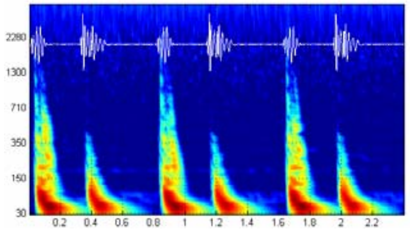
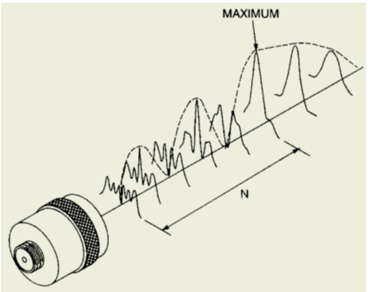
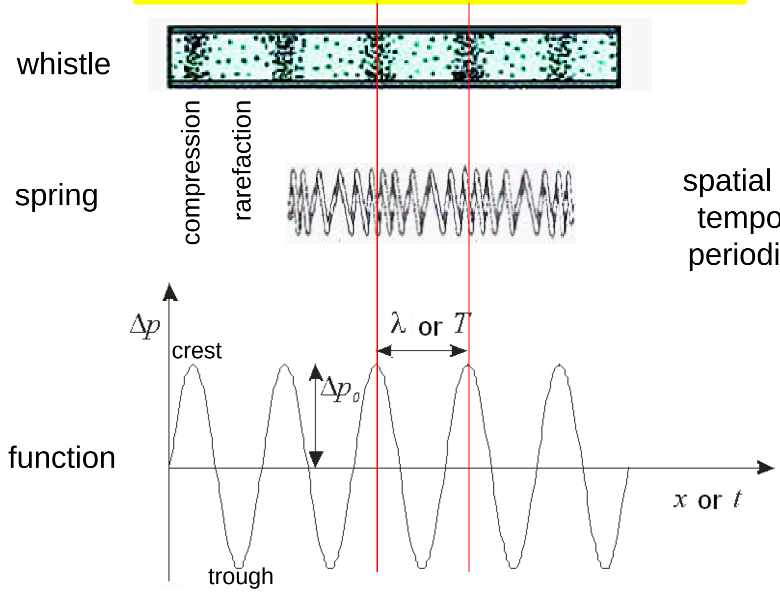
Physics of ultrasonography



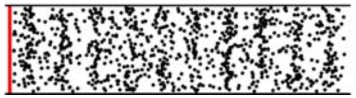


KAD 2013.02.13

Sound: mechanical wave (model)



spatial and temporal periodicity



longitudinal wave
(in the interior of liquids and gases only this type)

transverse wave

hydrostatic pressure   pressure change, sound pressure

$p_{total} = p_{hydrostat} + \Delta p$

pressure DC + AC

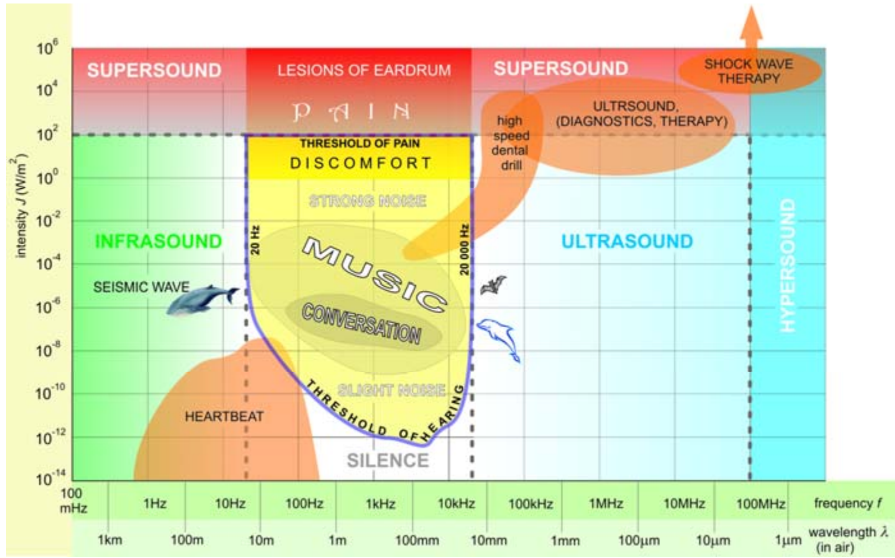
amplitude   phase

$$\Delta p(t,x) = \Delta p_{max} \sin\left[2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\right]$$

$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$

Biophysics textbook, Fig. II.46.

Frequency and intensity regions of sounds



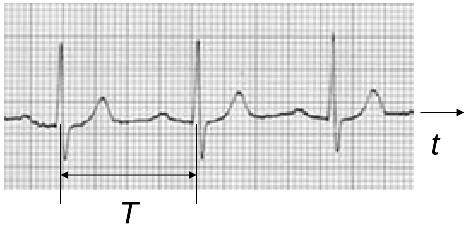
Laboratory manual, Audiometry

Fourier's theorem for periodic functions (signals)

all periodic functions can be expressed as a sum of sine (cosine) functions from the fundamental frequency and the overtones

periodic function:
there is a period, *T*

$\frac{1}{T} = f,$ where *f* is the frequency



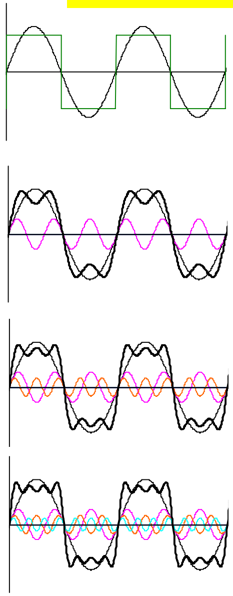
the sine function, which has the same frequency as the periodic function: **fundamental frequency**

2*f*, 3*f*, 4*f*, ... : **overtones** (line spectrum)

Fourier'theorem for non-periodic functions (signals)

all functions can be expressed as a sum of sine (cosine) functions (spectrum: continuous)

function | spectrum

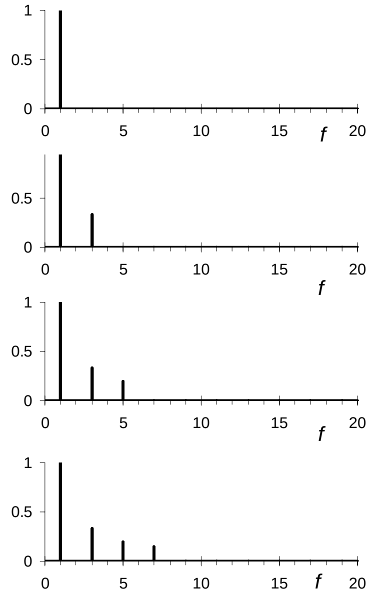


square pulse train
fundamental fr(equency)

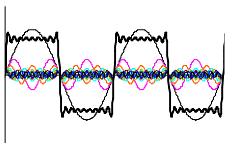
fundamental fr.+ 3rd overtone

fundamental fr.+ 3rd overtone + 5th overtone

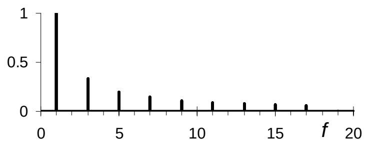
fundamental fr.+ 3rd overtone + 5th overtone + 7th overtone



function | spectrum



fundamental fr.+ 3rd overtone + + ... + 17th overtone



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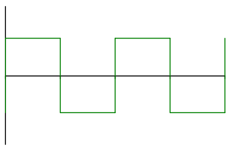
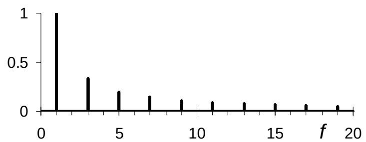
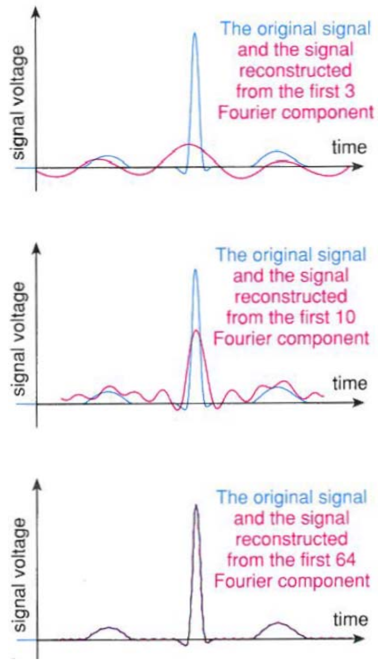


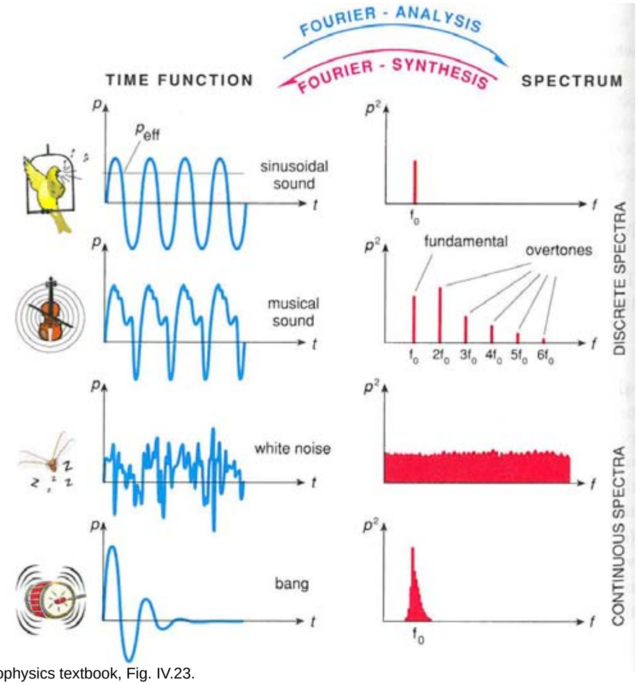
fundamental fr.+ 3rd overtone + + ... + 17th overtone + + ...



Creating an ECG signal from sine functions



Textbook, Figure VII.3.

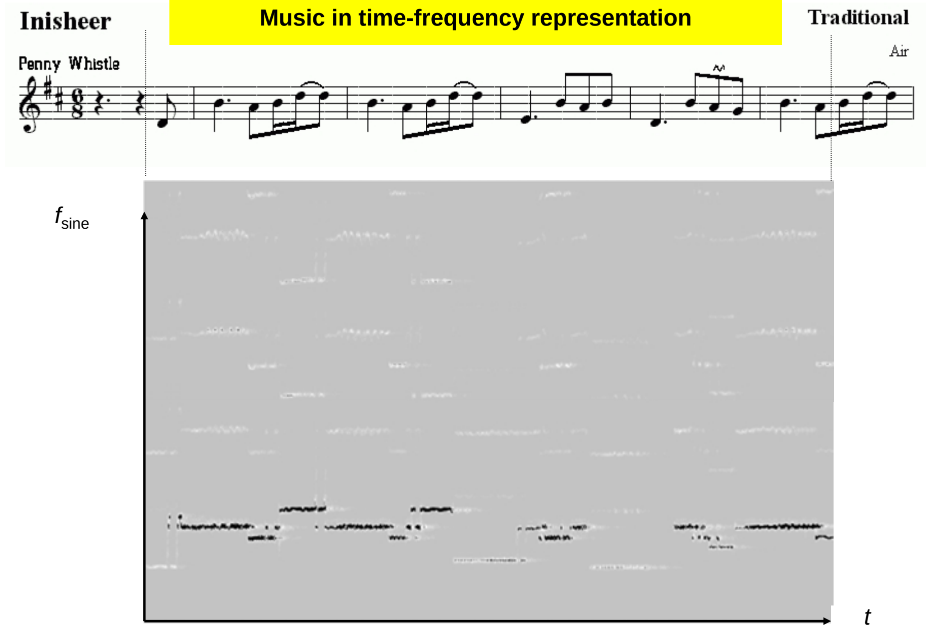


pitch:
frequency of the fundamental

timbre (tone colour):
relative strengths of overtones/harmonics (spectrum)

Biophysics textbook, Fig. IV.23.

Music in time-frequency representation



The role of elastic medium

mechanical waves require a medium in order to transport their energy from one location to another

$$\kappa = \frac{-\frac{\Delta V}{V}}{\Delta p}$$

compressibility
relative volume decrease over pressure

$$c = \frac{1}{\sqrt{\rho\kappa}}$$

speed of sound

$$Z = \frac{p}{v} = \frac{p_{max}}{v_{max}}$$

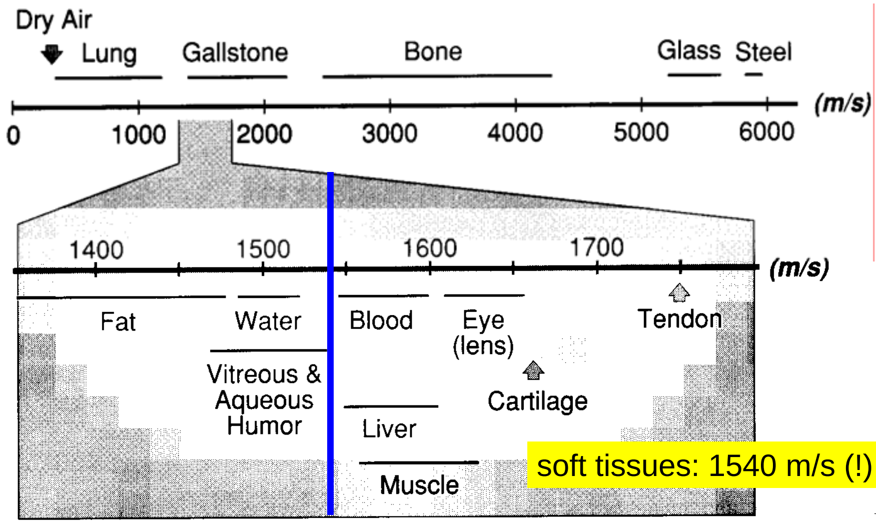
acoustic **impedance** (definition)

$$Z_{el} = \frac{U}{I}$$

$$\boxed{Z = c\rho} = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance** (useful form)

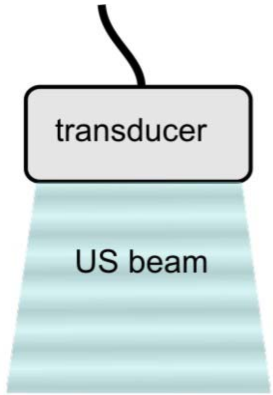
Speed of sound/US in different media



soft tissues: 1540 m/s (!)

Acoustic properties of materials

Medium	Density (kg/m³)	Speed of Sound (m/s)	Acoustic Impedance (kg/m².s) x10⁶
Air	1.2	333	0.0004
Blood	1060	1566	1.66
Bone	1380-1810	2070-5350	3.75-7.38
Brain	1030	1505-1612	1.55-1.66
Fat	920	1446	1.33
Kidney	1040	1567	1.62
Lung	400	650	0.26
Liver	1060	1566	1.66
Muscle	1070	1542-1626	1.65-1.74
Water	1000	1480	1.48

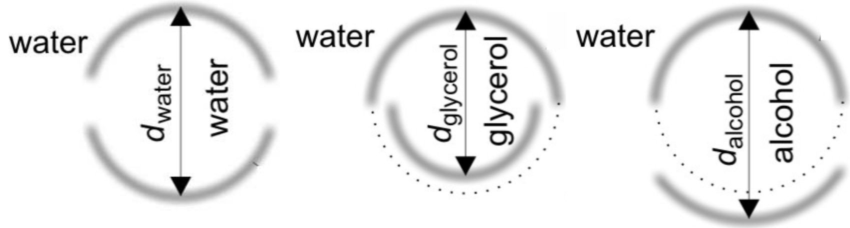


Assuming constant speed of US ➡ Artefacts

The image of the back-wall reflection appears in different distances, depending on the material in the finger of the rubber gloves

C_{water}= 1540 m/s, $C_{glycerol}$= 1900 m/s, $C_{alcohol}$= 1200 m/s

contours of the rubber glove finger on the screen



Laboratory manual, US, Figure 15

Intensity of US

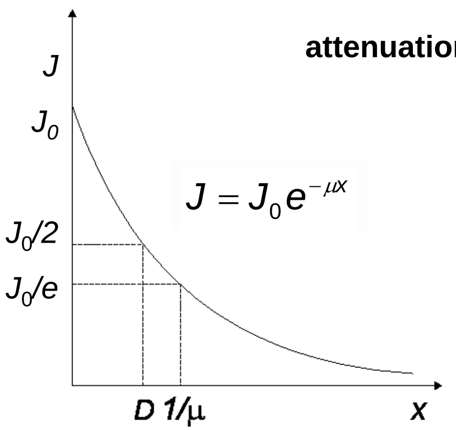
$$J = \frac{1}{Z}\Delta p_{eff}^{\ 2}$$

intensity = energy-current density

$$P_{el} = \frac{1}{Z_{el}}U_{eff}^{\ 2}$$

electric analogy

Loss of energy during propagation (absorption)



attenuation: $\alpha = 10 \cdot \lg \frac{J_0}{J}$ dB

$\alpha = 10 \cdot \mu \cdot x \cdot \lg e$ dB

μ is **proportional to frequency** in the diagnostic range

specific attenuation: $\frac{\alpha}{f \cdot x}$

μ is proportional to frequency in the diagnostic range

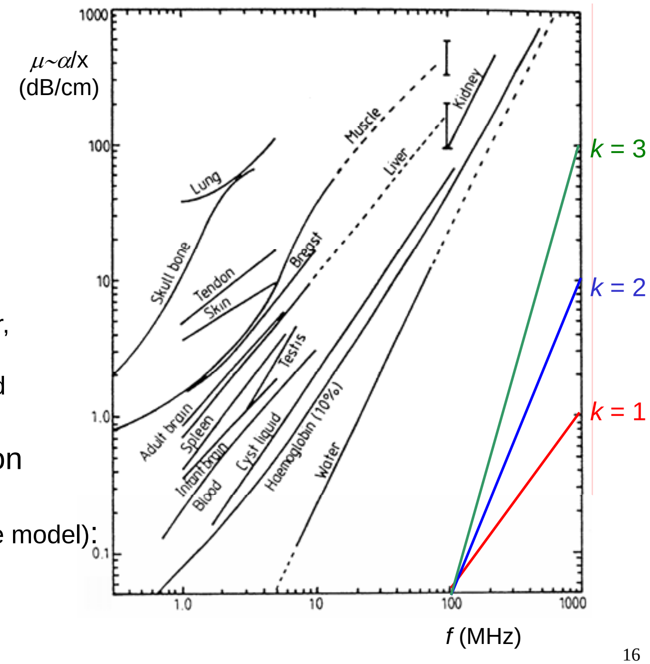
$\mu \sim f^k, \quad k \sim 1(?)$

$\log \mu \sim k \log f$

if the graph is a linear, the power function approximation is valid

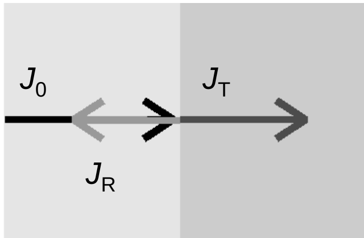
specific attenuation for soft tissues (homogeneous tissue model):

$$\frac{\alpha}{f\,x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$

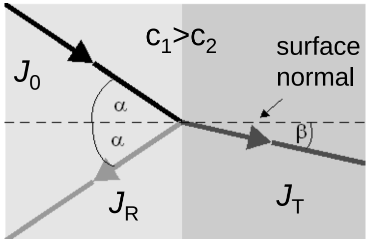


Phenomena at the boundary of different media

normal/perpendicular incidence



skew incidence



$J_0 = J_R + J_T$

reflection and transmission (penetration)

$$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

Biophysics textbook, Fig. II.47.

Reflection (normal incidence)

reflectivity:

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

boundary surface	*R*
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

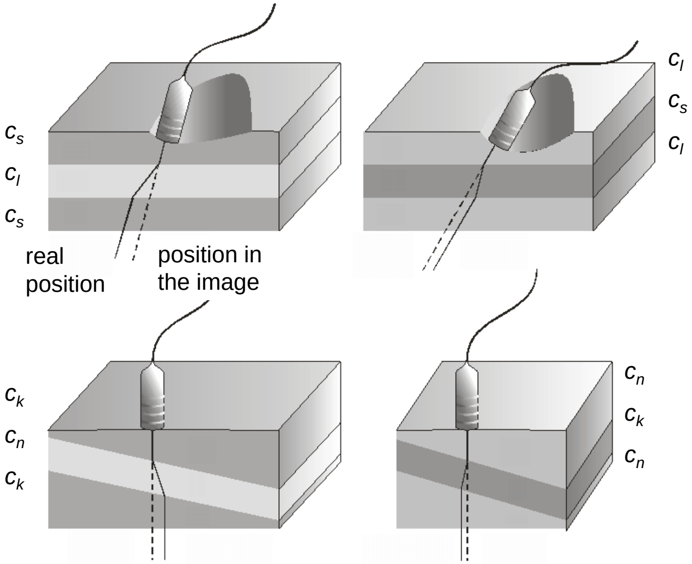
"full" reflection:

$$Z_1 << Z_2, \quad R \approx 1$$

optimal coupling:

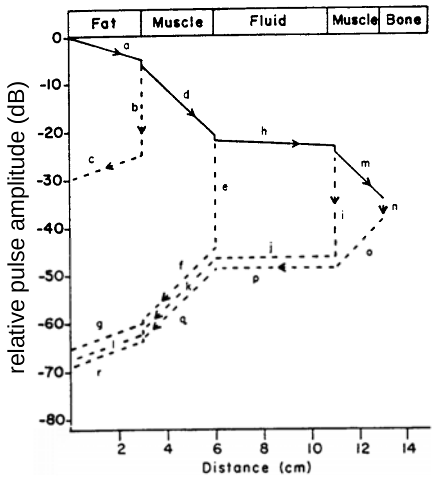
$$Z_{\text{connecting}} \approx \sqrt{Z_{\text{source}} Z_{\text{skin}}}$$

Phenomenon of skew incidence or normal incidence and skew boundaries



Biophysics textbook, Fig. on pg. 153

Absorption and reflection



the later comes back the reflection, the deeper lays the reflecting surface and the weaker is the intensity

run time dependent amplification

TGC: time gain compensation

DGC: depth gain control

boundary surface	*R*	10lg*R* (dB)	*T*	10lg*T* (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

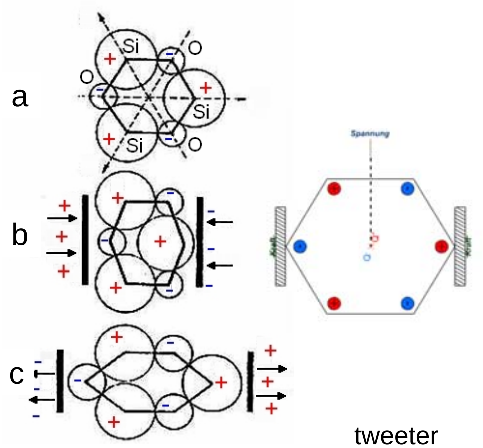
Generation of US. Piezoelectric effect

production: inverse ~
detection: direct ~

source of electric signal (sine wave oscillator)+ transducer (piezo-crystal)

(a) Center of charge of positive and negative charges coincides.

(b) and (c) As a result of pressure, the charge centers are separated, i.e. a potential difference arises (direct ~).
The crystal is deformed when voltage is applied (inverse ~).



at home: gas igniter

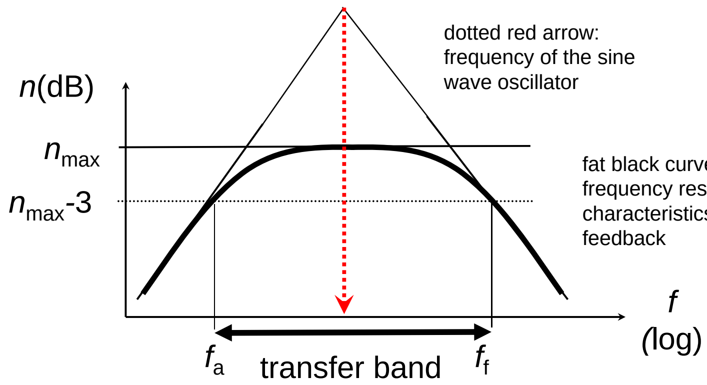
tweeter

Source of electric signal : sine wave oscillator

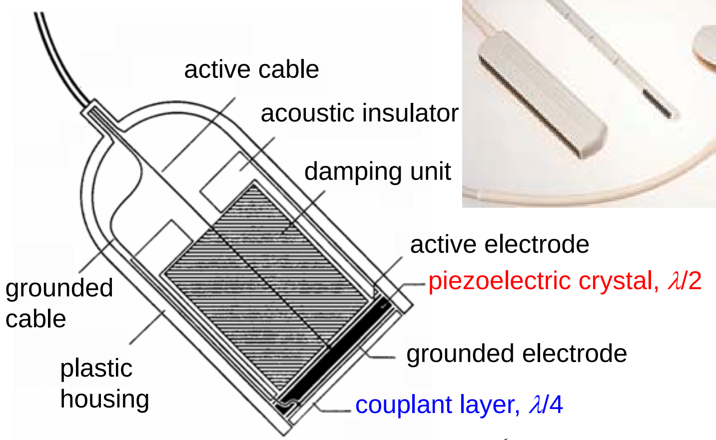
amplifier with positive feedback

$$A_{U,\text{ feedback}} = \frac{A_U}{1-\beta A_U}$$

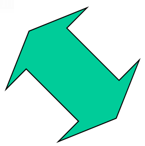
βA_U=1, amplification = „infinity" ➡ sine wave oscillator
no input signal, output signal: sine voltage



Ultrasound transducer



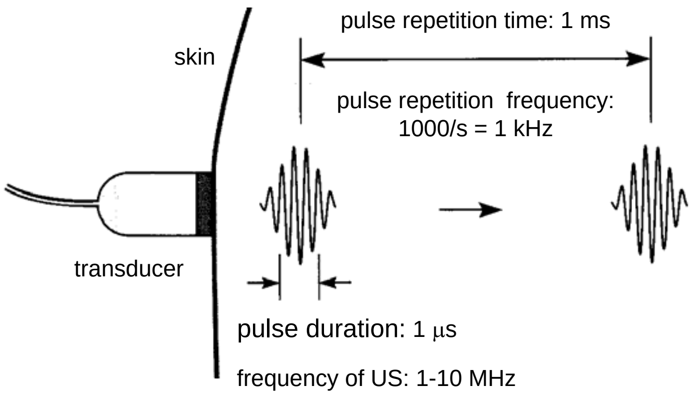
receiver



transmitter

Characteristic of US pulses

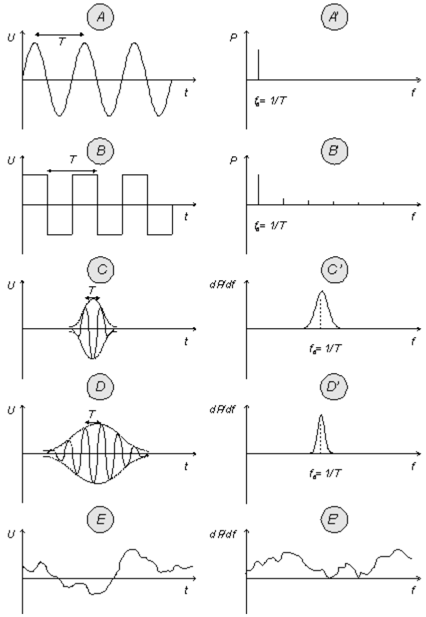
transducer: transmitter and receiver is the same unit
time sharing mode: pulses instead of continuous wave US



speed of propagation of US (in soft tissue): 1540 m/s

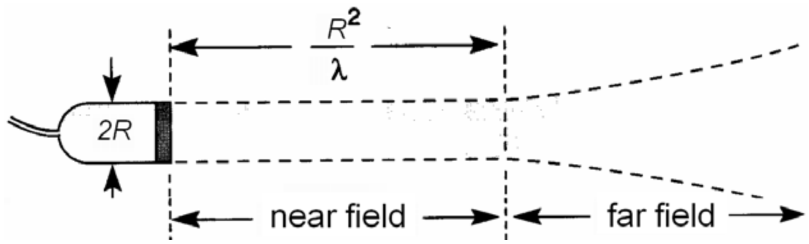
Biophysics textbook, Fig. VIII.32.

Time function **Spectrum**



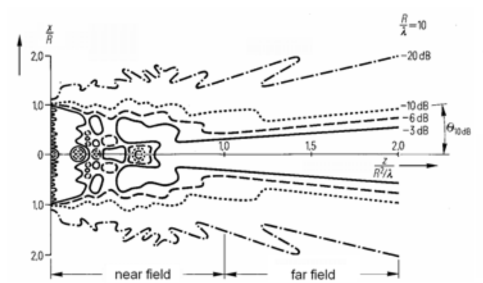
sine function	A / A'	line spectrum (1 line)
square function	B / B'	line spectrum
sine wave pocket (some „periods”)	C / C'	band spectrum
sine wave pocket (several „periods”)	D / D'	**US pulse** band spectrum
aperiodic function	E / E'	continuous spectrum

US beam shape (simplified version)

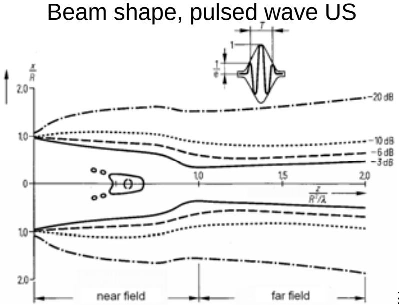


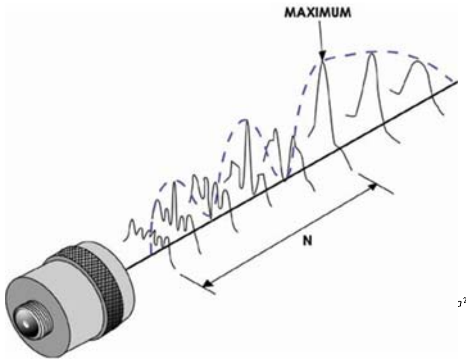
(Fresnel zone) (Fraunhofer zone)

Beam shape, continuous wave US

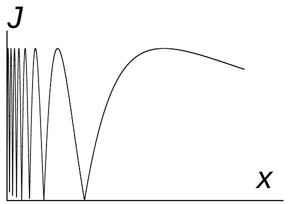


Beam shape, pulsed wave US

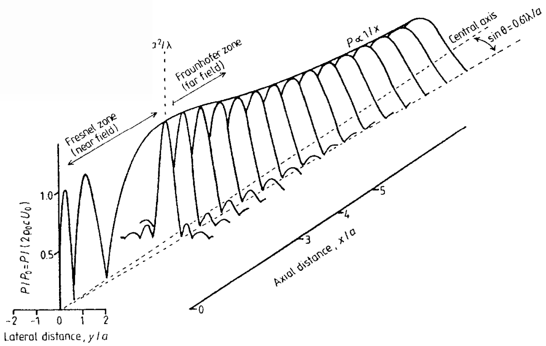




Perspective view of the US beam



change of the intensity in the axial direction



cf. Textbook. Fig. on p.505

Resolving limit, resolution

Resolving limit is the distance between two object details which can be just resolved as distinct objects (the smaller the better).
Resolution (resolving power): the reciprocal of the resolving limit (the greater the better)

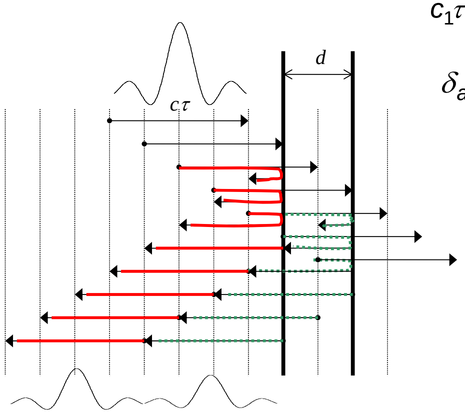
Axial resolving limit depends on the pulse length. Pulse length is inversly proportional to the frequency.

Lateral resolving limit is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

Typical values

frequency (MHz):	2	15
wavelength (in muscle) (mm):	0.78	0.1
penetration depth (cm):	12	1.6
lateral resolving limit (mm):	3.0	0.4
axial resolving limit (mm):	0.8	0.15

Axial resolving limit



τ : pulse duration

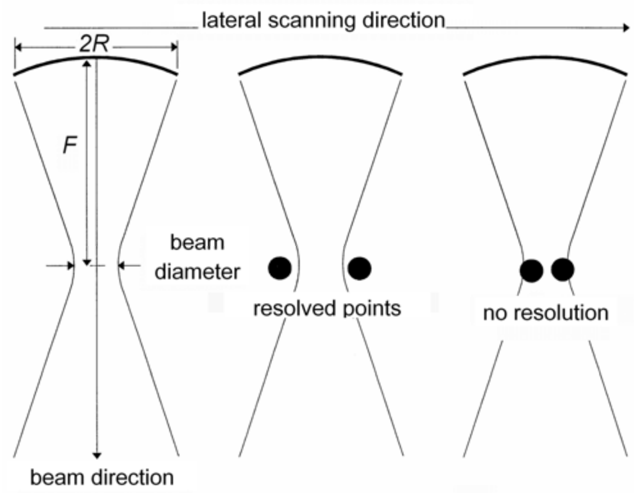
$c_1\tau \cong c_2\tau = c\tau$ pulse length

$\delta_{ax} = d = \frac{c\tau}{2}$ resolving limit

The axial resolving limit is the half of the pulse length. The echos from the adjacent surfaces in this case just hit another.

$$\tau \sim T = \frac{1}{f}$$

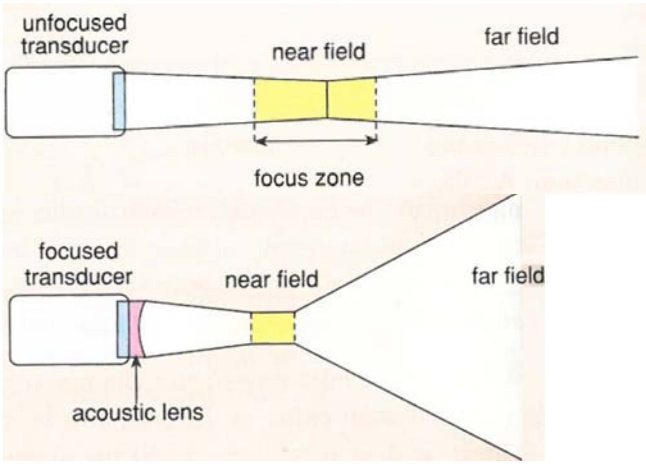
Lateral resolving limit



$\left(\delta_{\text{lat}} \sim \frac{F}{2R} \cdot \lambda\right)$

F: focal length
2*R*: diameter of the transducer
λ: wavelength

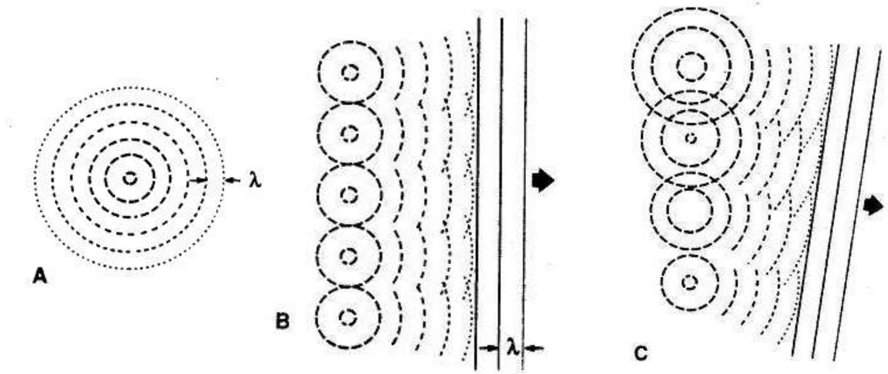
Focusing of the beam



Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

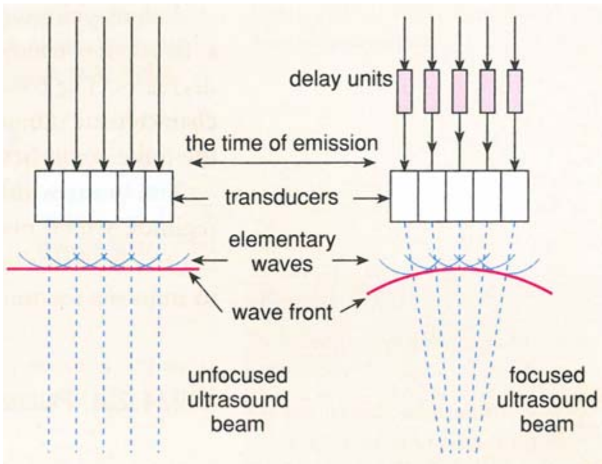
cf. Textbook Fig. on p.506

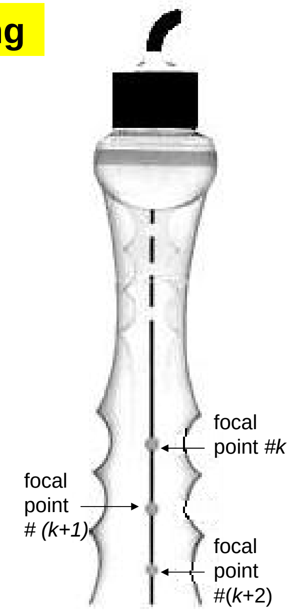
Huygens' principle



Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.

Electronic focusing

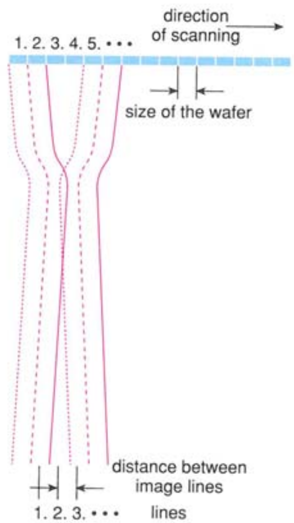




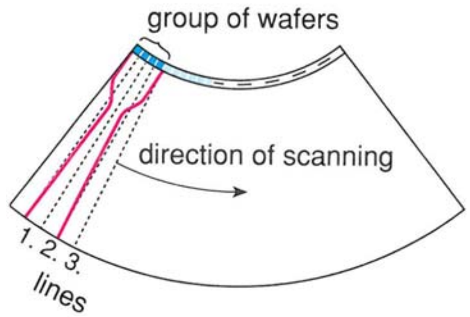
cf. Textbook Fig. on p.507

Scanning

multi unit linear array



multi unit curved array



cf. Textbook Fig. VII. 36-37

Safety

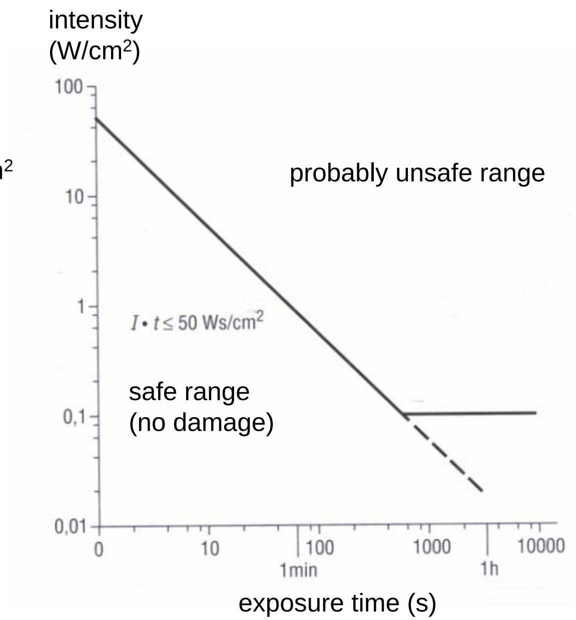
in the diagnostics:

10 mW/cm^2 = 100 W/m^2

cf. pain threshold: 10 W/m^2

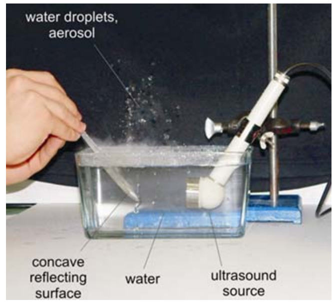
in the therapy: 1 W/cm^2

spatial average temporal average (SATA) intensity;
spatial peak temporal peak (SPTP) intensity;
spatial peak temporal average (SPTA) intensity;
spatial peak pulse average (SPPA) intensity
spatial average pulse average (SAPA) intensity



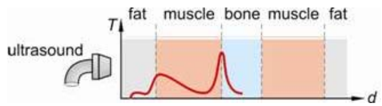
Effects of US

Aerosol effect: small water droplets (aerosol) can be formed by US



Cavitation effect:
formation of short-lived cavities (small bubbles, 100um) upon the breakage of intermolecular cohesion forces, extreme temperatures upto 10000 K

Thermal effect



Mechanical effect: cleaning, calculus or tartar removal (scaling)

Chemical effect: reactions may be induced

Biological effect: complex bactericidal, fungicidal, anit-viral etc.