# MEDICAL STATISTICS

Physiology

Anatomy

Chemistry

…

No any doubt

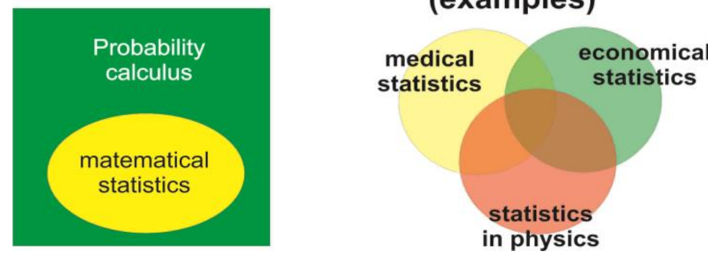
Statistics

?

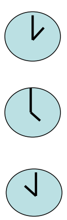
# Medical statistics

**Theory: matematics**

**Practice: applied statistics (examples)**




# Example: body temperature



36.7 °C

36.9 °C

36.6 °C

36.7 °C

36.9 °C

36.5 °C

1. Inaccuracy of the measurement.
2. ***Daily fluctuation!!!***

***3. Biological variability!!!***

The measured value is not constant!

Measured value: 37.0 °C.

Is it healthy or not?

# Another examples

RBC: $4.5 \times 10^{12}$ 1/l (<u>$3.9\text{-}5 \times 10^{12}$ 1/l</u>) → normal range?

The new method in therapy is better then the old one or not?

How can we prove that a medicine decreases the fewer or not?

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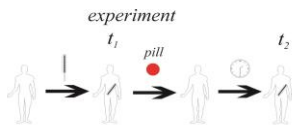
**Questions!**

# How can we answer?




$x = \Delta t = t_2 - t_1$

**1. Δt < 0**

Medicine is effective.

***Medicine is not effective***: fluctuation, patient will be better, etc.

**?**

**2. Δt >= 0**

**Consequence**:

The answer is not so simply!

(Not only the medicine influence the body temperature.)

Medicine is not effective.

***Medicine is effective***: fluctuation, doesn't increase further, etc.

# Variables

| variable | range | type | variable type | |
|---|---|---|---|---|
| height | ~50 cm … ~250 cm | real number | numerical | continuous |
| no. of teeth | 0 .. 32 | integer | | discrete |
| blood type | A, B, AB, 0 | letters | categorical | nominal |
| severity of cancer | 1 … 4 | integer | | ordinal |

**Descriptive statistics!**

# Description of a variable

- Type
- Possible values
- Occurrence of the values

# Numerical variables

| Name | ***Continuous*** | ***Discrete*** |
|---|---|---|
| Definition | Infinitely large no. of values in a certain range | Only finite number of values |
| Example | Height, temperature, pressure … | No. of teeth, no. of children … |

## Categorical variables

| Name | ***Nominal*** | ***Ordinal*** |
|---|---|---|
| Definition | No order among the values | There is a certain order |
| Example | Gender, blood-type … | Severity of the illness, strength of pain … |

## Determination of the possible values

- Continuous : giving a possible range.
  - » e.g.: height from ~50 cm - to ~ 250 cm
- Another : listing the values, if it is possible
  - » E.g.: blood type: A, B, AB, 0

## Occurence

***Observation***: The occurrence of the values are not the same!

↓

Trial: experiment, observation, data collection.

*Deal with only the case, when the trial may be repeated!*

Outcome: result of one trial. (e.g.: height of a student)

## Population

How many people?

↓

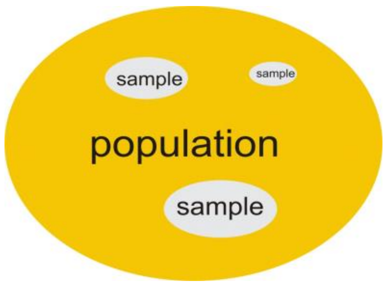
As many as possible.

↓

Ideal case: All of the people → ***population***

# Sample

A smaller portion of the population.




*n*: no. of the elements (people) in the sample.

*x*: the tested variable (quantity)

$x_i$: i-th element from the sample

# Selection of the sample

Main principle: <u>**Random sample**</u>

Medical statistics: <u>if there is no any reason to exclude,</u>
must be random!

# Occurrence

***Frequency*** (*k*): no. of occurrence in the sample.

$k_i$: no. of occurrence of the i-th value in the sample.

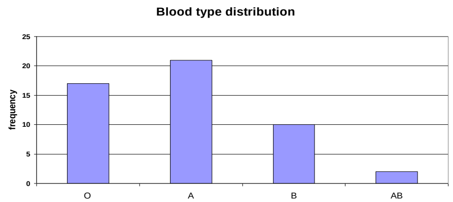
$$n = \sum_i k_i$$

# Frequency distribution

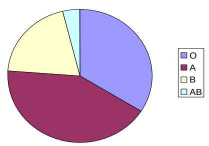
Frequency as the function of the possible values.

| Blood-type | **0** | **A** | **B** | **AB** | total |
|---|---|---|---|---|---|
| frequency | 17 | 21 | 10 | 2 | 50 |

# Presentation




Bar-chart




Pie-chart

# Relative frequency, proportion

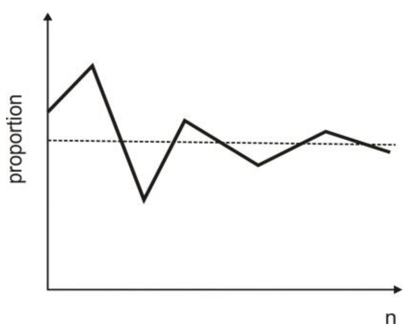
The ratio of the frequency and the total no. of the elements.

$$\sum_i \frac{k_i}{n} = \frac{1}{n} \sum_i k_i = \frac{1}{n} \times n = 1$$

Frequently it is given as percentage:

$$\frac{k_i}{n} \times 100\%$$

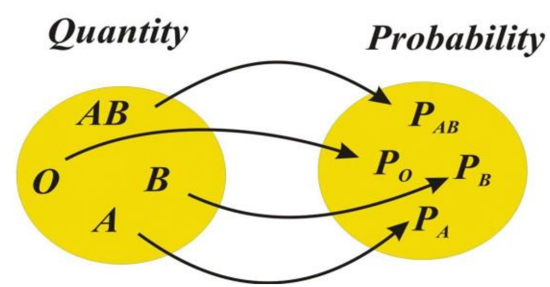
# Probability (*P*)




If *n* is infinite the name of the proportion is the probability.

Probability (P): proportion in the population.

# Probability distribution



## Properties of the probability

$$0 \le P \le 1$$ → P = 0 - never occur
P = 1 - always occur

Example: blood- type

$$P_A + P_B + P_{AB} + P_0 = 1$$ → $$\sum_i P_i = 1$$

(exclusive events)

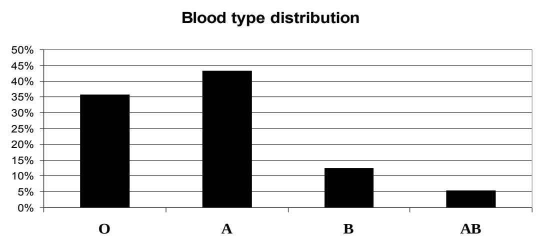
## Probability and proportion

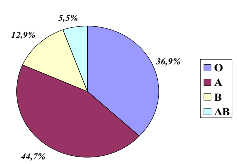
| Sample | Population |
|---|---|
| n is finite | $n = \infty$ |
| proportion | probability |

**Probability very frequently is unknown!**

We usually use proportion instead of the probability.

## Presentation




## Continuous quantity

Infinite no. of possible values!!!

**Class**: a short interval in the whole range.

**Class-width**: the length of the class.

Frequency: no. of elements in the given class.

↓

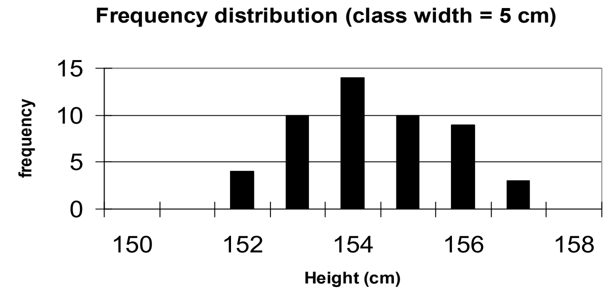
Like a discrete value!

## Example

| | |
|---|---|
| 1 | 160 cm |
| 2 | 181 cm |
| 3 | 175 cm |
| 4 | 163 cm |
| 5 | 165 cm |
| 6 | 179 cm |
| 7 | 164 cm |
| 8 | 185 cm |
| 9 | 177 cm |
| 10 | 168 cm |

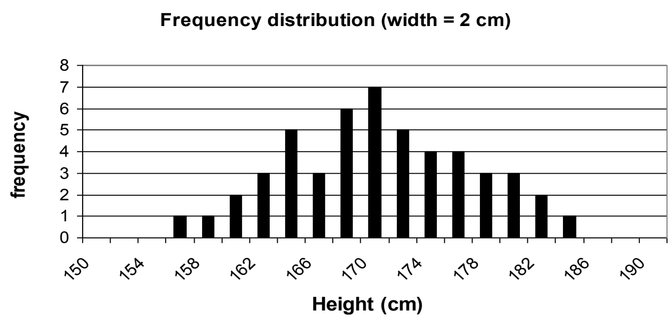
| class | $k_i$ |
|---|---|
| 160-164 | 3 |
| 165-169 | 2 |
| 170-174 | 0 |
| 175-179 | 3 |
| 180-184 | 1 |
| 185-189 | 1 |

## Presentation



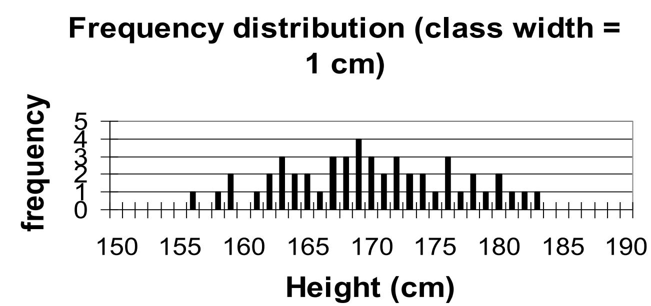

5 cm is too large!

## Decrease the width!




Observation: frequency decreases!

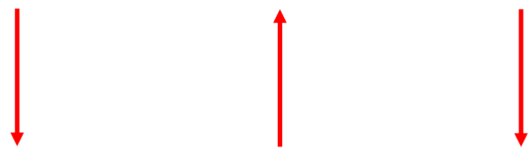
## Presentation




Reason: n is too small!

## Consequence

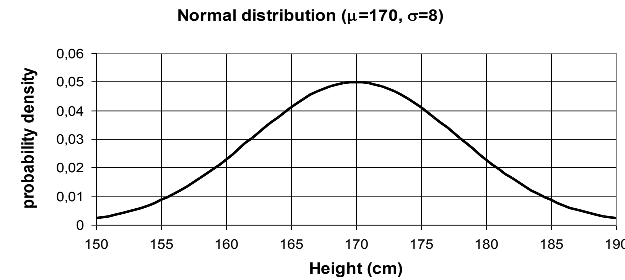
Class-width | No. of classes | Frequencies



We must increase the no. of the elements!

## Normal distribution

If $n$ and no. of classes are infinite!




## Theoretical description

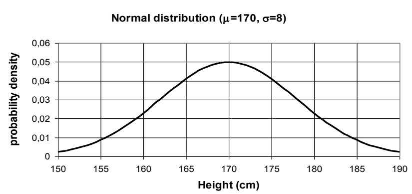
Normal or Gauss-distribution

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Parameters:

μ – expected value or mean

σ – theoretical standard deviation

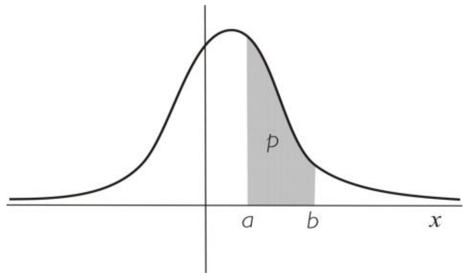



## Meaning of the parameters

μ **(mean):**
the value belonging to the maximum of the curve.

**σ (theoretical standard deviation):**
the average deviation of the data from the μ.

## Probability



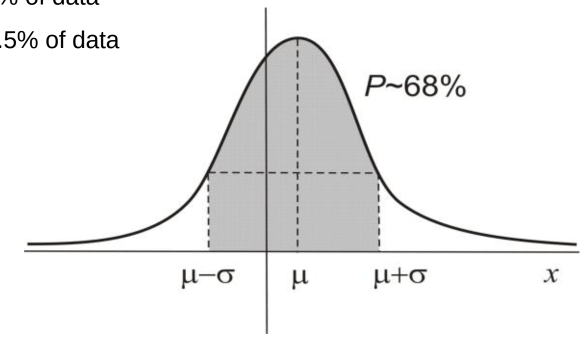

***P* is the probability that x is in the (a,b) interval.**

## Standard deviation

(μ±σ) ~ 68% of data

(μ±2σ) ~ 95% of data

(μ±3σ) ~ 99.5% of data



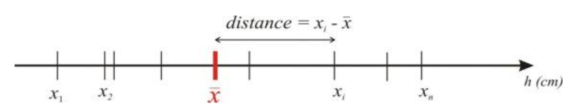

## Normal distribution

***Theoretical distribution*** that describes the population. In practice usually we don’t know the parameters of this.

We usually have a ***random sample*** from the population.

We must estimate the parameters!

## Estimation of the μ



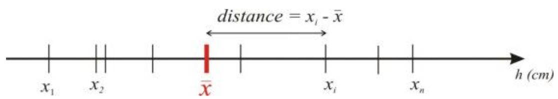

***average***: must be in the center of the data range.

$$\sum_i (x_i - \bar{x}) = 0 \longrightarrow \bar{x} = \frac{\sum_i x_i}{n}$$

## Estimation of the σ

σ = average deviation of the data from the μ.

**s (standard deviation)** = average deviation of the elements from the average.




$$Q_x = \sum_i (x_i - \bar{x})^2 \geq 0$$

## Standard deviation

$$s = \sqrt{\frac{Q_x}{n-1}}$$

*s*: the average deviation of the elements from the average.

($\bar{x}$±s) ~ 68%

($\bar{x}$±2s) ~ 95%

($\bar{x}$±3s) ~ 99.5%

## Relation of parameters

| Sample | n → ∞ | Population |
| --- | --- | --- |
| average | ⟶ | μ |
| *s* | ⟶ | σ |