



Physical Bases of Dental Material Science 7.

Mechanical properties of materials 1. Elasticity

Keywords:

- ❖ 1 Stress-strain
- ❖ 2 Elasticity
- ❖ 3 Deformations

E-book
chapter 14, 15.

Problems: 4.1,
4.3, 4.4, 4.5, 4.6,
4.9, 4.11, 4.16,
4.17, 4.18, 4.21,
4.22, 4.23, 4.24.

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Mechanical properties of materials

Stiffness, elasticity, toughness, hardness are the most important parameters when considering mastication or orthodontics.

Mechanical stress (force) is applied on material.

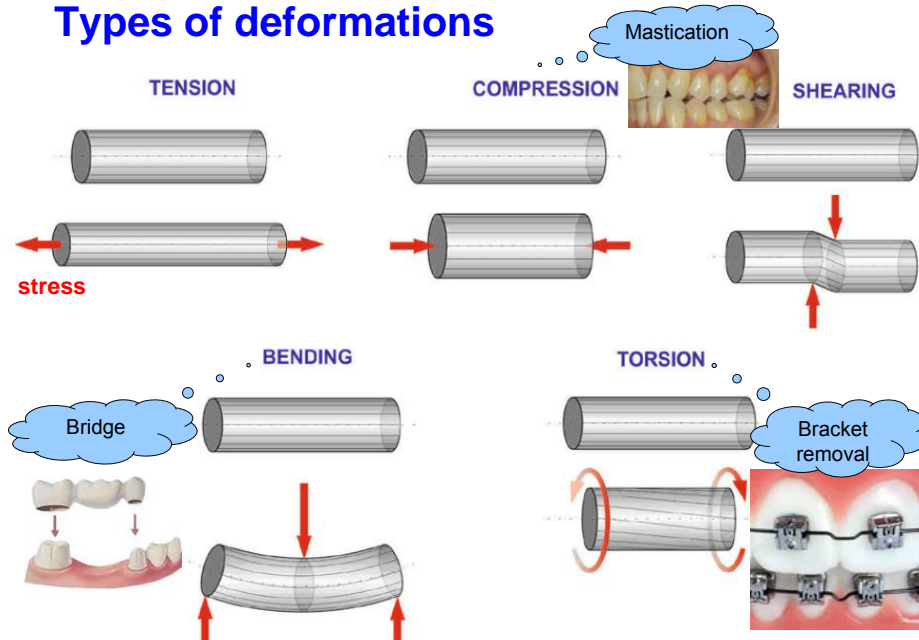


The consequence is usually some kind of deformation.
(change in shape depends on the direction and point of application)



Depending on the material's mechanical properties, deformation may be reversible or irreversible.

Types of deformations

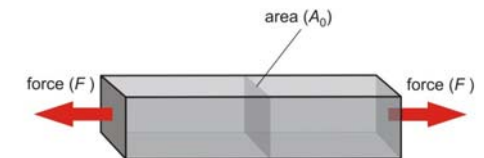


Basic concepts

Stress

$$\sigma = \frac{F}{A_0}$$

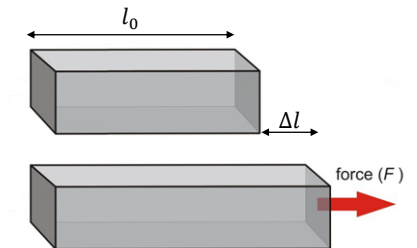
Dimension: $\left[\frac{N}{m^2} = Pa \right]$



Strain (deformation)

$$\varepsilon = \frac{\Delta l}{l_0}$$

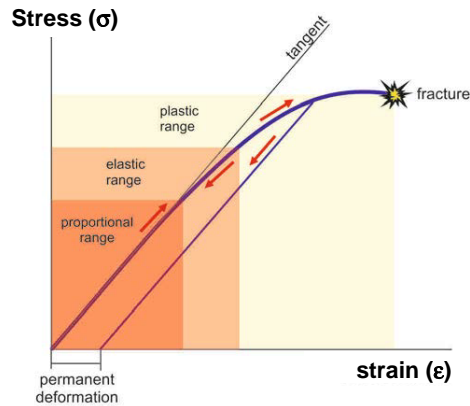
Dimensionless: $\left[\frac{m}{m} \right]$



Strain is proportional to stress!

$$\sigma \sim \varepsilon$$

Stress-strain diagram



Elastic range

Range of reversible deformation. Unloaded length (l_0) recovers when released. Hysteresis may occur.

Proportional range (part of elastic range)

Deformation is linearly proportional to the load. No hysteresis. (see Hooke's law: Biophysics, Rasonance lab)

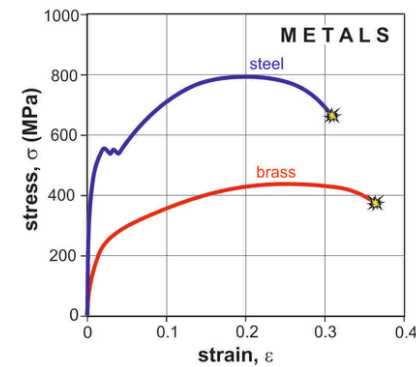
Plastic range

After a critical stress value, object undergoes irreversible change of its structure. Unloaded length (l_0) does not recover. Permanent deformation of object.

Fracture

Desintegration of object.

Typical stress-strain curves Metals



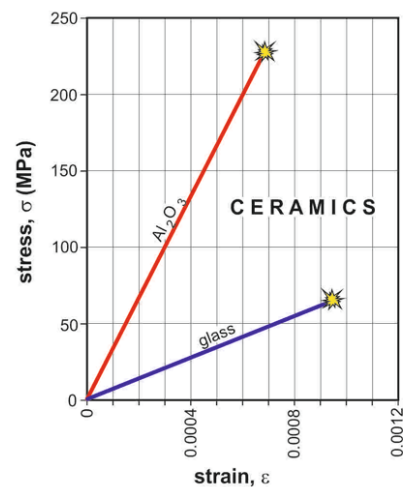
Short elastic range

Serious plastic range

Metals are highly deformable
 $\epsilon_{\max} \sim 0.3$ (~30%)

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Typical stress-strain curves Ceramics



Extremely short elastic range
 $\epsilon_{\max} < 0.001$ (less than ~0.1%)

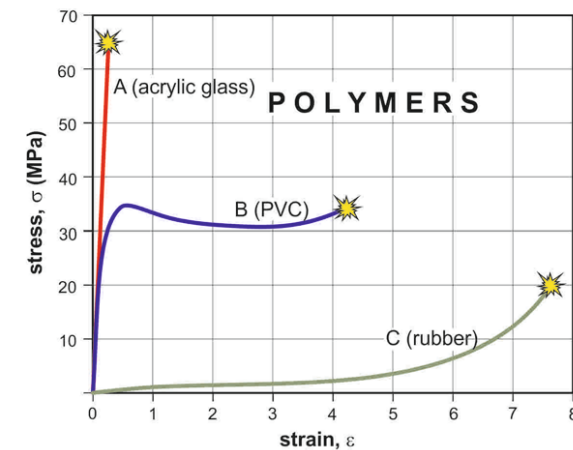
No plastic range!

Ceramics are brittle, they rupture after a short extension....



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Typical stress-strain curves Polymers



Polymers show various behaviours

Acrylic glass
No plastic range - brittle

PVC
Large plastic range
Plastic polymer

Rubber
Elastic polymer
 ϵ_{\max} up to 7-8 (~700-800%)

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Additional factors determining the stress-strain diagram

The main determinant is the material structure (e.g. amorphous or crystalline)

1. Type of stress. (For example the behavior of ceramics is different in the case of tension or elongation.)
2. The shape, size commonly the geometry of the object. (It is more difficult to bend a thicker rod than a thin.)
3. The run-time of the stress. (For example the glass is broken into pieces using a hammer but a bullet launched by a gun produces a small hole only.)
Mechanical properties – introduction
4. Temperature. (Some metals are very plastic and tough at room temperature but become more brittle cooling them.)
These conditions must be presented in the stress-strain diagram.

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Hooke's law of elasticity

Strain is proportional to stress!

$$\sigma \sim \varepsilon$$

$$\sigma = E\varepsilon$$

$$\frac{F}{A_0} = E \frac{\Delta l}{l_0}$$

$$F = \frac{EA_0}{l_0} \Delta l$$

$$F = D\Delta l$$

Proportionality coefficients:

Young's modulus

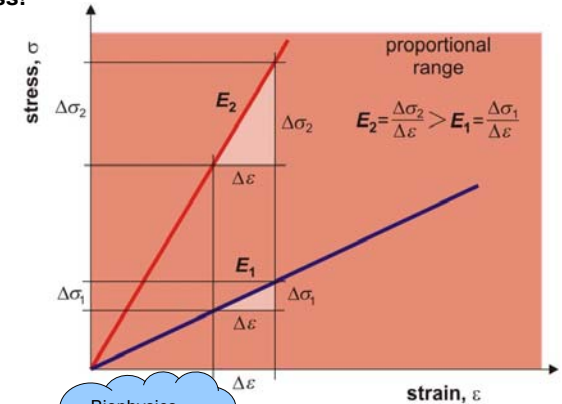
(material stiffness)

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{A_0} \frac{l_0}{\Delta l} \quad E = \left[\frac{N}{m^2} = Pa \right]$$

Body stiffness

$$D = \frac{F}{\Delta l} = \frac{EA_0}{l_0} \quad D = \left[\frac{N}{m} \right]$$

Biophysics uses k for spring constant!



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Quantification of stiffness

Young's modulus of various materials

$$GPa = 10^9 N/m^2$$

Spring constant

Easy to measure
Depends on size and shape of object

Young's modulus

Hard to measure (we need to know the object's dimensions)
Shape independent

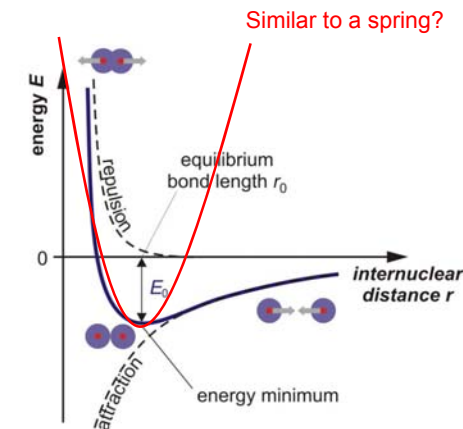
1/E – reciprocal of E is used to express elasticity!

material	E (GPa)
dentine	≈ 15
enamel	≈ 100
silicon rubber	≈ 0.0003
steel	200-230
amalgam	50-60
glass	60-90
porcelain	60-110
gold	79
gold alloys	75-110
Pd-Ag alloys	100-120
titanium	110
titanium alloys	105-120
Co-Cr alloys	120-220
Ni-Cr alloys	140-190

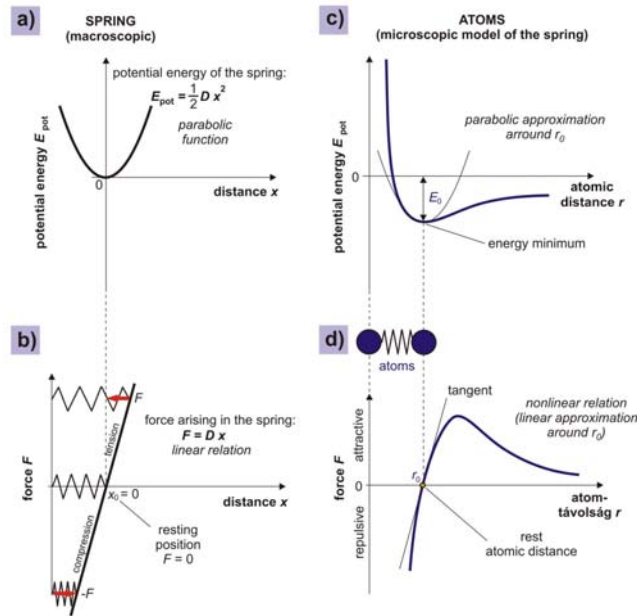
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Repetition

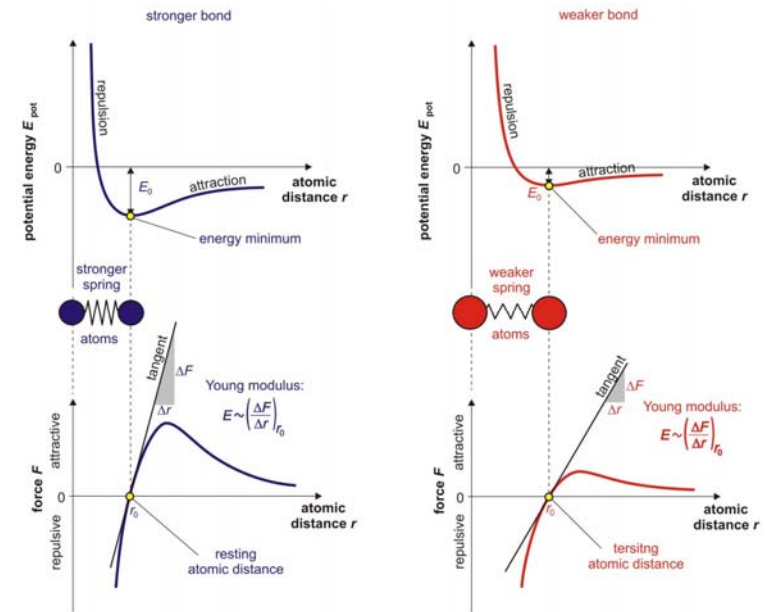
Potential energy of chemical bonds



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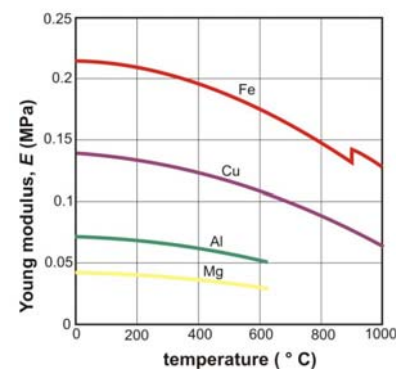
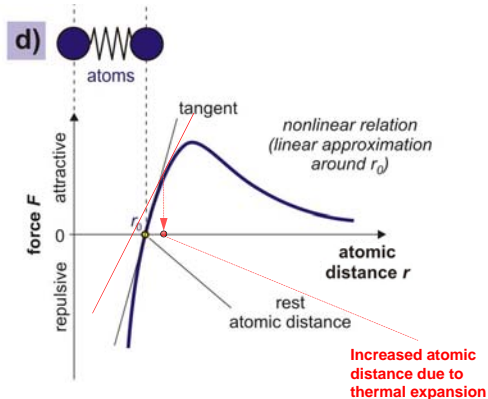
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Young's modulus

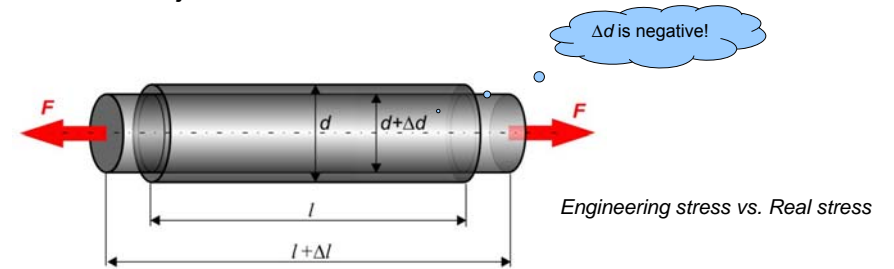
Young's modulus decreases with temperature increasing



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Changes in the cross-sectional dimension

Stretched objects become narrower or... ?



$$\frac{\Delta d}{d} = -\mu \frac{\Delta l}{l} \quad \mu: \text{Poisson's ratio (no dimension)} \\ \mu_{\text{max}} = 0.5 \text{ (usually between 0.3-0.4)}$$

Auxetic materials have negative Poisson's number: They become thicker when stretched. (bullet proof vests, compression bandages)

<http://www.youtube.com/watch?v=PLDbSWSm5i8>

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Deformations in other directions

Compression

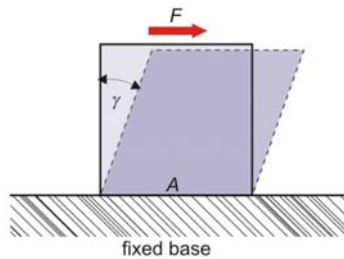
Same as stretch, but signs of stress and strain are opposite, Hooke's law is valid.

Shear

Force is parallel to the surface,

Shear stress: $\sigma_{\text{shear}} = \frac{F_{\text{shear}}}{A}$

Strain is characterized by the γ angle



$$\sigma_{\text{shear}} = G\gamma \quad G: \text{Shear modulus}$$

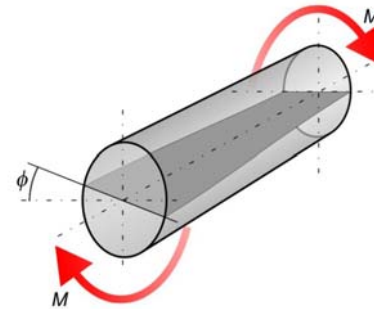
$$G = \frac{E}{2(1 + \mu)} \quad \text{Depends on Young's modulus and Poisson's ratio!}$$

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Deformations in other directions

Torsion

Can be derived from shear. Torque (M) is applied on the object. Angle of torsion (ϕ) is measured in radians!



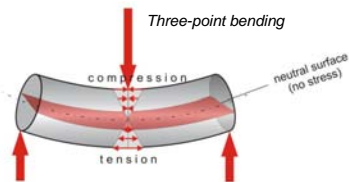
$$M = G \frac{r^4 \pi}{2l} \phi$$

G: Shear modulus
(contains E and μ)

r: radius
 ϕ : angle of torsion
l: length

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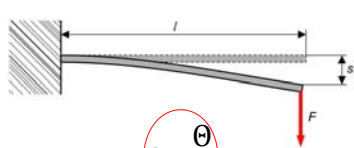
Deformations in other directions



Bending

Compression on the side of force applied, tension on the opposite side of object. Neutral surface at the middle – does not deform

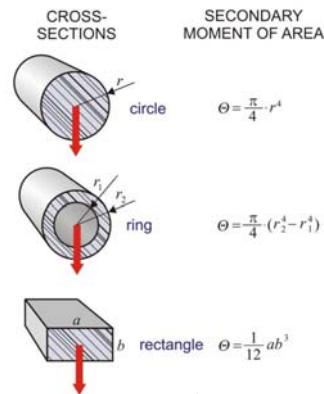
Cantilever deflection



$$F = 3E \frac{\theta}{l^3} s$$

G: Force
E: Young's modulus
 θ : secondary moment of area
s: deflection
l: length

Body stiffness
in case of
deflection



In case of bending the force required depends stronger on the shape!

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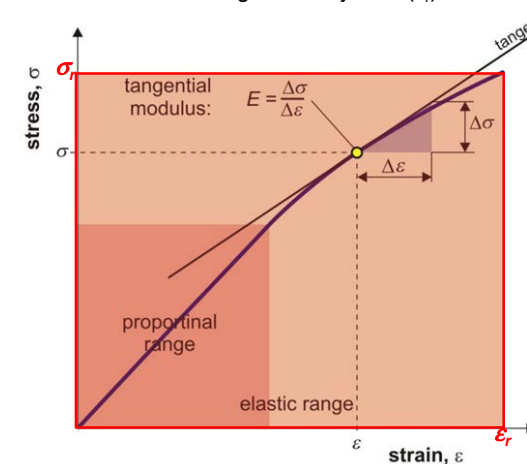
Elasticity beyond the proportional range

Elastic limit

End of the elastic range on the y-axis (σ_r) – maximum reversible strain

Elastic strain recovery

End of the elastic range on the x-axis (ϵ_r) – maximum reversible stress



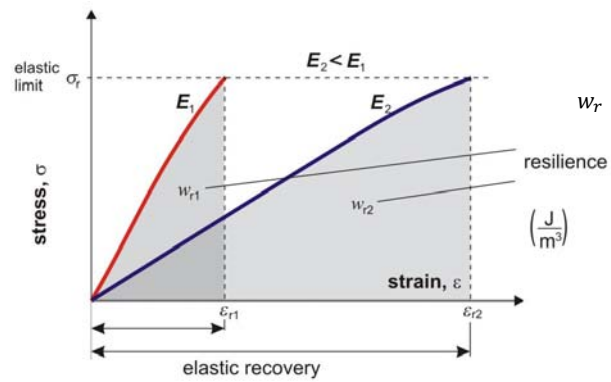
Tangential Young's modulus

$$E = \frac{\Delta \sigma}{\Delta \epsilon}$$

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Resilience

Work done on the material per unit volume till the elastic limit.



$$w_r \approx \frac{1}{2} \sigma_r \epsilon_r = \frac{1}{2} E \sigma_r^2 = \frac{1}{2E} \epsilon_r^2$$

$$\sigma_r = E \epsilon_r$$

$$w_r = \left[\frac{N}{m^2} = \frac{Nm}{m^3} = \frac{J}{m^3} \right]$$

Problems: 4.1, 4.3, 4.4, 4.5, 4.6, 4.9, 4.11, 4.16, 4.17, 4.18, 4.21, 4.22, 4.23, 4.24.