

# MEDICAL STATISTICS

Physiology

Anatomy

Chemistry

...

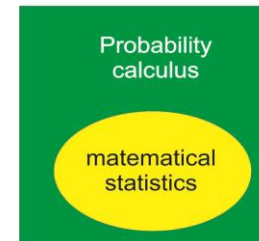
Statistics



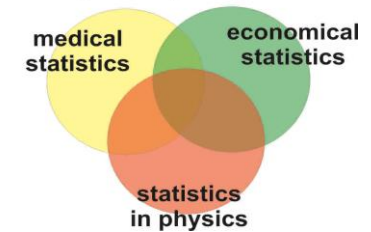
No any doubt

# Medical statistics

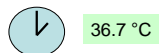
Theory:  
matematics



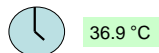
Practice:  
applied statistics  
(examples)



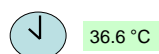
# Example: body temperature



36.7 °C



36.9 °C



36.6 °C



36.7 °C



36.9 °C



36.5 °C



1. Inaccuracy of the measurement.

2. Daily fluctuation!!!

3. Biological variability!!!

The measured value is not constant!

Measured value: 37.0 °C.

Is it healthy or not?

# Another examples

RBC:  $4.5 \times 10^{12} \text{ 1/l}$  ( $3.9\text{-}5 \times 10^{12} \text{ 1/l}$ ) → normal range?

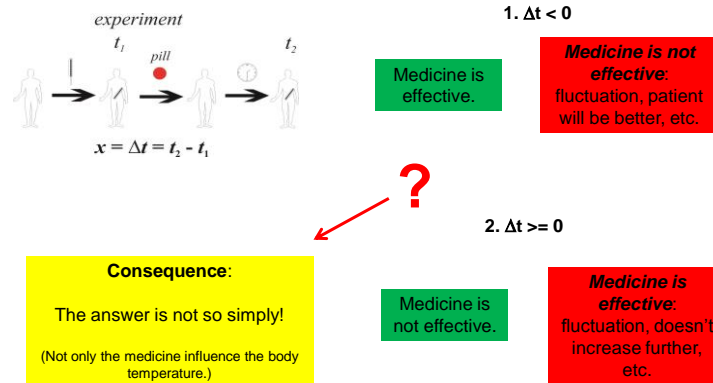
The new method in therapy is better then the old one or not?

How can we prove that a medicine decreases the fewer or not?



Questions!

## How can we answer?



## Variables

| variable           | range              | type        | variable type |            |
|--------------------|--------------------|-------------|---------------|------------|
| height             | ~50 cm ... ~250 cm | real number | numerical     | continuous |
| no. of teeth       | 0 .. 32            | integer     |               | discrete   |
| blood type         | A, B, AB, 0        | letters     |               | nominal    |
| severity of cancer | 1 ... 4            | integer     | categorical   | ordinal    |

**Descriptive statistics!**

## Description of a variable

- Type
- Possible values
- Occurrence of the values

## Numerical variables

| Name       | <i>Continuous</i>                                 | <i>Discrete</i>                   |
|------------|---|-----------------------------------|
| Definition | Infinitely large no. of values in a certain range | Only finite number of values      |
| Example    | Height, temperature, pressure ...                 | No. of teeth, no. of children ... |

## Categorical variables

| Name       | <i>Nominal</i>            | <i>Ordinal</i>                                |
|------------|---------------------------|---|
| Definition | No order among the values | There is a certain order                      |
| Example    | Gender, blood-type ...    | Severity of the illness, strength of pain ... |

## Determination of the possible values

- Continuous : giving a possible range.  
» e.g.: height from ~50 cm - to ~ 250 cm
- Another : listing the values, if it is possible  
» E.g.: blood type: A, B, AB, 0

## Occurence

**Observation:** The occurrence of the values are not the same!



Trial: experiment, observation, data collection.

*Deal with only the case, when the trial may be repeated!*

Outcome: result of one trial. (e.g.: height of a student)

## Population

How many people?



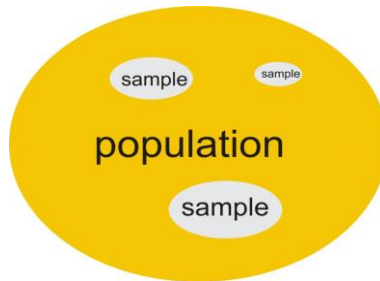
As many as possible.



Ideal case: All of the people → **population**

## Sample

A smaller portion of the population.



- $n$ : no. of the elements (people) in the sample.
- $x$ : the tested variable (quantity)
- $x_i$ :  $i$ -th element from the sample

## Selection of the sample

Main principle: **Random sample**

Medical statistics: if there is no any reason to exclude,  
must be random!

## Occurrence

**Frequency** ( $k$ ): no. of occurrence in the sample.

$k_i$ : no. of occurrence of the  $i$ -th value in the sample.

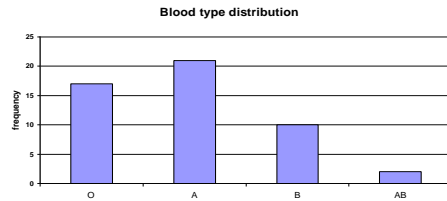
$$n = \sum_i k_i$$

## Frequency distribution

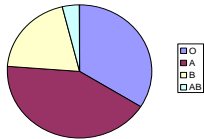
Frequency as the function of the possible values.

|            |          |          |          |           |       |
|------------|----------|----------|----------|-----------|-------|
| Blood-type | <b>0</b> | <b>A</b> | <b>B</b> | <b>AB</b> | total |
| frequency  | 17       | 21       | 10       | 2         | 50    |

## Presentation



Bar-chart



Pie-chart

## Relative frequency, proportion

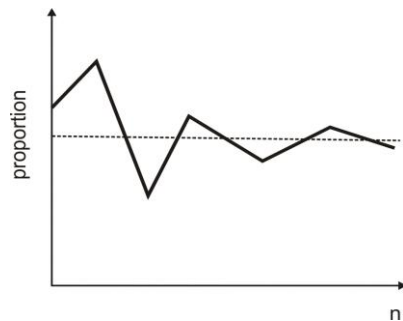
The ratio of the frequency and the total no. of the elements.

$$\sum_i \frac{k_i}{n} = \frac{1}{n} \sum_i k_i = \frac{1}{n} \times n = 1$$

Frequently it is given as percentage:

$$\frac{k_i}{n} \times 100\%$$

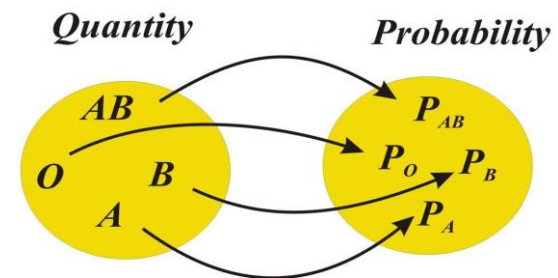
## Probability ( $P$ )



If  $n$  is infinite the name of the proportion is the probability.

Probability ( $P$ ): proportion in the population.

## Probability distribution



## Properties of the probability

$$0 \leq P \leq 1$$

$P = 0$  - never occur  
 $P = 1$  - always occur

Example: blood- type

$$P_A + P_B + P_{AB} + P_O = 1$$

(exclusive events)

$$\sum_i P_i = 1$$

## Probability and proportion

### Sample

$n$  is finite

proportion

### Population

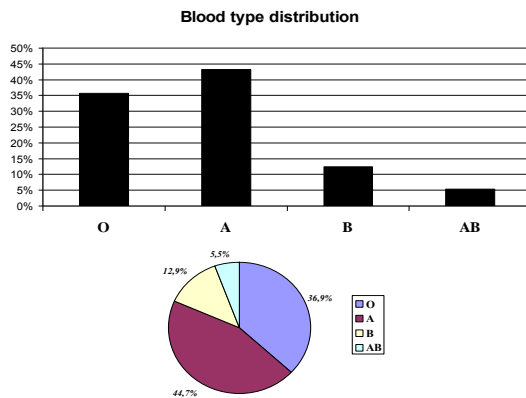
$n = \infty$

probability

**Probability very frequently is unknown!**

We usually use proportion instead of the probability.

## Presentation



## Continuous quantity

**Infinite no. of possible values!!!**

**Class:** a short interval in the whole range.

**Class-width:** the length of the class.

Frequency: no. of elements in the given class.

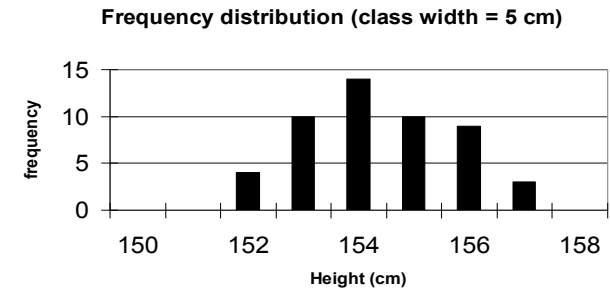
Like a discrete value!

## Example

|    |        |
|----|--------|
| 1  | 160 cm |
| 2  | 181 cm |
| 3  | 175 cm |
| 4  | 163 cm |
| 5  | 165 cm |
| 6  | 179 cm |
| 7  | 164 cm |
| 8  | 185 cm |
| 9  | 177 cm |
| 10 | 168 cm |

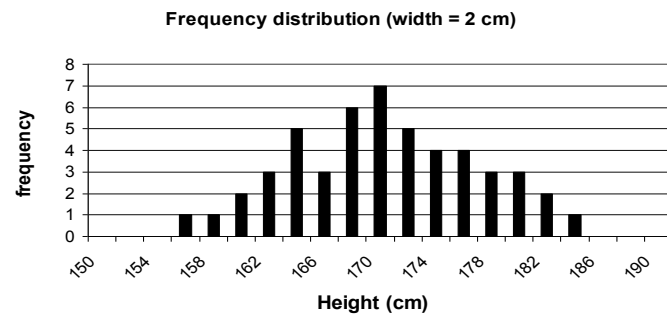
| class   | $k_i$ |
|---------|-------|
| 160-164 | 3     |
| 165-169 | 2     |
| 170-174 | 0     |
| 175-179 | 3     |
| 180-184 | 1     |
| 185-189 | 1     |

## Presentation



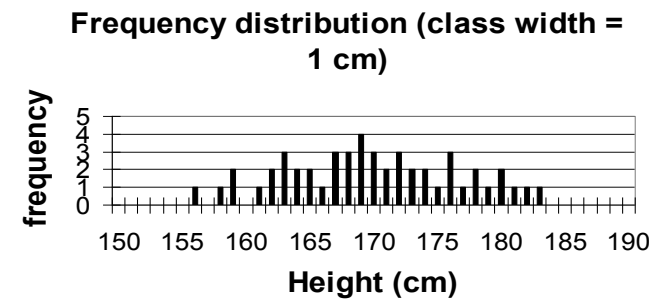
5 cm is too large!

## Decrease the width!



Observation: frequency decreases!

## Presentation



Reason: n is too small!

## Consequence

Class-width



No. of classes



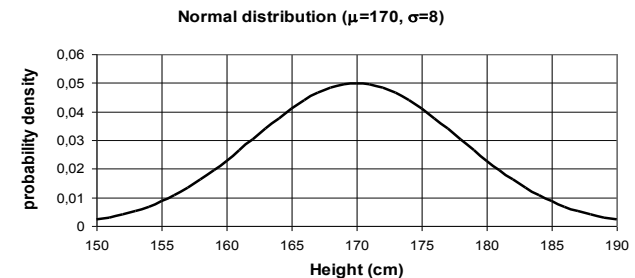
Frequencies



We must increase the no. of the elements!

## Normal distribution

If  $n$  and no. of classes are infinite!



## Theoretical description

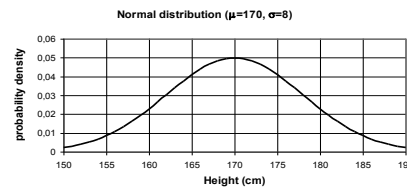
Normal or Gauss-distribution

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Parameters:

$\mu$  – expected value or mean

$\sigma$  – theoretical standard deviation



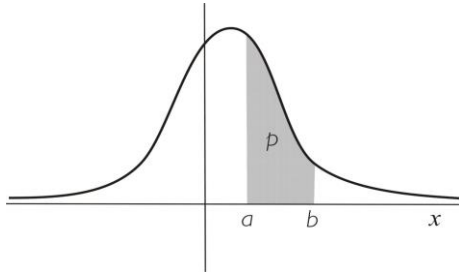
## Meaning of the parameters

$\mu$  **(mean):**  
the value belonging to the maximum of the curve.

$\sigma$  **(theoretical standard deviation):**  
the average deviation of the data from the  $\mu$ .



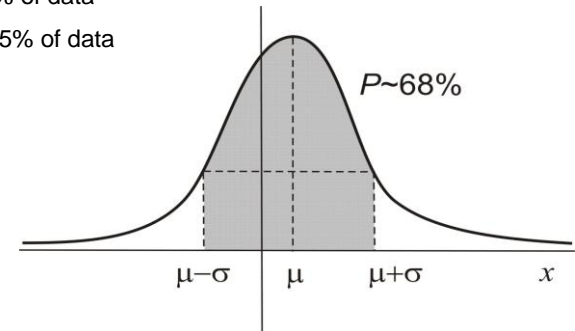
## Probability



$P$  is the probability that  $x$  is in the  $(a,b)$  interval.

## Standard deviation

$(\mu \pm \sigma)$  ~ 68% of data  
 $(\mu \pm 2\sigma)$  ~ 95% of data  
 $(\mu \pm 3\sigma)$  ~ 99.5% of data



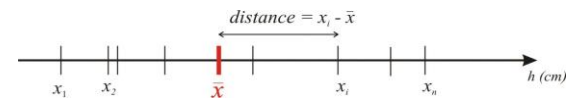
## Normal distribution

**Theoretical distribution** that describes the population. In practice usually we don't know the parameters of this.



We usually have a **random sample** from the population.  
 We must estimate the parameters!

## Estimation of the $\mu$



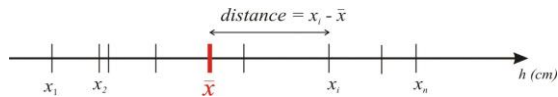
**average**: must be in the center of the data range.

$$\sum_i (x_i - \bar{x}) = 0 \quad \longrightarrow \quad \bar{x} = \frac{\sum_i x_i}{n}$$

## Estimation of the $\sigma$

$\sigma$  = average deviation of the data from the  $\mu$ .

**s (standard deviation)** = average deviation of the elements from the average.



$$Q_x = \sum_i (x_i - \bar{x})^2 \geq 0$$

## Standard deviation

$$s = \sqrt{\frac{Q_x}{n-1}}$$

s: the average deviation of the elements from the average.

$$(\bar{x} \pm s) \sim 68\%$$

$$(\bar{x} \pm 2s) \sim 95\%$$

$$(\bar{x} \pm 3s) \sim 99.5\%$$

## Relation of parameters

| Sample  | $n \rightarrow \infty$ | Population |
|---------|------------------------|------------|
| average | $\longrightarrow$      | $\mu$      |
| s       | $\longrightarrow$      | $\sigma$   |

## Question of the week!

How can we estimate the  $\mu$  and the  $\sigma$ ?