

# Dynamics

## Motion, deformation and their background

One of the fundamental physical quantities of the topic is the **momentum** ( $p$ ).

In classical case it is the product of the **mass** ( $m$ ) and the **velocity** ( $v$ ) of the body.

$$p = mv$$

vector quantity

## Newton's laws of motion

II. For the **change of momentum** needs **force** ( $F$ ).

$$\frac{\Delta mv}{\Delta t} = m \frac{\Delta v}{\Delta t} = ma = F$$

If no forces are exerted (or  $F = 0$ )

$\Delta mv = 0$ , means  $p = mv = \text{constant}$ .

I. **Momentum is conserved** (momentum conservation)

law of inertia

III.  $F = -F_{\text{versus}}$  interaction

A single force cannot exist.

Forces are always directed to contrary parts.

law of action and reaction

Application e.g.:

at the pressure of ideal gases, (see later)

at the explanation of annihilation (see in PET).

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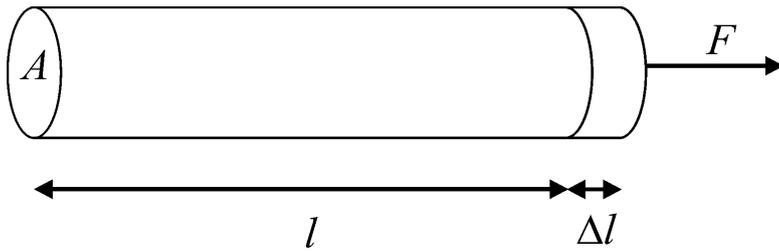
**But**

**force** may result in **deformation**.

Simplest deformation is the **elongation**.

tensile **strain**:  $\Delta/l$

## Hooke's law



$$F = AE \frac{\Delta l}{l}$$

$$\frac{F}{A} = E \frac{\Delta l}{l}$$

$F/A$  the **stress** (tensile stress), but

it could be compressive stress or **pressure** ( $p$ [Pa])

Coefficient: **Young's modulus** ( $E$ [Pa])

Similar to the case of spring:  $F_{\text{spring}} = Dx$  (if  $x \equiv \Delta l$ , and  $D \equiv AE/l$ )

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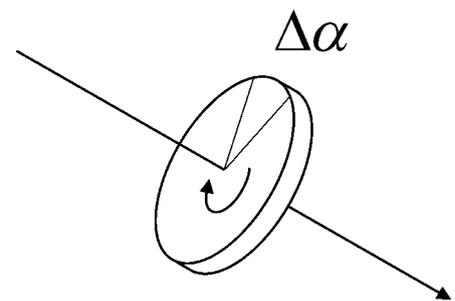
## Newton's laws for rotation

similarly to the momentum ( $mv$ ) here the fundamental physical quantity is the **angular momentum** ( $\Theta\omega$ ), where

$\Theta$  is **moment of inertia**, rotational analog of the mass,  
 $\omega$  is **angular velocity**,

$$\omega = \frac{\Delta\alpha}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

**period** ( $T$ ), **frequency** ( $f$ )  
 ( $\omega$  **angular frequency**)



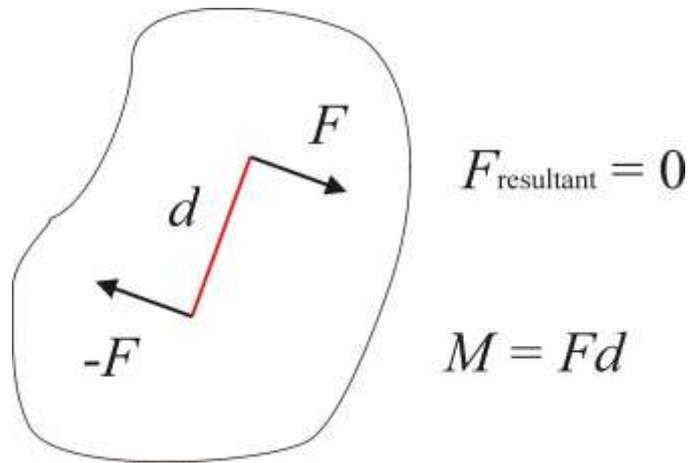
- I.  $\Theta\omega = \text{constant}$  (conservation of angular momentum)  
 (see: **rotating skater**)
- II. For the **change of angular momentum** needs **torque** ( $M$ ).

$$\frac{\Delta\Theta\omega}{\Delta t} = M$$

## Equilibrium, if

$F_{\text{resultant}} = 0$  **and**  $M_{\text{resultant}} = 0$   
simultaneously.

Then:  $m\mathbf{v} = \text{constant}$   
and  $\Theta\omega = \text{constant}$



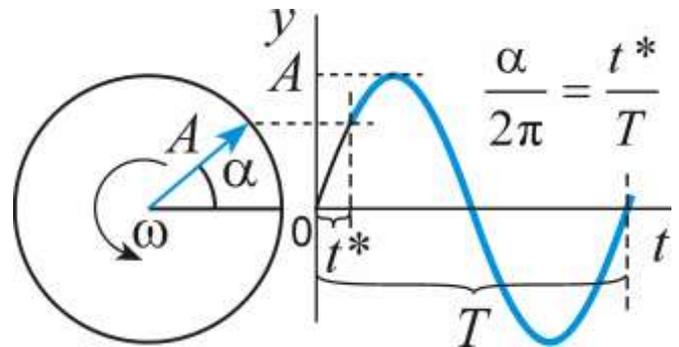
## Statics

See: orthopaedy

## Harmonic motion

Projection of  
uniform circular motion  
( $\alpha = \omega t = 2\pi t/T = 2\pi f t$ )

$$y = A \sin \omega t$$



Dynamical condition:  $F = -Dx = ma$

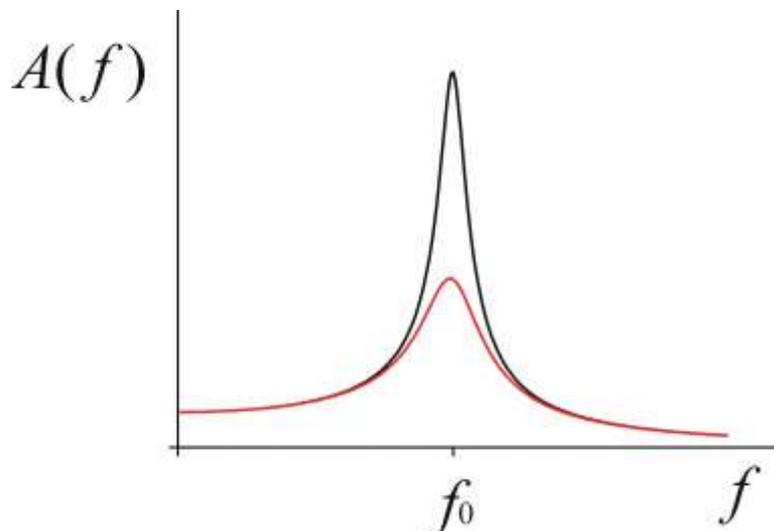
$$\omega = \sqrt{\frac{D}{m}}$$

## Forced oscillation, resonance

( $\omega = 2\pi f$ )

$$A(f) \sim \frac{1}{(f - f_0)^2 + K}$$

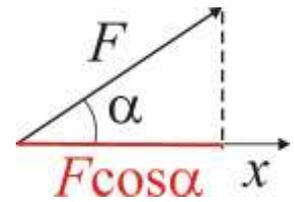
$K$  is characteristic for  
damping



**Application e.g.:** at the interpretation of different spectra  
(ESR, NMR); at the explanation of AFM and **MRI**

## Work

**Work** ( $W$ ) is a product of **displacement** ( $\Delta x$ ) and the projection of force ( $F$ ) to the direction of displacement.



$$W = \Delta x F \cos \alpha \quad [\text{Nm}] \text{ or } [\text{J}]$$

Permanent acting force without displacement ( $\Delta x = 0$ );  
or  $\alpha = \pi/2$  (means  $\cos \alpha = 0$ ), then  $W = 0$  (in mechanics)

### Work-energy theorem

Force is constant (and  $\alpha = 0$ ).

$$W = F\Delta x = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \Delta E_{\text{kin}}$$

**kinetic energy** ( $E_{\text{kin}}$ )

Result of work  $\rightarrow$  bigger  $E_{\text{kin}}$ .

**Application e.g.:** at the discussion of x-ray tube or electron-microscope.

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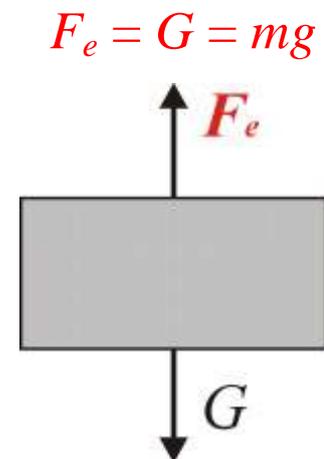
### Work done against another force

e.g. elevation against **force of gravity** ( $G$ )  
( $g$  acceleration of gravity)

Result of work  $\rightarrow$  „storable”

**potential energy** ( $E_{\text{pot}}$ )

In gravitational field:  $\Delta E_{\text{pot}} = mg\Delta h$ ;



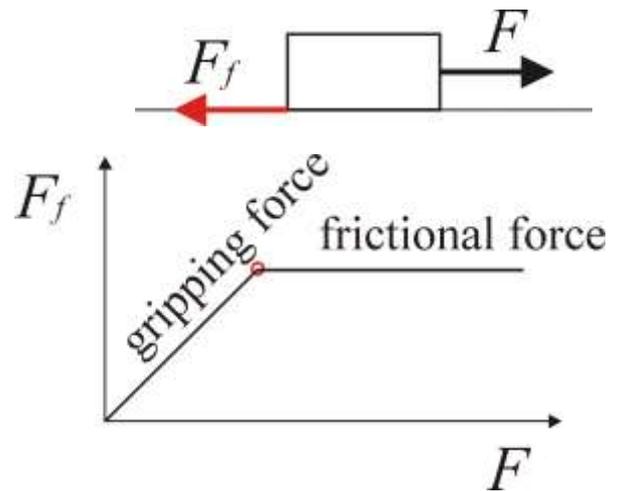
**Power** (“speed” of work done):

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} \quad [\text{W}] = [\text{J/s}]$$

## Friction

Is **not** the energy conservation valid?

(see [thermodynamics](#))



**Static fluids** (and gases)

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**hydrostatics**

## Pascal's principle

Pressure is transmitted undiminished in fluids because they are incompressible.

(hydraulic jack, brakes)

**Hydrostatic pressure** (originates from the weight of fluid)

In a static fluid on the Earth (simplest case):

$$mg = V\rho g = Ah\rho g = F_{\text{weight}}$$

( $\rho$  density)

$$p = F_{\text{weight}}/A = \rho gh$$

Its consequence is the buoyant force ( $F_b$ ):

## Archimedes' principle

A body that is submerged in a fluid is buoyed up by a force:

$$F_b = \rho_{\text{fluid}}gV$$

