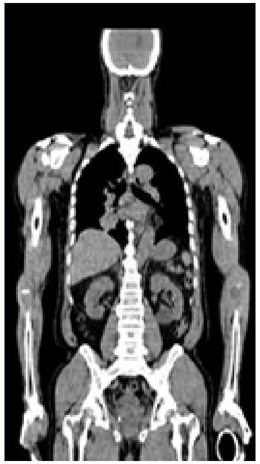
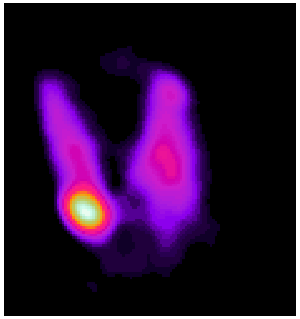
# The atomic nucleus.
# Radioactivity. Nuclear radiations

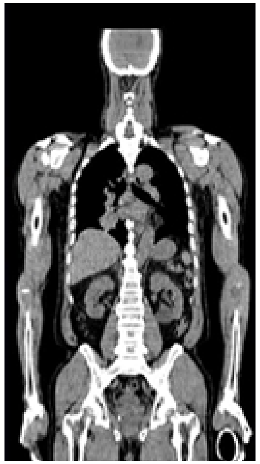


László Smeller



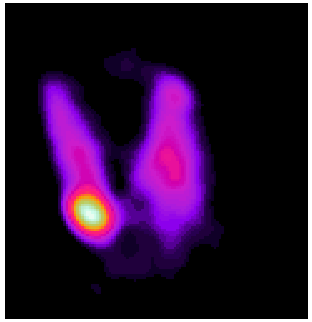
# Why?

Medical application of nuclear radiation:

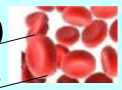
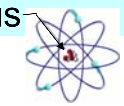


Nuclear imaging

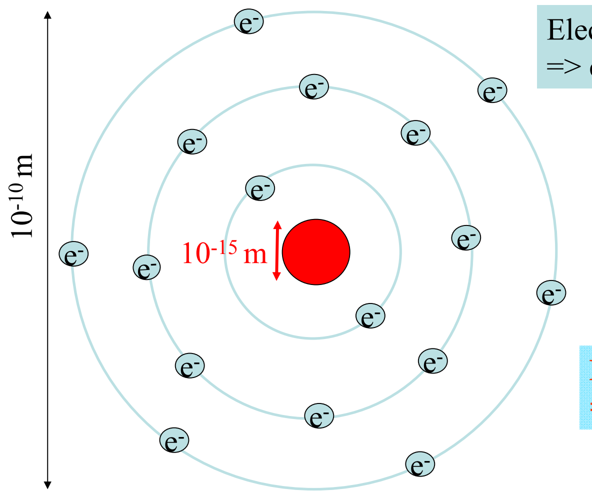
Radiotherapy



# Length scale of the nature

| m | | |
|---|---|---|
| $10^{0}$ | meter | men  |
| $10^{-3}$ | millimeter | size of letters you can read |
| $10^{-6}$ | micrometer | size of a cell (e.g. erytrocyte)  ∅ 7μm |
| $10^{-9}$ | nanometer | protein  |
| $10^{-10}$ | – angstrom | diameter of an atom, bond length<br>H atom ∅ ≈ 1 angstrom (Å)  |
| $10^{-12}$ | picometer | wavelength of the X-ray  |
| $10^{-15}$ | femtometer | size of the nucleus  |

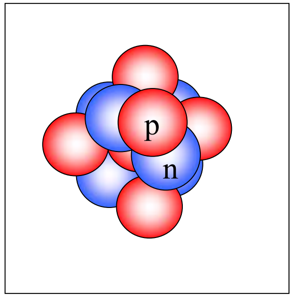
# The electrons and the nucleus




Electrons:
=> chemical properties

**Nucleus:**
**=> radioactivity**

# Structure of the nucleus




Elementary charge=$1{,}6 \cdot 10^{-19}$ C

| | charge | mass |
|---|---|---|
| **proton** | **+1 e** | **1 atomic mass unit** |
| **neutron** | **0** | **1 atomic mass unit** |

A (mass number) = number of protons + number of neutrons

Z (atomic number) = number of protons

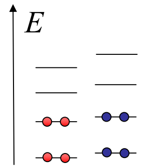
$^{99}_{43}\mathbf{Tc}$

99 nucleon, 43 proton and 56 neutron

# Stability of the nucleus

- Coulomb force: destabilization
  (electrostatic repulsion between the protons)
- Nuclear force: very strong
  attractive force
  acts only on short range (~fm)
  independent on the charge

- Quantized energy levels for the nucleus.
- Typical binding energy is in the MeV range
  eV=$1{,}6 \cdot 10^{-19}$ J




# Isotopes

Number of protons is the same

Number of neutrons is different

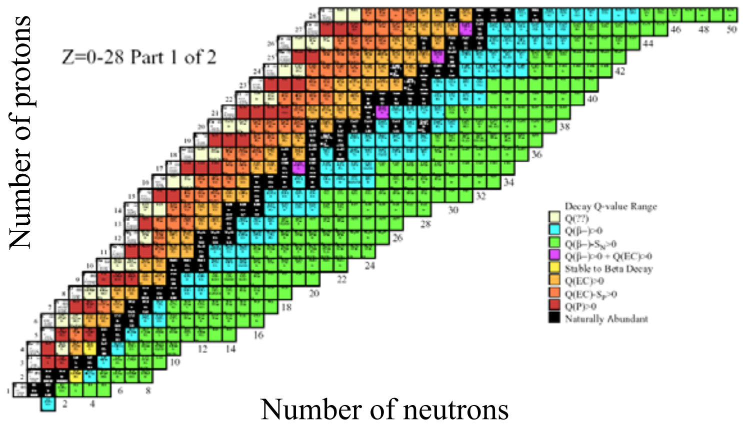
Variants of the same element

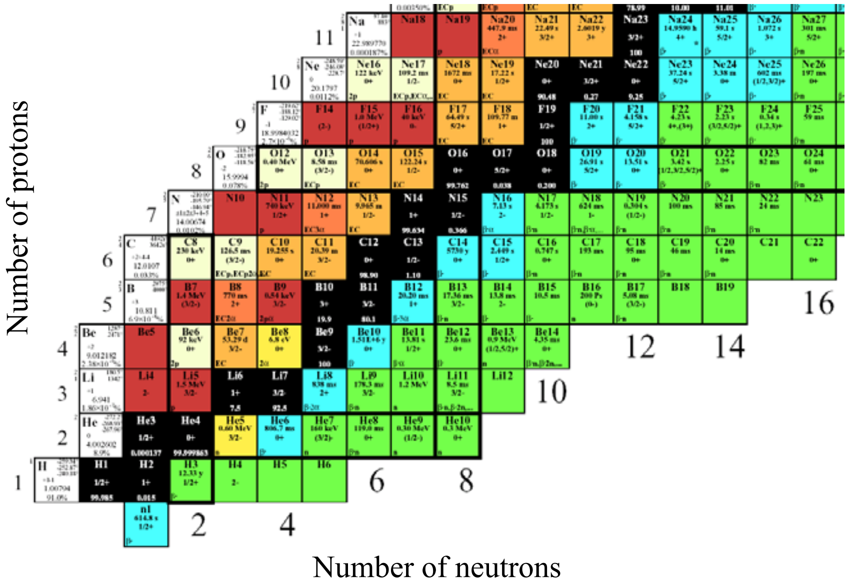
$\Rightarrow$ the chemical properties are identical.

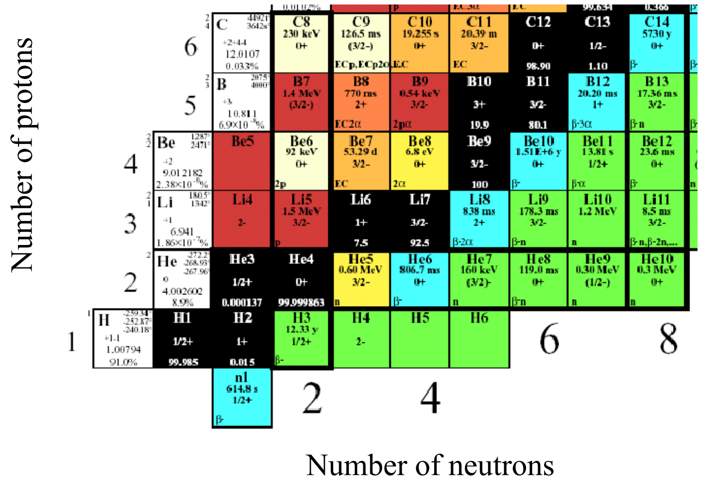
Pl: $^{18}_{9}\mathbf{F}$ unstable (radioactive) $^{19}_{9}\mathbf{F}$ stable $^{20}_{9}\mathbf{F}$ unstable (radioactive)

isotope <-> radioactive isotope

# Table of isotopes






## Radioactive decays and particles

| | | |
|---|---|---|
| α - decay | α - particle = | $^{4}_{2}$He nucleus |
| β - decay : $\beta^-$ | $\beta^-$ particle = | electron |
| $\beta^+$ | $\beta^+$ particle = | positron |
| Isomeric transition | | γ-ray |
| K-electron capture | | characteristic x-ray photon |

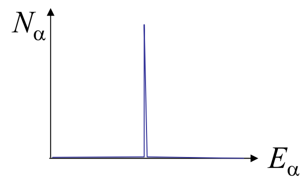
## α - decay

α - decay: an α particle ($^4$He nucleus) will be emitted typical for the heavy atoms

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$$

e.g. $$^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\alpha$$

Line spectrum
$E_\alpha \sim$ MeV



# $\beta^-$ - decay

neutron surplus

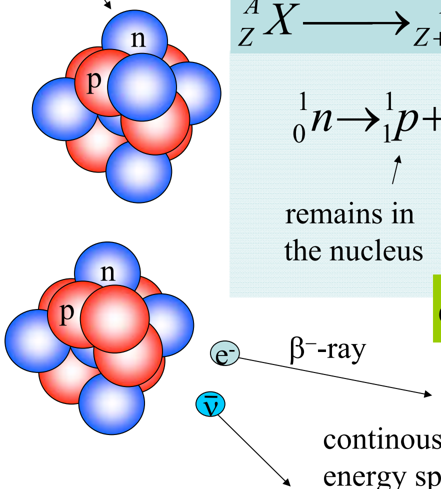
$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\nu}$$

$${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}\beta + \bar{\nu}$$

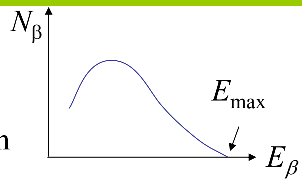
remains in the nucleus (${}^{1}_{1}p$); leave the nucleus (${}^{0}_{-1}\beta$, $\bar{\nu}$)

e.g.: ${}^{20}_{9}F$, ${}^{32}_{15}P$, ${}^{59}_{26}Fe$, ${}^{131}_{53}I$

e.g.: ${}^{32}_{15}\mathrm{P} \longrightarrow {}^{32}_{16}\mathrm{S} + {}^{0}_{-1}\beta + \bar{\nu}$




continous energy spectrum




$\beta^- = {}^{0}_{-1}\beta = e^-$

# $\beta^+$ - decay

proton surplus

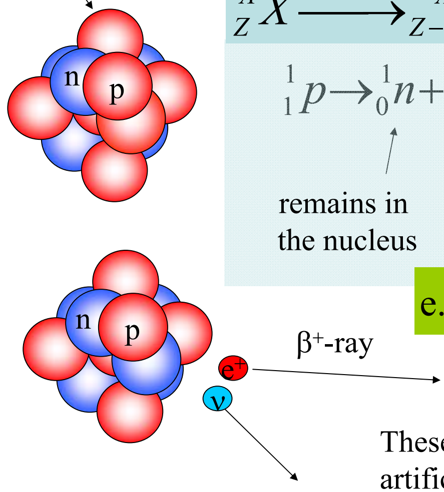
$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta + \nu$$

$${}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}\beta + \nu$$

remains in the nucleus (${}^{1}_{0}n$); leave the nucleus (${}^{0}_{+1}\beta$, $\nu$)

e.g.: ${}^{11}_{6}C$, ${}^{15}_{8}O$, ${}^{18}_{9}F$, ${}^{52}_{26}Fe$

e.g.: ${}^{30}_{15}\mathrm{P} \longrightarrow {}^{30}_{14}\mathrm{Si} + {}^{0}_{+1}\beta + \nu$



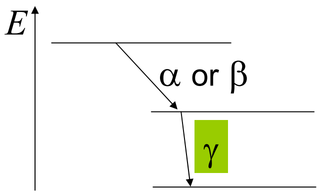

continous energy spectrum

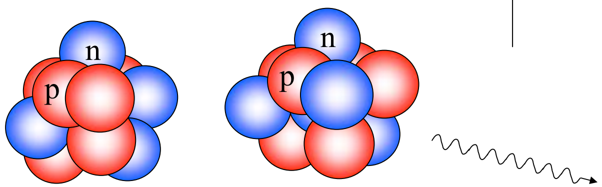
These isotopes must be produced artificially (e.g. in cyclotron)

# Prompt $\gamma$-radiation

**The daughter nucleus has an energetically unfavoured arrangement of nucleons.**

The surplus energy will be immediately (<ps) emitted in form of the $\gamma$ radiation




Atomic number, mass number are unchanged.

# Isomeric transition

If the excited state of the daughter nucleus is metastable, the $\gamma$-radiation will be emitted later.

The parent and daughter atoms can be separated**:** the daughter atom emits **only $\gamma$-radiation!**

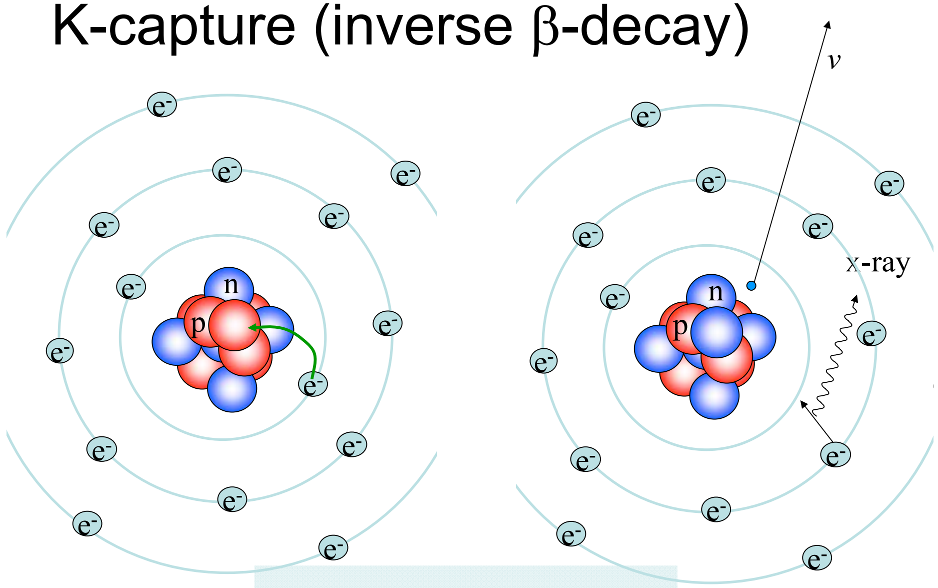
=> **Isotope diagnostics (nuclear imaging)**

E.g.: ${}^{99m}\mathrm{Tc}$

$${}^{99}_{42}Mo \xrightarrow{\beta^-} {}^{99m}_{43}Tc \xrightarrow{\gamma} {}^{99}_{43}Tc$$

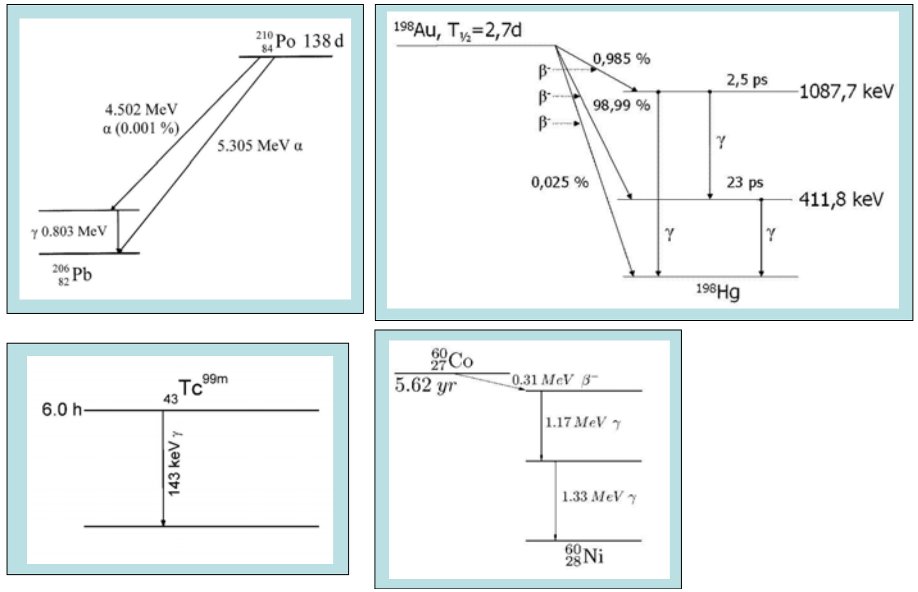
66 h     6 h

# K-capture (inverse β-decay)




$$ {}^{1}_{1}p + {}^{\;0}_{-1}\beta \rightarrow {}^{1}_{0}n + \nu $$

# Some examples of the decay paths




# Characteristics of radioactive decays in general

| | |
|---|---|
| activity | characterizes the source |
| half life time | characterizes the speed of the decay* |
| particle energy | characterizes the radiation* |

*depends on the type of the isotope

# Activity (Λ)

$$ \Lambda = \left|\frac{dN}{dt}\right| \quad \left( = \left|\frac{\Delta N}{\Delta t}\right| \right) $$

$N$ = Number of undecayed atoms
$t$ = time
$\Delta N$ = Number of decays during $\Delta t$ time

Activity= number of decays in a unit time

unit: becquerel Bq
1 Bq= 1 decay/sec

kBq, MBq, GBq, TBq

immeasurably small

**level of natural activity**

**in vivo diagn.**

**careful work with it!**

**activity applied in radiotherapy**

## Law of radioactive decay

$\Delta N = -\lambda N \Delta t$ N: Number of undecayed nuclei

$\frac{dN}{dt} = -\lambda N$ λ: decay constant (probability of the decay [1/s])

$1/\lambda = \tau$ average lifetime

Differential equation

solution: $N(t) = N_0 e^{-\lambda t}$ Exponential decrease

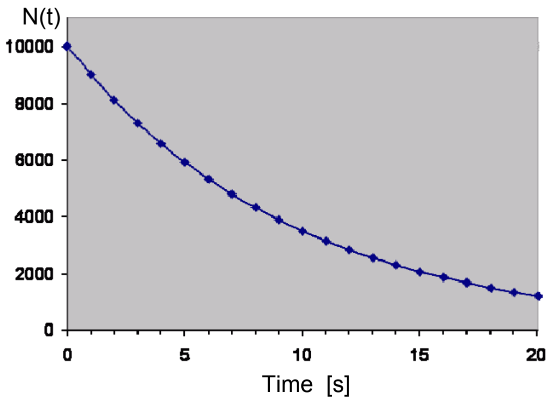
number of undecayed atoms at $t$=0

## An example

- At start: $N_0$=10000 (number of atoms
  $\lambda$=0,1 $^1/_s$ (decay probability)
- after 1 sec: 9000 (10000x0,1=1000 decayed)
- after 2 sec: 8100 (9000x0,1=900 decayed)
- after 3 sec: 7290 (8100x0,1=810 decayed)
- after 4 sec: 6561 (7290x0,1=729 decayed)
- ….

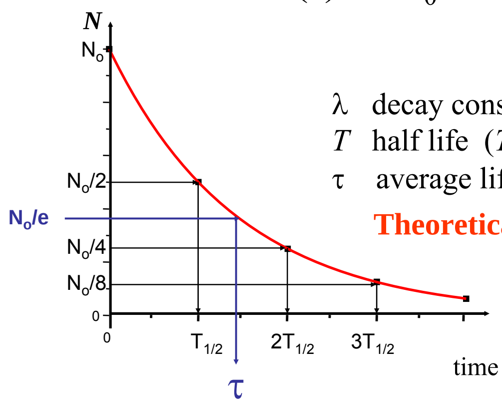
## An example

- At start: $N_0$=10000 (number of atoms
  $\lambda$=0,1 $^1/_s$ (decay probability)
- after 1 s: 9000
- after 2 s: 8100
- after 3 s: 7290
- after 4 s: 6561
- ….




## Law of radioactive decay

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}}$$




λ decay constant (probability of the decay)

$T$ half life ($T_{1/2}$)

τ average lifetime

**Theoretically never decreases to zero !**

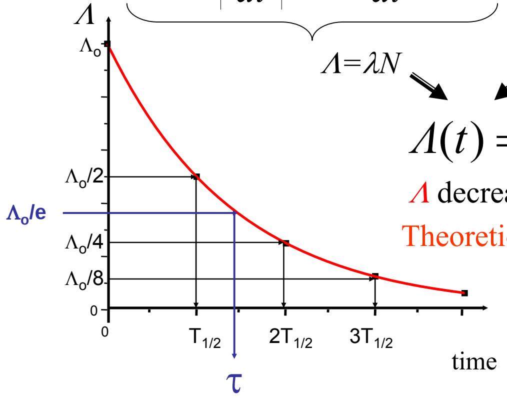
$$\lambda = \frac{\ln 2}{T} = \frac{0{,}693}{T}$$

## Decrease of the activity as a function of time

$$\Lambda = \left|\frac{dN}{dt}\right| \qquad \frac{dN}{dt} = -\lambda N \qquad N(t) = N_0 e^{-\lambda t}$$

$\Lambda = \lambda N$

$$\Lambda(t) = \Lambda_0 e^{-\lambda t} = \Lambda_0 2^{-\frac{t}{T}}$$



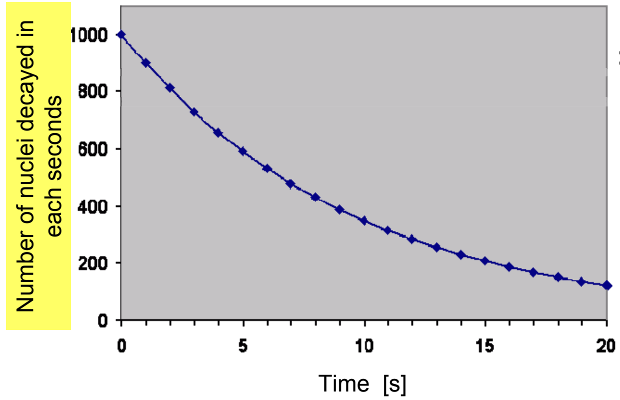

$\Lambda$ decreases on the same mode as N!

Theoretically never decreases to zero!

During about 10 $T$ the activity decreases to its 1/1000 (e.g GBq->MBq)

## An example

- At start: $N_0$=10000 (number of atoms)
  λ=0,1 $^1/_s$ (decay probability)




=1000 decayed)

=900 decayed)

=810 decayed)

=729 decayed)

## Few examples for half life

| | |
|---|---|
| $^{232}$Th | 1,4·10$^{10}$ y |
| $^{238}$U | 4,5 ·10$^{9}$ y |
| $^{40}$K | 1,3 ·10$^{9}$ y |
| $^{14}$C | 5736 y |
| $^{137}$Cs | 30 y |
| $^{3}$H | 12,3 y |

| | |
|---|---|
| $^{60}$Co | 5,3 y |
| $^{59}$Fe | 1,5 m |
| $^{56}$Cr | 1 m (28 d) |
| $^{131}$I | 8 d |
| $^{99m}$Tc | 6 h |
| $^{18}$F | 110 min |
| $^{11}$C | 20 min |
| $^{15}$O | 2 min |
| $^{222}$Th | 2,8 ms |

**Don't learn these numbers!**

## Typical energy levels in the microworld

| Excitation of the outer electrons | Electron transition between inner electrons | Transformation of the nucleus (decay) |
|---|---|---|
| eV (aJ) | keV (fJ) | MeV (pJ) |
| light | X-ray | Nuclear radiation α, β, γ |

# Absorption of the nuclear radiation

α, β⁻, β⁺ – particles with electric charge – **direct ionisation**

different types of interactions with matter!

γ (x-ray) – uncharged particles (photons) – **indirect ionisation**

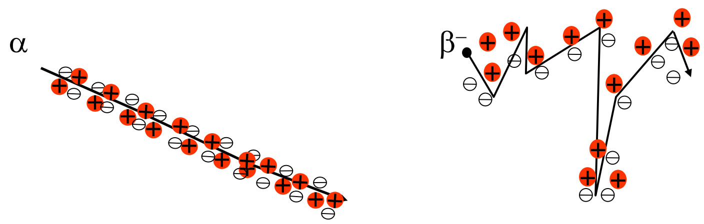
not a nuclear radiation, but its absorption is similar to that of the γ radiation

# Absorption of the charged particles

Ionizing during the path => continuous decrease of the particle energy
The energy after a given path length decreases to the thermal value

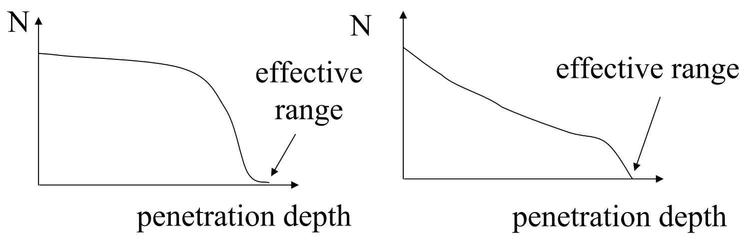
⇓

effective range



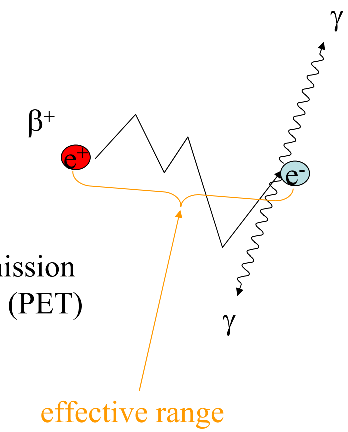

# Effective range

| | α-particle | β⁻-particle |
|---|---|---|
| in air | **few cm** | **~ m** |
| in tissue | **0,01-0,1 mm** | **~ cm** |




# β⁺-radiation

annihilation

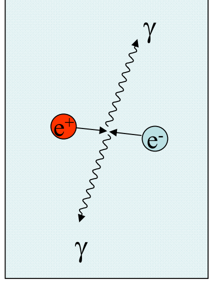


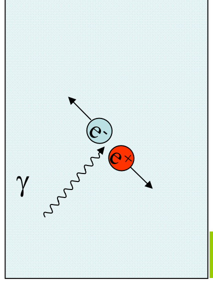

see: Positron Emission Tomography (PET)

# Electron and positron

- particle - antiparticle
- same mass,
- charge: same value, but differen[redacted]

annihilation and pair production



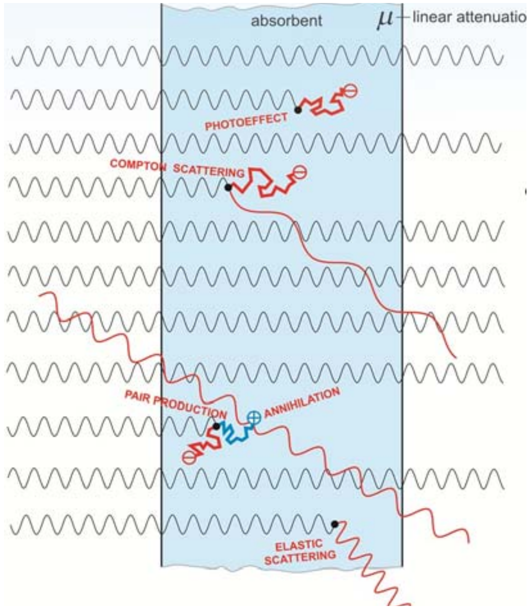

Einstein:

mass-energy ekvivalence

$$E=mc^2$$

$m_e c^2$=511 keV≈0,5 MeV

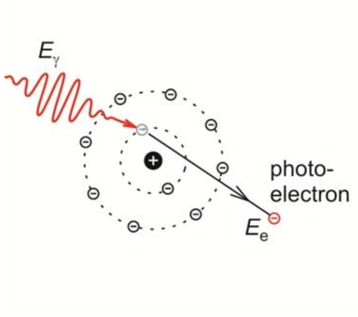
# Absorption of the γ-radiation (and x-ray)



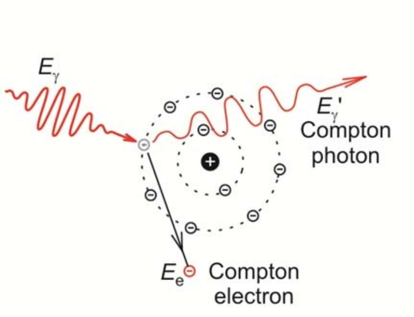

Absorption pricesses happen accidentally :

Photoeffect,
Compton-effect,
Pair production,
(elastic scattering)

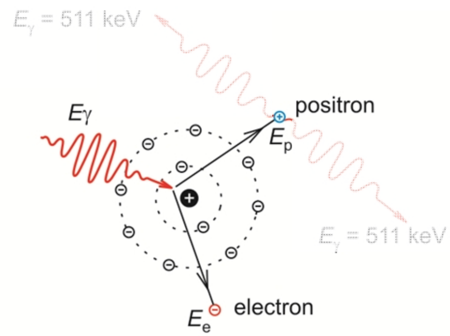
**Photoeffect**



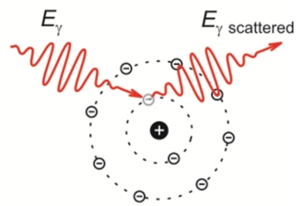

**Compton effect**
**Compton-scattering**



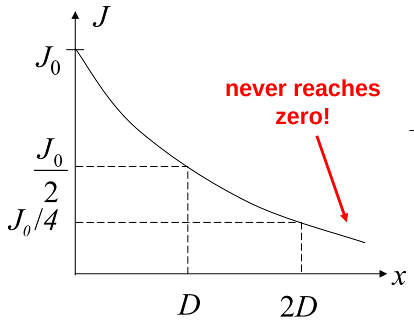

**Pair production**

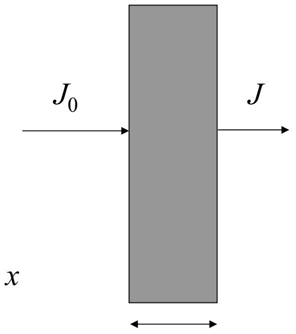



**elastic scattering**



# Attenuation
# of the γ–radiation and x-ray

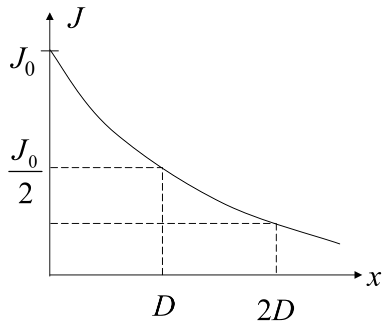



$$J = J_0 e^{-\mu x}$$

no limited effective range!

few rules of thumb: $x_{1/10}$=3,33 D $x_{1/1000}$=10 D




$$J = J_0 e^{-\mu x}$$

μ: (linear) attenuation coefficient

its units are: 1/m, 1/cm

$$\delta = \frac{1}{\mu}$$ „penetration depth”

Intensity decreases to the *e*-th part (c.a. 37%)

μ(material, number of absorbing centers, energy of the radiation)
=μ(material,ρ,$E_{photon}$) ~ ρ

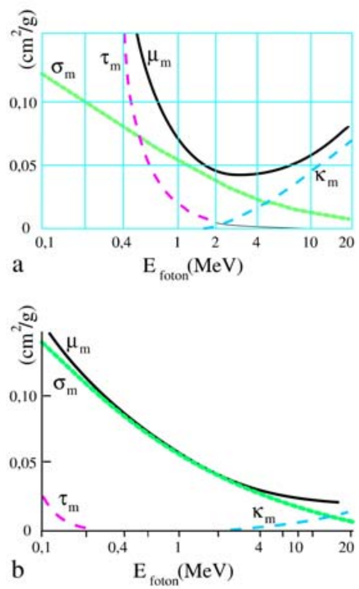
$$\mu_m = \frac{\mu}{\rho}$$ mass attenuation coefficient

mass attenuation coeff.

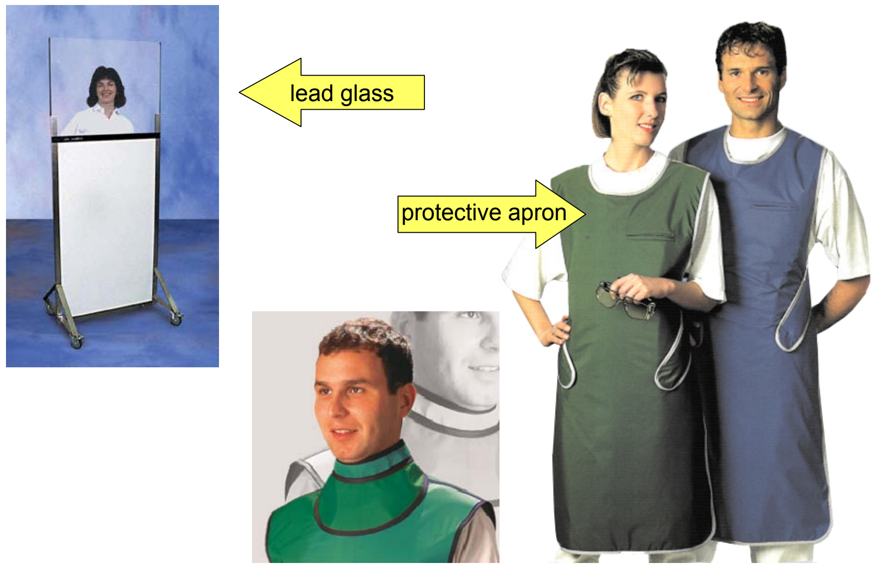
$$\mu_m = \frac{\mu}{\rho}$$

$$\mu_m = \tau_m + \sigma_m + \kappa_m$$

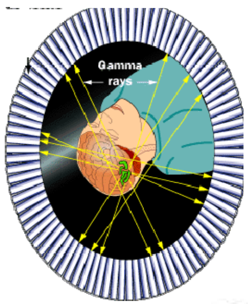
$$\tau_m = c\lambda^3 Z^3$$

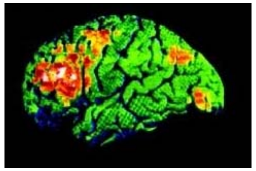



# Applications (attenuation)



# Applications: isotopes and nuclear radiation



# Thank you for your attention!