SEMMELWEIS UNIVERSITY

Dept. of Biophysics and Radiation Biology, Laboratory of Nanochemistry

Laboratory of soft matters

Bio-thermodynamics, entropy, equilibrium and processes

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Internal energy is the energy of an object or substance due to the molecular **kinetic** and **potential** energies associated with the random motions of all the particles that make it up.

1st law of thermodynamics:
(Conservation of energy)

$$\Delta U = T\Delta S - p\Delta V + \sum_{i=1}^{K} \mu_i \, \Delta n_i + \ldots +$$

2nd law of thermodynamics:
(Non-conservation of entropy)

During spontaneous processes entropy increases. Thermodynamic equilibrium is characterized by entropy maximum.

3rd law of thermodynamics:
(Absolute scale of entropy)

The entropy of one-component, crystallizing material at 0 K temperature is 0

enthalpy $H=U+PV$ ⟹ heat

free energy $F=U-TS$ ⟹ work

**free enthalpy** $G=H-TS$ ⟹ chemistry

(Gibbs free energy)

BIO-THERMODYNAMIC SYSTEM

O_2

radiation

food

BODY

Energy storage

work done
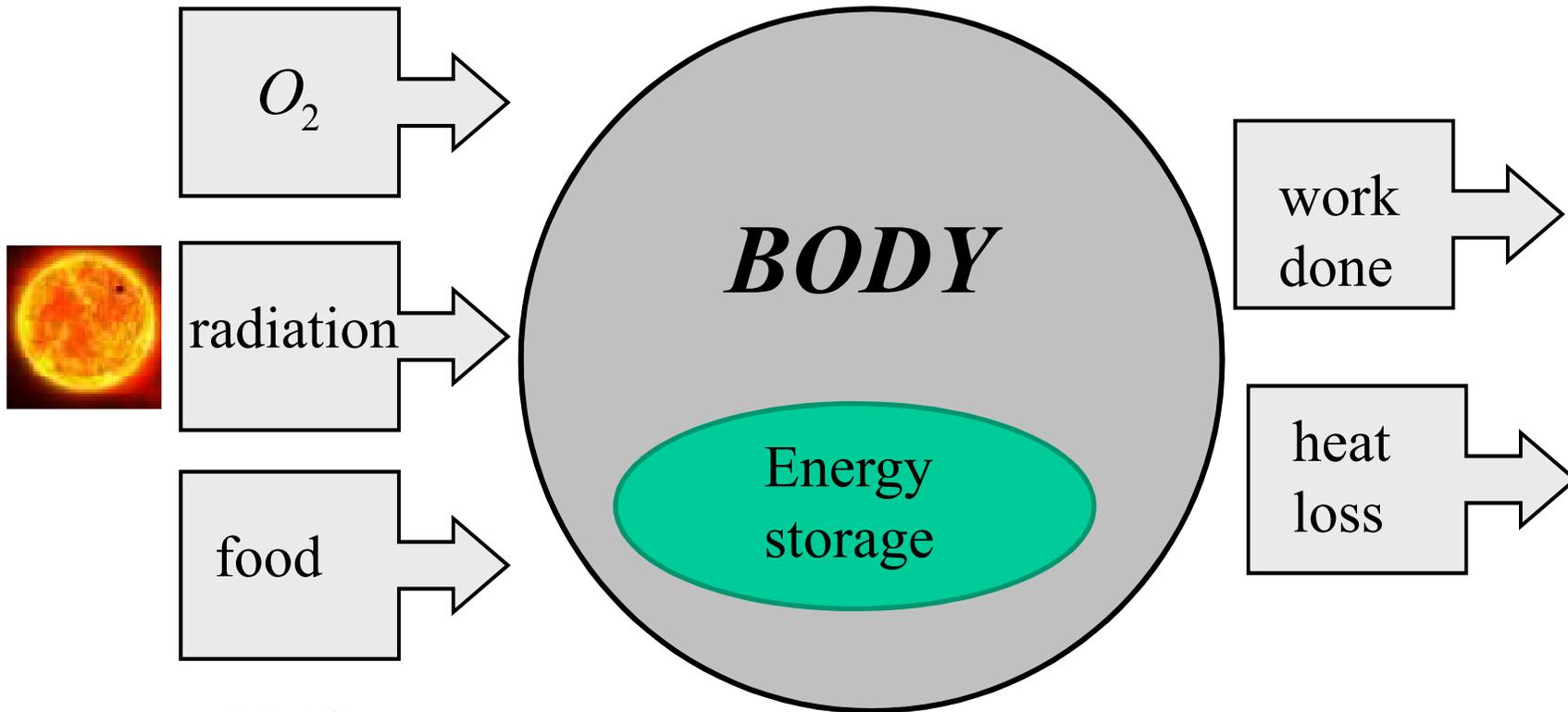
heat loss

Energy flow into and from the body.

Heat has unique role in biology !

$0\ C^o \longrightarrow T \longrightarrow 42\ C^o$

Biological Thermodynamics
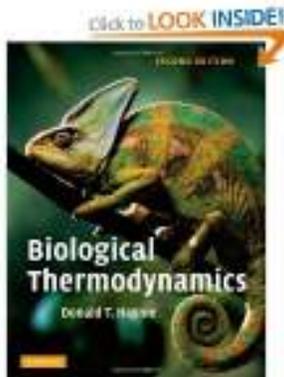
First law of bio-thermodynamics

change in stored energy

$$\Delta U = \Delta Q + \Delta W_{mech} + \Delta W_{chem}$$

heat of metabolism

heat loss

bio-synthesis

Mechanical work

physical

biological

Radiation

Sweat evaporation

Convection

Conduction

Sweat gland   Heat in blood

Heat produced in muscles
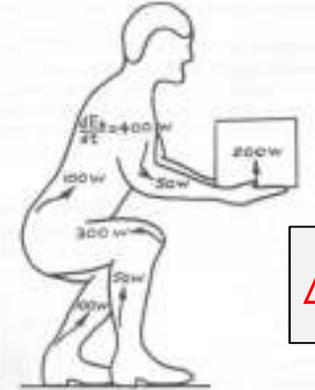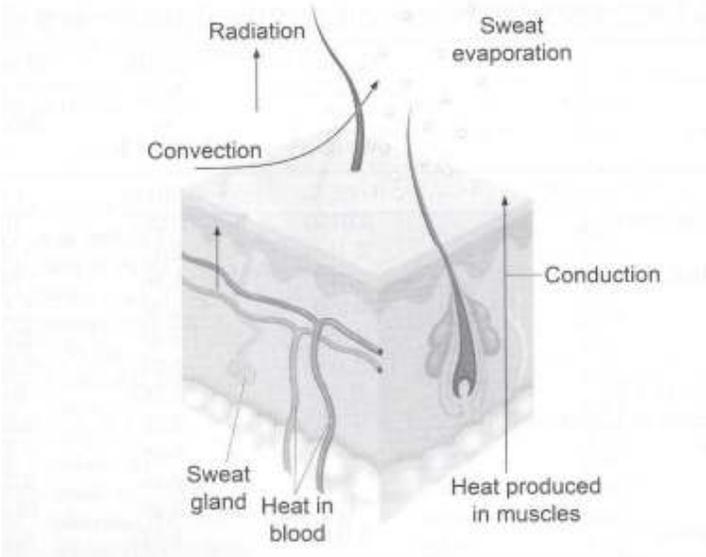
$$\Delta W_{mech} < 0$$

Primary source of energy
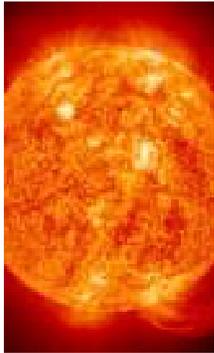
$5 \cdot 10^{18} \ MJ \ / \ year$

$1.7 \cdot 10^{17} \ J \ / \ s$

LIFE

Stored in clouds

Scattered by clouds

Absorbed and converted to heat
photosynthesis
0.025%

$$6CO_2 + 6H_2O = C_6H_{12}O_6 + 6O_2$$
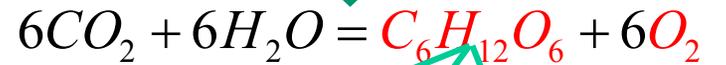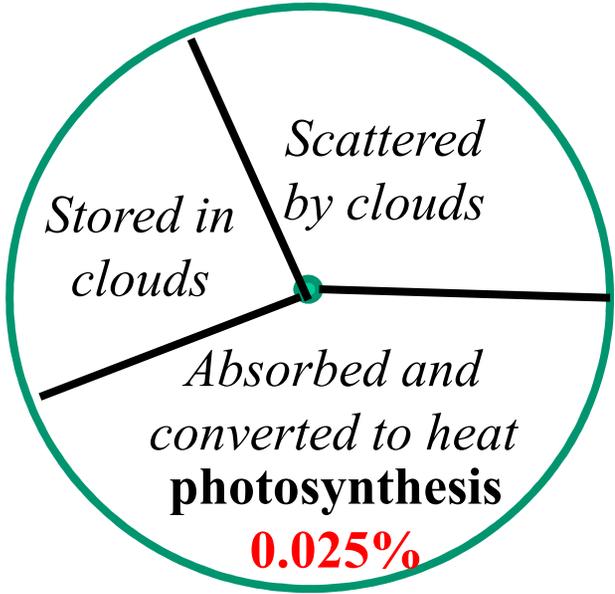
Biological macromolecules ← *starch* | *glycogen* | cellulose
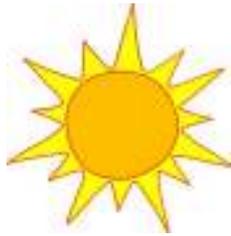
| Light, H_2O, CO_2 | → | carbohydrates, plant tissues | → | fat, animal tissues |

Metabolism is a term that is used to describe all chemical reactions involved in maintaining the living state of the cells and the organism. Metabolism can be conveniently divided into two categories:
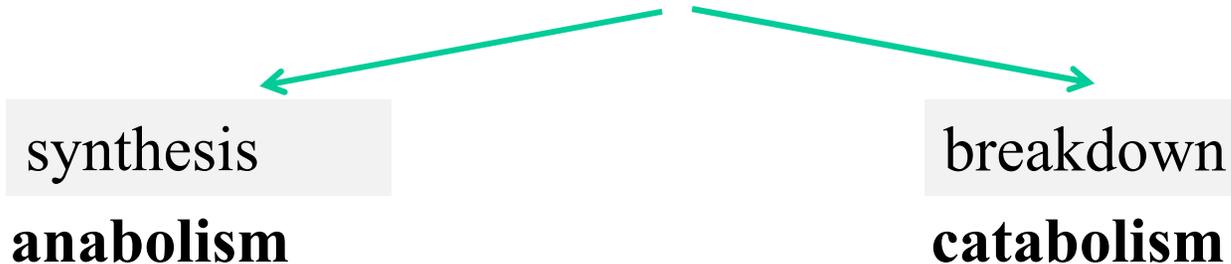
synthesis

anabolism

the synthesis of all compounds needed by the cells

breakdown

catabolism

the breakdown of molecules to obtain energy

Anabolism: The set of metabolic pathways that construct complex materials of living tissue from smaller units:

**photo**tropism

simple inorganic material

↓

complex organic materials


chemotropism

organic materials
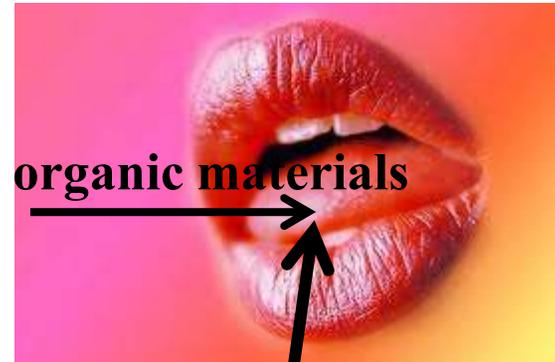
Catabolism: The breakdown of molecules to obtain energy.

polysacharides,
lipids,
proteins,
nucleic acids

$\longrightarrow$

monosacharides,
fat acids,
nucleotides,
amino acids
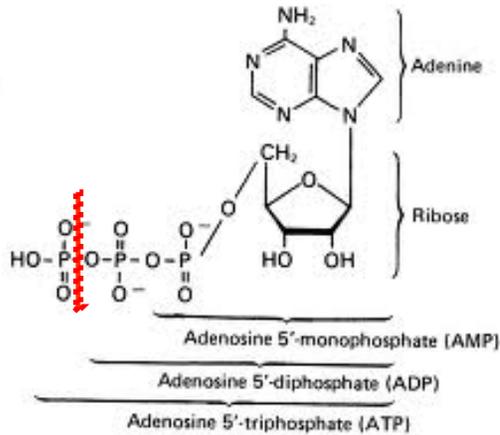
Q

ADP

CH3-CH(OH)-COOH
acetic acid
ammonium
carbamide

ATP $+ H_2O =$ **ADP** $+$ phosphate $+$ **energy**
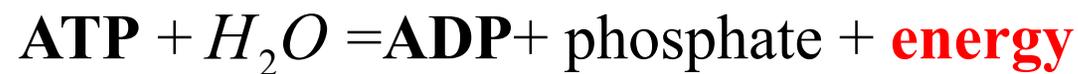
58 - 59 kJ/mol

Direct calorimetry

$$\Delta S_{term} = 0 \qquad Q = \Delta H > 0$$

final state

Hess law:

initial state
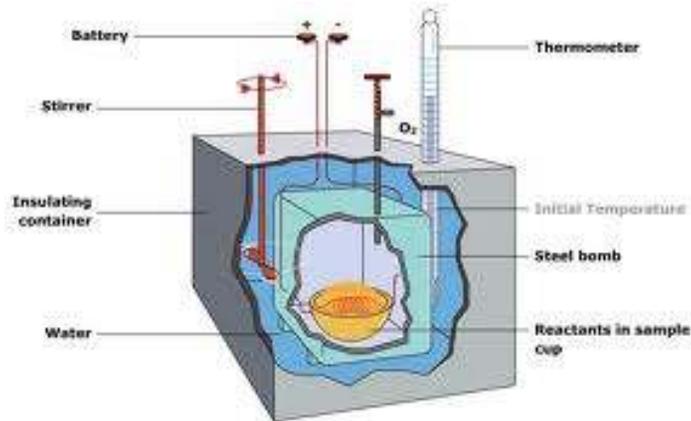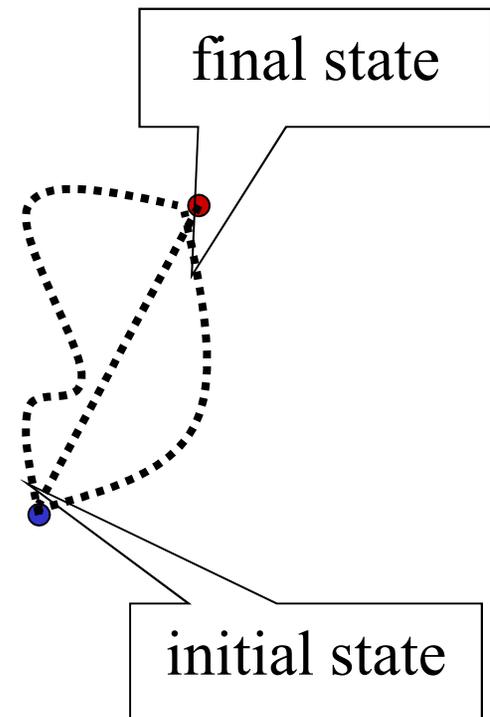
chemical fuel	calorimetric energy kJ/g
carbohydrate	17.1
protein	23.6
ethanol	29.7
fat	39.6

Energy content of body fuel

Chemical fuel	Storage energy density [J/Kg]
ATP	$1.0 \cdot 10^5$
H_2 gas at 10^3 atm.	$1.2 \cdot 10^8$
glycin (amino acid)	$6.5 \cdot 10^6$
fat	$3.9 \cdot 10^7$
carbohydrate	$1.7 \cdot 10^7$
glucose	$1.6 \cdot 10^7$

Metabolizable energy (ME): U_{food} - $U_{endproducts}$

apparently digested energy

$$\Delta U = \Delta(ME) + \Delta Q + \Delta U_{stored} + \Delta W_{mechanical}$$

bond energies → heat
→ work
→ endproducts
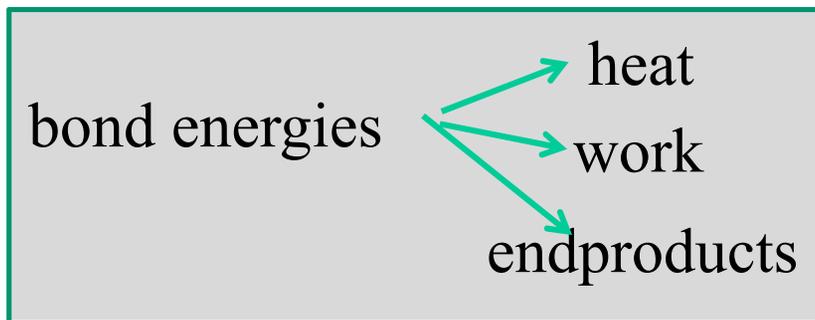
$$\Delta(ME) = 0 \implies \Delta U < 0$$

Meaurement of biological heat flow

Direct calorimetry

Indirect calorimetry

Rate of heat is proportional to the rate CO2 production or rate O2 consumtion.

$$\Delta Q = Q_{metabolic} + Q_{loss}$$

$$Q_{loss} = Q_{radiation} + Q_{convective} + Q_{conductive} + Q_{evaporation} + Q_{respiration}$$

25 %

54-60 %

7 %

14 %

Direct and indirect calorimetry

$$C_6H_{12}O_6 + 6O_2 = 6CO_2 + 6H_2O \qquad \Delta_c H = +2817\text{kJ}$$

$$C_6H_{12}O_6 + 36ATP = 36ADP + P_i \qquad \Delta_m H = +1757\text{kJ}$$
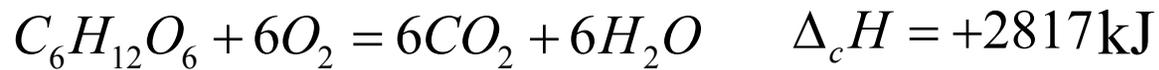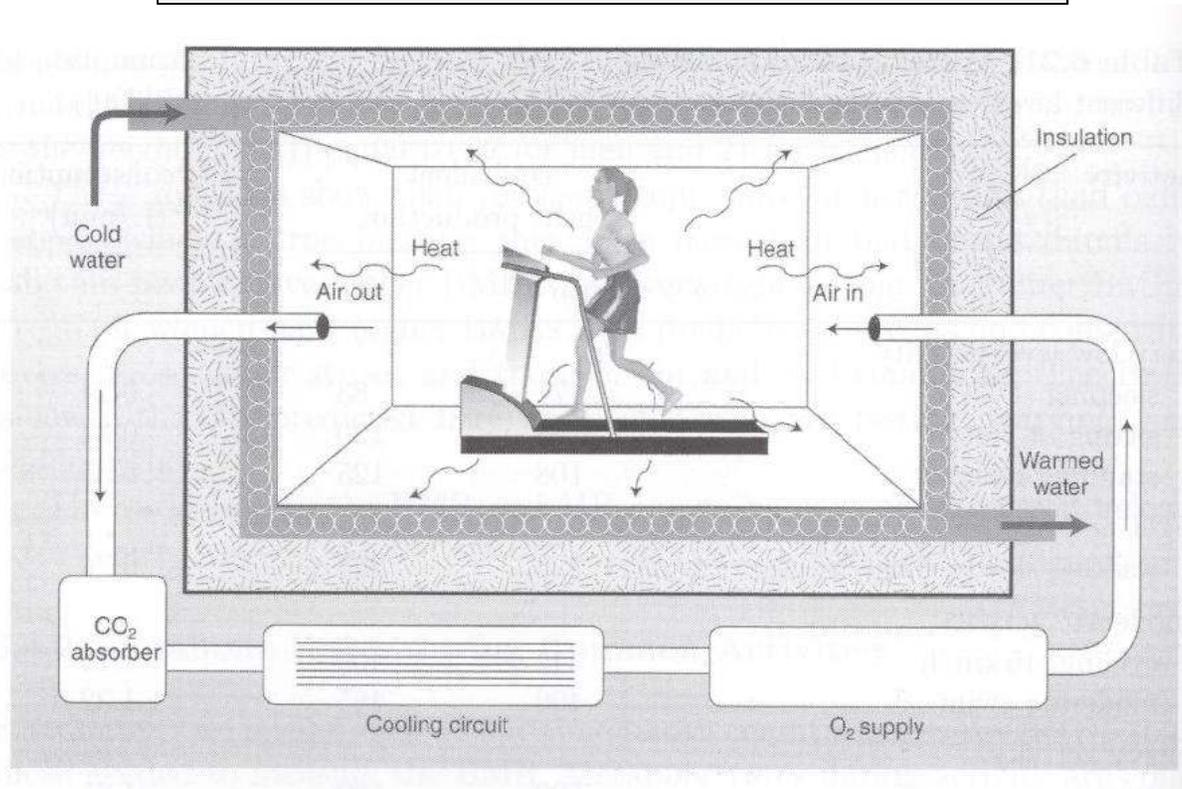
$$\eta = 61-65\,\%$$

calorimetric > metabolic

(V=const.)   (p=const.)

Indirect calorimetry

1 mol glucose + 6 mol=134,46 l oxygen!

$$C_6H_{12}O_6 + 6O_2 = 6CO_2 + 6H_2O \qquad \Delta_c H = +2817 \text{ kJ}$$
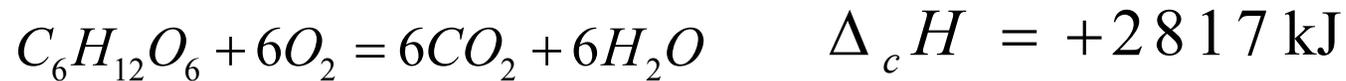
Oxygen energia equivalence:

1 l oxygen corresponds to 21 kJ energy !

Body fuel	Calorimertic energy kJ/g	Oxygen equivalence kJ/l	Carbondioxide equivalence kJ/l
carbohydrate	17.1	21.1	21.1
protein	23.6	18.7	23.3
ethanol	29.7	20.3	30.3
fat	39.6	19.8	27.9

| **Basal metabolic rate : BMR** |

$$BMR = \left.\frac{\Delta Q}{\Delta t}\right|_{at\ rest}$$

$$BMR \propto m_b^{3/4}$$ Kleiber law

BMR *depends on age*

$m_b = 70$ kg   7029 kJ/day   293 kJ/hour   81 W man
                                                     60 W women

| **Metabolic rate :(MR) and oxygen consumption** |

sleeping	83 W	O_2 :	0.24 l/min
walking	265 W	O_2 :	0.76 l/min
cycling	400 W	O_2 :	1.13 l/min

Average human body, average composition and thermal characteristics:

Composition of the human body. Protein accounts for about half of the dry mass of the body. On the level of individual elements, carbon is by far the largest component, followed by nitrogen, oxygen, hydrogen and other elements. It is interesting that the elements contributing the most to the dry mass of the body are also the major components of air, earth, water, and carbon-based combustible matter. Based on data from Freiden (1972).
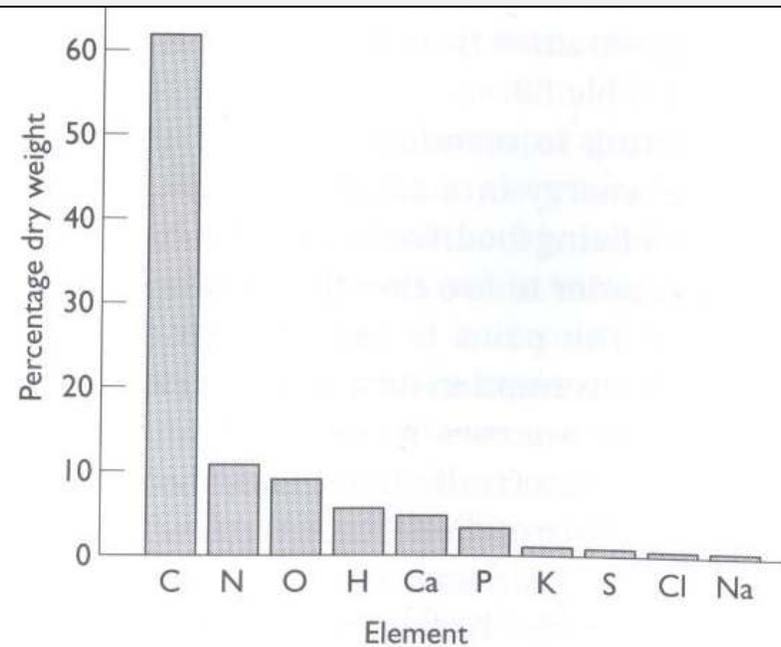
Specific heat: 3.47 kJ/kgK

70 kg human **heat capacity**: 243 kJ/ C^o

$$Q = C \cdot m_b \cdot \Delta T$$

$$\frac{\Delta Q}{\Delta t} = C \cdot m_b \cdot \frac{\Delta T}{\Delta t}$$

$$\frac{\Delta T}{\Delta t} = \frac{1}{C \cdot m_b} \cdot \frac{\Delta Q}{\Delta t} = \frac{1}{C \cdot m_b} \cdot BMR$$

$$\frac{\Delta T}{\Delta t} = \frac{BMR}{C \cdot m_b}$$

$\Longrightarrow$ $$\frac{\Delta T}{\Delta t} = 1.2 \ C^o/h$$ *If there were no heat loss!*

During physical activities ➡️

$$\frac{\Delta Q}{\Delta t} = f \cdot BMR$$

$$\frac{\Delta T}{\Delta t} = f \cdot \frac{BMR}{C \cdot m_b} \approx 1.2 \, f \, C^o / h$$

$$0 < f < 20$$

exercise

activity	f
sleeping	1
sitting	1.5
standing	1.7
walking	4.7

Eating és heat production at rest

	Big eater	Small eater	ratio
weight (kg)	54.2	52.7	1.03
Food energy (kJ/day)	9916	6485	1.54
Daily heat production (kJ/day)	9079	5815	1.55
Heat production by night (kJ/day)	7196	4602	1.56

but….

$$\frac{dQ}{dt} = f \cdot BMR$$

$$0 < f < 20$$
Physical activity

Energy flow and movement

Dependence of released heat on the walking speed.
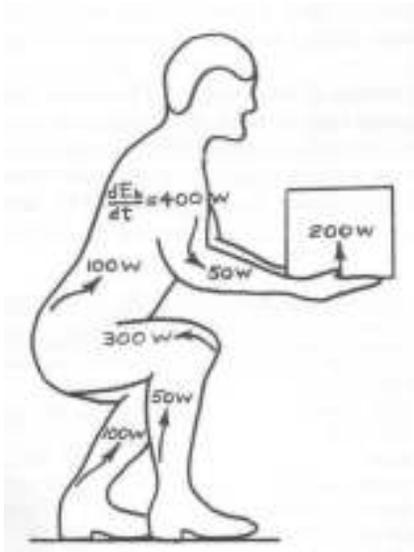
work *(physical)* ≠ work *(biological)*

external   internal

$$\Delta W_{mech} < 0$$
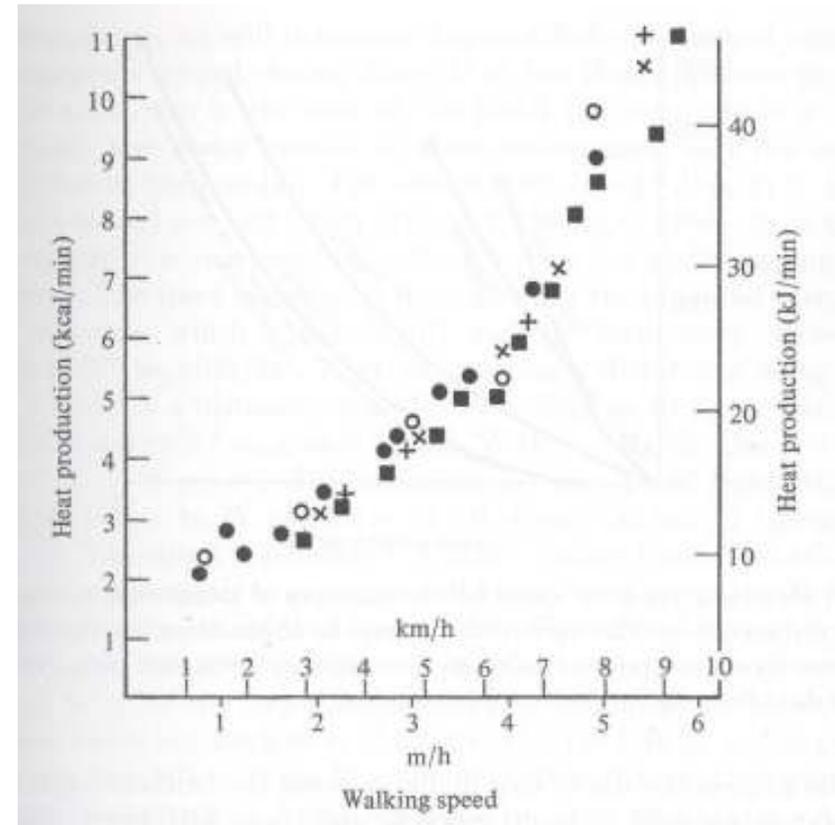
At 8.6 km/h speed the metabolic power needed to run is 14% less than that needed to walk.

Back to thermodynamics

*Dependence of **entropy** on the temperature*

$$\Delta_b S = \frac{\Delta H_b}{T_p}$$

$$\Delta_m S = \frac{\Delta H_m}{T_m}$$

$S(T)$

T_{op} T_{fp} T
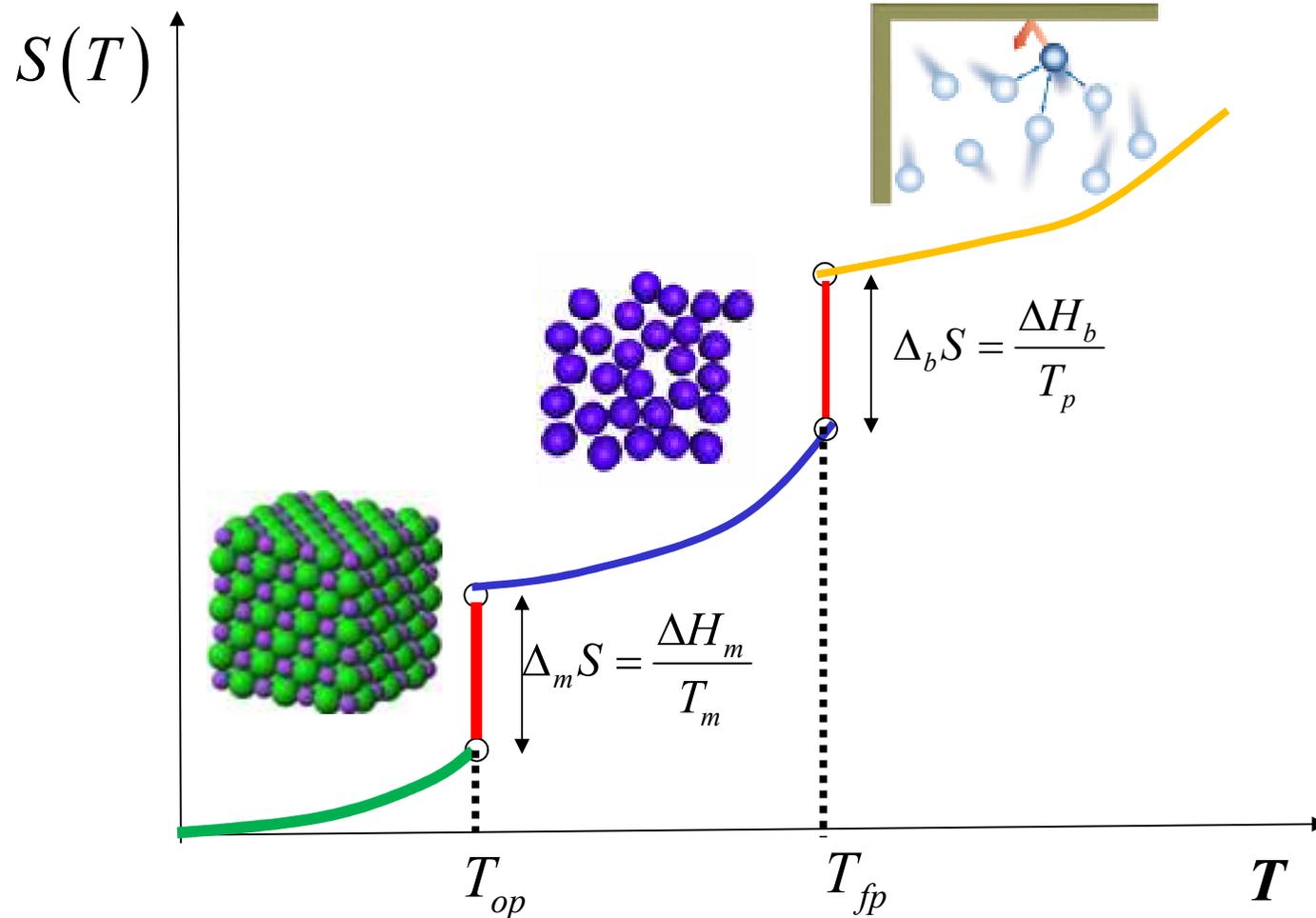
Entropy increases

Related to order – disorder transition?

YES!

Boltzmann equation:

$$S = k_B \ln W$$

$$k_B = 1.38 \cdot 10^{-23} \text{ J/K} \longrightarrow k_B = \frac{R}{N_{Av}}$$

W: thermodynamic probability
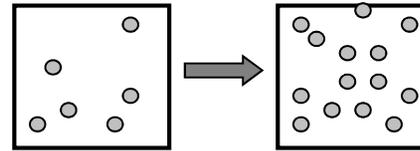
Increase of particle number
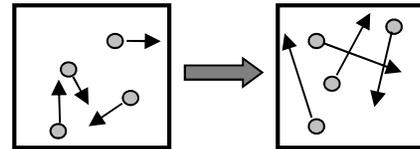
Increase of temperature
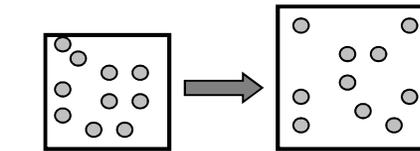
Increase of volume
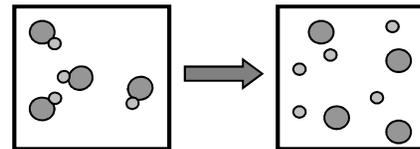
Dissociation
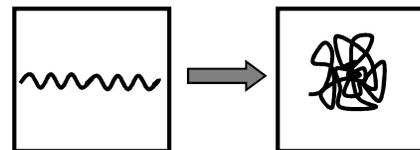
Helix-coil transition
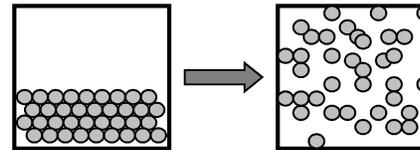
Melting
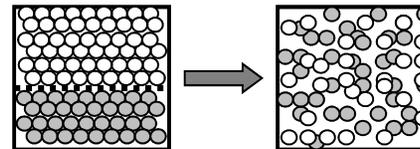
Mixing

W thermodynamic probability gives the number of microstates for a given macrostate.

1. example:

macrostate: concentration
microstate: No. of arrangement

$$W = \frac{16!}{8!8!} = 12870$$

$x_A = 8/16$
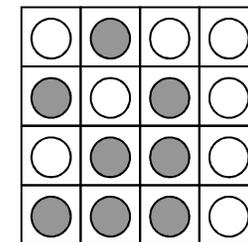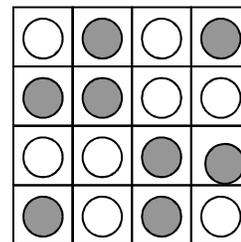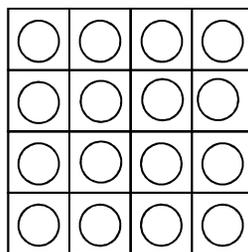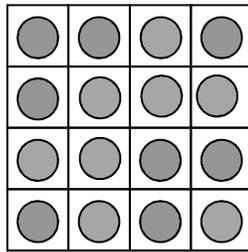
$\Delta S > 0$

$S = k_B \ln W$

$W_A = 1$
$S_A = 0$

$W_B = 1$
$S_B = 0$

$W_{A,B} = 12870$
$S_{A,B} = k_B T \ln(12870)$

$$W_{A,B} = \frac{(N_A + N_B)!}{N_A! N_B!} = \frac{16!}{8!8!} = 12870$$
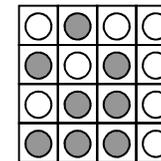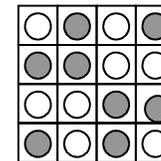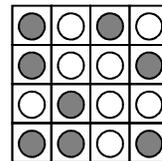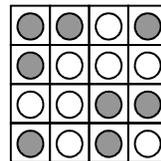
Four possible microstates form the overall 12870 states.

W **thermodynamic probability gives the number of microstates for a given macrostate.**

2. example: **macro**state : end-to-end distance
microstate: number of conformation

coiled, c

extended, e
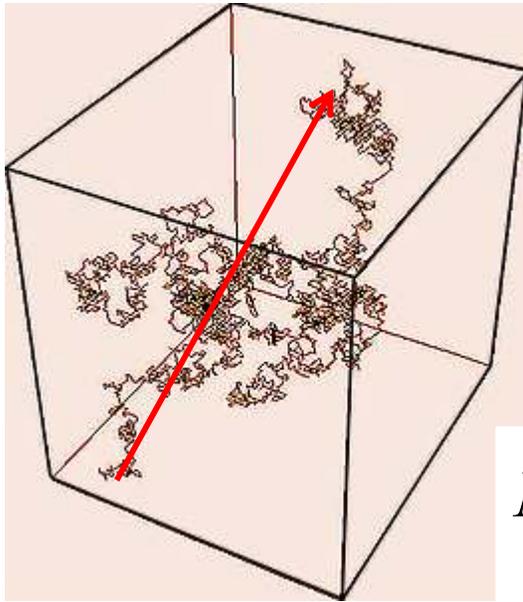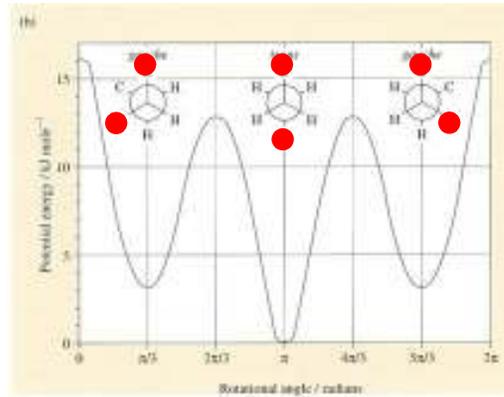
$$N_{C-C} = 10^4$$
$$W_c = 3^{10000}$$
$$S_g = k_B T \ln(3^{10000})$$
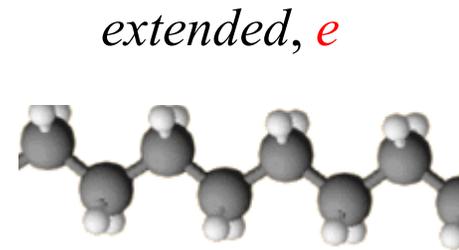
$$N_{C-C} = 10^4$$
$$W_e = 1$$
$$S_{ny} = k_B T \ln 1$$

$$\boxed{\Delta S > 0}$$

$$\boxed{S = k_B \ln W}$$

$$\boxed{S_c = 10^4 k_B T \ln 3}$$

$$\boxed{S_e = 0}$$

THERMODYNAMICS OF A SYSTEM

TD
system

time independent   time dependent

equilibrium   **steady state**   **non-equilibrium**   **reactive**

Transport processes   *Reaction kinetics*
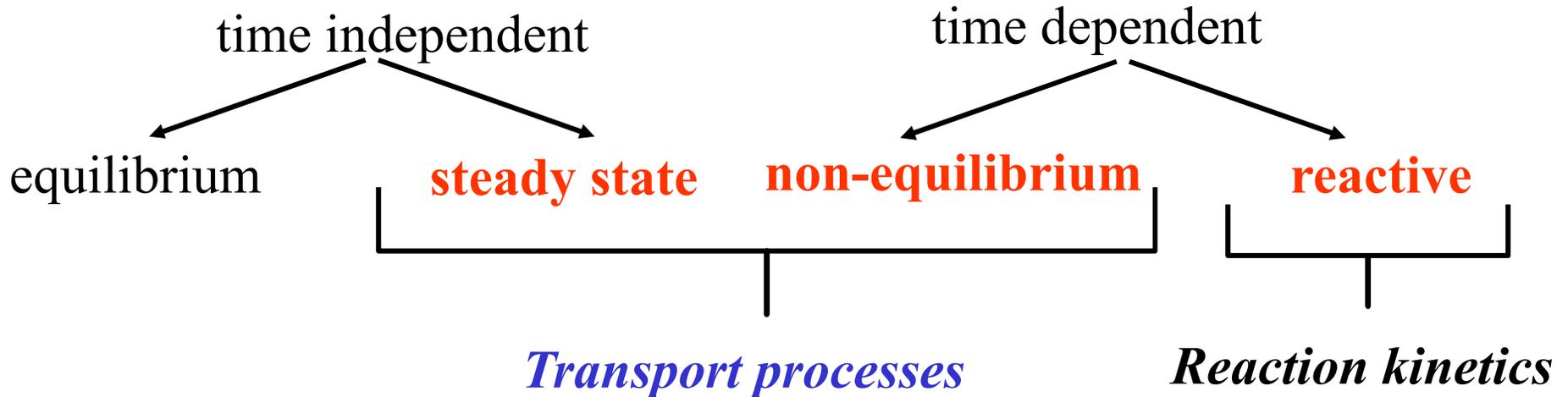
Driving force: **going to equilibrium**

During spontaneous process $\Delta S > 0$ $\Delta F < 0$ $\Delta G < 0$

$F = U - TS$ $G = H - TS$

Driving force of chemical and biochemical reactions

$$\Delta_r G < 0$$

$$\Delta_r G = \Delta_r H - T\Delta_r S$$

Heat of reaction

Entropy of reaction

endotherm   exotherm

Change in molecular order

$$\Delta_r H > 0$$ $$\Delta_r H < 0$$

decomposition   ring formation

$$\Delta_r S > 0$$ $$\Delta_r S < 0$$
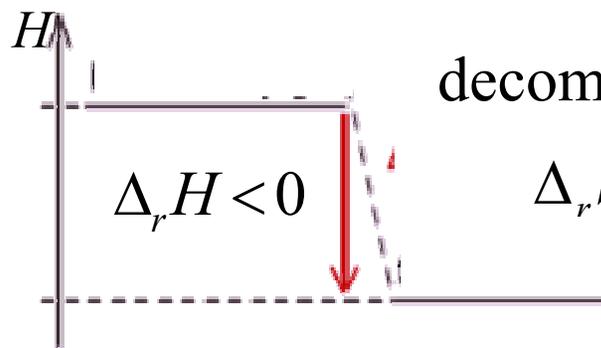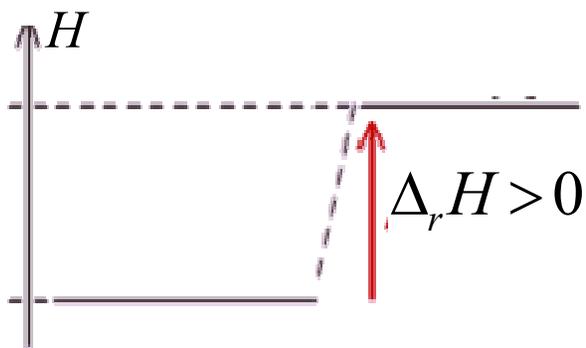
Significance of entropy and gibbs Free energy

- Spontaneous processes are those, during which Gibbs free energy decreases (ΔG < 0) and the entropy of the universe increases ($S_{tot} > 0$).

Decrease in free energy: stabilizes
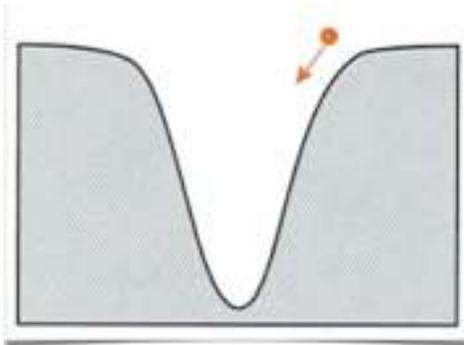
Increase in entropy: randomizes
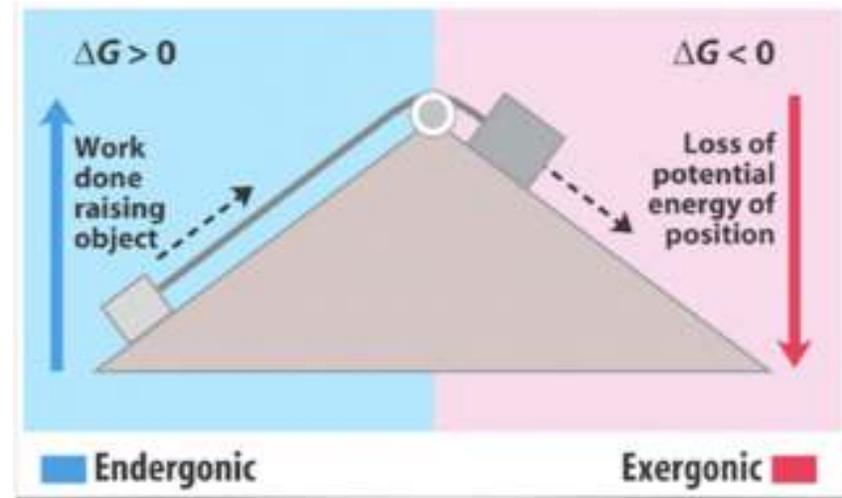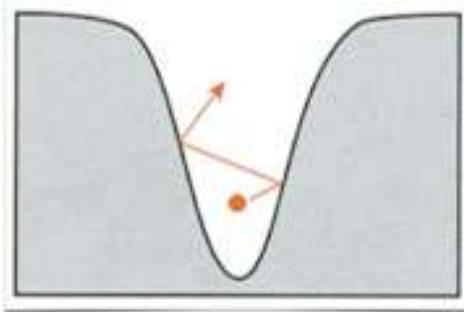
- Useful work is done by exergonic processes.
- Endergonic processes can be driven by coupling to exergonic processes.
- Entropy can be decreased locally.
- Life consumes entropy (its entropy is decreased at the expense of increasing total entropy).

CHEMICAL COUPLING

endergonic reaction → **exergonic reaction**

$$\Delta_r G_1 > 0$$

$$\Delta_r G_1 + \Delta_r G_2 < 0$$

idea

↓

mechanical coupling

i n t e n t i o n
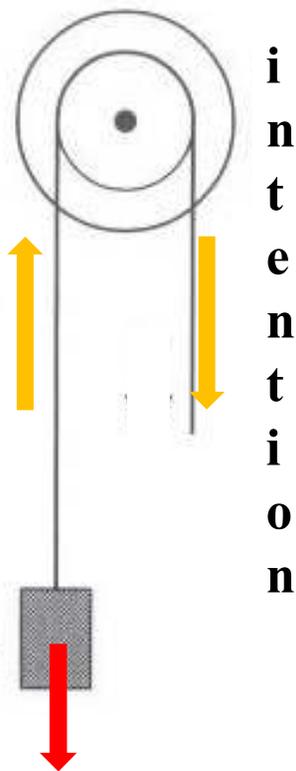
$\Delta_r G_1 > 0$ $\Delta_r G_1 < 0$
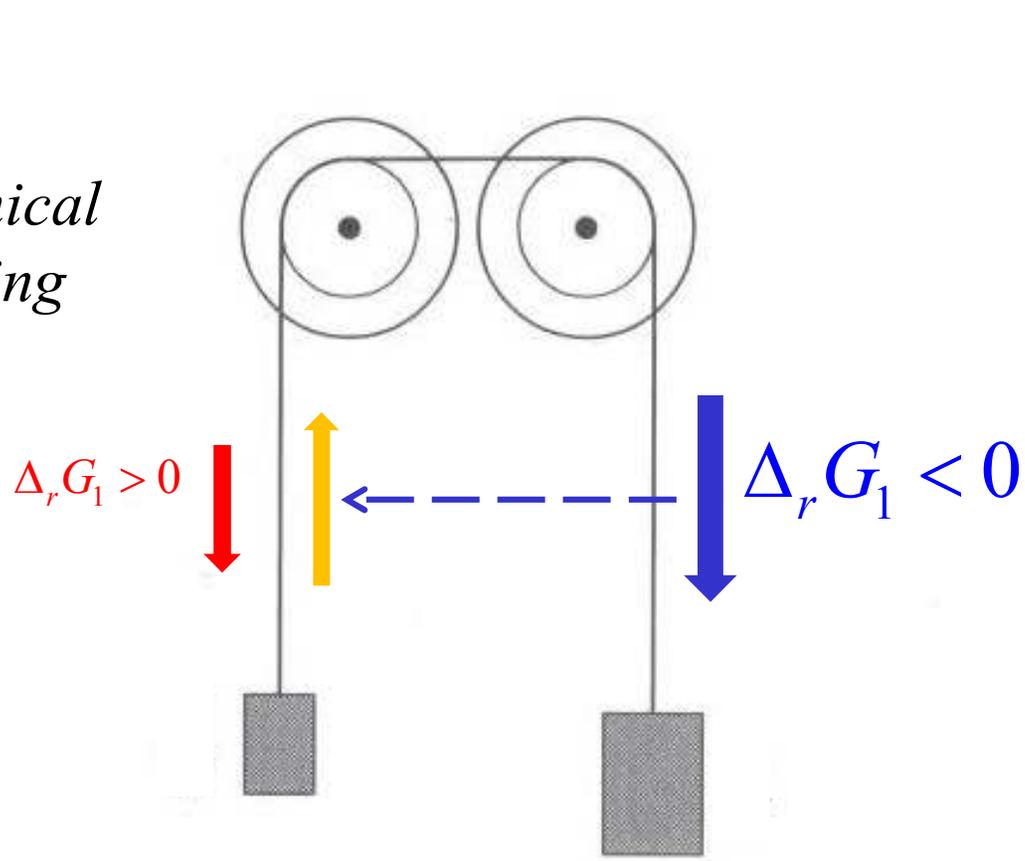
Coupled chemical reactions

$$A+B \rightleftharpoons C+D \quad \Delta_r G^o = +20 \text{ kJ/mol} \quad K_{a,1} = 0.0003$$

$$C \rightleftharpoons E \quad \Delta_r G^o = -30 \text{ kJ/mol} \quad K_{a,2} = 18100$$

$$A+B = E+D \quad \Delta_r G^o = -10 \text{ kJ/mol} \quad K_\Sigma = 56$$
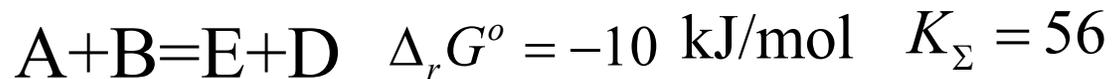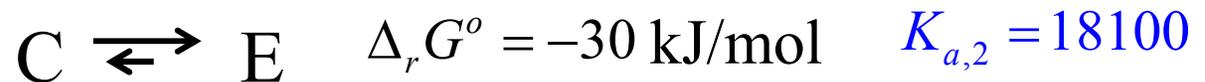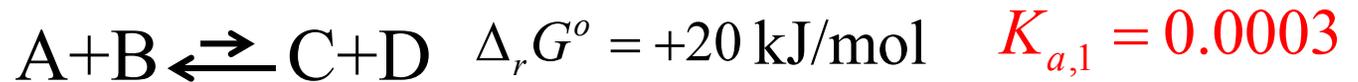
Thermodynamically unfavorable endergonic reaction $\Delta_r G > 0$ can be powered by an exergonic reaction $\Delta_r G < 0$ if the two reactions are chemically coupled.

Biochemical process

$$\text{Glucose} + \textbf{ATP} \longrightarrow \text{Glucose -6-phosphate} + \textbf{ADP} \quad \Delta_r G^o = -17.2 \text{ kJ/mol}$$

ENERGIA

ATP   **ADP**   **P**
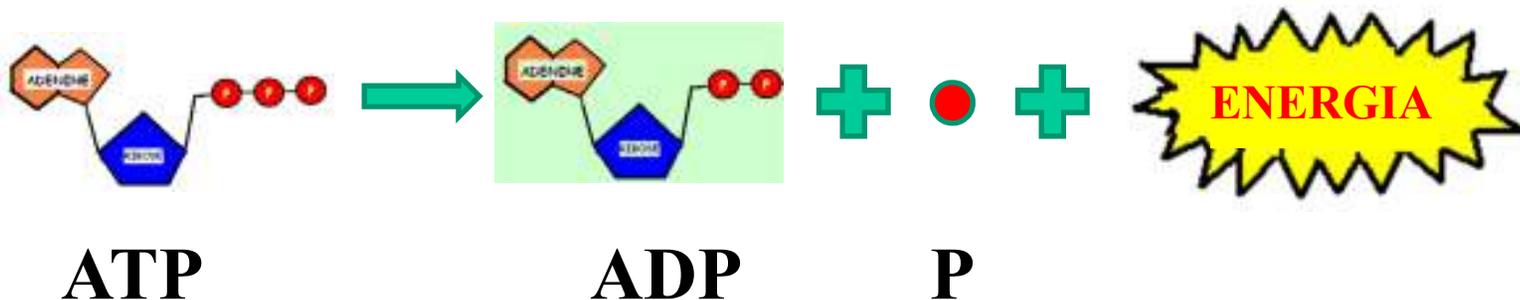
Stable and nonstable equilibrium condition

$\Delta_r G$

Non-equilibrium

Equilibrium —
unstable

Equilibrium —
local minimum

Equilibrium —
global minimum
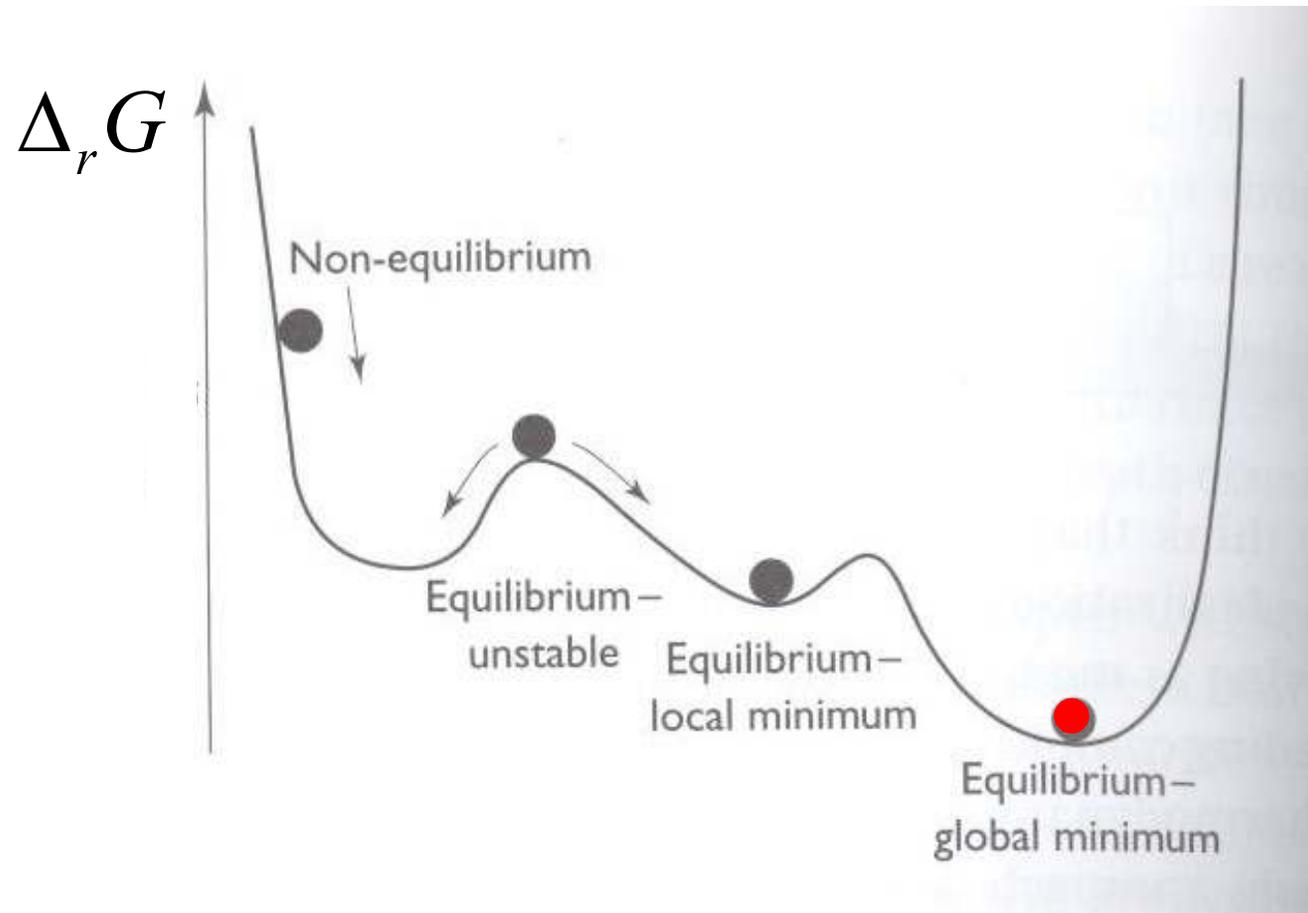
| Equilibrium condition	$\Delta_r G = \Delta_r H - T\Delta_r S < 0$

$$N_2O_4 \rightleftharpoons 2NO_2$$
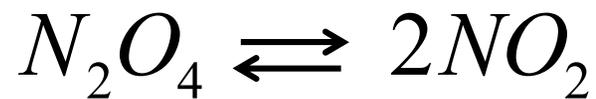
$$\Delta_r G = \Delta_r G^o + RT \ln \frac{[NO_2]^2}{[N_2O_4]}$$

Equilibrium const. $\quad K = \dfrac{[NO_2]^2}{[N_2O_4]}$
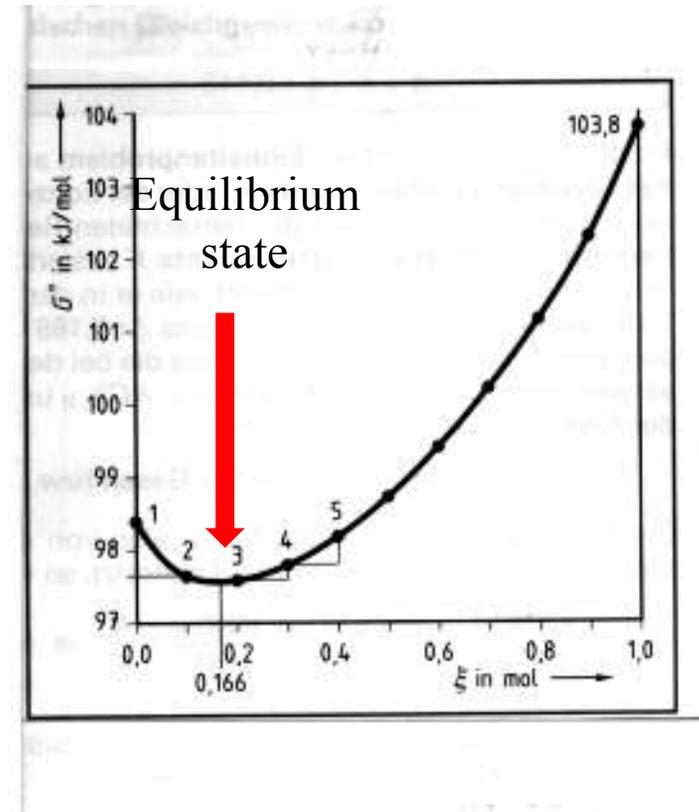
DRIVING FORCE OF MIXING

$$\Delta_e G < 0$$

$$\Delta_e G = \Delta_e H - T \Delta_e S$$

heat of mixing

entropy of mixing

endother

athermic

exotherm

$$\Delta_e H > 0$$

$$\Delta_e H = 0$$

$$\Delta_e H < 0$$

Change in molecular order
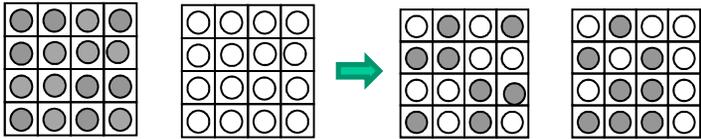
$$\Delta_e S > 0$$

Why do small molecules disolve better than macromolecules?

$$\Delta_e G = \Delta_e H - T\Delta_e S$$

Athermic solution: $\Delta_e G \cong -T\Delta_e S$

monomer

$$S = k_B \ln W$$

polymer

$c = \dfrac{10}{64}$
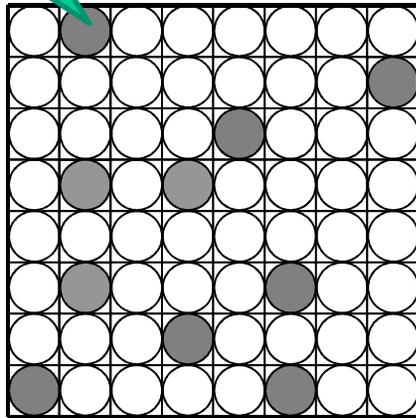
$c = \dfrac{10}{64}$
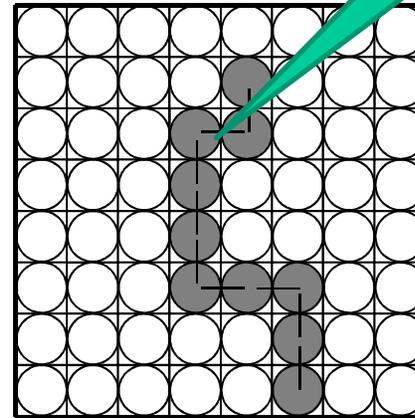
$$W_{A,B} = \frac{(N_A + N_B)!}{N_A!N_B!} = \frac{64!}{10!54!} = 1.51 \cdot 10^{11}$$

$$W_{A,B} = 64 \cdot 4 \cdot 3^8 = 6144$$

$$S_{A,B} = k_B \ln(1.51 \cdot 10^{11})$$

$$S_{P,A} = k_B \ln(6144)$$

$$\Delta G_{A,B} = -k_B T \ln(1.51 \cdot 10^{11})$$

$$\Delta G_{P,A} = -k_B T \ln(6144)$$

Bigger TD driving force !

Smaller TD driving force !

Conformation and entropy

constitution - configuration - conformation

Random coil   Ordered conformations

Higher conformational entropy   **Much smaller conformational entropy**


$$W=1 \quad \longrightarrow \quad S_{konf} = 0$$

Flexibility of macromolecules

Entropic elasticity

$$\Delta S_{konfig} < 0$$

$$\Delta S = \Delta S_{konfig} + \Delta S_{term} = 0 \implies \Delta S_{term} > 0$$

adiabatic

$$\Delta T > 0$$

$$\Delta S_{term} \implies \Delta S_{konfig}$$

$$\Delta S_{konfig} > 0$$

streched $\implies$ coiled

shrinking

$$Q > 0$$

heating

$$\Delta l < 0$$

M

M
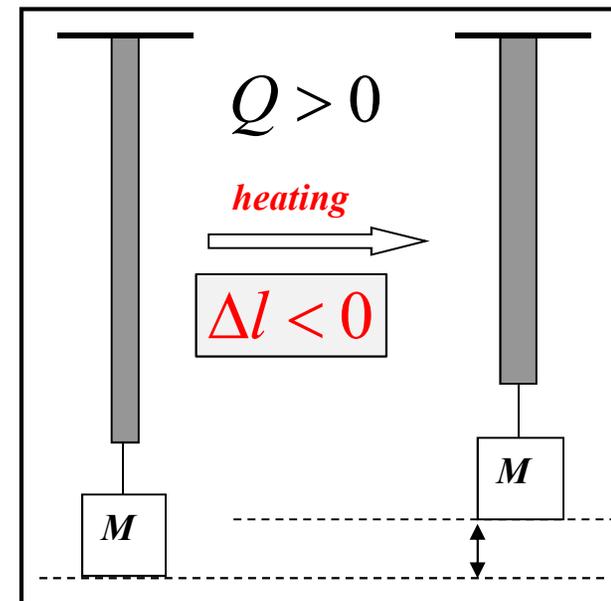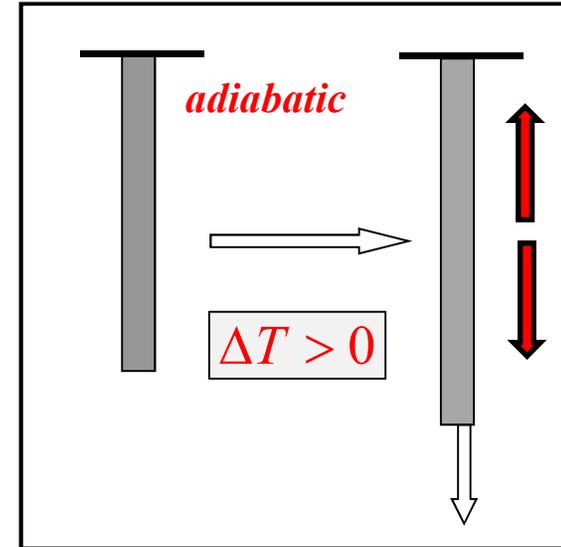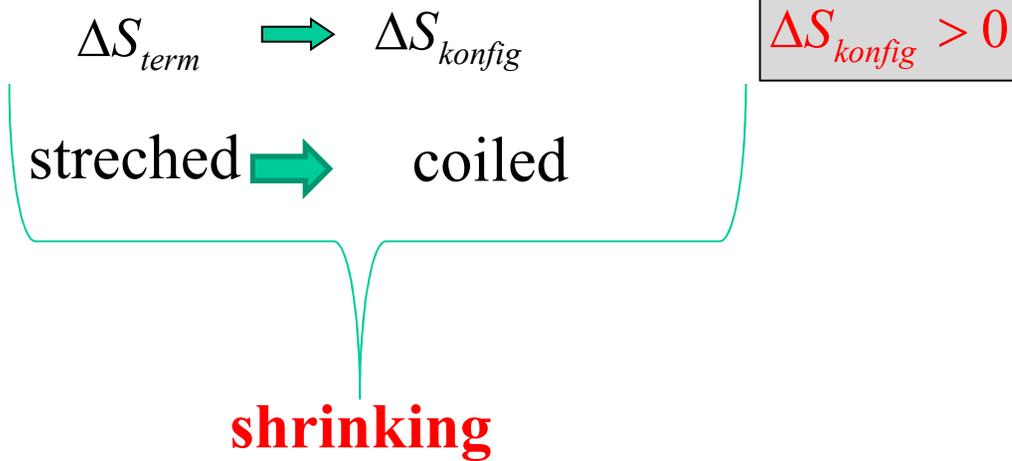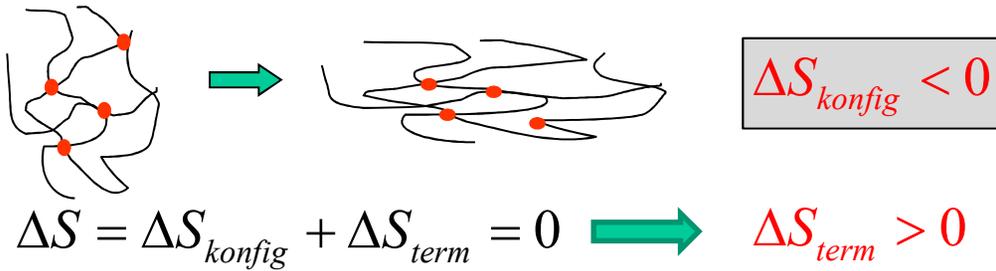
Thermodynamics of happiness *(W. Ostwald):*

B :   Measure of happiness (like entropy)

E_o :   Optional energy

E_{no} :   Non optional energy

$(E_o + E_{no})$ total energy

$(E_o - E_{no})$ Energy for happyness

$$B = (E_o + E_{no})(E_o - E_{no}) = E_o^2 - E_{no}^2$$

$B \rightarrow \infty$ Ultimate goal!