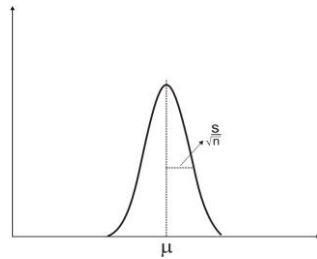


μ and the average

sample	average
1	170
2	168
3	166
4	173

Averages fluctuate, deviate around the μ .



Standard error

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Average deviation of the averages around the μ !

Confidence interval for μ .

$$(\bar{x} \pm s_{\bar{x}}) \sim 68\%$$

~68% is the probability that μ is in this range.
(~32% that isn't)

Estimation of the μ

Average

Confidence interval

Point estimation

A simple value.

Interval estimation

A range and the probability that the mean is in this range.

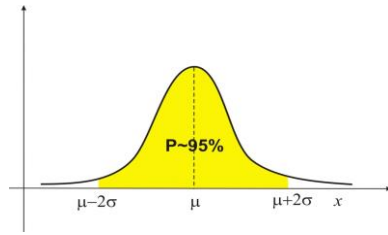
Information

	interval	probability	information content
$(\bar{x} \pm s_{\bar{x}}) \sim 68\%$			
$(\bar{x} \pm 2s_{\bar{x}}) \sim 95\%$			
$(\bar{x} \pm 3s_{\bar{x}}) \sim 99.5\%$			
$(\bar{x} \pm \infty) = 100\%$	∞	$P = 1$	$= 0$

Normal (reference) range

Normal distribution

Other quantiles



A range, that contains the 95% of the possible values.

But: 5% is the chance being out!!!

Hypothesis test

Questions

(example)

Is the medicine effective?

How can we answer?



literature

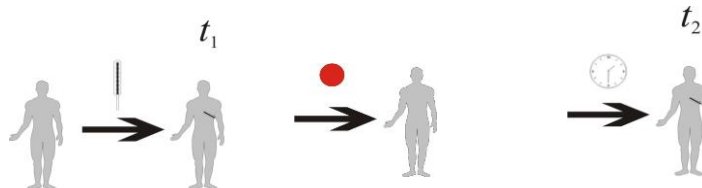


experiments

An example

Question: Does the medicine decrease the fever?

experiment



$$x = \Delta t = t_2 - t_1$$

Hypotheses

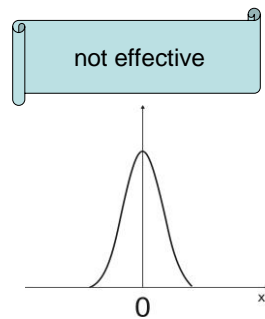
The medicine isn't effective.

The medicine is effective.

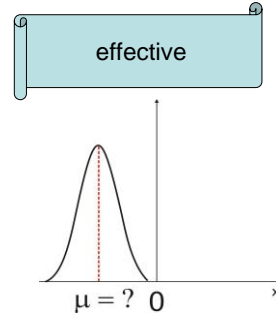
Exclusive statements,
enough to test one of them!

Which is better?

The distribution of the observed quantity



The result of the random effects is that $\mu = 0$.



How much is the effect?



If we know the population!!! (we are able to calculate μ)

result:

$$\mu = 0$$



conclusion:

The medicine isn't effective.

$$\mu < 0$$



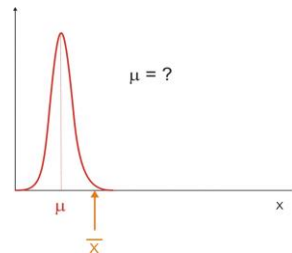
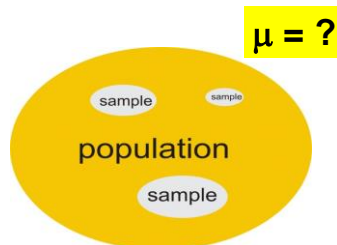
The medicine is effective and μ characterizes the effect.

The situation is more difficult

Normally the population is unknown.

The sample differs from the population! (**sampling error**)

E.g. the averages fluctuate around the μ !



What is the reason of the deviation?

Sampling error, **random fluctuation**.
(Our hypothesis is right!)



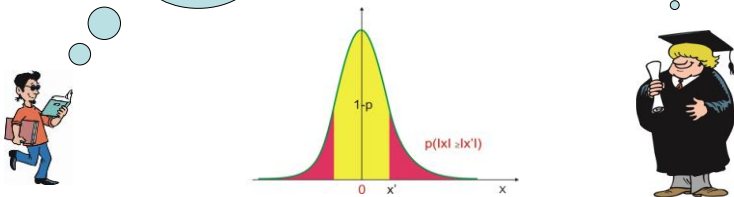
The hypothesis is false (**mistake!**).
The deviation is non-random.



What is the base of the decision?

How much is the chance that the sample derives from the given population?

We must know the parameters of the distribution!



Null hypothesis: (H_0)

The deviation of the sample or samples from the population or populations is a random deviation due to the sampling error. Frequently it is a negative answer. (e.g.: the medicine is not effective.)



Alternative hypothesis: (H_1)

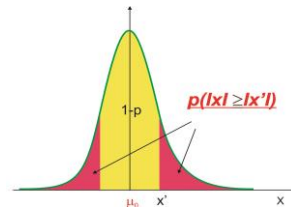
The deviation of the sample or samples from the population or populations is not a random deviation. (e.g.: the medicine is effective)

Null hypothesis

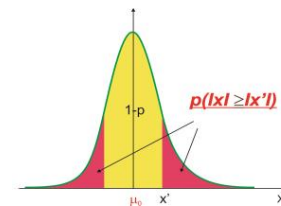
How much is the probability of the random deviation?

In the case of known distribution we are able to determine!

(The shape of the distribution not always gaussian, but known!)



Significant?



If the **p** is enough large, may be random, if the **p** is enough small we consider the difference being significant!

p is the probability being random!



Significance level

Enough large,
enough small?

Select a value as limit!
This is the significance
level.



Symbol: α .
In medical practice this value
is frequently 5%.



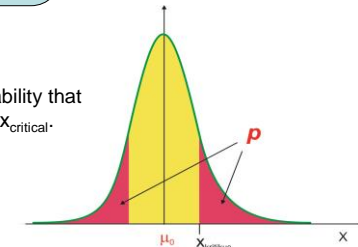
The base of the decision

If the p is enough small, there is a
big chance, that the nullhypothesis
is not true. So the alternative
hypothesis is more probable.

x_{critical} : the value
belonging to the
significance level

$x_{\text{calculated}}$: the value
calculated from the
sample

p is the probability that
 $x_{\text{calculated}} \geq x_{\text{critical}}$.



Decision

- 1. If the probability of the random deviation is small ($p(|x| \geq |x_{\text{crit}}|) \leq 5\%$) – we **reject** the nullhypothesis.
- 2. If the probability of the random deviation is large ($p(|x| \geq |x_{\text{crit}}|) > 5\%$) – we **accept** the nullhypothesis.

The answer is newer yes – no or true - false!!!

The possibility of the error

decision:
the nullhypothesis is

accepted

rejected

true

Right decision

I. Type error (α)

false

II. Type error (β)

Right decision

reality:
the nullhypothesis

Test in one sample: 1-sample t-test

Question: On the base of the sample the parameter of the population may be a given value?

example: The medicine effective or not?



Null hypothesis: not! $\mu_0 = 0$. But the average is not 0!

sample	Average
1.	-0.2 °C
2.	-1 °C
3.	-1.5 °C



If the difference is bigger,
it seems to be more
probable being non
random.

What is the big difference?

What is the measure of the difference?

Standard error: the average deviations of the averages from the μ .

$(\bar{x} \pm s_{\bar{x}})$ ~ 68% - confidence interval.

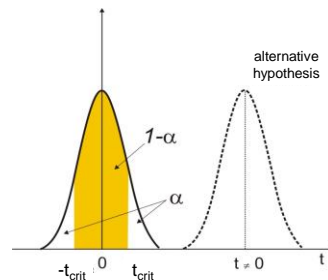
t-value

$$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}}$$

Compare the difference to the
standard error!
(μ_0 very frequently = 0)

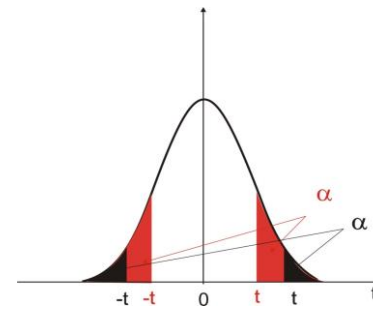
The averages fluctuate
around the μ_0 so the
 t -values deviate around
the 0.

(providing, that the
null hypothesis is true!)



Why is the t-value is better?

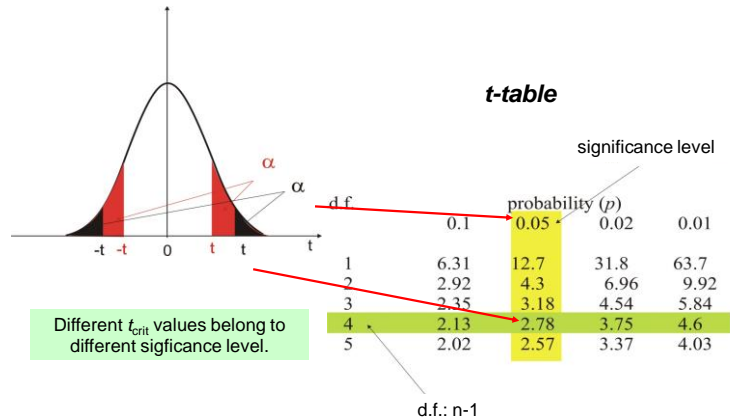
We are able to calculate the probabilities on the base of
this distribution!!! (Student- or t -distribution)



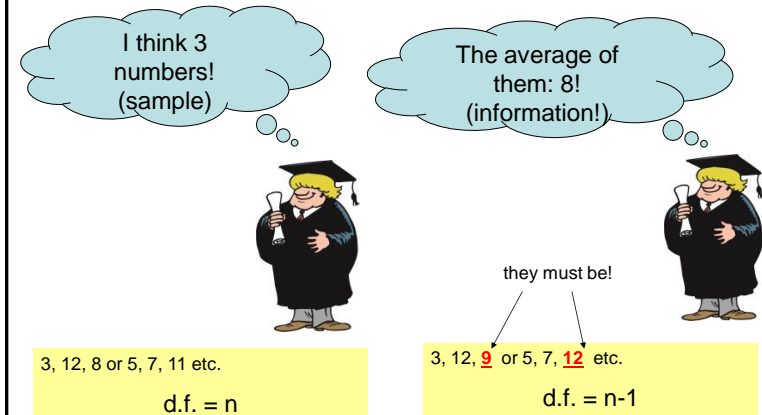
It describes only the
random deviations of
the t -values!

The shape of the
distribution depends
on the no. of
elements.

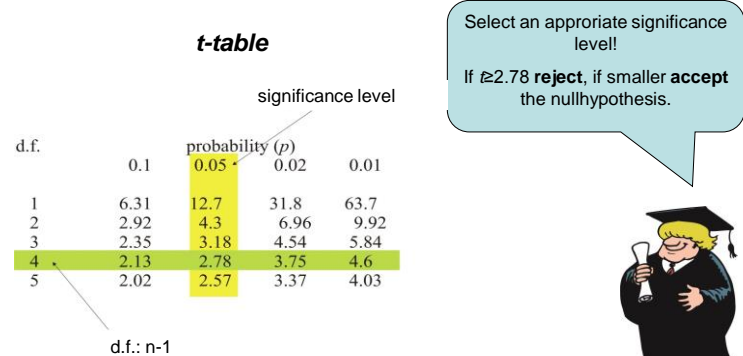
The t-table



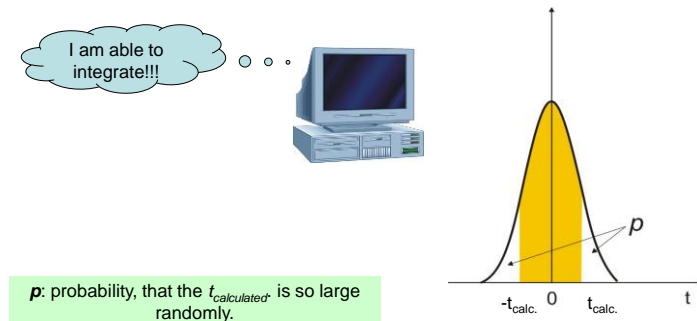
Degree of freedom (d.f.)



Decision the base of t-table



Decision using computer



Decision

- 1. If the probability of random deviation is small ($p(|t| \geq t_{\text{krit}}) \leq 5\%$) – **reject** the nullhypothesis.
- 2. If the probability of random deviation is large ($p(|t| \geq t_{\text{krit}}) > 5\%$) – **accept** the nullhypothesis.

Conditions

- Question: May be a parameter equal to a certain value?
- The variable must have **normal distribution**.



Test in two groups

Question: May the samples derive from the same population or the parameters of the two populations are the same?

$$\mu_1 = \mu_2 ?$$

Nullhypothesis: $\mu_1 = \mu_2$

(but normally $\bar{x}_1 \neq \bar{x}_2$)

2-sample t-test

2-sample t-test

$$\bar{x}_1 \neq \bar{x}_2$$



Known distribution is necessary!

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s^* \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$s^* = \sqrt{\frac{Q_1 + Q_2}{n_1 + n_2 - 2}}$$

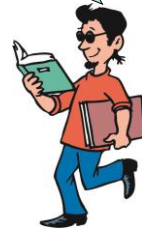
Test

The t-value is same!

How much is the d.f.?

$$d.f. = n_1 + n_2 - 2$$

$$((n_1 - 1) + (n_2 - 1))$$



Conditions for the test

- task: comparison of two **independent** sample.
- The quantity must have **normal distribution**.
- The sd are **same** in the two groups.



The last one is new!
How is it possible to prove?

Test for standard deviations

How can I do?

Nullhypothesis: the two sd are the same, the difference is random (sampling error).



It is similar to the hypothesis testing!

F-test

$$F = \frac{s_1^2}{s_2^2}$$

A so-called F-distribution belongs to the nullhypothesis.

But what sd is in the nominator?



The larger variance is in the denominator! ($F \geq 1$)



Decision

- 1. if the probability of the random deviation is small ($p(F \geq F_{\text{crit}}) \leq 5\%$) – **reject** the nullhypothesis.
- 2. if the probability of the random deviation is large ($p(F \geq F_{\text{crit}}) > 5\%$) – **accept** the nullhypothesis.

Two or more variables

Correlation and regression

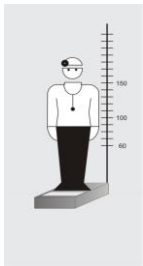
The relationship between two variables.

Method for estimating the relationships among variables.

Correlation of two variables

Example:
Is there any relationship between the height and weight?

Experiment:



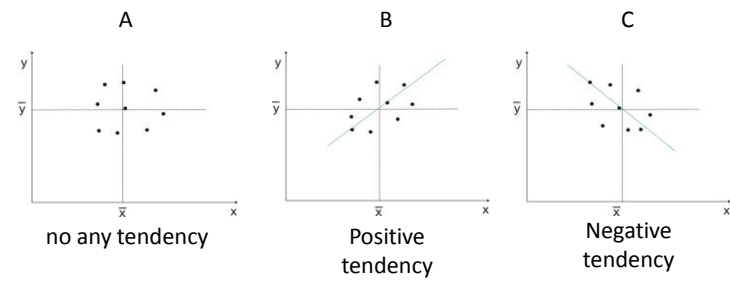
Data pairs:

No.	Height (cm)	Weight (kg)
1	150	61
2	170	70
3	166	75
4	174	70
5	180	72
6	155	50
7	172	65
8	161	59
9	177	81

Graphic representation

E.g.: height is the x and weight is the y.

Possible situations:



Pearson's correlation coefficient

$$r = \frac{\text{cov}(x, y)}{s_x \cdot s_y} = \frac{Q_{xy}}{\sqrt{Q_x \cdot Q_y}}$$

$$Q_{xy} = \sum_i [(x_i - \bar{x})(y_i - \bar{y})]$$

$$Q_x = \sum_i (x_i - \bar{x})^2$$

$$Q_y = \sum_i (y_i - \bar{y})^2$$

Possible range
for r :

$$-1 \leq r \leq 1$$

In the population:

$r = 0$ no correlation,

$r \neq 0$ correlation (strength is proportional to the actual value of r .)

Coefficient of determination

$$r^2$$

The coefficient of the determination tells us how strong is the relationship.
Expresses how much percent of the variability of the y values may be accounted by the variability of the independent variable or variables.

Correlation t -test

Calculated r is only an estimation of the r in the population.
This fluctuates around the theoretical value.
(e.g. $r_{\text{calc}} = 0.1$?)

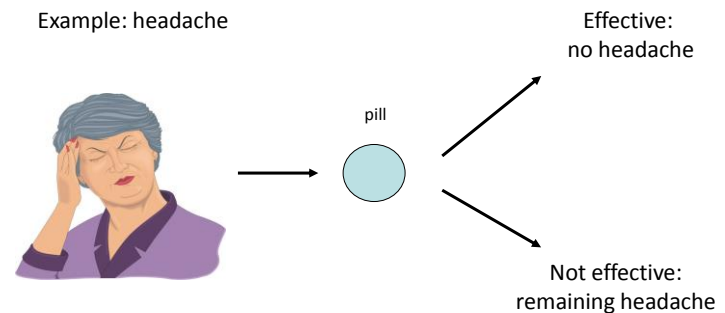
$$H_0: r = 0! \longrightarrow t = r \sqrt{\frac{n-2}{1-r^2}} \longrightarrow \text{d.f.: } n-2$$

Decision: based on t -value. Look previous cases!

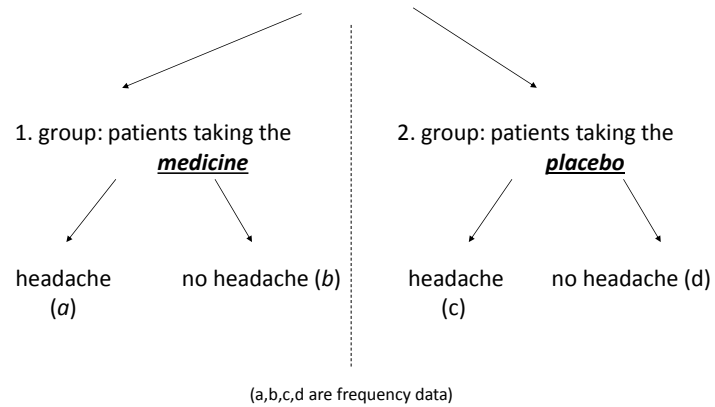
Condition: at least one of the variable has normal distribution.

Chi-square test (analyzing frequency data)

Example: headache



Experiment



Contingency table

	headache	no headache	Total
1. group	a	b	a+b
2. group	c	d	c+d
total	a+c	b+d	n

So-called 2 x 2 table.

Nullhypothesis

If the effect is independent from the medicine, we expect:

$$\frac{a}{b} = \frac{c}{d}$$



$$a \times d = b \times c$$

Nullhypothesis: the effect is independent from the medicine is due to the placebo effect only.

Chi-Square test for independence.

χ^2 -distribution

Shortcut formula
for 2 x 2 tables:

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

Nullhypothesis: χ^2 -value is 0, the difference is only due to the sampling error.

χ^2 -distribution describes the random deviations of the χ^2 -value.

Decision

Same, then in the case of t -distribution. We use χ^2 -distribution.

Expected value is 0, if the null hypothesis is true.

if $\chi^2_{\text{calc}} \geq \chi^2_{\text{crit}}$ - reject the null hypothesis else accept.

or $p(\chi^2 \geq \chi^2_{\text{calc}}) \leq 5\%$ - reject the null hypothesis else accept.

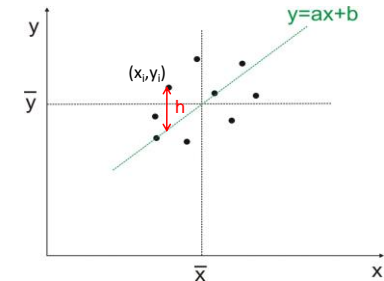
degree of freedom: in this special case = 1.

In general:

d.f. = $(r-1)(c-1)$, where r - no. of rows
 c - no. of columns

Linear regression

If the variables have normal distribution, the relationship is linear, and we can describe it with a straightline.



$$y_i = ax_i + b + h_i$$

y : dependent variable

x : independent (explanatory) variable

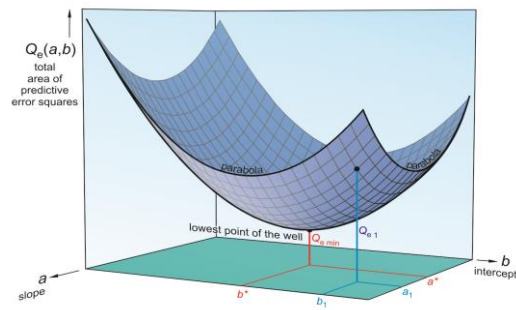
h_i : error term = $y_i - (ax_i + b)$.

(the difference between the actual value and the predicted value.)

Least-squares method

$$Q_e = \sum_i h_i^2 = \sum_i (y_i - (ax_i + b))^2$$

The x_i and y_i measured values.
 Unknown are the a and b !



Which is the best straightline?

Q_e has minimum!

$$a^* = \frac{Q_{xy}}{Q_{xx}} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad b^* = \bar{y} - a^* \bar{x}$$

r^2 : coefficient of determination.

How much part of the variance of y is explained by x .

Prediction of insulin sensitivity by BMI.

indep.	regr. coeff.	st. error	t	p	decision
BMI	-0.077	0.018	-4.25	0.0011	significant
r^2	0.6				

Question of the week

Conditions for 2-sample t-test.