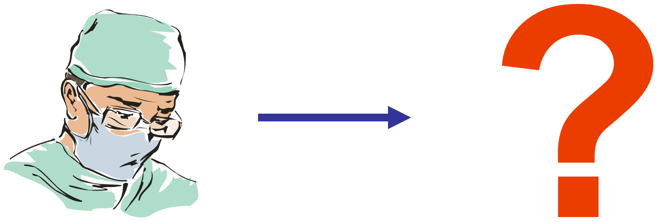
Hypothesis testing



Question → Answer: hypothesis → Test: true or not?

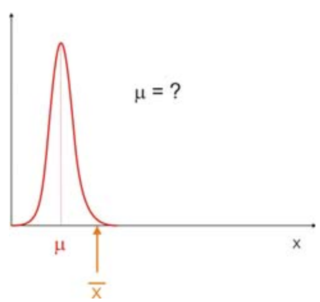
Question

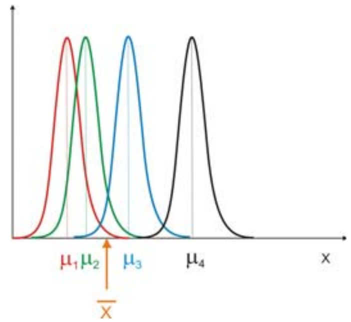
<u>Very frequently:</u> what is the possible value of μ?

Sample: we know only the average!

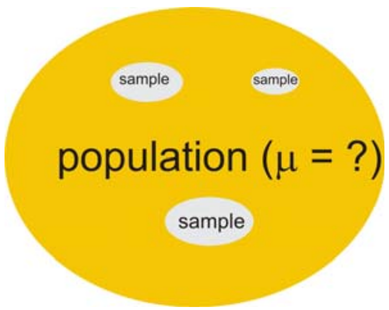
$\mu \neq$ average. Random deviation or not?

<u>Standard error</u>: the measure of the random deviation.





Average is not the mean!



Why?

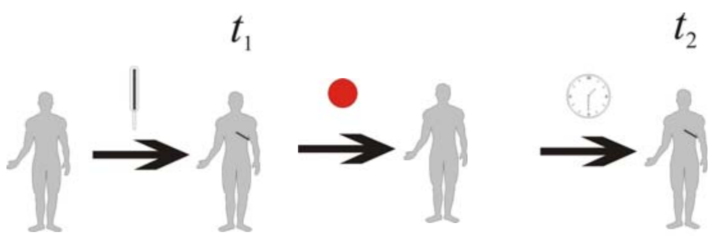
1. Finite no. of elements (random deviation)

2. Systematic deviation. (the hypothesis is not true, the value of the μ is different)

Example

Question: The medicine decreases the fewer or not?

experiment



$$x = \Delta t = t_2 - t_1$$

How many trials?

Outcome: 1. $\Delta t > 0$; 2. $\Delta t = 0$; 3. $\Delta t < 0$

Only 1?

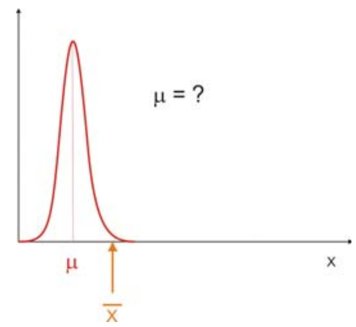
Not only the medicine influences the fewer!

We suppose that other effects are random events!

__Population__:

1. Effective $\mu \neq 0$!
2. Not effective $\mu = 0$!

Sample



The average deviates around the mean!

The average normally is not 0! Why?

1. ***__Random event__***, due to the finite no. of the elements.
2. Our ***__base hypothesis is not true__***.

Which case is true?

Nullhypothesis

__Population__: 1. Effective $\mu \neq 0$! 2. Not effective $\mu = 0$!

__Sample__: average $\neq 0$ | average should be 0

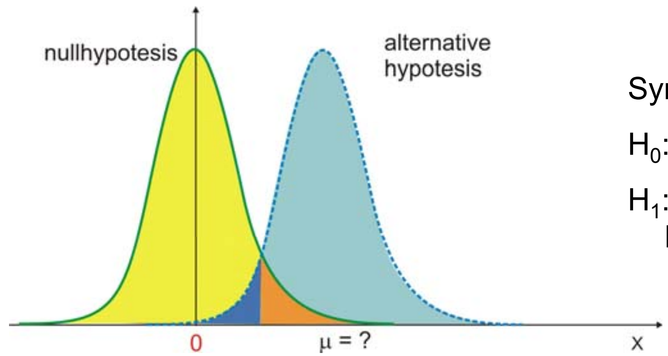
Alternative hypothesis

μ = ?

__Nullhypothesis!__

Random deviation (finite no. of elements)

Why the nullhypothesis?



Symbols.

H_0: nullhypothesis

H_1: alternative hypothesis

Nullhypothesis: the mathematical statistics is able to calculate the possible ***__random deviation__*** of the average from the μ!!!

Example

Medicine decreases the fewer or not?

Nullhypothesis: not! μ is 0. But the average is not 0.

sample	average
1.	-0.2 °C
2.	-1 °C
3.	-1.5 °C

If the difference is larger,

seems to be more sure.

Large difference?

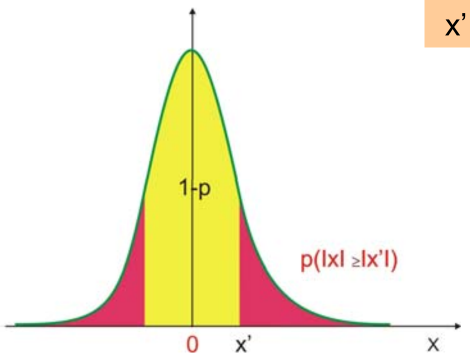
What is the measure of the difference?

Standard error: average deviation of the averages around the μ.

$(\overline{x} \pm s_{\overline{x}})$ ~ 68% - confidence interval

Base: nullhypothesis!

Question:
x' average may be 0?



Condition:

Known distribution is necessary.

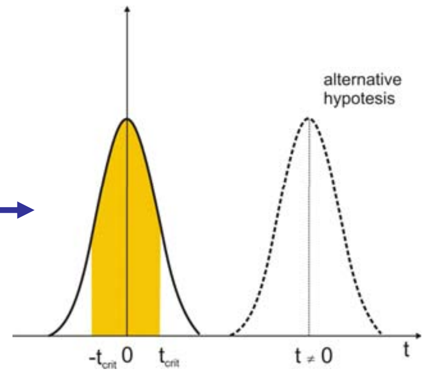
t-value

$$t = \frac{\overline{x} - \mu}{s_{\overline{x}}}$$

Compare the difference to the standard error.
(μ is frequently 0)

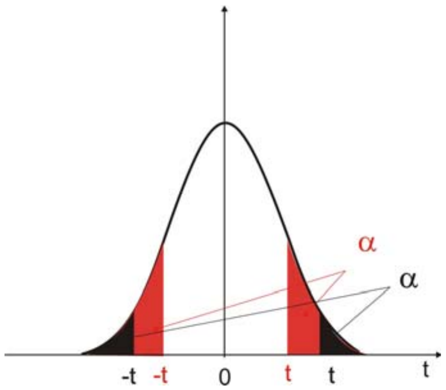
The average deviates around the μ, so the *t*-value deviates around the 0.

(if the nullhypothesis is true!)



Why is it better?

We are able to calculate the ***random distribution*** of the *t* values!!! (Student's *t*-distribution)



This describes the random deviation of the *t* from the 0!!!

Shape of the distribution depends on the no. of the elements.

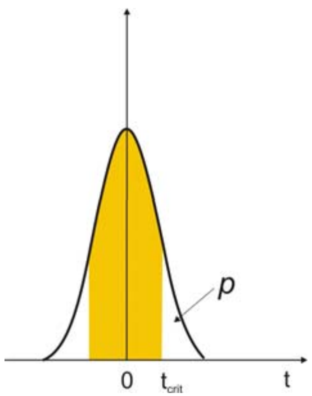
One-sample t-test

Question about the possible value of the μ.

Nullhypothesis: the difference of the average is random.

t = 0, the difference is due to the finite no. of elements.

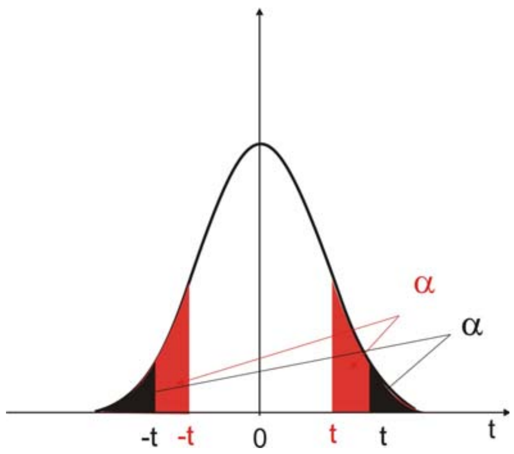
Base of decision



p is the probability, that $t_{calc} \geq t_{crit}$ randomly.

If *p* is enough small, more probable that the nullhypothesis is not true.

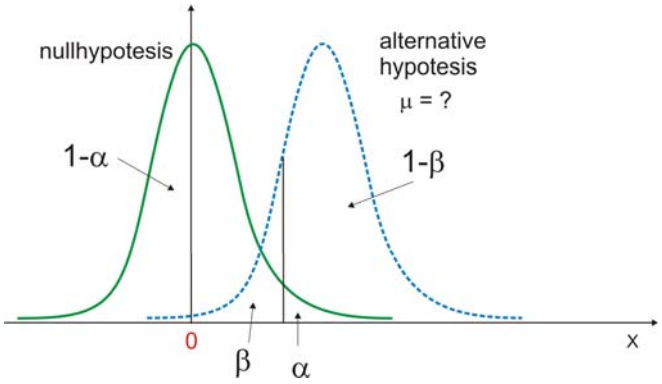
t-table



Different t_{crit} value belong to different significance level.

α – is the significance level: the probability that *t*-value is randomly out an interval.

Base of the decision



Decision

- 1. if the probability of the random event is small ($p(|t| \geq t_{crit})$ is $\leq 5\%$) – ***reject*** the nullhypothesis.
- 2. if the probability of the random event is high ($p(|t| \geq t_{crit})$ is $> 5\%$) – ***accept*** the nullhypothesis.

The answer is not yes or no,
true or false!!!

Decision using *t*-table

t-table

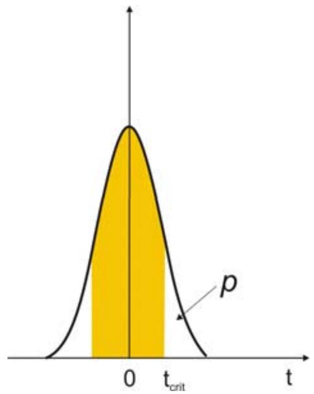
d.f.: degree of freedom

significance level

d.f.	probability (p)			
	0.1	0.05	0.02	0.01
1	6.31	12.7	31.8	63.7
2	2.92	4.3	6.96	9.92
3	2.35	3.18	4.54	5.84
4	2.13	2.78	3.75	4.6
5	2.02	2.57	3.37	4.03

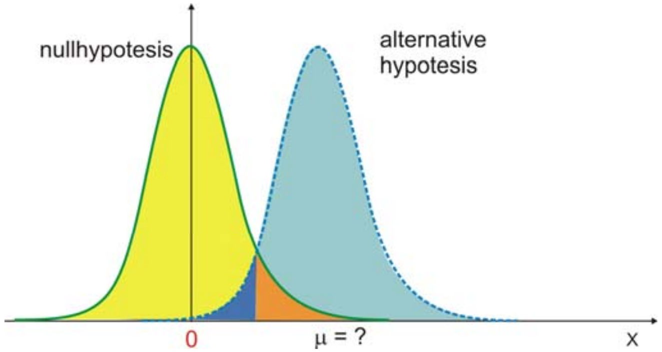
degree of freedom: (n-1)

Decision using computer



p: the probability of random event that the $t \geq t_{calculated}$.

Possibility of the mistake



Decision

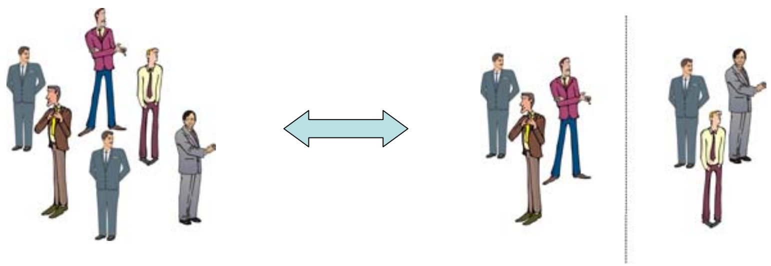
Decision: the nullhypothesis

Nullhypothesis	accepted	rejected
true	**Right decision**	I. Type error (α)
false	II. Type error (β)	**Right decision**

Conditions

- 1. Question about the mean of the population.
- 2. Variable has <u>normal distribution</u>.

Test in two groups

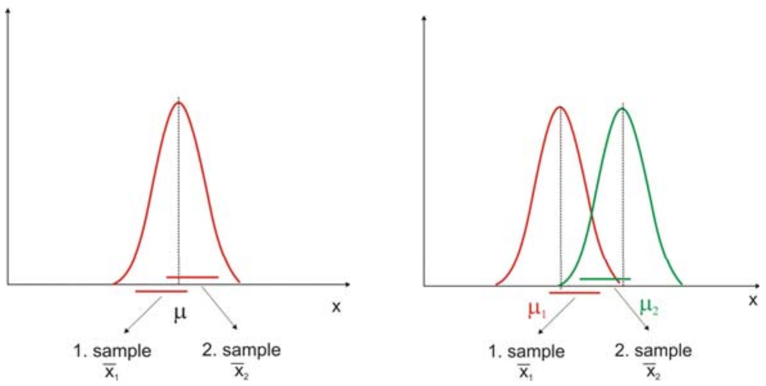


<u>Conditions</u>:

1. Variable has <u>normal distribution</u>.
2. The standard deviation is same in the two groups, the difference is due to the sampling.
3. The samples are independent from each other.

Nullhypothesis: samples derive from the same population.

Alternative hypothesis: samples derive from different populations.



The nullhypotesis

Samples derive from the same population.

But, due to the sampling: $\longrightarrow$ $\bar{x}_1 - \bar{x}_2 \neq 0$

The expected value of the difference is zero.

The observed difference is due to the sampling only.

The common standard error

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{Q_1+Q_2}{n_1+n_2-2}} \cdot \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

If the standard deviation is really common.
$\sigma_1 = \sigma_2$

Calculation of the t-value

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_{\bar{x}_1-\bar{x}_2}}$$

two-sample *t*-test

The degree of the freedom: n_1+n_2-2

Decision: look one-sample *t*-test!

$\sigma_1 = \sigma_2$?

Condition: $\sigma_1 = \sigma_2$ $\longrightarrow$ Question: $s_1 = s_2$?

Nullhypotesis: H_0: $\sigma_1 = \sigma_2$.

Alternative hypothesis: H_1: not.

F-test!

F-test

$$F = \frac{s_1^2}{s_2^2}$$

The expected value belonging to the nullhypotesis: 1.

degree of freedom:
nominator: $n_1 - 1$.
denominator : $n_2 - 1$.

(Calculation: $s_1 > s_2$. the greater is in the nominator.)

Decision

Table: (α = 0,05)

(1. line: d.f. of the nominator,
1. column: d.f. of the denominator)

	1	2	3	4	5
1	161,4	199,5	215,7	224,6	230,2
2	18,51	19,00	19,16	19,25	19,30
3	10,13	9,55	9,28	9,12	9,01
4	7,71	6,94	6,59	6,39	6,26
5	6,61	5,79	5,41	5,19	5,05

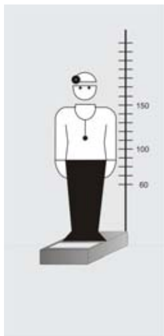
$F_{calc} \geq F_{table}$, reject.
$F_{calc} < F_{table}$, accept.

on the base of the p value:

p: the probability, that F_{calc} randomly over the F_{table}.

Correlation of two variables

Experiment:

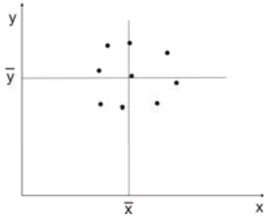


Data pairs:

No.	weight (kg)	Height (cm)
1		
2		
3		
4		
5		
6		
7		
8		
9		

Graphic representation

A

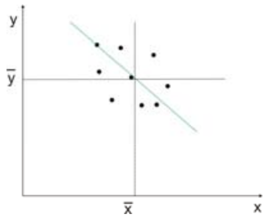


no any
tendency

B

Positive
tendency

C



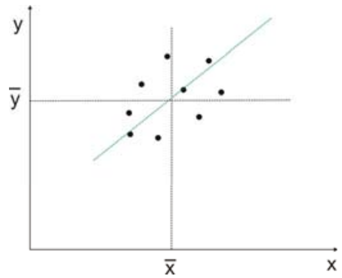
Negative
tendency

Covariance

$$Q_{xy} = \sum_i \left[(x_i - \bar{x}) \cdot (y_i - \bar{y}) \right]$$

$$\mathrm{Cov(x, y)} = \frac{Q_{xy}}{n-1}$$

Positive tendency:

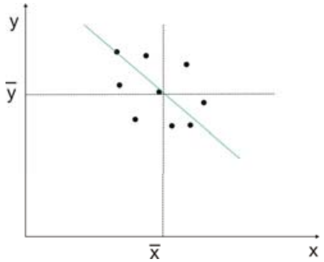


Frequently:

If $x_i > \bar{x}$, then $y_i > \bar{y}$ and
if $x_i < \bar{x}$, then $y_i < \bar{y}$.

Consequence: $Q_{xy} > 0$.

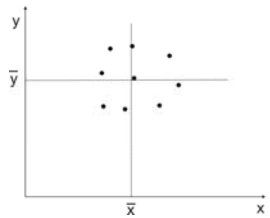
Negative tendency:



Frequently:
If $x_i > \bar{x}$, then $y_i < \bar{y}$ and
if $x_i < \bar{x}$, then $y_i > \bar{y}$.

Consequence: $Q_{xy} < 0$.

No tendency:



The y values are independent from the x-values.

Consequence: $Q_{xy} = 0$.
(if $n=\infty$)

Correlation coefficient

$$r = \frac{Q_{xy}}{\sqrt{Q_x \cdot Q_y}}$$

$$-1 \leq r \leq 1$$

Population: $r = 0$ no correlation,
$r \neq 0$ correlation (strength is proportional to the actual value of r.

Testing of the *r*

Sample: due to the finite no. of the elements we can calculate only the estimation of the *r*.

Question: Correlation or not?

Nullhypotesis: No correlation.
H_0: r = 0.

Correlation *t*-test

$$t = r\sqrt{\frac{n-2}{1-r^2}}$$

d.f.: n - 2

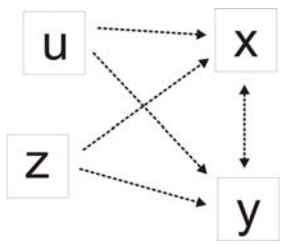
Decision: based on t-value. Look previous cases!

Condition: at least one of the variable has normal distribution.

Meaning of correlation

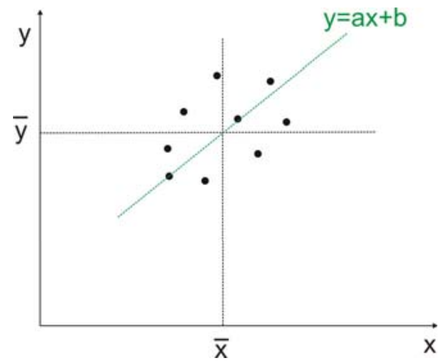
Not necessary being direct causality.
(But may be!)

In the background there may be quantities, effects that influence both measured variables.



Linear regression

If variables have normal distribution, the dependency is linear, and we can describe with straight line.



Frequency data

Has no normal distribution!

Example: headache

Effective: no headache

Not effective: headache

Experiment

1. group: patients introducing the ***medicine***

headache (a) | no headache (b)

2. group: patients introducing the ***placebo***

headache (c) | no headache (d)

Contingency table

	headache	no headache	Total
1. group	a	b	a+b
2. group	c	d	c+d
total	a+c	b+d	n

Nullhypothesis

If the effect is independent from the medicine, we expect:

$$\frac{a}{b} = \frac{c}{d} \longrightarrow a \times d = b \times c$$

Nullhypothesis: the effect is independent from the medicine.

χ^2-distribution

$$\chi^2 = \frac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

Nullhypothesis: χ^2-value is 0, the difference due to the random event only.

χ^2-distribution describes the random deviations of the χ^2-value.

Decision

Same, than in the case of *t*-distribution.
We use χ^2-distribution.

Expected value is 0, if the nullhypothesis is true.

if calc. ≥ crit. - reject the nullhypothesis else accept.

degree of freedom: in this special case = 1.

In general:

d.f.=(r-1)(c-1), where r – no. of rows
c – no. of columns