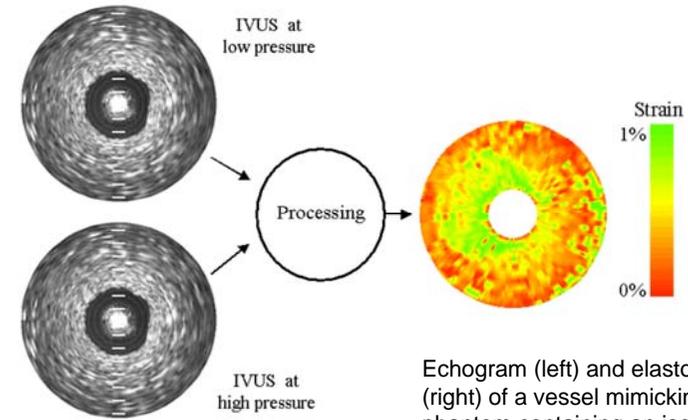


lumen of the coronary artery: yellow

outer elastic membrane: blue



## Intravascular sonoelastography



Echogram (left) and elastogram (right) of a vessel mimicking phantom containing an isoechoic soft lesion between 7 and 11 o'clock. The lesion is invisible in the echogram, while it is clearly depicted in the elastogram

## Safety

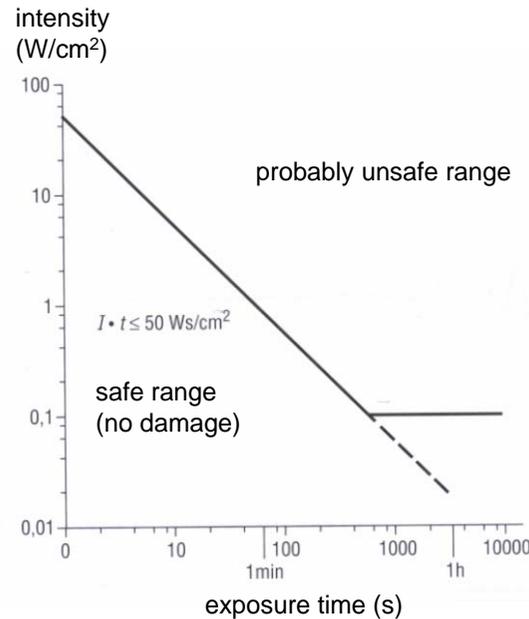
in the diagnostics:

$10 \text{ mW/cm}^2 = 100 \text{ W/m}^2$

cf. pain threshold:  $10 \text{ W/m}^2$

in the therapy:  $1 \text{ W/cm}^2$

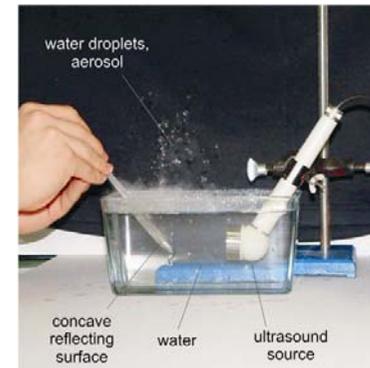
spatial average temporal average (SATA) intensity;  
 spatial peak temporal peak (SPTP) intensity;  
 spatial peak temporal average (SPTA) intensity;  
 spatial peak pulse average (SPPA) intensity  
 spatial average pulse average (SAPA) intensity



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more:

in practice



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