Two or more variables (one group)

Correlation and regression

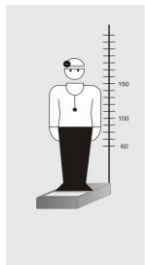
The relationship between two variables.

Method for estimating the relationship between variables.

Correlation of two variables

Example:
Is there any relationship between the height and weigth?

Experiment:



Data pairs:

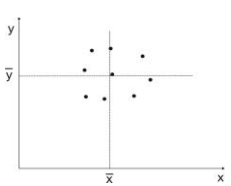
No.	Height (cm)	Weight (kg)
1	150	61
2	170	70
3	166	75
4	174	70
5	180	72
6	155	50
7	172	65
8	161	59
9	177	81

Graphic representation

E.g.: height is the **x** and weigth is the **y**.

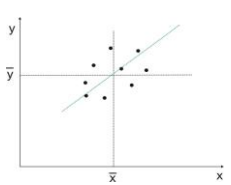
Possible situations:

A



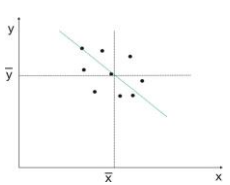
no any tendency

B



Positive tendency

C



Negative tendency

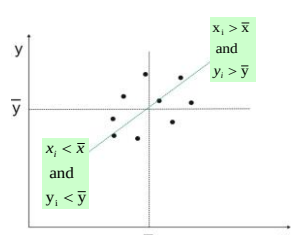
Covariance

$$Q_{xy} = \sum_i [(x_i - \bar{x}) \cdot (y_i - \bar{y})]$$

$$\mathrm{cov}(x, y) = \frac{Q_{xy}}{n-1}$$

(n: no. of elements)

Positive tendency:

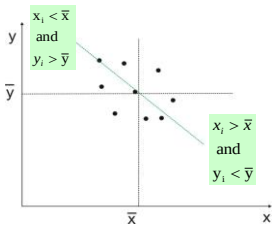


Frequently:

if $x_i < \bar{x}$ then $y_i < \bar{y}$
or $x_i > \bar{x}$ then $y_i > \bar{y}$

Consequence: $Q_{xy} > 0$.

Negative tendency:



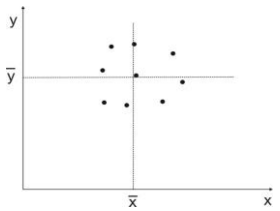
Frequently:

if $x_i < \bar{x}$ then $y_i > \bar{y}$

or $x_i > \bar{x}$ then $y_i < \bar{y}$

Consequence: $Q_{xy} < 0$.

No tendency:



The y values are independent from the x-values!

Consequence: $Q_{xy} = 0$.
(if n=∞)

Pearson's correlation coefficient

$$r = \frac{\mathrm{cov}(x,y)}{s_x \cdot s_y} = \frac{Q_{xy}}{\sqrt{Q_x \cdot Q_y}}$$

Possible range for r:

$$-1 \le r \le 1$$

Covariance divided by the squareroot of the product of the standard deviation of the two variables (= standardized covariance) .
The measure of the relationship.

In the population:

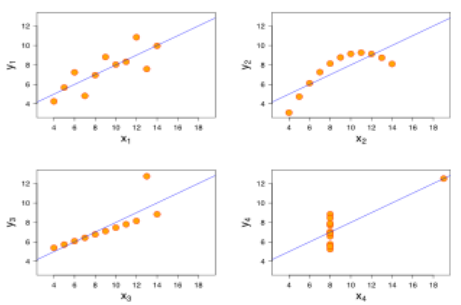
$r = 0$ no correlation,

$r \neq 0$ correlation (strength is proportional to the actual value of *r*.)

Dependency on distribution

r-value are equal to each other in these figures!

Pearson's r value is sensitive for the skewness of the distribution and for the extreme values. Characterizes well the relationship in the case of normal distribution.



http://en.wikipedia.org/wiki/Anscombe%27s_quartet

Coefficient of determination

r^2

The coefficient of the determination tells us how strong is the relationship. Expresses how much percent of the variability of the y values may be accounted by the variability of the independent variable or variables.

Correlation *t*-test

Calculated ***r*** is the estimation of the ***r*** in the population. This fluctuates around the theoretical value.
(e.g. r_{calc} = 0.1 ?)

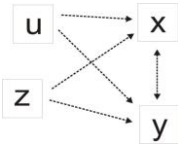
H_0: r = 0! ⟶ $t = r\sqrt{\frac{n-2}{1-r^2}}$ ⟶ d.f.: n - 2

Decision: based on t-value. Look previous cases!

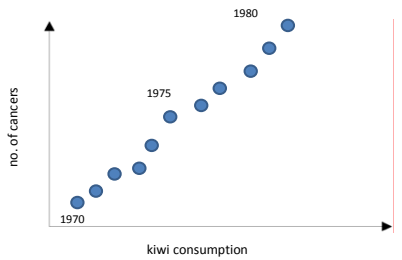
Condition: at least one of the variable has normal distribution.

Interpretation of the correlation

Not necessary being direct causality. (But may be!)
In the background there may be quantities, effects that influence both measured variables.



Example:
There is positive correlation, but we cann't suppose causality.



Non-normal distribution or ordinal data

Example: blood pressure measurements.
Relationship between the two methods.

The distribution is skewed, non-normal.
Pearson's r is false.

Spearman's rank-correlation

Example: diastolic pressure.
(only a few cases!)

case	Cuff	rank	finger	rank
1	80	4.5	58	1
2	65	2	79	5
3	70	3	66	2
4	80	4.5	93	6
5	60	1	75	4
6	82	6	71	3
…				

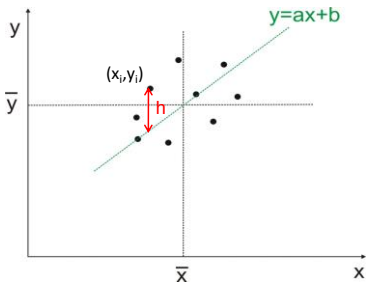
$$r_s = \frac{\sum(R_x - \bar{R}_x)(R_y - \bar{R}_y)}{\sqrt{\sum(R_x - \bar{R}_x)^2 \cdot \sum(R_y - \bar{R}_y)^2}}$$

R_x: rank of the x variable
R_y: rank of the y variable.

The test and the decision are the same as in the case of Pearson's r.

Linear regression

If the variables have normal distribution, the relationship is linear, and we can describe with straightline.



The regression model:
(to predict the y values)

$$y_i = ax_i + b + h_i$$

In regression analysis :

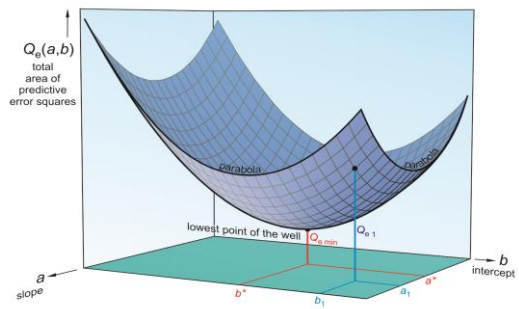
$$y_i = b_0 + bx_i + h_i$$

y: dependent variable
x: independent (explanatory) variable
$\mathbf{h_i}$: error term = $y_i - (ax_i+b)$.
(the difference between the actual value and the predicted value.)

Least-squares method

$$Q_e = \sum_i h_i^2 = \sum_i (y_i - (ax_i + b))^2$$

The $\boldsymbol{x_i}$ and $\boldsymbol{y_i}$ are measured values.
Unknown values are the ***a*** and ***b***!



Which is the best straightline?

Q_e has minimum! ⟶

$$a^* = \frac{Q_{xy}}{Q_{xx}} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$b^* = \bar{y} - a^*\bar{x}$$

Prediction of insulin sensitivity by BMI.

r^2: coefficient of determination.
How much part of the variance of y is explained by x.

indep.	regr. coeff.	st. error	t	p	decision
BMI	-0.077	0.018	-4.25	0.0011	significant
r^2	0.6				

Multilinear regression

Two or more independent variables.

The regression model:
(to predict the y values)

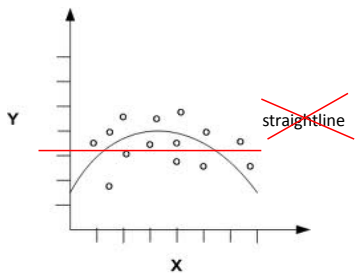
$$y_i = b_0 + \sum_j b_j x_{j,i} + h_i$$

Predict the insulin sensitivity (y)!
Conclusion: only the BMI influences the sensitivity. About the 64% of the variation of the sensitivity explained by the BMI.

indep.	regr. coeff.	st. error	t	p	decision
Age	-0.0045	0.0041	-1.09	0.3	not sign.
BMI	-0.068	0.02	-.344	0.0055	significant
r^2	0.639				

Non-linear regression

On the base of the model (e.g.):



Polinomial: $y = b_0 + \sum_i b_i x^i + h$

Exponential: $y = hab^x$

Power: $y = hax^b$

Chi-square test
Analyzing frequency data

Example: headache

Experiment

1. group: patients taking the ***medicine***

headache (*a*)

no headache (*b*)

2. group: patients taking the ***placebo***

headache (c)

no headache (d)

(a,b,c,d are frequency data)

Contingency table

	headache	no headache	Total
1. group	a	b	a+b
2. group	c	d	c+d
total	a+c	b+d	n

So-called 2 x 2 table.

Nullhypothesis

If the medicine is similar to the placebo, we expect:

$$\frac{a}{b}=\frac{c}{d} \longrightarrow a\times d = b\times c$$

Nullhypothesis: the medicine is similar to the placebo.

Chi-Square test for independence.

Independent case

Remember: P(AB) = P(A) x P(B) if A and B are independent from each other.
(P(AB), P(A) and P(B) are estimated by relative frequencies.)

$$\frac{a}{n}\approx\frac{a+b}{n}\times\frac{a+c}{n}$$

a/n ~ (P(AB))
(a+b)/n ~ (P(A))
(a+c)/n ~ (P(B))

Observed proportion: a/n

Expected proportion: $\frac{a+b}{n}\times\frac{a+c}{n}$

transformation

$$\chi^2=\sum\frac{(O-E)^2}{E}$$

χ^2-distribution

Shortcut formula for 2 x2 tables:

$$\chi^2=\frac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

Nullhypothesis: χ^2-value is 0, the difference due to the sampling error only.

χ^2-distribution describes the random deviations of the χ^2-value.

Decision

Same, then in the case of *t*-distribution. We use χ^2-distribution.

Expected value is 0, if the null hypothesis is true.

$p \le \alpha$ - reject the null hypothesis else accept.

degree of freedom: in this special case = 1.

In general:

d.f.=(r-1)(c-1), where r – no. of rows
c – no. of columns

Small expected frequencies

May not be used if:
1. An expected frequency is 2 or less.
2. More than 20% of the expected frequencies are less than 5.

Fisher's exact test may be used.
Calculates the exact probability for the given table.

$$P = \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{a!b!c!d!n!}$$

Remember!
n! = multiplying the integers from 1 to n.

Decision is based on the P.

The Chi-Square test
Goodness-of-Fit test

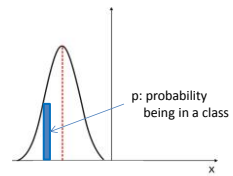
Example: testing normality of the larger diameter of the frog red blood cells.

Observed frequencies:

size (µm)

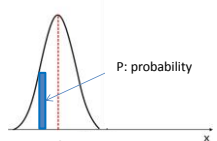
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	n
4	10	9	20	26	27	37	42	48	53	45	39	35	17	18	10	7	5	450

Nullhypothesis (H_0):
Data has normal distribution. Calculate the average and the sd from the sample!
Calculate expected frequency from the normal distribution!
in a class = np (see figure)



Chi-Square test

avg = 24.57 µm; sd = 3.62 µm



n_i ~ 450 x P

µ = 24.57 µm

Expected frequencies:

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
2.8	5.1	8.9	14.2	21	29	37	44	48	49	46.4	41	33	25	18	11	6.9	7.2

Degree of freedom = m – b – 1
m: no. of classes. (in example = 18)
b: no. of parameters (in example = 2)

Calculation:
p = 0.96 → We accept the nullhypothesis.

Chi-Square test
Test for homogenity

Example: The frequency of wearing glasses is the same in the groups of girls and boys or not?

H_0: There is no difference.
(independent!)

P(**W**ith) ~ 76/200; P(**B**oys) ~ 97/200
Independent case:
P(W and B) = P(W) x P(B)
expected freq. ~ n x (PW and B)
= 200 * 76/200 * 97/200 ~ 36.9

Observed frequencies

	with	without	
boys	48	49	97
girls	28	75	103
	76	124	200

Expected frequencies

	with	without	
boys	36.9	60.1	97
girls	39.1	63.9	103
	76	124	200

Calculation

$$\chi^2 = \frac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)} = \frac{200\cdot(48\cdot 75-49\cdot 28)^2}{76\cdot 124\cdot 97\cdot 103}$$

$\chi^2 \approx 10.5$ d.f. = 1 $p \approx 0.001$

Decision:
We reject the nullhypothesis. There are signifcant difference between boys and girls.

Conditions for tests

test	condition
One-sample t-test	One group, one variable, normal distribution
Two-sample t-test	Two independent groups, one variable, normal distribution, the standard deviations may be the same in the groups
ANOVA	3 or more independent groups, one variable, normal distribution
Sign test	One group, numerical or ordinal quantity
Wilcoxon's signed rank-test	One group, numerical or ordinal quantity
Mann-Whitney U-test	Two independent groups, numerical or ordinal quantity
Kruskall-Wallis test	3 or more groups, numerical quantity
Pearson's correlation test	One group, two variables, normal distribution
Spearman's correlation test	One group, two variables, numerical or ordinal quantity
Chi-Square test (independency)	Two or more groups, frequency data
Chi-Square test (homogenity)	Two or more groups, frequency data
Chi-Square test (fit)	One group, known distribution, frequency data