Biostatistics and Informatics

Lecture 2: Functions

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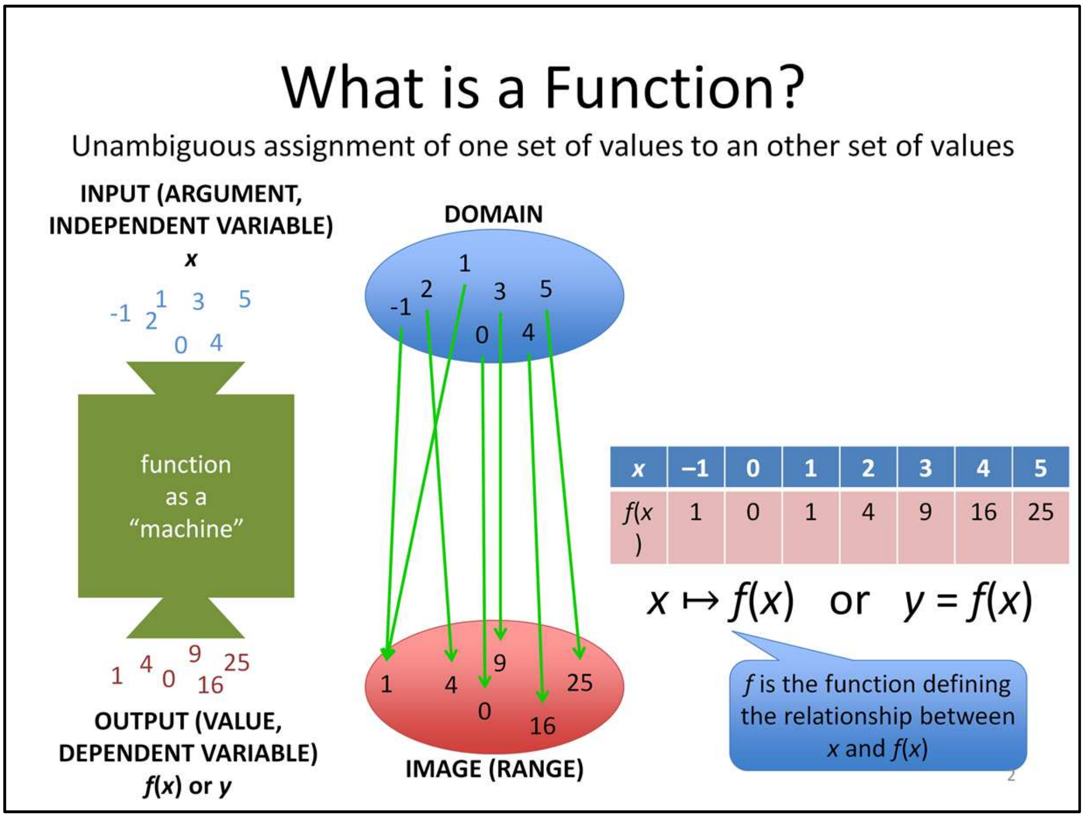
1

In this lecture we are discussing the elements of mathematical functions. This will be necessary both your statistics and biophysics studies.

In statistics, we will use functions to describe the distribution of the random variable, which in turn will be the foundation of hypothesis testing, the statistical tool for comparison. By the end of the course we will also learn the mathematical background of finding the best fitting function to measurement points. Statistical functions study primarily the random or stochastic change of variables (e.g. repeating the same measurement there may be variations in the outcome, which can be characterized by the distribution functions discussed in the previous lecture).

The basis of (bio)physics is the mathematical description of relationship between physical quantities as variables. Here we describe relationships where the value of one variable determines unambiguously the value of an other variable (e.g. the relationship between the temperature of and volume of a gas in a piston). Such relationships are called deterministic.

In reality, no purely deterministic phenomena exist, and purely stochastic are also rare, their combination is typical.

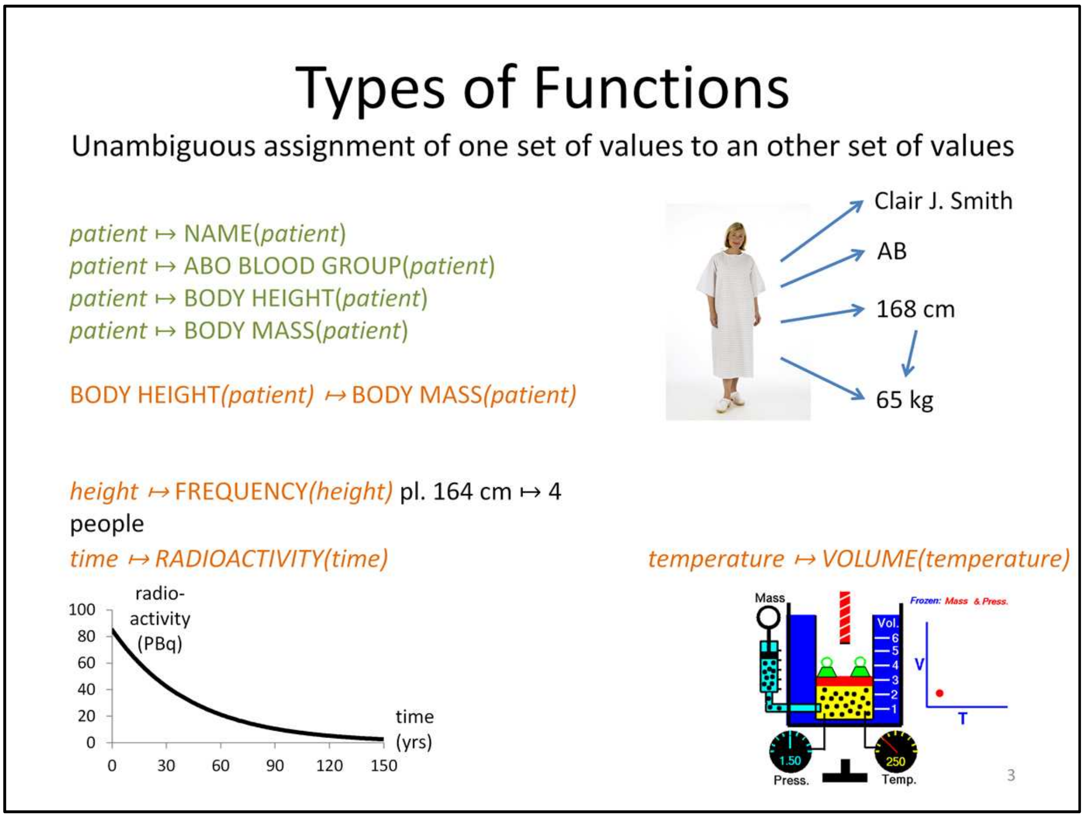


The short definition of function is “unambiguous assignment”. Function is a mathematical abstraction, so it makes sense to show examples through which we can make generalizations. On function is for example, if I give the name of each member of a patient group: in this case I assign a name to each person. I can do the same with age, body height, body mass etc. Naturally, I can also assign these data to each other: the blood group of the patient to the name of the patient, the body height to the name of the patient, the body mass of the patient to the body height of the patient and so on. From our previous mathematical studies we know the functions which assign number to number.

In the above example we assign the squares to the set of numbers {-1; 0; 1; 2; 3; 4; 5}. The inputs of the function are usually called the arguments or points of the function or independent variable. The values of these inputs are collectively called the domain of the function (denoted by D).

So we assign to every number its square, that is, the expectation toward a function is fulfilled: we assign to each value of the independent variable strictly one value of the dependent variable. It may happen that two input values produce the same output (in our example, the square of both –1 and +1 is one) but not vice versa: one input cannot produce two different outputs (i.e. one number has strictly a single square value), it is also unpermitted to have no output (i.e. every number has a square value). The output values are called the value of the function or dependent variable, they collectively form the range, or more precisely, the image set (denoted with R).

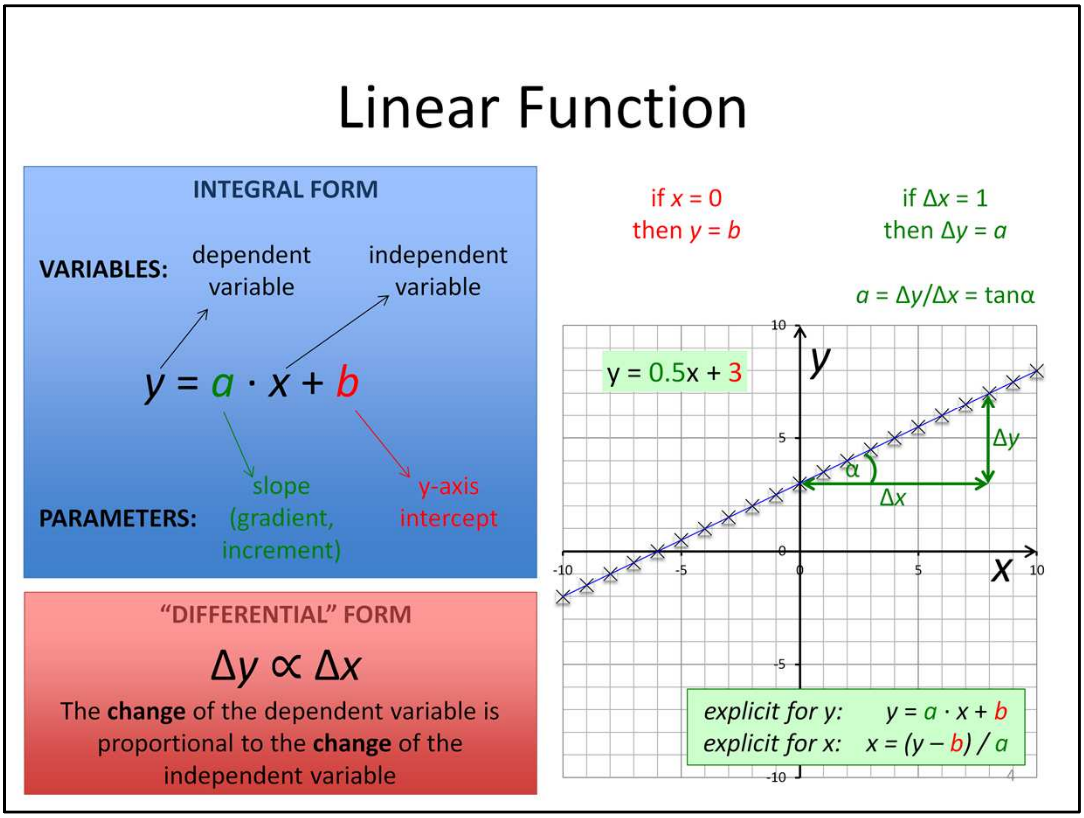
The relationship between the inputs and outputs can be given with tables, but if we assign numeric values to numeric values, we prefer to do it with a mathematical equation. In this case we use the $x \mapsto f(x)$ notation, where *f* stands for *functio* (latin for function). In univariate functions the dependent variable is generally and traditionally represented by *y* or *f*(*x*) while the independent variable is represented by *x*.



Before we would go on with the exact function types, lets refer to the variable types we learned on the previous lecture. It is enough to use the "first approach" classification this time. In cases where both the independent and the dependent variables are numerical, the relationship may probably be given with a mathematical equation, and the dependent variable–independent variable value pairs may be represented in a graph (in Descartes-coordinate system). If any of the variables is non-numerical, we can only deal with frequencies (how many people have a blood type "A"?), and logical relationships (**if** someone has acromegaly, **then** that person has a malfunctioning pituitary gland).

An other important thing is what we already mention in the introduction: the relationship between some variables may be a perfectly unambiguous function (the volume of an ideal gas is proportional to its temperature if pressure is kept constant), that is, to any temperature value we can assign unambiguously a volume – this kind of relationship is called **deterministic**. If we roll a die, however, the outcome is chiefly influenced by chance, such a variable is called random or **stochastic**. Relationships between real life phenomena usually carry both deterministic and stochastic traits.

First let us review the most important functions describing deterministic relationship between variables.



First consider the linear function. The general equation of the linear function is ***y = a * x + b*** (the symbols for the parameter may vary). *y* is the dependent, *x* is the independent variable, *a* and *b* are parameter.

If *y* is known, it may be important to express *x* (make the function explicit for *x*): **x = (y – b) / a** .

Before we go on, let us clarify some concepts. If a function is given with an equation, it always contains to types of letters: **parameters** and **variables**. Variables were characterized above. Parameters do not vary when a certain relationship is considered, while the variables take different values from D and R, respectively. The values of parameters may only change if the relationship between other variables is considered. Sometimes parameters are called **constants** but this naming should be avoided since the value of parameters is not constant *in the way* in which mathematical and physical ("universal") constants retain their value (π, e, k, R etc.), they remain unchanged only for a given case. So let us reserve the name «constant» for universal constants. Naturally, in practice the naming conventions are not strict at all. Other names are also in use: **coefficient** is used for prameters or numbers which multiply the variable (or an expression containing the variable), so it is also sometimes called a **multiplier** or a **factor**.

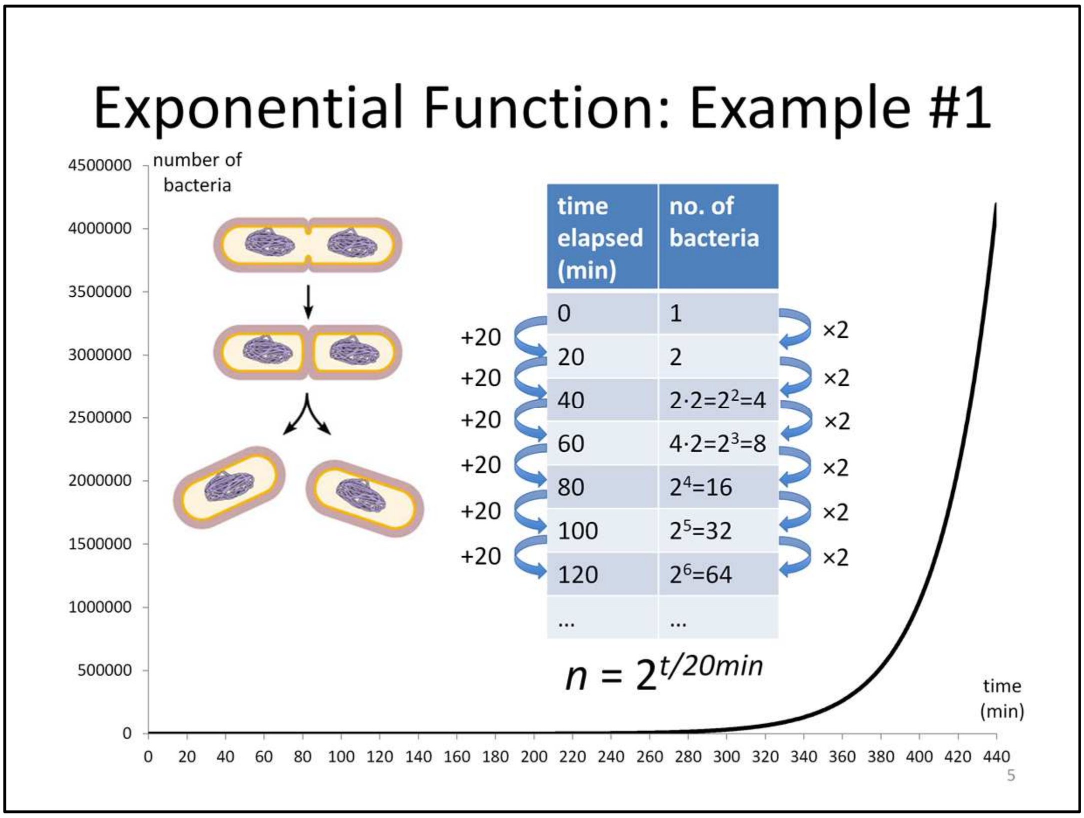
Now let us get back to the linear function. Using the graph of the function or substituting certain well-chosen values for the variables the demonstrative meaning of parameters can be determined. If, for example, $x = 0$ then $y = a * 0 + b = 0 + b = b$. That is, the graph of the function intersects the y axis at b. The name of the parameter b is therefore ***y*-axis intercept** or – sloppily – just **intercept**. Those linear relationships for which $b = 0$ (i.e. the line of the graph goes through the origin) are called **direct proportion**alities.

If x increases by 1, then y increases by a. If x increases by 2 then y increases by 2a, if x increases by 3, then y by 3a and so on. If we demonstrate these increases (changes) with line segments, we get right triangles. The horizontal cathetus (cathetus = one of the sides of a right triangle that forms the right angle) is the increase in x (i.e. its change, Δx), the vertical cathetus is the increase in y (i.e. its change, Δy), the hypotenuse is a segment of the line of the function's graph. It is clear from the previous examples that a is given as a ratio of $\Delta y/\Delta x$ this is also called the directional tangent, i.e., the tangent of the angle of inclination of the line. Therefore, the parameter a has many names: **slope**, **gradient**, **increment**, **inclination**, **directional tangent** and, in case of direct proportionality ($b = 0$), **proportionality constant**.

The linear function cannot only be given by the "**integral form**" formula, which shows how to calculate the y for any given x, but also with the "**differential form**", which shows the relationship between the change of y (i.e. Δy) and the change of x (i.e. Δx). As we mentioned above, that Δy is proportional to Δx (and the proportionality constant is a).

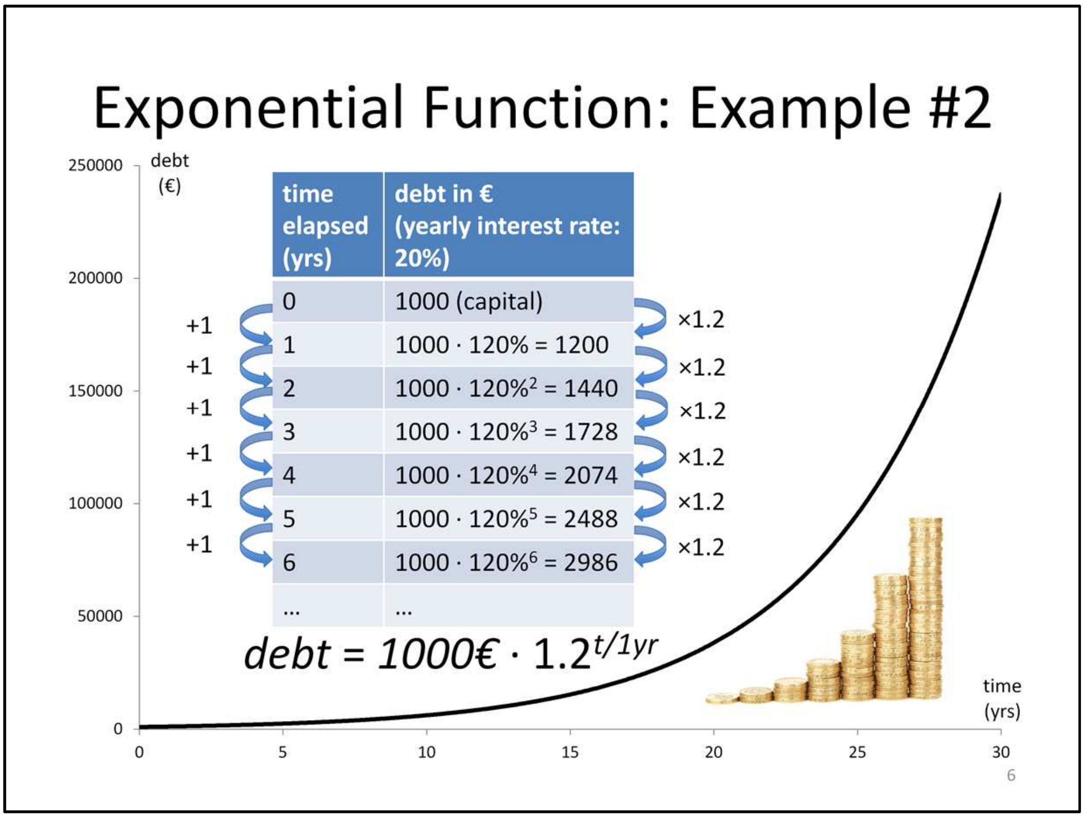
Homework: Look up those functions in the biophysics formula collection, where the relationship between the dependent and independent variables is linear! Make a schematic drawing of its graph and indicate the parameters in the graph.

Linear function plots are used in case of the following biophysics labs:
4. Refractometry [refractive index – concentration]
7. Polarimetry [angle of rotation – concentration, tube length]
11. Gamma energy [photopeak voltage – photonenergy]
21. Resonance [force – extension]
26. Sensor [action potential frequency – receptorpotential]

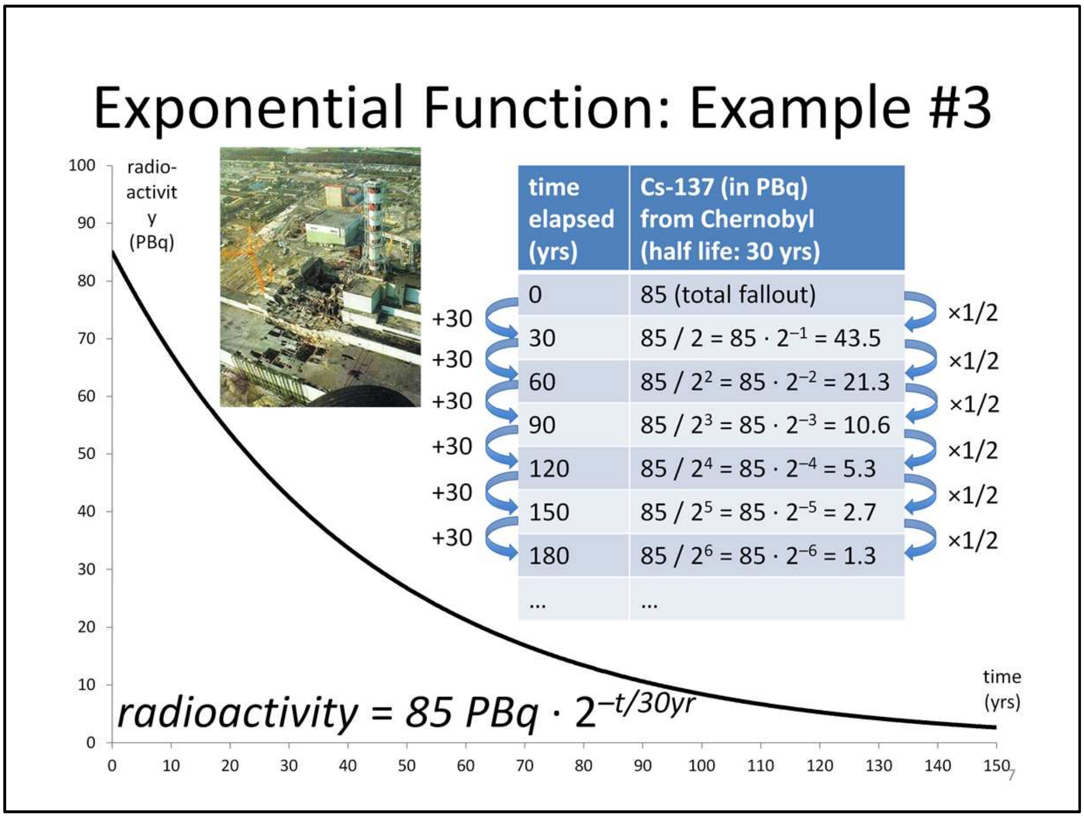


The exponential function is present in case of many natural phenomena, and even its definition is not that complicated, its understanding, however, is not profound enough in many cases. The discussion of some examples may probably help.

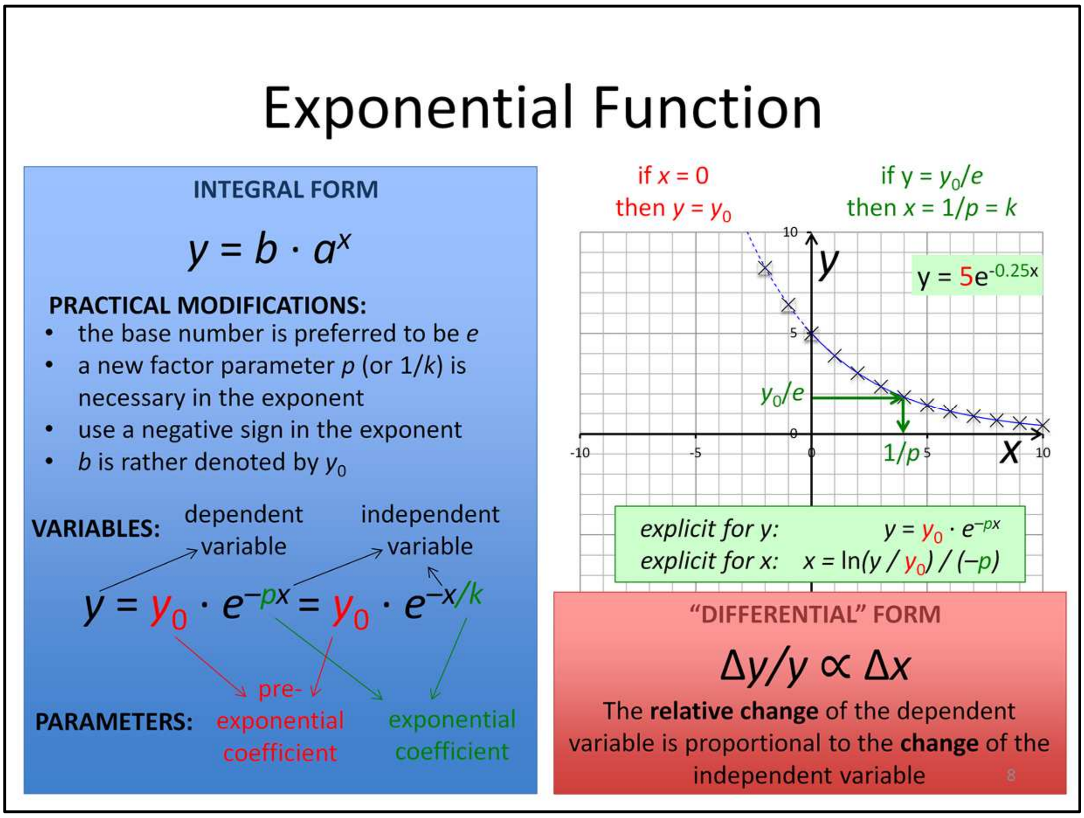
The time between two cell divisions of the bacterium *Escherichia coli* is approx. 20 minutes, that is, 20 minutes is the doubling time. If we suppose for the sake of simplicity that the cells keep on dividing synchronously, then the cell number indicated in the table may be observed. We can observe that every 20 minute absolute change in time results in the doubling, i.e., 200% relative change of the cell number. This will result in a very fast proliferation. (Naturally in real life the proliferation won't last forever, because the growth will be replaced by stagnation with the depletion of nutrients, and finally, the increasing concentration of excreta will cause the death of the cell colony.)



The next example shows the increase of a bank debt supposing 1000 euro initial capital and a 20% annual interest rate as well as one year maturity (and also supposing that the whole amount will be repaid at once). It is clear that an annual increase by 20% corresponds to a factor of 1.2 = 120%. The next year the sum increased with the previous year's interest will function as capital. The growth can be described as a series of multiplications with 1.2. As many yearly periods elapse, the same number of times will 1.2 used as a factor, i.e. after t years the capital is multiplied by 1.2^t (one point two raised to the power of t). The amount to be paid back increases rampantly, in four years for example the sum increases by two fold.



After examples from biology and economics, let us look at a physical case. In 26th April, 1986 there was an accident in the Chernobyl power plant (Ukraine), which led to the release of – among others – 85 PBq cesium–137 isotope. (PBq = petabecquerel; peta- is an SI prefix for the factor $\times 10^{15}$; becquerel [pronunciation: ˈbɛkrɛl] is the unit of activity, 1 Bq is one decay per second.) The temporal change in the activity of isotopes can be characterized by the half life: half of a certain amount of cesium–137 decays in 30 years, after another 30 years only the fourth of it is left, after another 30 years only an eighth and so on. Therefore we can get the actual "amount" (activity) of cesium–137 if we multiply the initial "amount" (activity) with 0.5 as many times as many 30-year-priods have elapsed. The repeated multiplication should, again, be written up as exponentiation: the number of 30-year-periods goes into the exponent. It is visible that the function decreases slower than a linear function would: even after 100 years approx. 8.4 PBq activity is left. Although the activity approaches 0 **beyond any limit**, it will – at least in theory – never reach it: the curve of the function approaches the x-axis **asymptotically**.



What is common in the previous example is that the independent variable (in the examples: the time) is in the exponent; such functions are called exponential functions. The parameters of a general exponential function are the base number (here: *a*) and usually a factor (sometimes called pre-exponential factor, here *b*). With the variation of these two parameters, all exponential functions can be given.

This general form, however, is not used very much in physics for various reasons. First, physicists prefer only a few base numbers, most importantly *e*, the natural unit (e = 2.718…) because of some mathematical reasons (the derivative of e^x remains e^x). We won't see too much of these advantages, but exponential functions between physical quantities appear almost everywhere in this form. Some other base numbers like 2 or 10 are also used sometimes.

Second, since the base number parameter is now fixed at *e*, not all exponential functions can be given, so a new parameter needs to be introduced in the exponent to replace the role of *a* somehow. This parameter may either be a factor (here denoted by *p*), or a divisor (here *k* which is obviously equal to 1/*p*) of the independent variable (*x*), depending on the actual case, the difference is rather practical.

Third, physics deals in most cases with exponential decreases, so this *p* factor (or *k* divisor) would almost always be negative (radioactivity, absorption of light, drop of atmospheric pressure with height etc.). To avoid using negative parameters, a negative sign is introduced in the exponent.

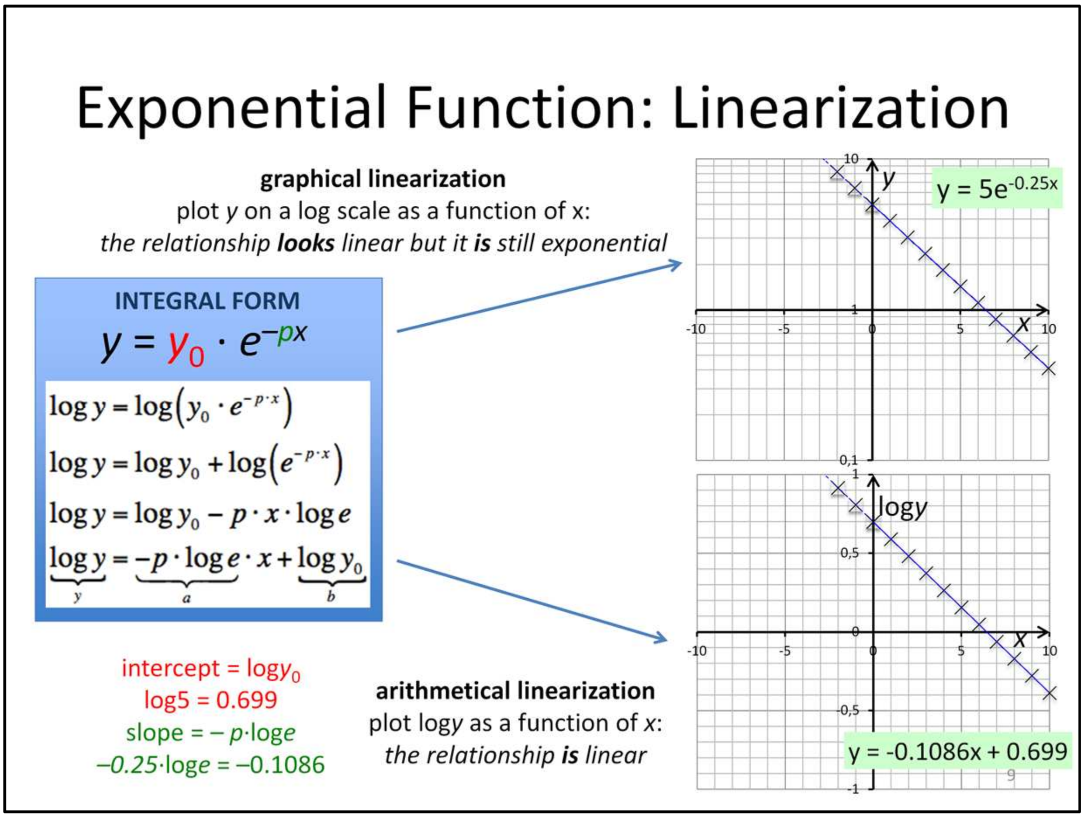
Finally, the b pre-exponential factor is mostly denoted by y_0 to indicate its actual meaning (i.e. the initial value of the function, at x = 0.)

After clarifying the form of the exponential function used in physics, let us examine the meaning of the parameters. The y_0 (that is *b*) can again be interpreted easily: if x = 0, then $y = y_0 * e^{(-p*0)} = y_0 * e^0 = y_0 * 1 = y_0$. The more problematic case is to interpret *p* (that is 1/*k*) parameter. This is somehow in relationship with the "rate of decreasing", but this concept is not that easy to define in case of a function with ever changing slope. Consequently we have to find a **trivial case** which would elucidate the meaning or role of *p*. In case of the linear function we already saw that trivial cases were those where the value of *x* was either 0 or it increased by 1. To interpret the meaning of y_0 for the exponential function, we again used the *x* = 0 case. In general we can say that trivial cases occur if *x* is equal to 0; +1; –1; +∞; or –∞. In our case, however, we need to look for something else. The strategy would be that the value of x should "cancel" the p parameter. Since *x* is a factor of *p*, it can be cancelled if *x* = 1/*p*. After substituting 1/*p* for *x* we get: $y = y_0 * e^{(-p/p)} = y_0 * e^{(-1)} = y_0 * (1/e) = y_0/e$. Consequently, the value of the function (*y*) reaches the *e*-th part of y_0, where *x* reaches 1/*p* (or *k*, depending on notation).

Using a different approach one can say that in case of an exponential function the **relative change** of the dependent variable (Δy/y) is proportional to the **absolute change** of the independent variable (Δ*x*).

Homework: Look up those functions in the biophysics formula collection, where the relationship between the dependent and independent variables is exponential. Make a schematic drawing of its graph and indicate the parameters in the graph.

Exponential or linearized exponential function plots are used in case of the following biophysics labs:
6. Light absorption: [absorbance – concentration, layer thickness]
10. Gamma absorption: [pulse count – layer thickness]
12. Isotope diagnostics: [(biological/physical/effective) activity – time]
14. CAT-SCAN: [X-ray intensity – layer thickness]
21. Resonance: [damped free oscillation amplitude – time]
22. Pulse generator: [discharged RC-circuit voltage – time]
29. Diffusion: [amount of KCl in the gel cylinder – time]



After understanding the behavior of the exponential function we need to face one more problem. Our aim during measurements is usually to fit a function to the measured data. If the plotting of the graph and the fitting of the function are both executed manually, then we can only fit a linear (we do not have any "curved" ruler), moreover, we are only able to detect linear tendencies with the naked eye. Our eye is not sensitive for the type of the curve, we can only tell that it is curved but usually we cannot estimate reliably whether the curved line belongs to an exponential, quadratic, or sine function, or it is just n arc belonging to an ellipse; we just simply see that it is a curve although the aforementioned functions differ fundamentally.

We can overcome the difficulties of fitting a curvilinear function manually or our inability to judge the nature of curvature with the naked eye, if we could "stretch" the function somehow, what we call **linearization**. In order to do this, take the logarithm of both sides of the equation. After transformations it is visible that is we plot the logarithm of the dependent variable (log*y*) as a function of the independent variable (*x*), we get a linear function with an intercept of logy_0 and a slope of –*p**log*e*.

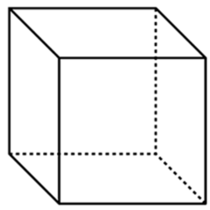
A faster method is if we plot original y values on a logarithmic vertical scale. But what is a logarithmic scale? Let us first understand the linear scale in depth. Let us consider a segment between 0 and 2 on the *x* axis: its length is 2 units (= 2–0).

We get similar segments between 3 and 5, 6 and 8, because the **difference** is 2 in all three cases. That means that on the linear scale a certain distance corresponds to a certain difference. – The logarithmic (or simply log) scale is different: the distance between 1 and 2 is the same as between 2 and 4, or 3 and 6, or 4 and 8, or 5 and 10: here the segment length corresponds to a certain **ratio** (here: ½). Naturally the "physical" (measured) distance always means difference, it is the special outlay of the log scale that makes transforms this into ratio. It is easy to see it if we consider that: log(a/b) = loga – logb, i.e. the logarithm of a ratio is a difference.

Getting back to linearization: if we make the y axis instead of the y variable logarithmic, we again get a linear graph. It is important, however, that here only a "graphical" transformation happened: the relationship between the variables indicated on the axes remains exponential (e.g. we have to use exponential function if we want to fit the points in Excel).

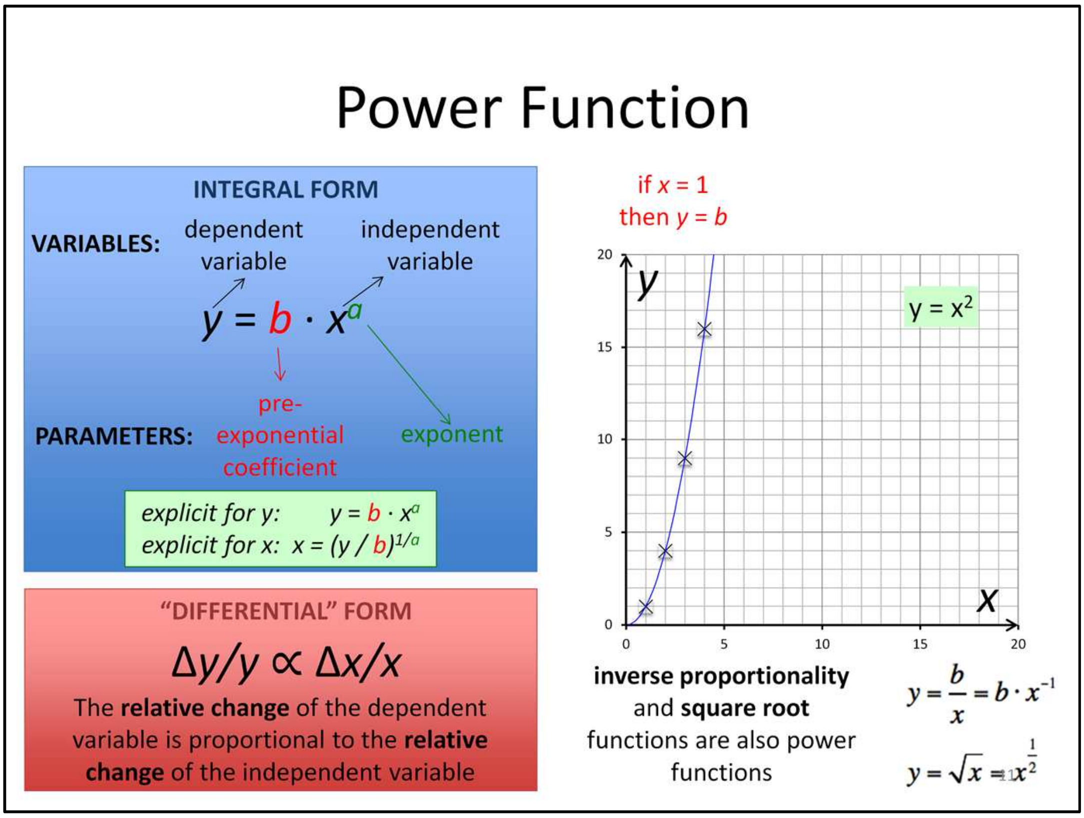
Power Function: Example

mass $\propto$ volume $\propto$ [body]length^3
surface area $\propto$ [body]length^2



The next function is the power function. For this I only give a geometrical example first: the surface area of a cube (and in general: of any 3D body) is proportional to the square of the length, while its volume is proportional to its third power. That is, if a body is linearly magnified by a factor two, then its linear measures will double ($\times 2^1$), its surface area will quadruple ($\times 2^2$), and its volume (and its mass, which is proportional to volume) will be eight times greater ($\times 2^3$).

Because the numerous physiological parameters of animals are related either to length, area, volume, or most commonly, to a certain combination of these, we may expect power function relationships between them.



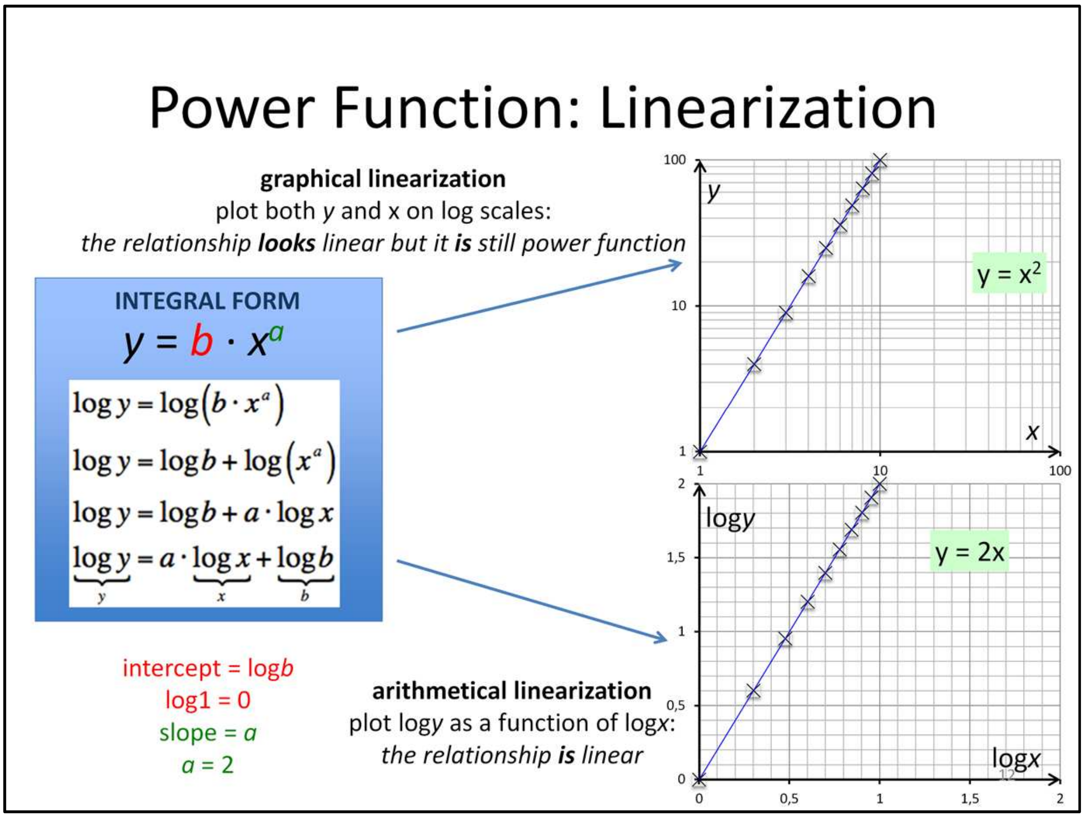
In case of a power function, the independent variable is again part of a power expression, but in this case – in contrast with the exponential function – it is in the base. Its general formula contains two parameters: the exponent (here: *a*) and the pre-exponential coefficient (here: *b*) standing in front of the power expression. Notice that the function assumes the value of *b* if $x = 1$ because $y=b*x^a=b*1^a=b*1=b$. Remember, that radical function (e.g. square root) are also power functions (the *n*-th radical can be transcribed as the 1/*n*-th power), as well as that the inverse proportionality is also a power function (a division with a number is the same as multiplication by its reciprocal, i.e., its –1 power).

If we want to elucidate the change in the function: we can say, that the **relative change** in the dependent variable ($\Delta y/y$) is proportional to the **relative change** in the independent variable ($\Delta x/x$).

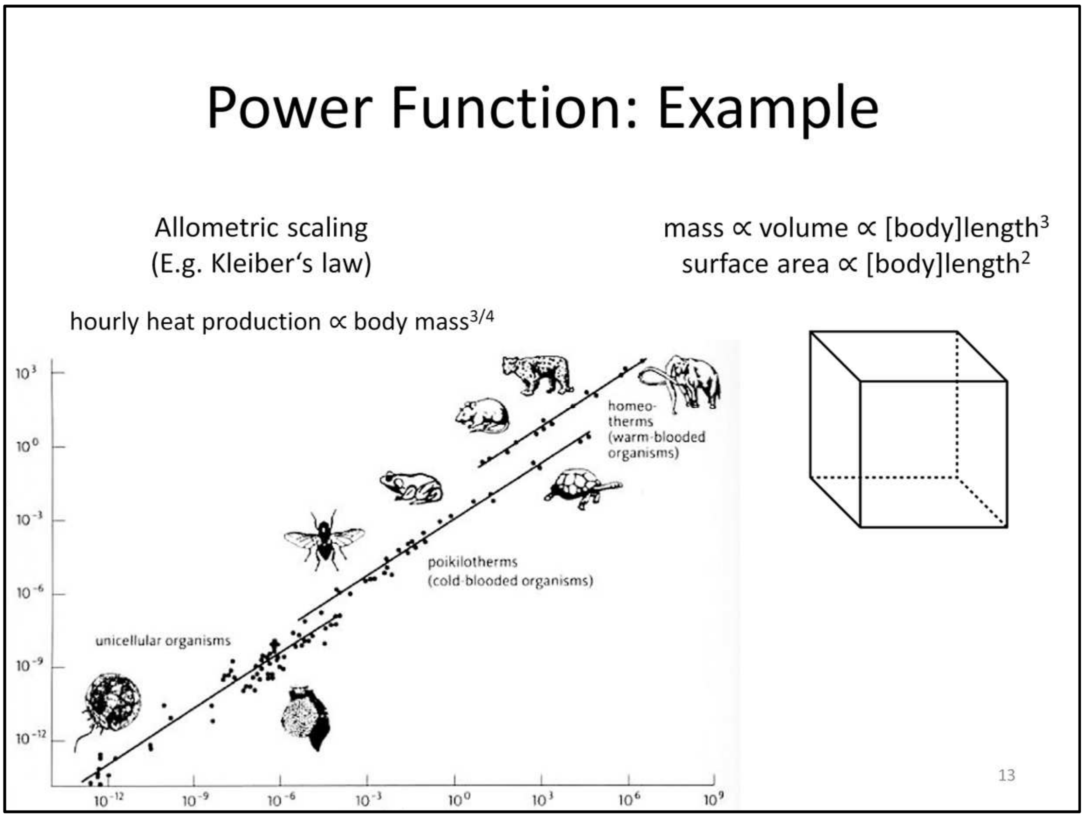
Homework: Look up those functions in the biophysics formula collection, where the relationship between the dependent and independent variables is power function. Make a schematic drawing of its graph and indicate the parameters in the graph.

Power or linearized power function plots are used in case of the following biophysics labs:
13. X-ray: [cut-off wavelength of Bremsstrahlung spectrum – accelerating voltage]
13. X-ray: [partial mass attenuation coefficient of photoeffect – atomic number of absorbent]

15. Dosimetry: [dose or dose rate – distance from isotope]
18. Amplifier: [lower and upper limit of band pass filter: voltage – frequency]
19. Sine wave oscillator: [eigenfrequency – capacitance, inductance]
21. Resonance: [eigenfrequency of oscillation – mass]
24. Skin impedance: [capacitive reactance – frequency]
25. Audiometry: [sone scale – sound intensity]
26. Sensory function: [receptor potential – illuminance]
26. Sensory function: [action potential frequency – illuminance]
26. Sensory function: [according to Stevens’ law: sensation intensity – stimulus intensity]



The practical problem caused by the “curvedness” of the graph also arises in case of power functions. The logarithmic transformation may be used again. After derivation it is visible, that log*y* depends on log*x* linearly; the intercept is log*b*, the slope is *a*. If we do not want to transform variables, we can “graphically” linearize the function by using log scale on both axes.



Let us get back to the biological-medical use of power functions. A number of physiological variables show a power function relationship with body mass. For example the metabolic rate ("hourly heat production at rest") is proportional to the 0.75-power of body mass (this is Kleiber's law). The frequency of respiration or the heart, the diameter of the aorta also show a power function relationship with body mass. From the medical aspect it is very important that the different variables related to the metabolism of drugs also follow a power function mass-dependence. Consequently, if the given metabolic parameter is measured for mouse, rat, guinea-pig, rabbit, cat, dog, goat, and horse (as well as the body mass of the subjected animals), the metabolic value – mass data pairs may be fitted with a power function, which makes it possible to estimate the value for humans before carrying out actual human experiments.

Derivative and Integral: Example #1

x	$y = x^2$	$y' = \Delta y/\Delta x$	$y'' = \Delta(\Delta y/\Delta x)/\Delta x$
0	0		
1	1	1	
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	2
6	36	11	2
7	49	13	2
8	64	15	2
9	81	17	2
10	100	19	2

14

As a next step let us have a short introduction to calculus, that is **integrals** and **derivatives**.

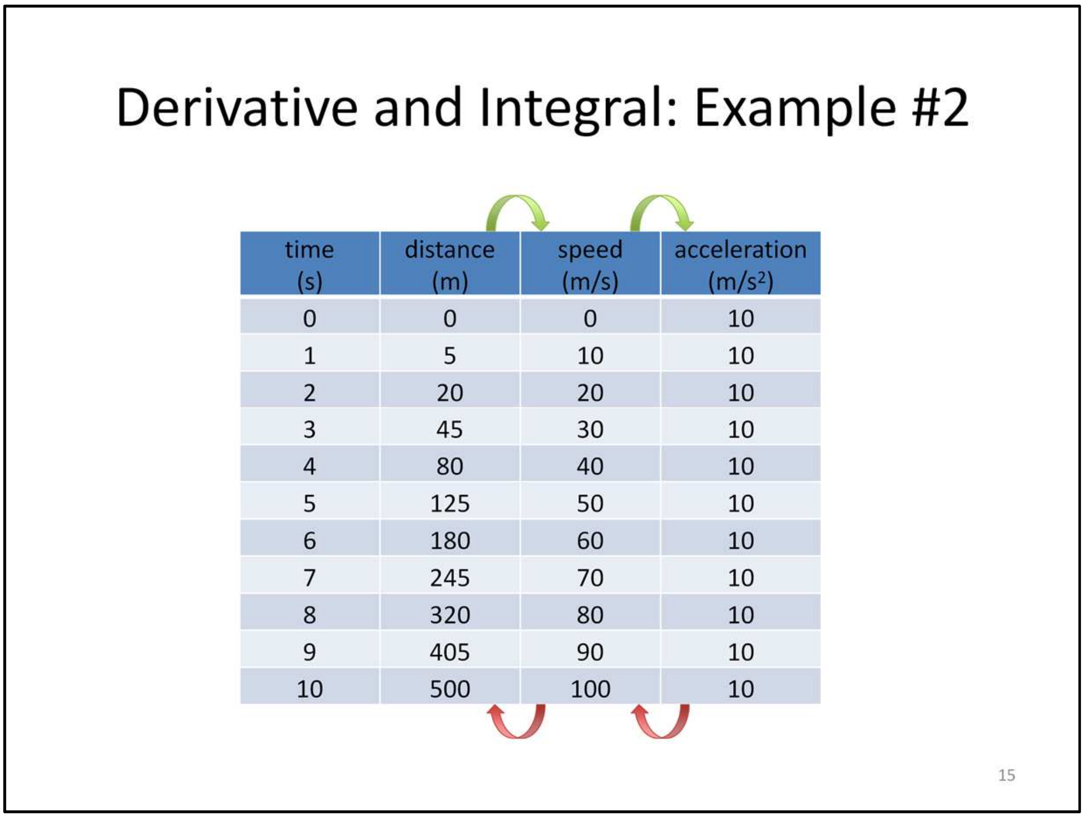
It is important, that YOU DON'T NEED TO DO SUCH CALCULATIONS, but we need a simplified understanding of the demonstrative meaning of derivative and integral is needed.

In the above example I wrote the square number under one another (y is the square of x). Notice that the difference between the consecutive numbers (y') is not random but changes regularly: increases by 2. That is, these differences change linearly: they increase every time by +2.

Now let us see the difference between the differences (y'')This value is always the same, 2, that is, constant.

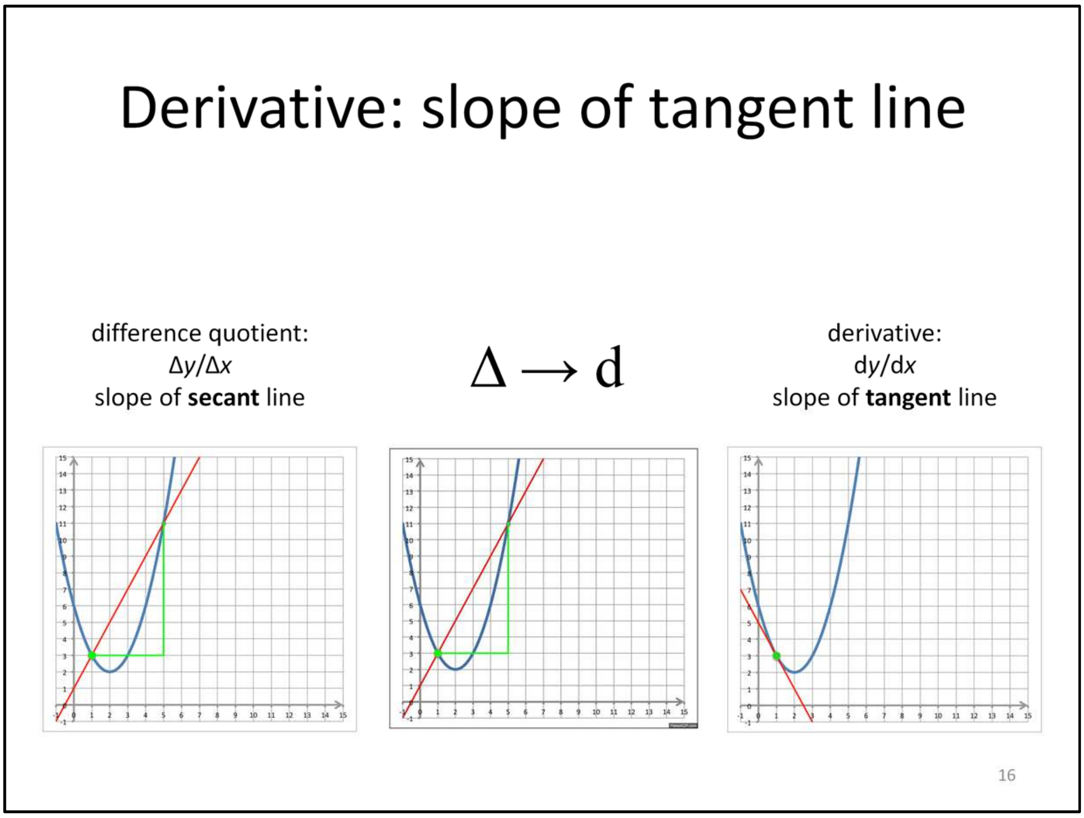
If we do the same thing the other way round, we can recreate by addition of constants (2; 2; 2; 2 …) the linear function (1; 3; 5; 7; 9 …), and from the addition of these values, the series of square numbers (1; 4; 9; 16; 25 …).

The change of functions, that is the change of y in case of a given (very little) change of x, is the field of interest of calculus.



Let us see a practical example:

The change in distance per unit time is called speed (or velocity), the change of velocity per unit time is acceleration. If we drop an object from some high place, it will fall and will have an acceleration of approx. 10 m/s^2, which increases its speed by 10 m/s in every second, and the distance increases by a quadratic function. However, the speed cannot be derived from the path with the previously explained subtraction (differential) method, since the speed does not increase in **steps** but **continuously** (this is also true for the summation method in the other direction). If we inspect the change in finite steps, we introduce a bias into the calculations: we will get the *average* change in the given step. The bias is less if the step is smaller, optimally, it should be as close to zero as possible.

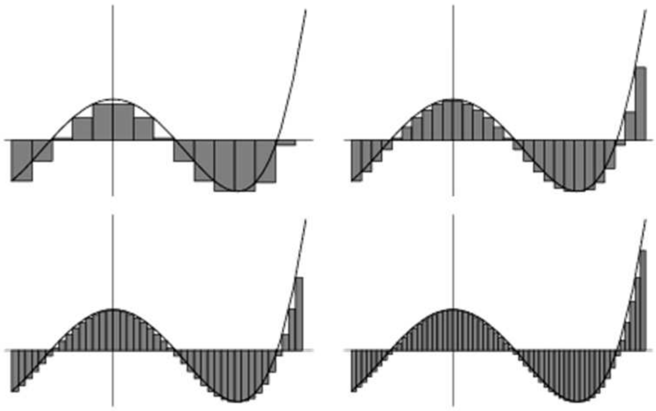


To make the previous explanation clearer, let us inspect the following figures (the GIF animation in the middle can be seen at http://makeagif.com/9KJyhD).In the left image, we take a +4 step with x which results in the +8 step of y. The $\Delta y/\Delta x$ ratio (difference quotient), which characterizes the "rate" of change, geometrically corresponds to the slope of the secant passing through the two points. To reduce bias coming from the Δx finite step, decrease Δx: the $\Delta y/\Delta x$ ratio changes together with the slope of the secant as shown in the middle animation (see the weblink given above). As the two points of the secant are getting closer they will practically unite: both the Δy and Δx will be very-very close to zero (to indicate this, we use d instead of delta), and the secant turns into a tangent. In case of such an infinitesimally small step. In case of such a small step the bias coming from the step length will be negligible, so the dy/dx ratio gives the **local slope of the function**, and the name of this ratio is the derivative.

The pratical example for the difference quotient is average speed (speed for a given time interval) and for the derivative is the instantaneous speed (speed at a given point in time).

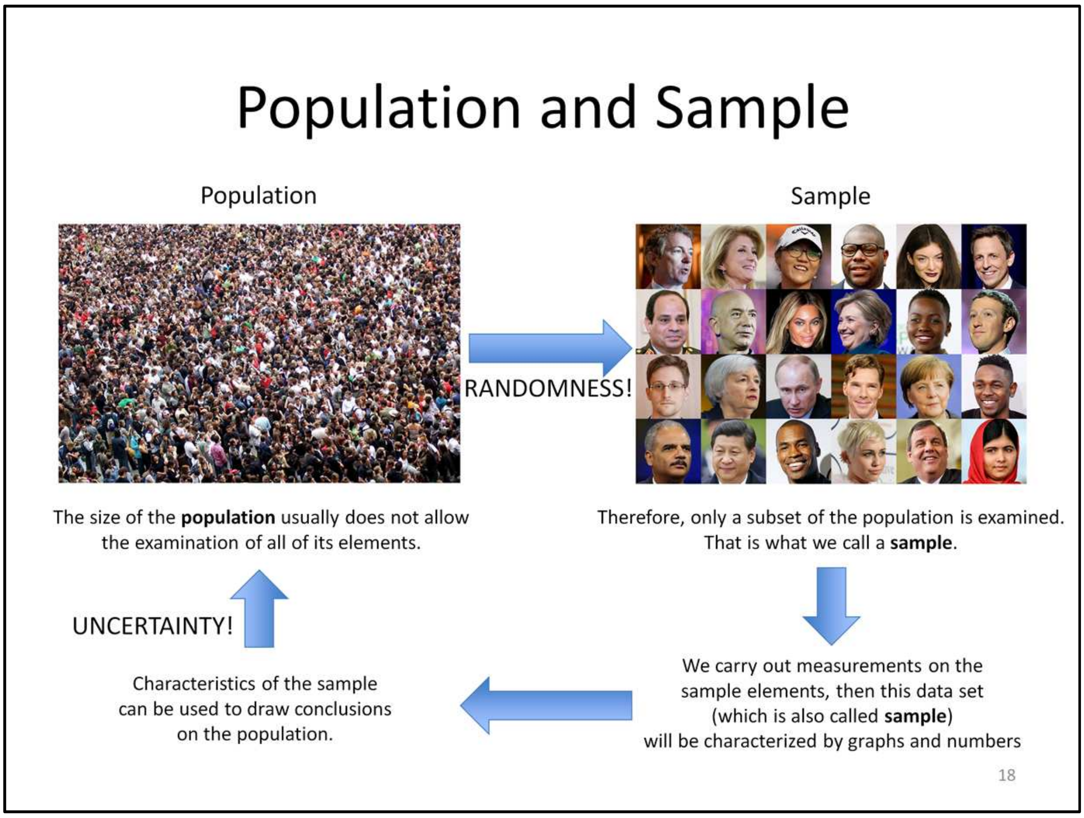
Integral: Area Under the Curve (AUC)

$$\Sigma \rightarrow \int$$

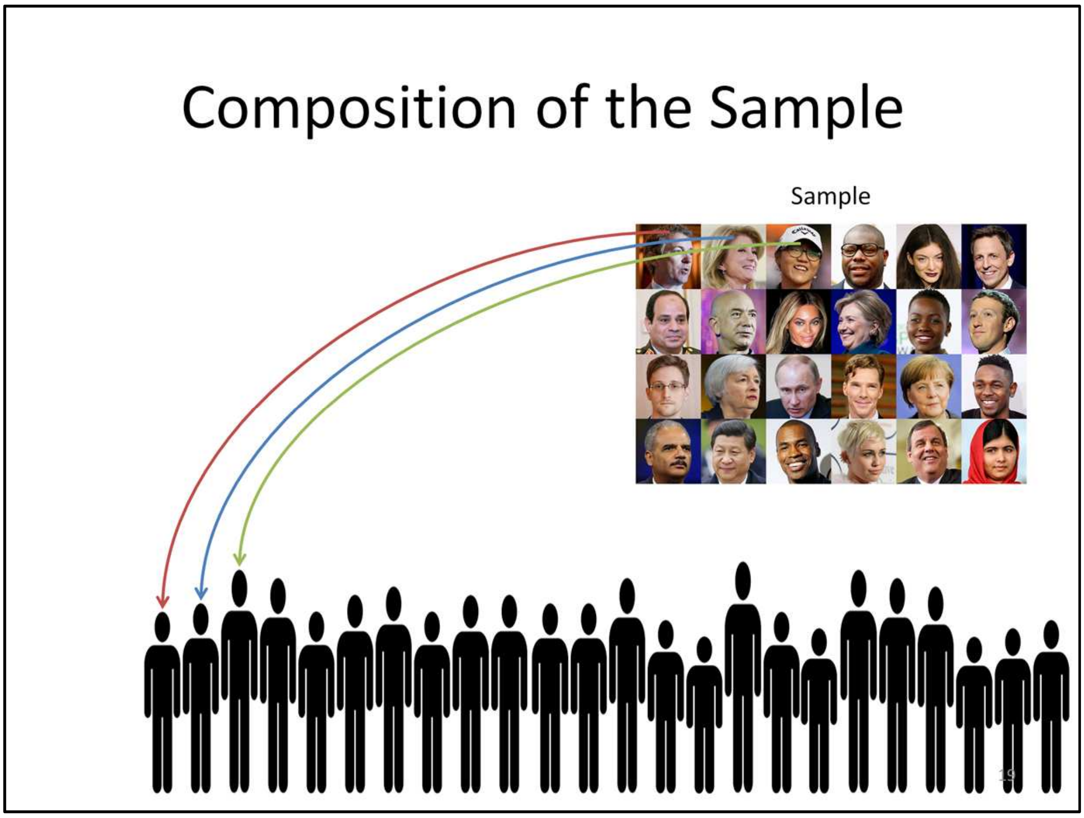


17

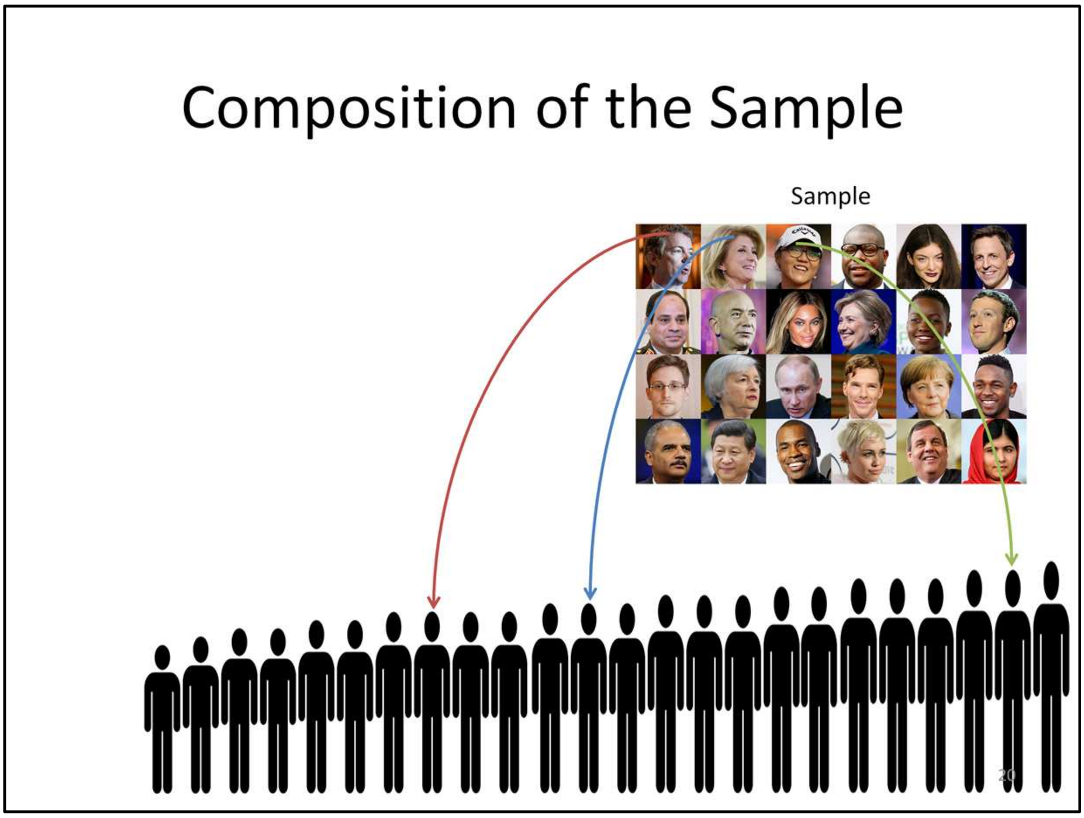
The integral can be derived from series of summations, principally as a reversal of differentiation: the integral of a function at a given *x* point can be given as the sum of the function values (*y*) weighted (that is, multiplied) with the step length (Δx) from negative infinity to *x*. The product of the function value and the step length ($\Delta x * y$) geometrically corresponds to a rectangle: basically, the area under the curve is approximated with step-length rectangles. Naturally, the step length causes a bias in this case as well: the smaller the step length, the less the bias of approximation. If the step length becomes very close to zero (that is, the d*x* introduced in case of the derivative), the bias will be minimal: in that case we speak about integral instead of summation. The geometrical meaning of the integral therefore is the **area under the curve**.



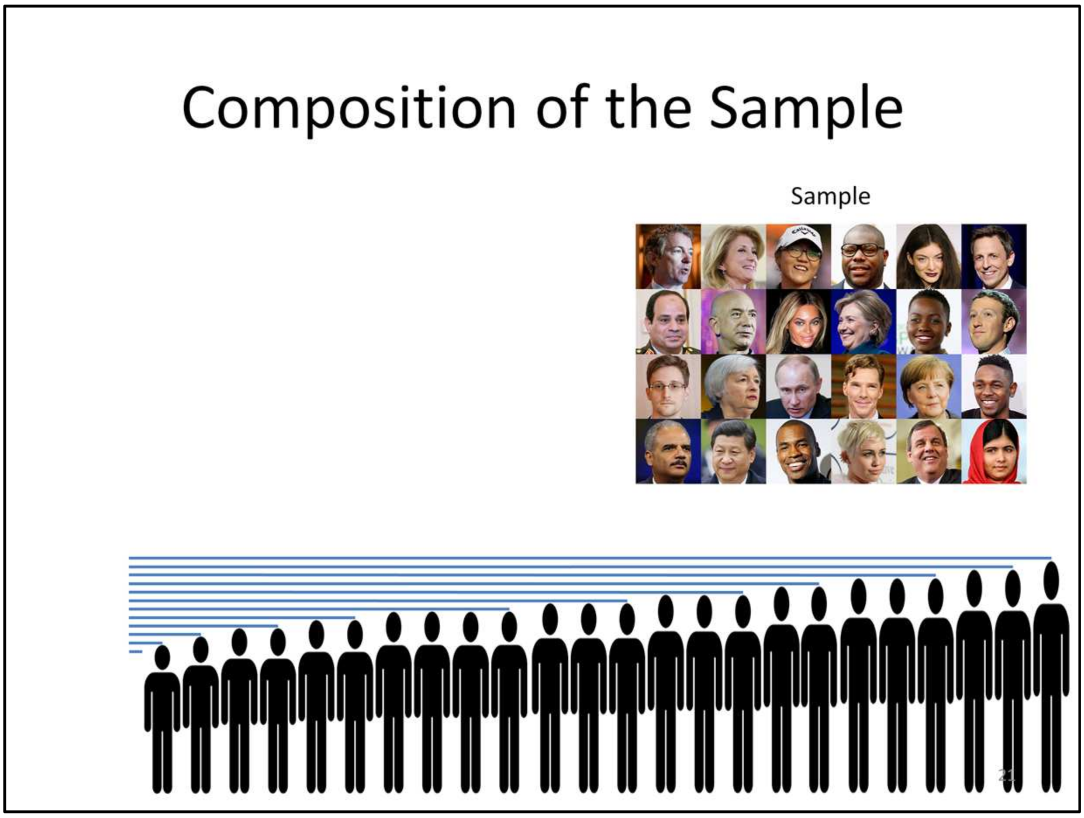
After discussing the most important functions describing deterministic relationships, let us return to stochastic variables which are at the focus of statistics. In case of deterministic phenomena, to every input of a series of measurements (x) there is an unambiguous output (y) (e.g. after 30 years (= x) the activity of the isotope is halved (= y)); however, stochastic phenomena have multiple possible outcomes for a given input due to randomness (e.g. if the length of an egg is measured repeatedly the result will vary somewhat due to measurement uncertainty). This could mean that the most important condition of the function is not met (i.e. unambiguous assignment) but the situation can be handled: we just need to review what should be considered as independent and dependent variable: in case of stochastic variables the frequency of occurrence (y) is assigned to a given outcome (x) of a certain measurement. This will produce the frequency distribution of the outcomes of the measurement results (i.e., the sample), this was mentioned in the previous lecture. Since we have reviewed the basic characteristics of functions, we can discuss them again.



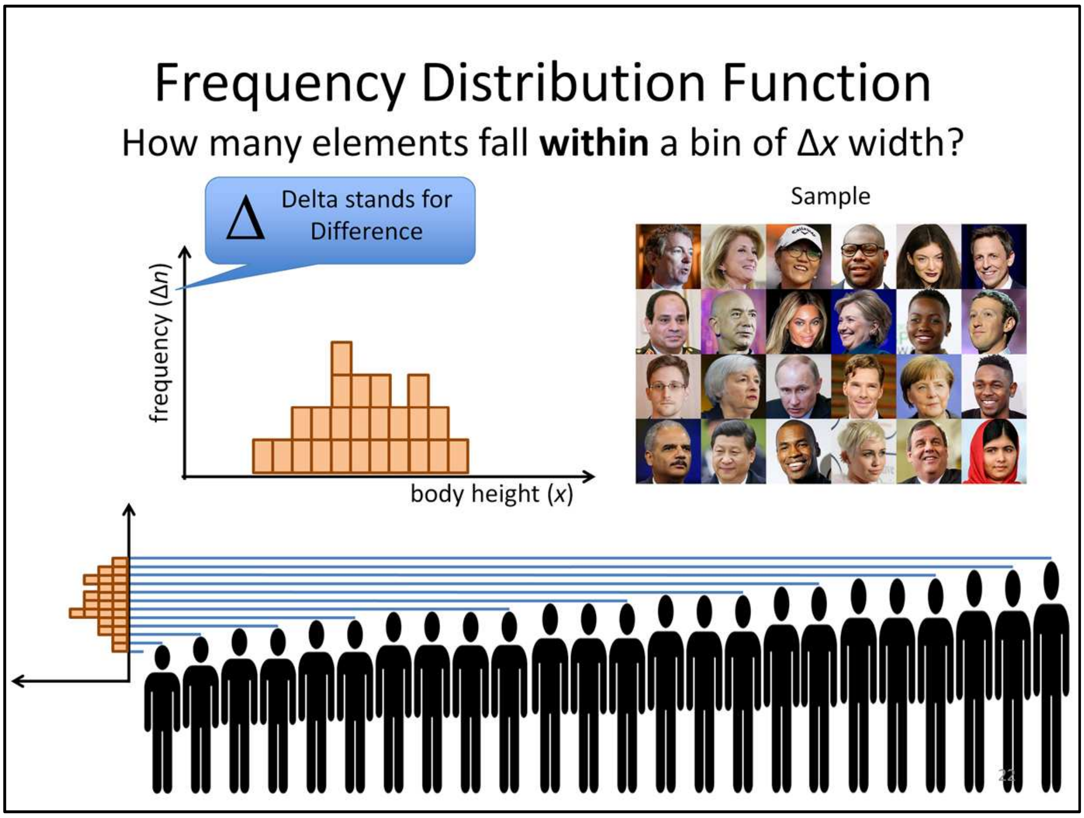
First, let us consider a measurement task: we would like to characterize the stature (body height) of a group of 24.



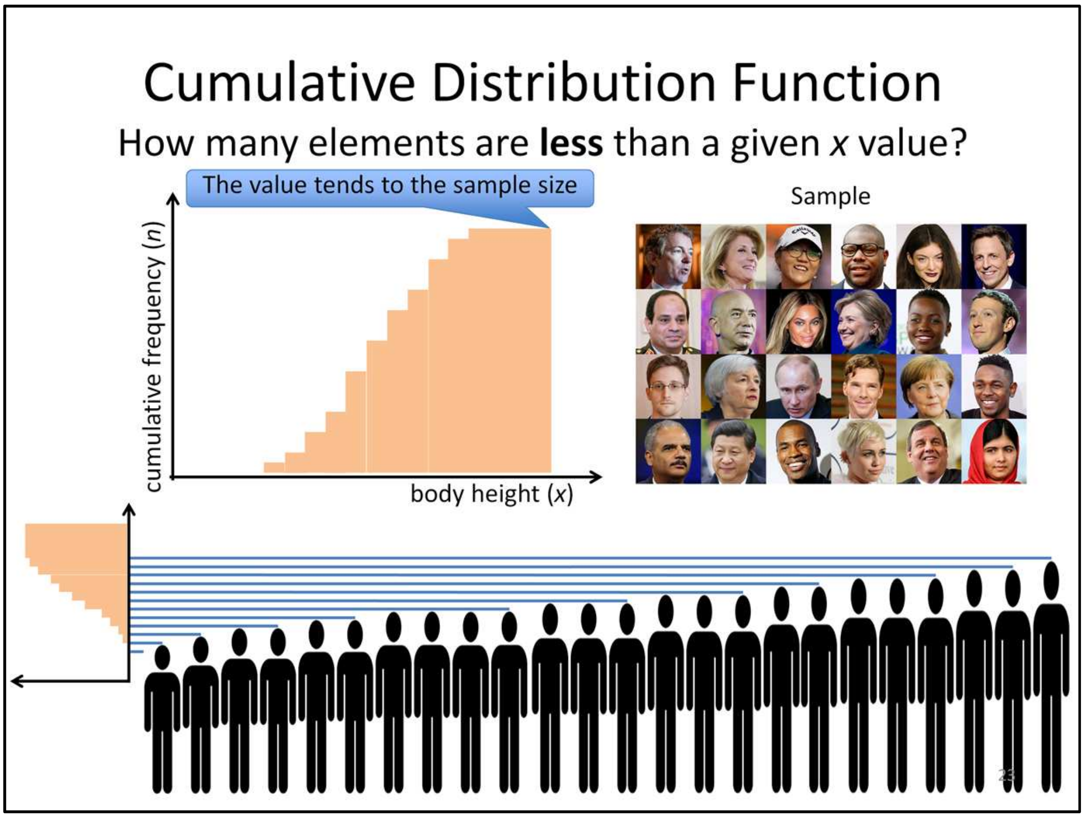
It is more demonstrative to show the values sorted by magnitude.



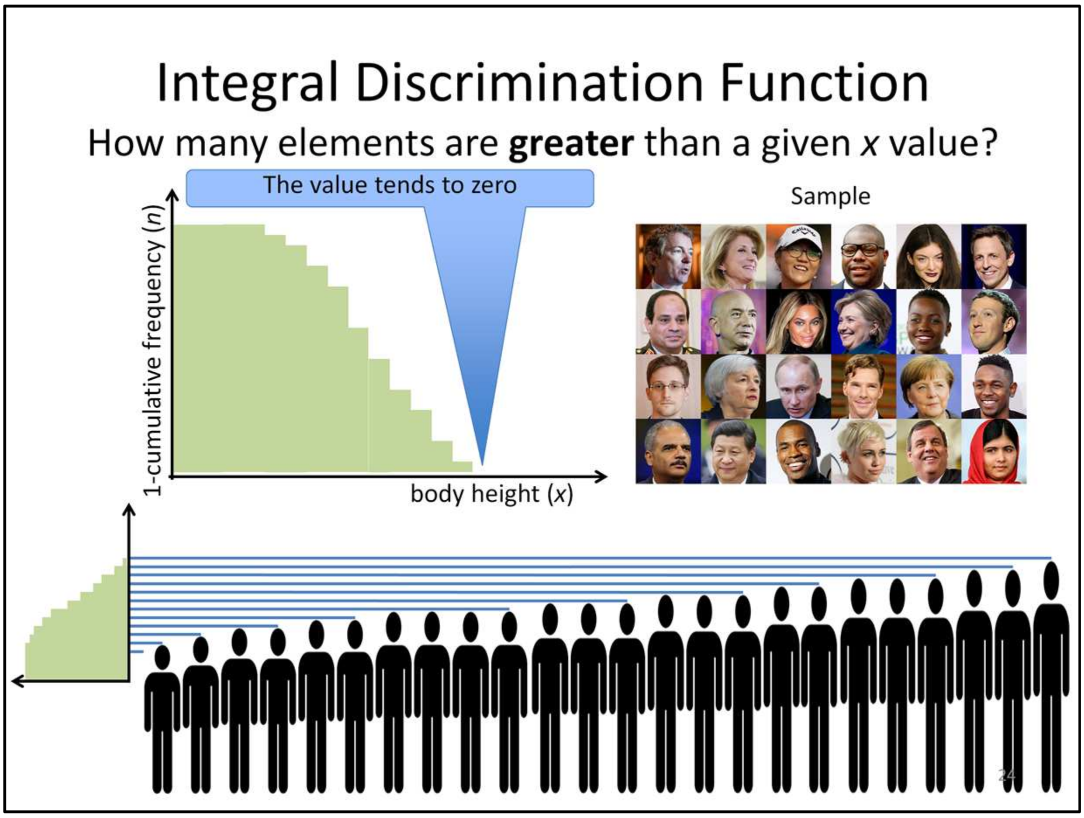
Then set intervals, within which data are assign to the same bin (cathegory).



Next, prepare the frequency distribution in the well–known way: count the number of elements belonging to a bin, then indicate this number with small boxes in the function. Obviously, the total number of boxes is equal to the number of elements in the sample (i.e. sample size).



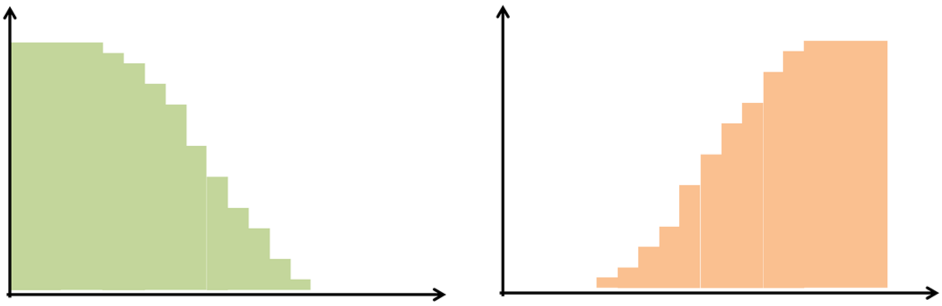
Now let us see an other method for counting: count the number of elements less than a given x value (these "levels" of x are indicated with the blue lines). There is no one below the lowest line. Below the next line there is one person, below the next line there are altogether two persons, then four persons and so on. Finally there will be a height below which all the elements (all the 24) will be found, that is, this function tends to the sample size if x increases. Since with increasing x more and more elements are included, this is called **cumulative frequency distribution** (cumulate = pile up, heap up) .



We can reverse the previous method of counting elements: let us count now how many elements are greater than a given x “level (blue line in the slide). Everyone is taller than the lowest line, so the value of the function is the sample size. In case of the next line one element “falls out”, then another one (two altogether), then again two (four altogether) and so on. Finally no elements will be greater than x: the value of this function tends to zero as x is increased. This type of distribution is called **integral discrimination distribution**.

Three Functions – Same Information?

Only if Bins are Arbitrary in All Three Cases

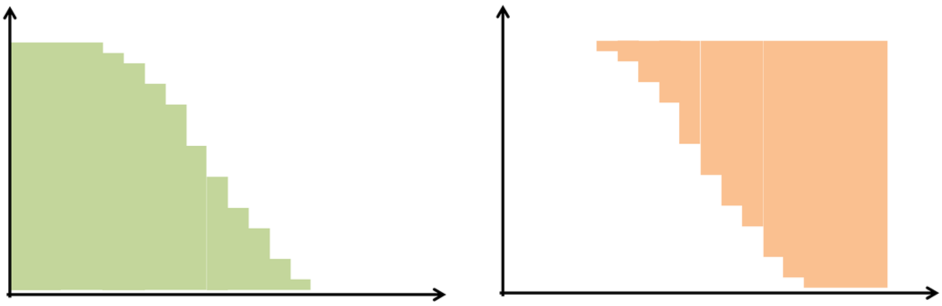


25

Finally let us summarize the different kinds of frequency distributions, with special attention to the relationships between them.

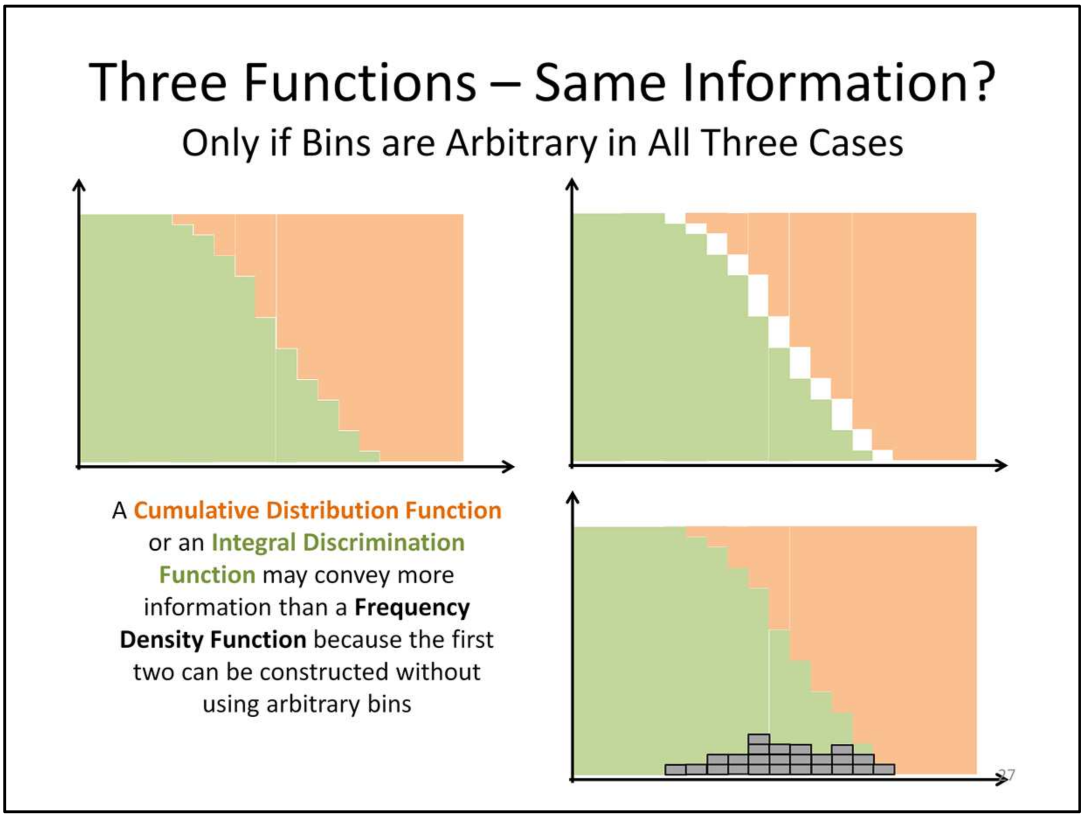
Three Functions – Same Information?

Only if Bins are Arbitrary in All Three Cases



26

It was clear during the explanation that there is a tight relationship between the different types of distributions: e.g. the cumulative frequency (orange) and the integral discrimination frequency (green) belonging to a given x “level” adds up to the sample size because the former counts values less than x, the latter those greater than the same x. This may be demonstrated f the cumulative frequency distribution is mirrored to the x-axis: the graph of the two functions maybe shifted into each other.



The relationship with the frequency distribution is also clear: if we consider the change of either the cumulative or the integral discrimination function, we get the frequency belonging to the corresponding bin. I.e. the stepwise change in the cumulative (i.e. its derivative) gives the frequency distribution. This is also true in the other way round: the cumulative summation (i.e. integral) of the frequencies of the frequency distribution returns – depending on the direction of the operation – the cumulative distribution, as well as the integral discrimination distribution.

Consequently, the from the cumulative distribution (as well as from the integral discrimination spectrum) one can create the frequency distribution by calculating the differences ("derivative"); from the frequency distribution one can derive the cumulative frequency distribution (or integral discrimination frequency) by summation ("integral").

Test Questions #1

- Give the definition of function.
- What is a variable? What are the types of variables?
- How can a function be given?
- What is the meaning of the following terms: constant, coefficient, parameter, factor.
- What do the terms stochastic event and deterministic event stand for? Which type prevails in real life?
- What is the general equation of the linear function? What is the demonstrative meaning of its parameters? What trivial examples can demonstrate these meanings?
- How can the relationship between the *change of the variables* of the linear function be expressed?
- Give example for linearly related physical quantities.
- What is a direct proportionality?
- When do we call a linear function linear proportionality?
- A geometrical distance (line segment), could be described as what arithmetically?
- How can the equation of a line be determined if only two of its points are known?
- Give example for exponentially related physical quantities.
- What is the essence (or definition) of exponential change?
- What is the general equation of the exponential function (at first approach)?
- What transformations are made on this general equation to make it fit for practical (physical) use?
- What are the meanings of the parameters of the practical form of the exponential function?
- What trivial cases can be used to elucidate the meaning of the parameters of the practical form of the exponential function?
- How does the exponential function change if y_0, and if p is increased?

The following questions may be answered using lecture material, consultation with practice teacher, or your own investigation (on the library or the internet). These test questions are examples for multiple choice items that may occur in the midterm and exam tests.

Test Questions #2

- Why is the linearization of exponential functions necessary?
- How can the practical form of the exponential function be linearized? What is the relationship between the parameters of the original and the linearized functions?
- How can exponential functions be linearized graphically?
- How can the relationship between the *change of the variables* of the exponential function be described?
- What is the meaning of the word *exponent*?
- Give geometrical, physical, and biological examples for the power function.
- What is the essence (or definition) of the power function?
- What type of function is the square root function?
- What type of function is inverse proportionality?
- How can the power function be linearized? What is the relationship between the parameters of the original and the linearized functions?
- How can power functions be linearized graphically?
- The following data were obtained when we observed the free fall of an object:

time (s)	distance (m)
0	0
1	4,905
2	19,62
3	44,145
4	78,48

  What is the reason that a power function can *not* be fitted to these data points in Excel?
- We would like to plot numerical data pairs in a co-ordinate system. Should the points be connected with line segments or not? Why?

Test Questions #3

- Consider the increasing exponential and power functions. Which of them grows faster? Give a demonstrative proof (for any base or exponent parameter, respectively).
- Where does the $y=2e^{-px}$ function tend if p = 1.3 and x tends to infinity ($x \rightarrow +\infty$)?
- The slope of a linearized power function is 2. What kind of power function was it originally? What type of curve is the graph of the original function?
- The slope of a linearized power function is 0.5. What kind of power function was it originally?
- The slope of a linearized power function is –1. What kind of power function was it originally? What type of curve is the graph of the original function?
- What sort of regularity can be seen in the sequence of squared natural numbers?
- What is the meaning of difference quotient, and derivative? What is their graphical interpretation?
- What is integral? What is its graphical interpretation?
- What is the relationship between the displacement–time, velocity–time, and acceleration–time functions?
- What is the relationship between the linear and the square functions?
- What is the definition of the frequency distribution, cumulative frequency distribution, and the integral discrimination distribution?
- Where does the value of the integral discrimination distribution tend if $x \rightarrow +\infty$?
- Where does the value of the integral discrimination distribution tend if $x \rightarrow -\infty$?
- What is the relationship between the cumulative frequency distribution and the frequency distribution?
- Which diagram type should be used in Excel if we would like to plot numerical data pairs? Why?

Test Questions #4

- What is the general equation of the linear function if we want to express *x* (in case if the *y* values are given)?
- What is the general equation of the practical form of the exponential function if we want to express *x* (in case if the *y* values are given)?
- What is the general equation of the power function if we want to express *x* (in case if the *y* values are given)?