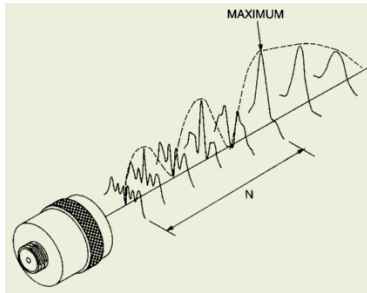
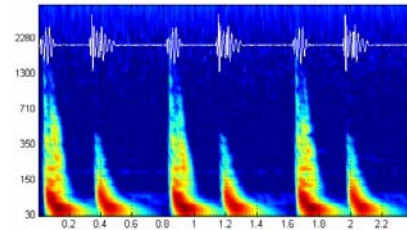


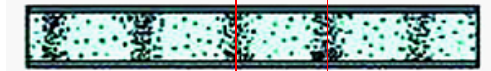
Physics of ultrasonography



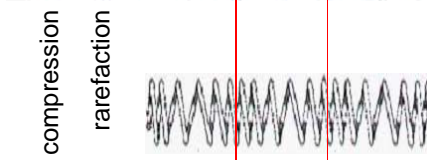
KAD 2014.02.18

Sound: mechanical wave (model)

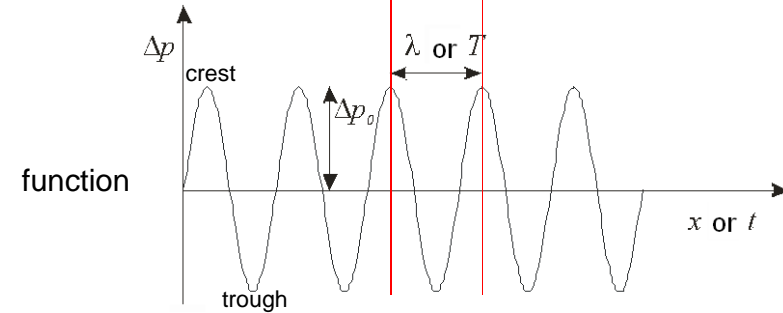
whistle



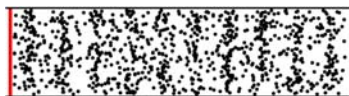
spring



spatial and temporal periodicity



2



longitudinal wave
(in the interior of liquids and gases only this type)



transverse wave

hydrostatic pressure pressure change, sound pressure

$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

pressure DC + AC amplitude phase

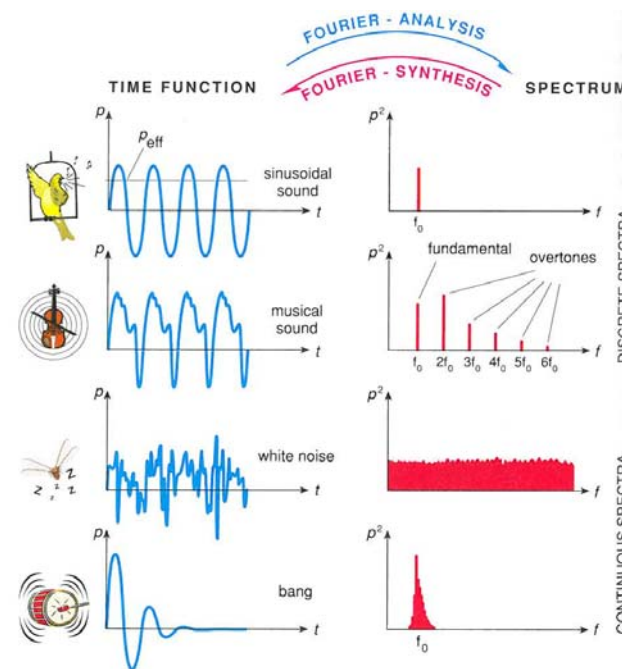
$$\Delta p(t, x) = \Delta p_{\text{max}} \sin \left[2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \right]$$

$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$



3

Textbook, Fig. II.46.



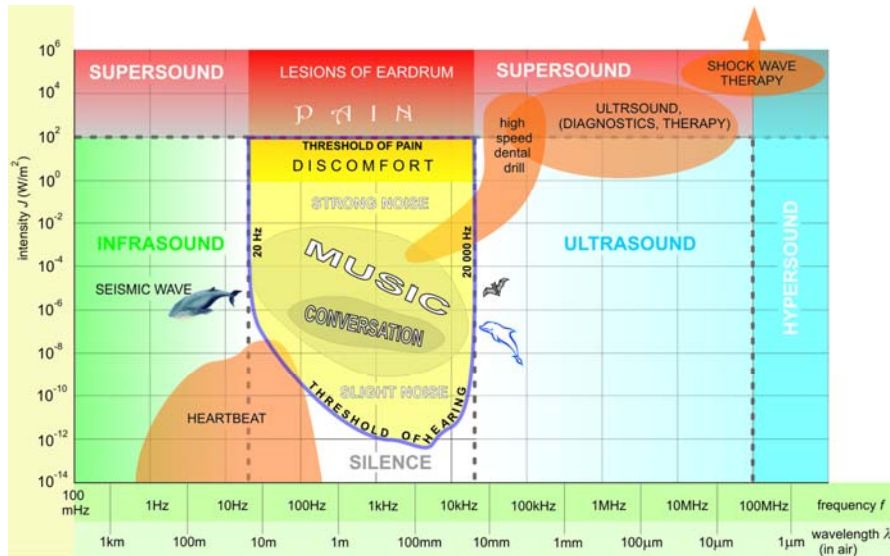
pitch:
frequency of the fundamental

timbre (tone colour):
relative strengths of overtones/harmonics (spectrum)

Textbook, Fig. IV.23.

4

Frequency and intensity regions of sounds



Lab. manual, Audiometry.

5

The role of elastic medium

$$\kappa = -\frac{\frac{\Delta V}{V}}{\Delta p}$$

compressibility
relative volume decrease
over pressure

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

speed of sound

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

acoustic impedance
(definition)

$$Z_{\text{el}} = \frac{U}{I}$$

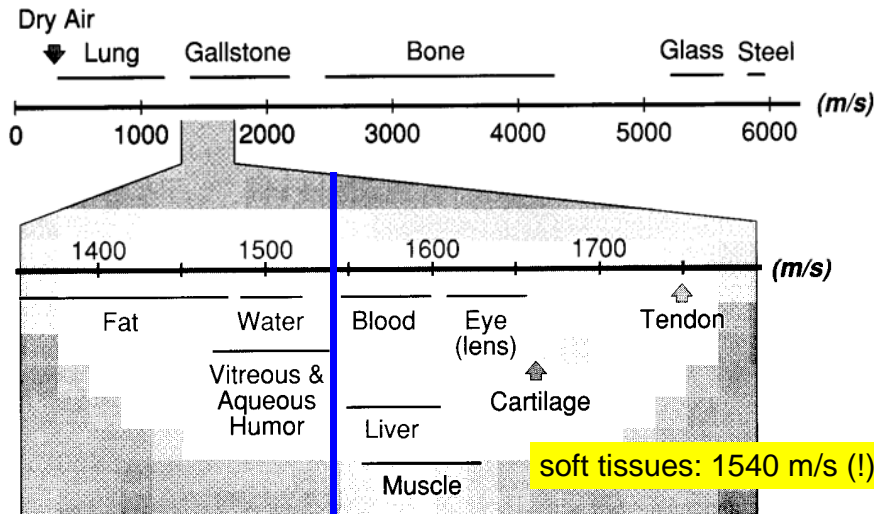
$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

acoustic impedance
(useful form)



6

Speed of sound/US in different media



7

Intensity of US

$$J = \frac{1}{Z} \Delta p_{\text{eff}}^2$$

$$P_{\text{el}} = \frac{1}{Z_{\text{el}}} U_{\text{eff}}^2$$

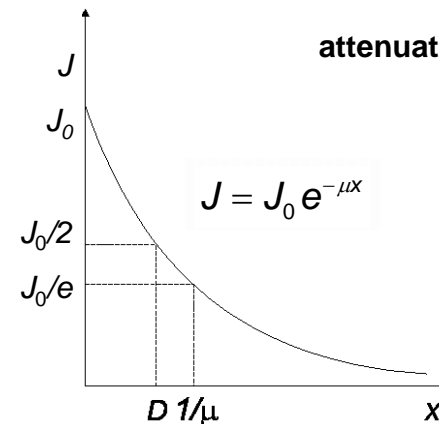
intensity =
energy-current density electric analogy

Loss of energy during propagation (absorption)

$$\text{attenuation: } \alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

μ is proportional to
frequency in the
diagnostic range



**specific
attenuation:**

$$\frac{\alpha}{f \cdot x}$$

8

μ is proportional to frequency in the diagnostic range

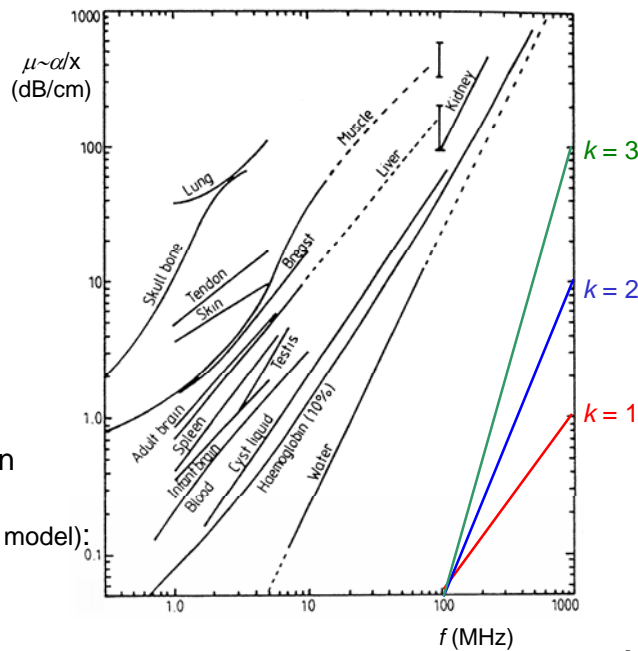
$$\mu \sim f^k, \quad k \sim 1(?)$$

$$\log \mu \sim k \log f$$

if the graph is a linear, the power function approximation is valid

specific attenuation for soft tissues (homogeneous tissue model):

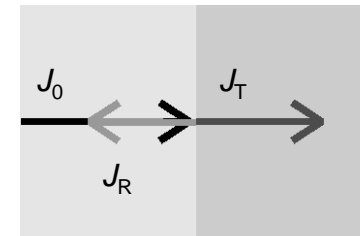
$$\frac{\alpha}{f x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$



9

Phenomena at the boundary of different media

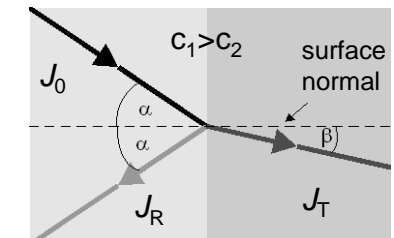
normal/perpendicular incidence



$$J_0 = J_R + J_T$$

reflection and transmission (penetration)

skew incidence



$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

Textbook, Fig. II.47.

10

Reflection (normal incidence)

reflectivity:

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

“full” reflection:

$$Z_1 \ll Z_2, \quad R \approx 1$$

optimal coupling:

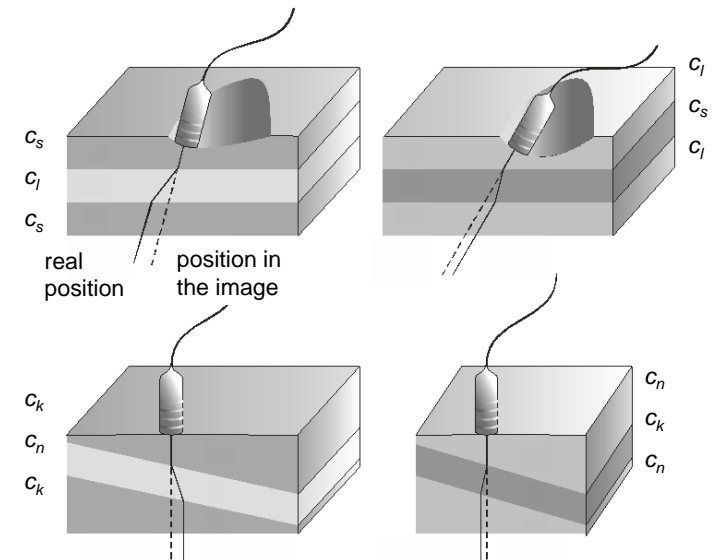
$$Z_{\text{connecting}} \approx \sqrt{Z_{\text{source}} Z_{\text{skin}}}$$



boundary surface	R
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

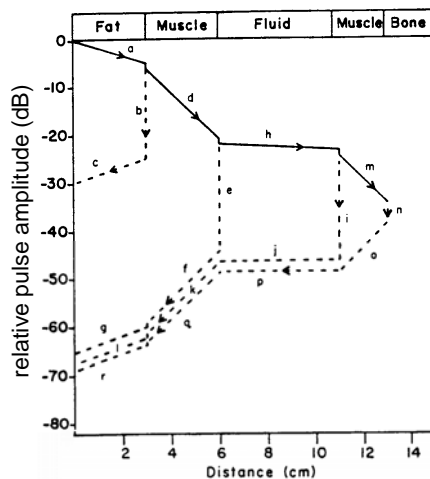
11

Phenomenon of skew incidence or normal incidence and skew boundaries



Textbook, Fig. on pg. 153

12



Absorption and reflection

the later comes back the reflection, the deeper lays the reflecting surface and the weaker is the intensity

run time dependent amplification

TGC: time gain compensation

DGC: depth gain control

boundary surface	R	$10\lg R$ (dB)	T	$10\lg T$ (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

13

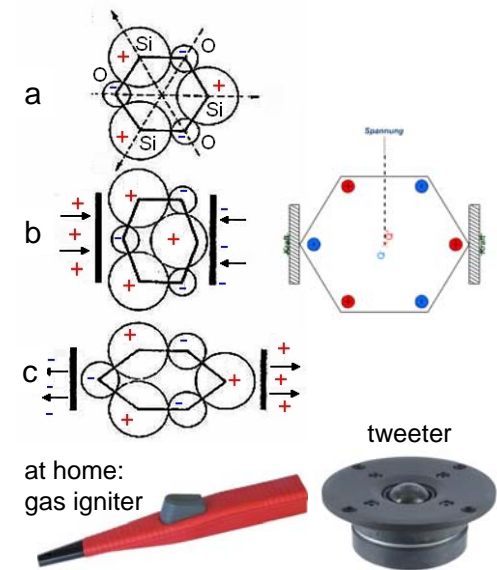
Generation of US. Piezoelectric effect

source of electric signal
(sine wave oscillator)+
transducer (piezo-crystal)

(a) Center of charge of positive and negative charges coincides.

(b) and (c) As a result of pressure, the charge centers are separated, i.e. a potential difference arises (direct ~).

The crystal is deformed when voltage is applied (inverse ~).

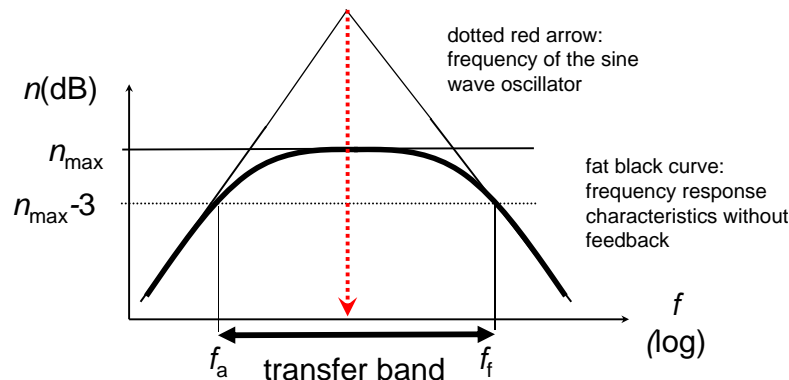


Source of electric signal : sine wave oscillator

amplifier with positive feedback

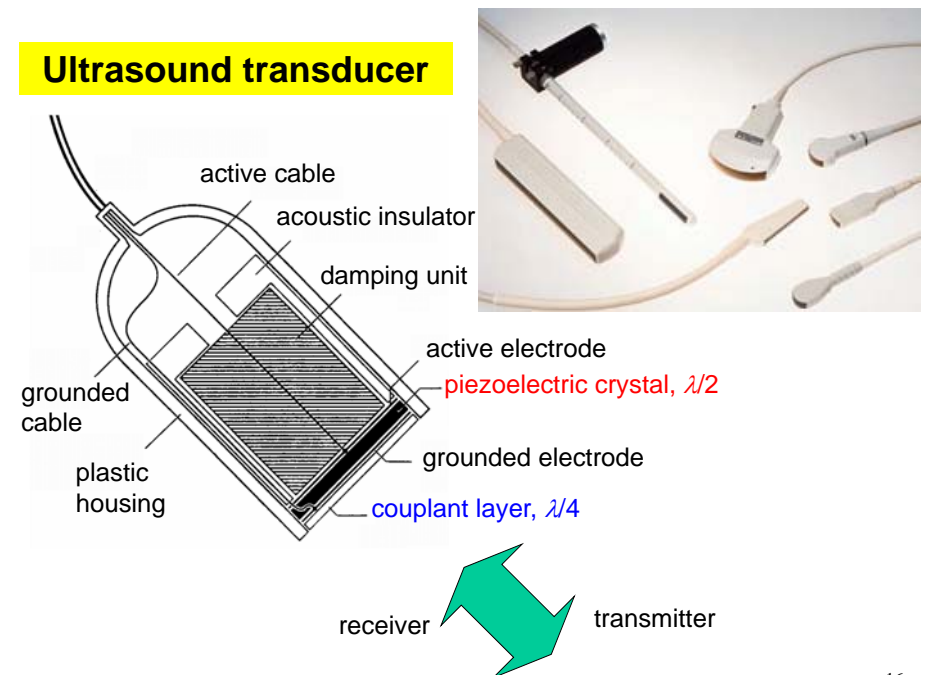
$$A_{U, \text{feedback}} = \frac{A_U}{1 - \beta A_U}$$

$\beta A_U = 1$, amplification = „infinity“ → sine wave oscillator
no input signal, output signal: sine voltage



15

Ultrasound transducer

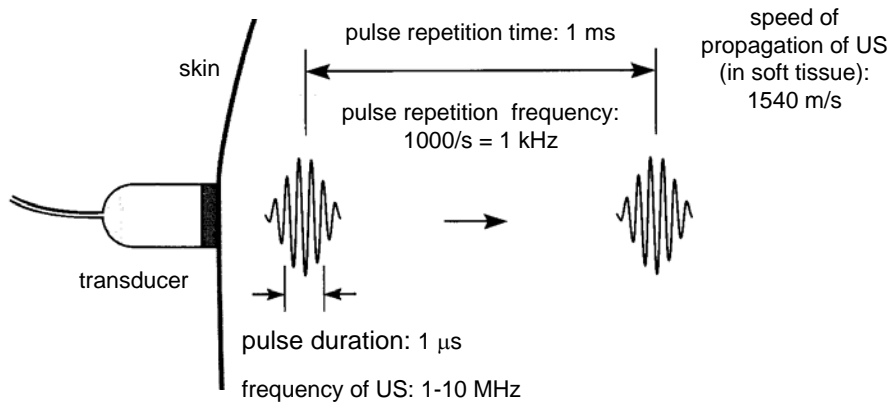


16

Characteristic of US pulses

transducer: transmitter and receiver is the same unit

time sharing mode: pulses instead of continuous wave US



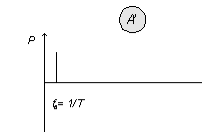
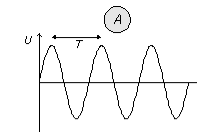
Textbook, Fig. VIII.32.

17

Time function

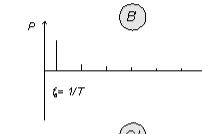
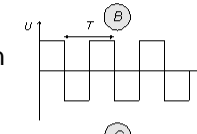
Spectrum

sine function



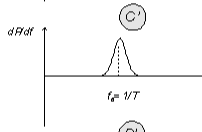
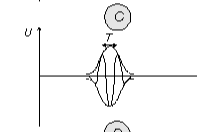
line spectrum (1 line)

square function



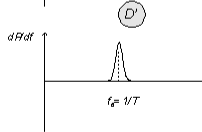
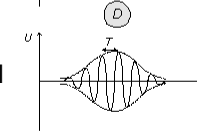
line spectrum

sine wave pocket (some „periods”)



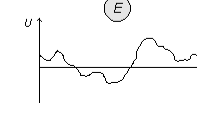
band spectrum

sine wave pocket (several „periods”)



band spectrum

aperiodic function

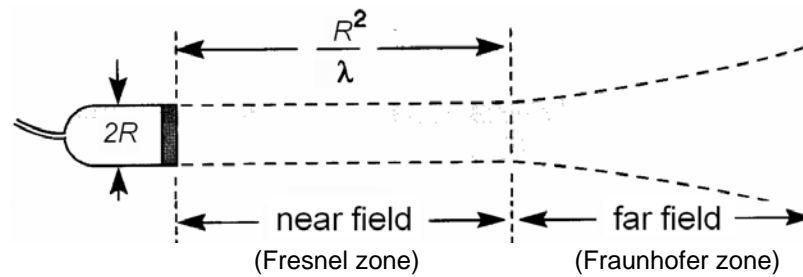


continuous spectrum

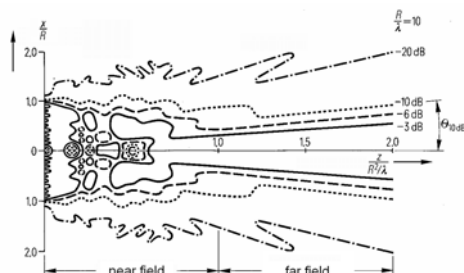
US pulse

18

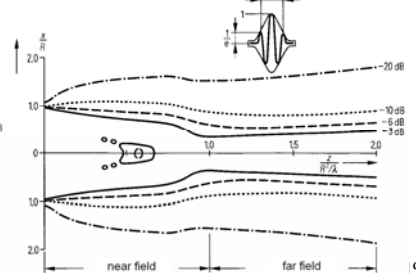
US beam shape (simplified version)



Beam shape, continuous wave US

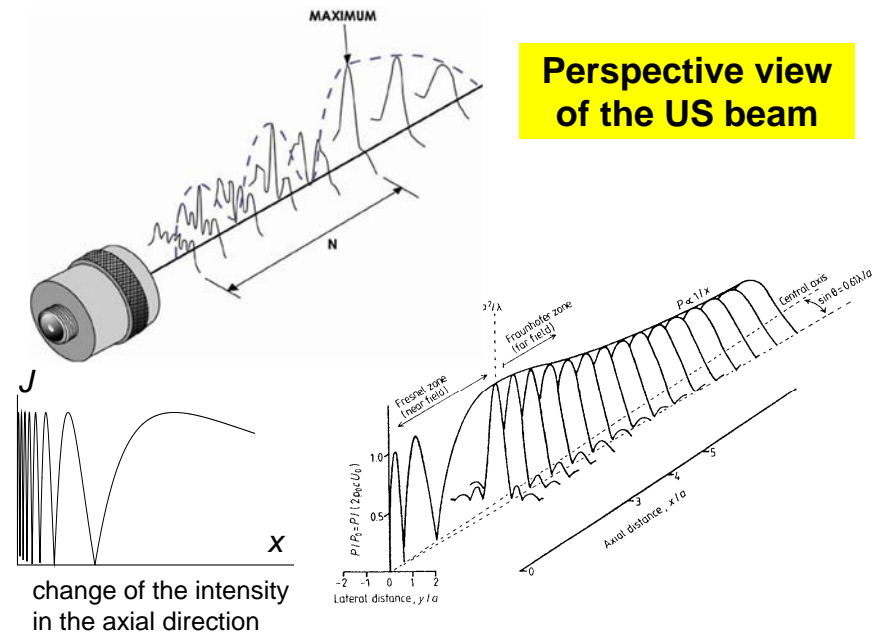


Beam shape, pulsed wave US



19

Perspective view of the US beam



cf. Textbook. Fig. on p.505

20

Resolving limit, resolution

Resolving limit is the distance between two object details which can be just resolved as distinct objects (the smaller the better).

Resolution (resolving power): the reciprocal of the resolving limit (the greater the better)

Axial resolving limit depends on the pulse length. Pulse length is inversely proportional to the frequency.

Lateral resolving limit is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

Typical values	frequency (MHz):	2	15
	wavelength (in muscle) (mm):	0.78	0.1
	penetration depth (cm):	12	1.6
	lateral resolving limit (mm):	3.0	0.4
	axial resolving limit (mm):	0.8	0.15

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Axial resolving limit

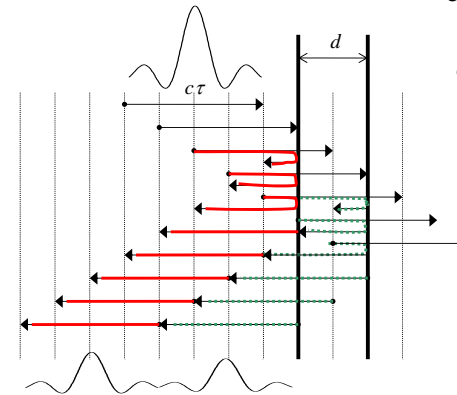
τ : pulse duration

$c_1\tau \cong c_2\tau = c\tau$ pulse length

$\delta_{ax} = d = \frac{c\tau}{2}$ resolving limit

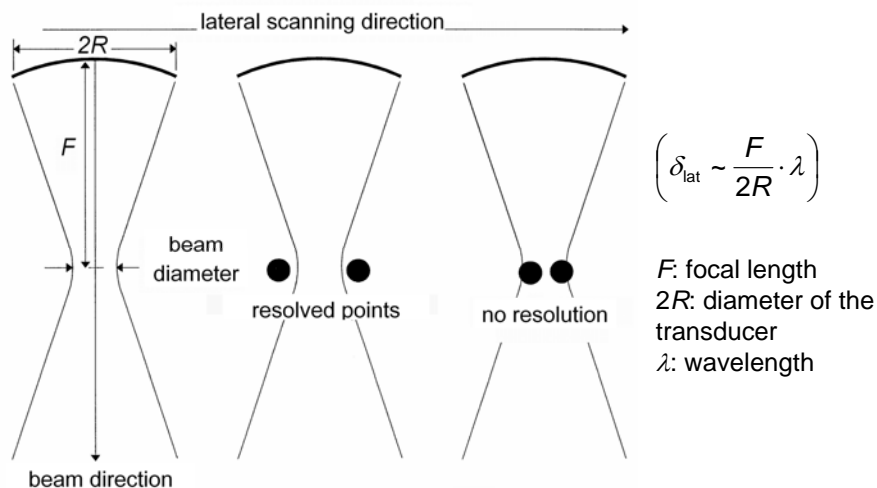
The axial resolving limit is the half of the pulse length. The echos from the adjacent surfaces in this case just hit another.

$$\tau \sim T = \frac{1}{f}$$



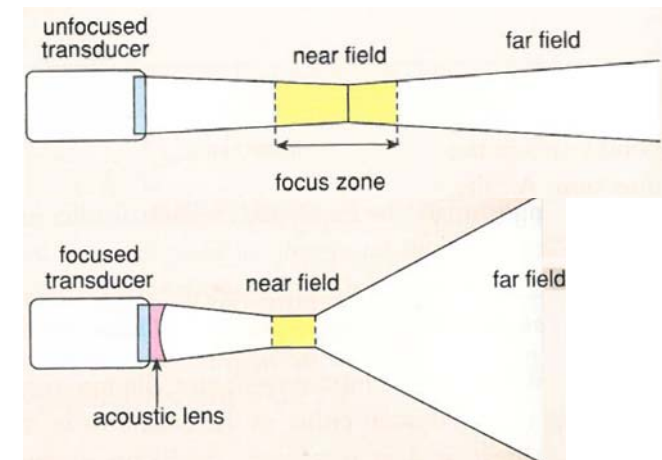
22

Lateral resolving limit



23

Focusing of the beam

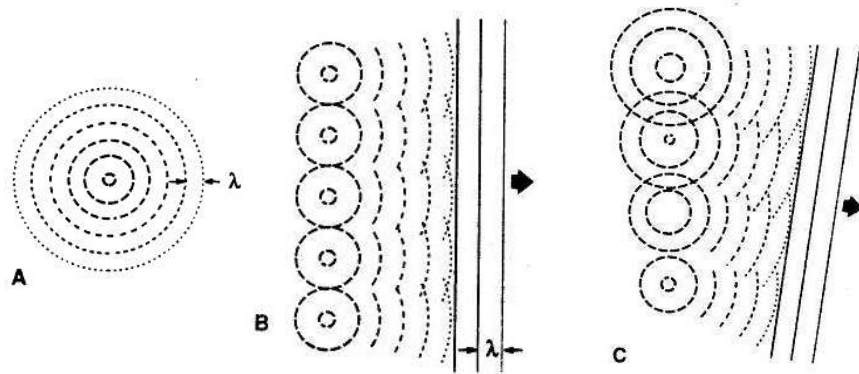


Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

cf. Textbook Fig. on p.506

24

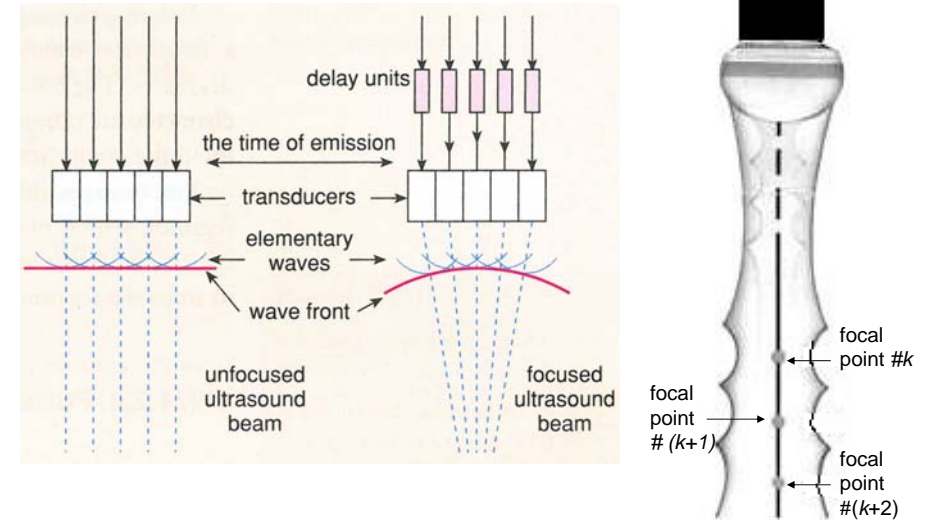
Huygens' principle



Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.

25

Electronic focusing

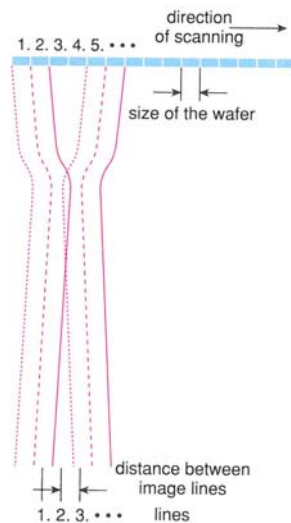


cf. Textbook Fig. on p.507

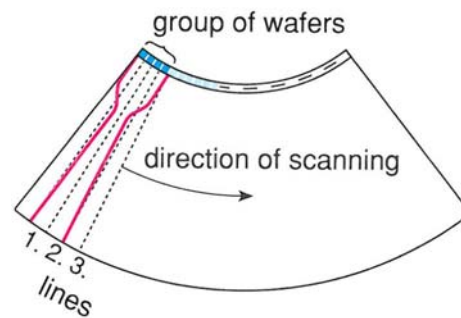
26

Scanning

multi unit linear array



multi unit curved array



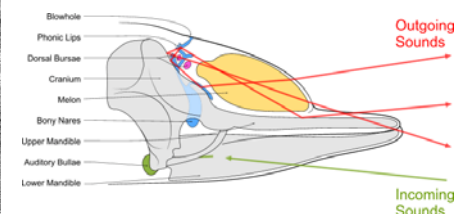
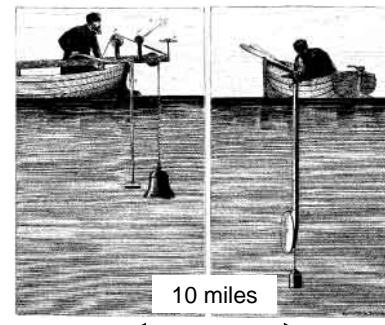
cf. Textbook Fig. VII. 36-37

27

Echo principle

1794 Spallanzani:
bat's navigation

1822 Colladen
measured the speed of
sound in water

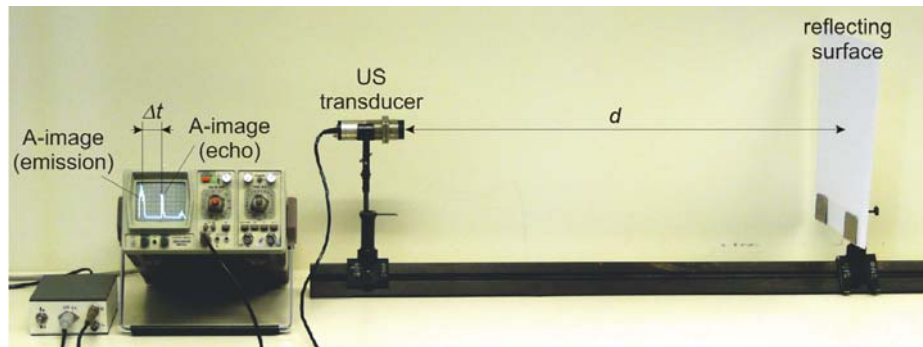


bottlenose dolphin

28

Echo principle

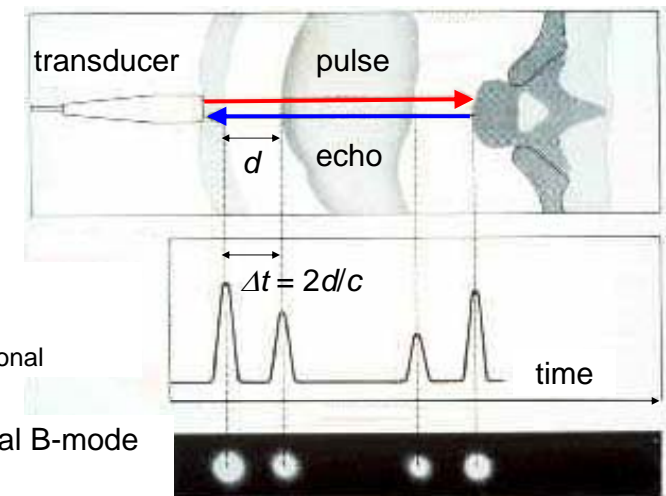
using a special US-head, short pulses are emitted in the air towards a reflecting surface, and the same US-head detects the echo signal



$$c\Delta t = d + d = 2d$$

29

Receiving the echos



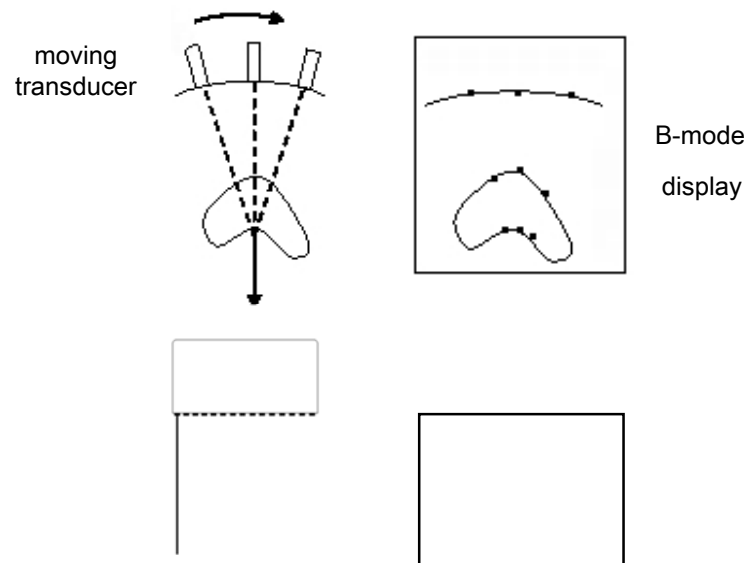
A-mode
(**A**mplitude)
only 1-dimensional

1-dimensional B-mode
(**B**rightness)

cf. Textbook Fig. VIII.33

30

2-dimensional B-mode

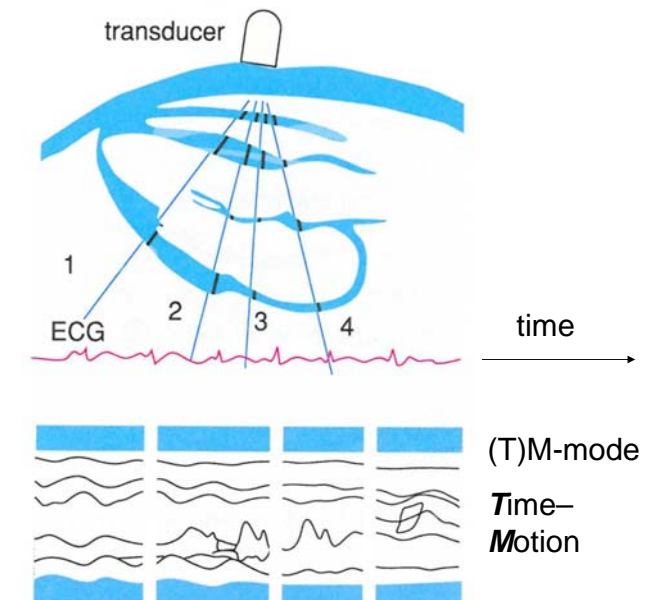


31

TM-mode

ECG signal
for reference

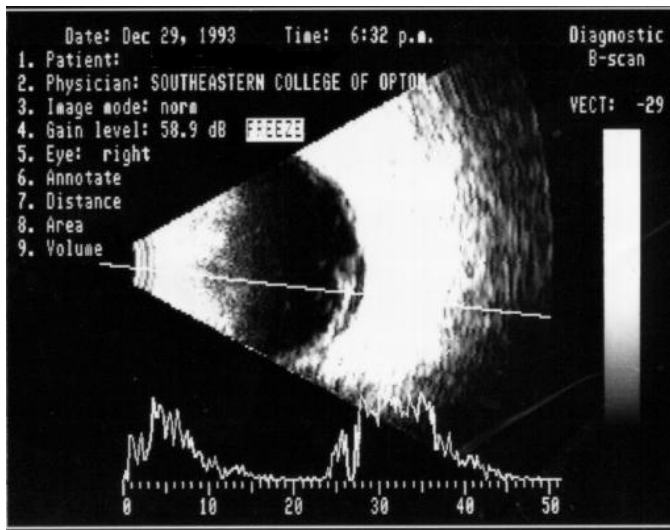
(vertical)
time-dependent
1-dimensional
B-mode



Textbook Fig. VIII.34

32

2-dimensional B-mode and A-mode (used in ophthalmology)



real speed of propagation for the accurate determination of distances:

cornea: 1641 m/s

aqueous humour: 1532 m/s

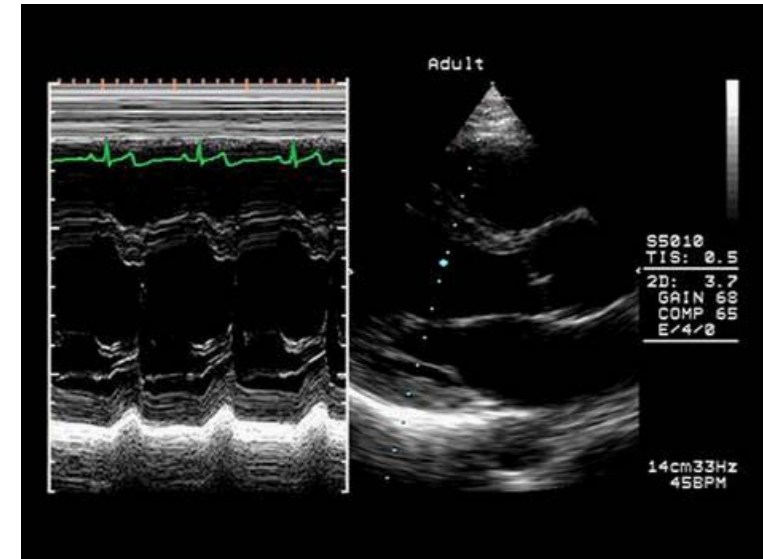
crystalline lens: 1641 m/s

vitreous body: 1532 m/s

33

TM-mode

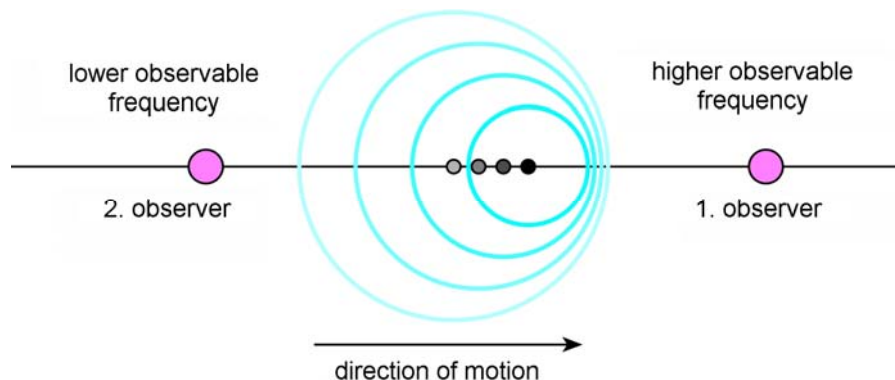
B-mode



34

Doppler phenomenon

„The pitch of a train whistle seems to get higher as it approaches, then seems to lower as the train whistle moves away.” (C. Doppler, 1842)



35

f' : **observed frequency**, f : original frequency

- (a) standing source and moving observer (v_o)
- +: observer approaches the source
- : observer moves away from the source

$$f' = f \left(1 \pm \frac{v_o}{c} \right)$$

- (b) moving source and standing observer
(if $v_s \ll c$, then „same” as (a))

$$f' = \frac{f}{1 \mp \frac{v_s}{c}}$$

- (c) moving source and moving observer

$$f' = f \frac{1 \pm \frac{v_o}{c}}{1 \mp \frac{v_s}{c}}$$

- (d) moving reflecting object (surface),
(if $v_R \ll c$)

$$f' = f \left(1 \pm \frac{2v_R}{c} \right)$$

36

Doppler frequency = frequency change = frequency shift

if $v_i, v_R \ll c$ (i= S or O)

rearranging equation (a)
moving source or observer:

$$\Delta f = f_D = \pm \frac{v_i}{c} f$$

rearranging equation (d)
**moving reflecting object
or surface:**

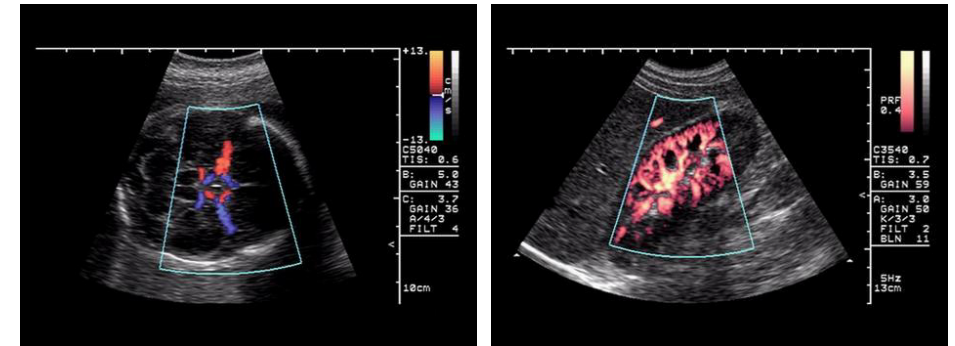
$$\Delta f = f_D = \pm 2 \frac{v_R}{c} f$$

if v and c are not parallel, then $v \cos \theta$ should be used
instead of v (remark: if $\theta = 90^\circ$, $f_D = 0$)

37

Colour coding

towards the transducer: warm colours
away from the transducer: cold colours

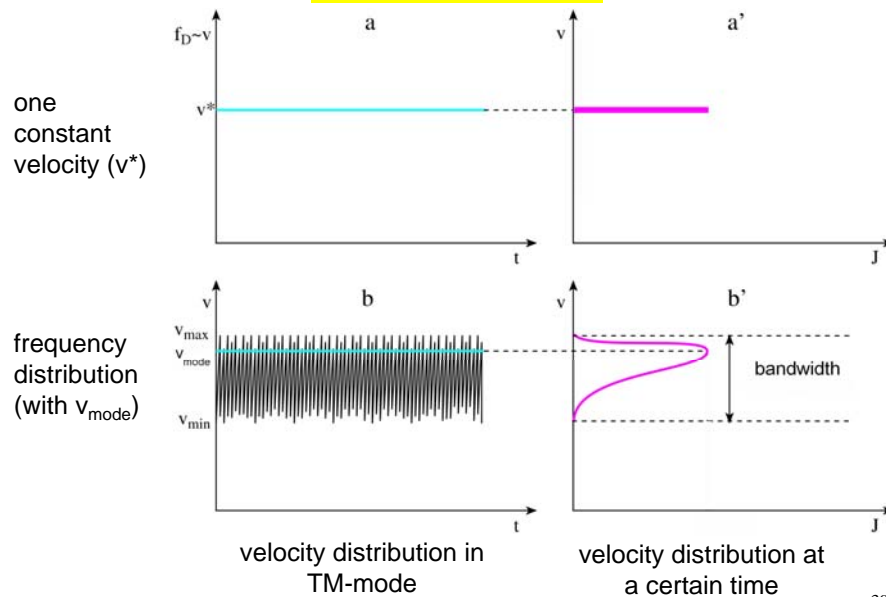


BART: Blue Away Red Towards

power Doppler

38

Doppler curves

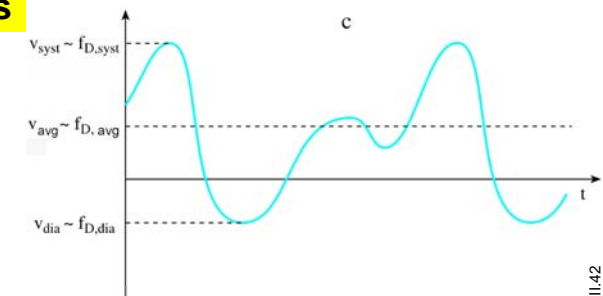


Textbook Fig. VIII.42

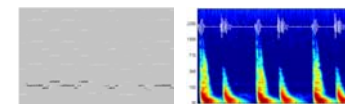
39

Doppler curves

flow can be represented by one velocity in each moment



flow can be represented by a velocity distribution in each moment

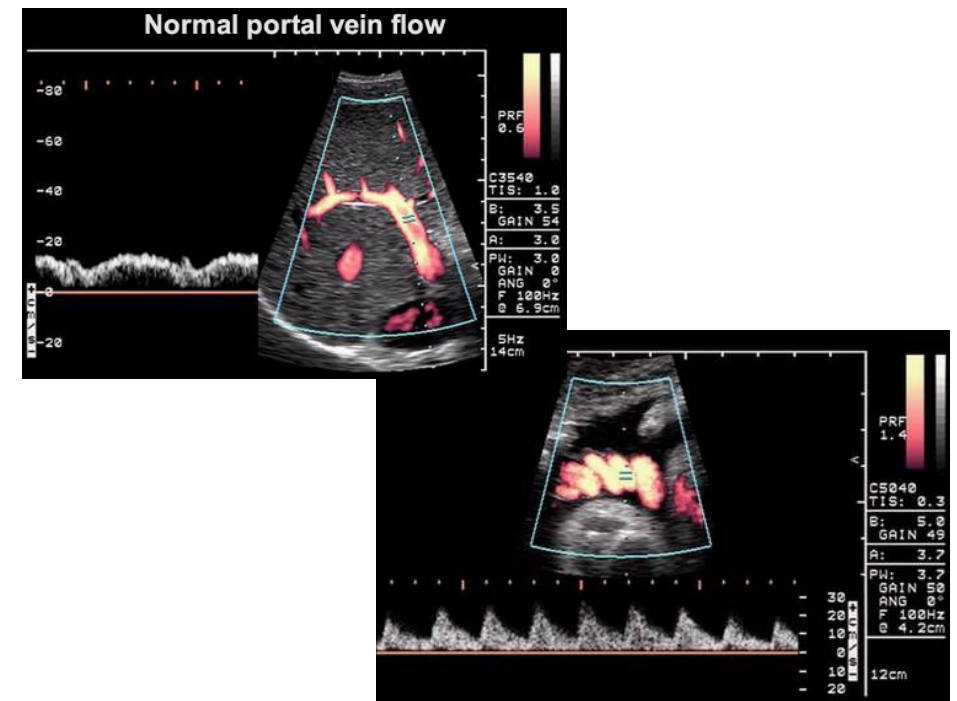
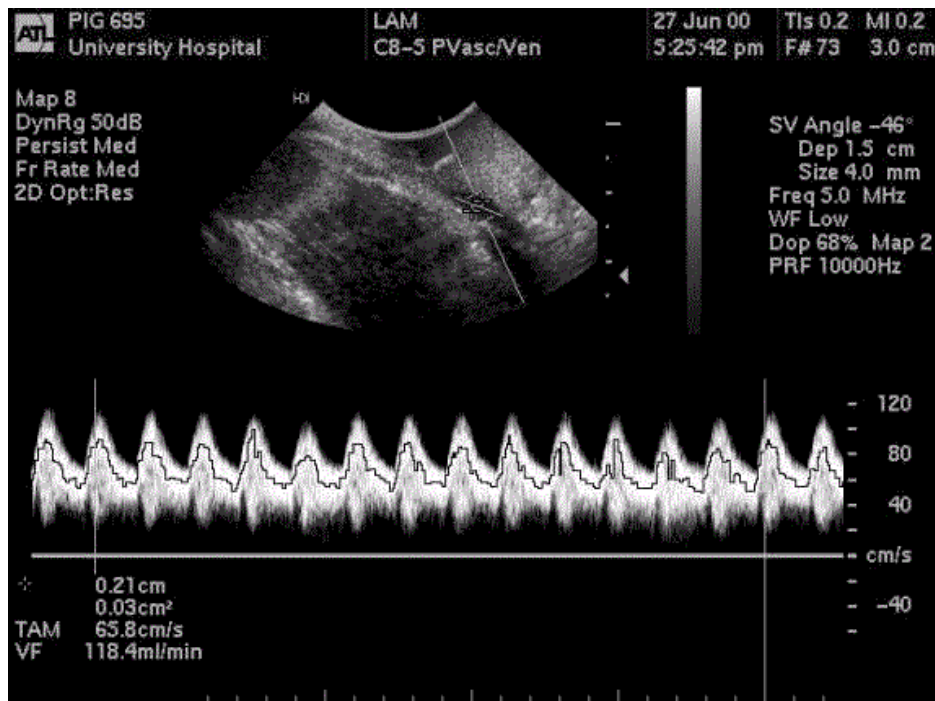


cf. voiceprint, music/heart beats
in time-frequency representation

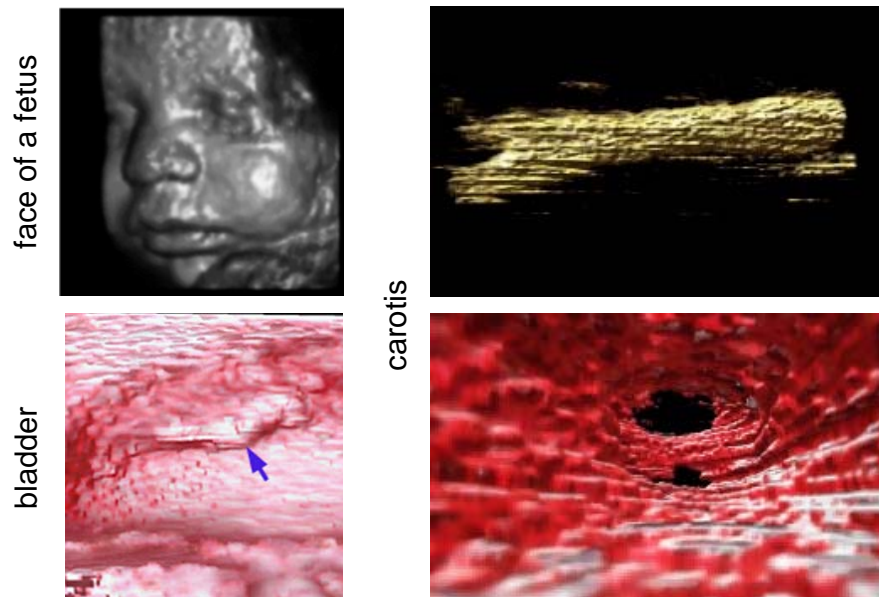
velocity distribution in TM-mode

Textbook Fig. VIII.42

40



3D reconstruction



Safety

in the diagnostics:
 $10 \text{ mW/cm}^2 = 100 \text{ W/m}^2$
 cf. pain threshold: 10 W/m^2

in the therapy: 1 W/cm^2

spatial average temporal
 average (SATA) intensity;
 spatial peak temporal peak
 (SPTP) intensity;
 spatial peak temporal average
 (SPTA) intensity;
 spatial peak pulse average
 (SPPA) intensity
 spatial average pulse average
 (SAPA) intensity

