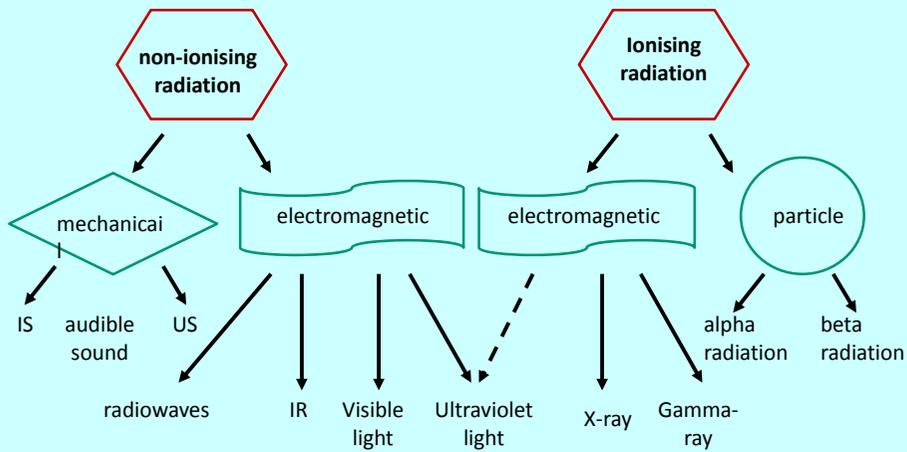


# Radiation



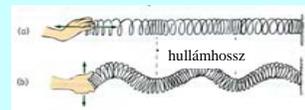
# Sound and

# ultrasound



# Physics of sound

## Longitudinal vs. Transverse wave



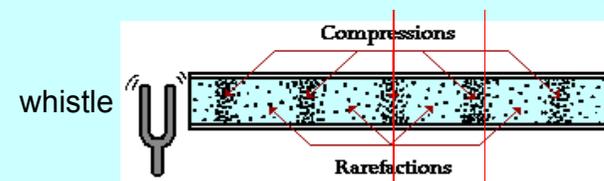
**longitudinal wave**  
(in the interior of liquids and gases only this type)



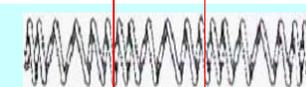
**transverse wave**  
can generated in solid materials and at liquid surfaces

# Physics of sound

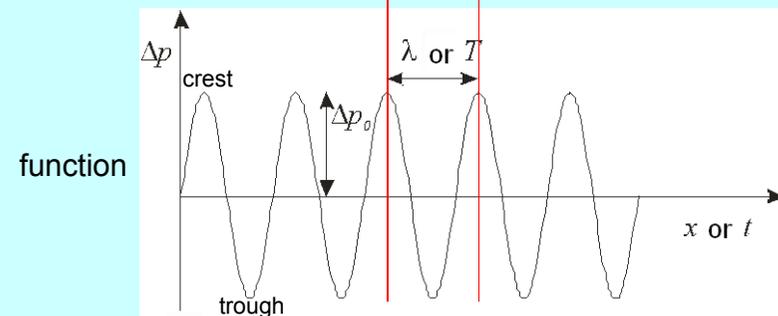
Sound: mechanical wave (model)

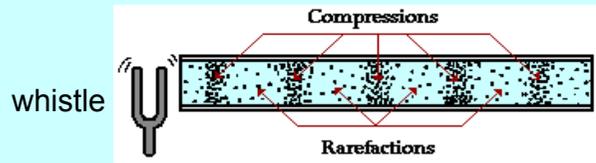


spring



spatial and temporal periodicity





hydrostatic pressure + pressure change, sound pressure

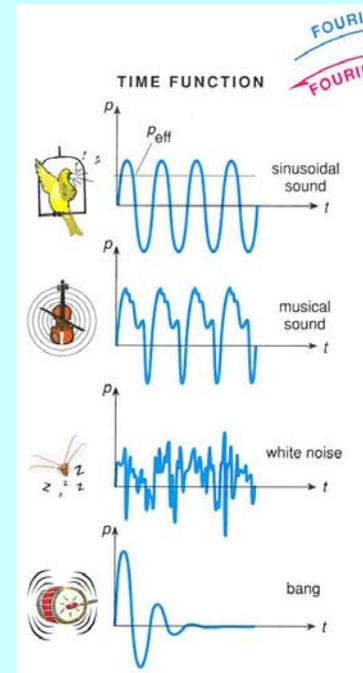
$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

pressure DC + AC amplitude phase

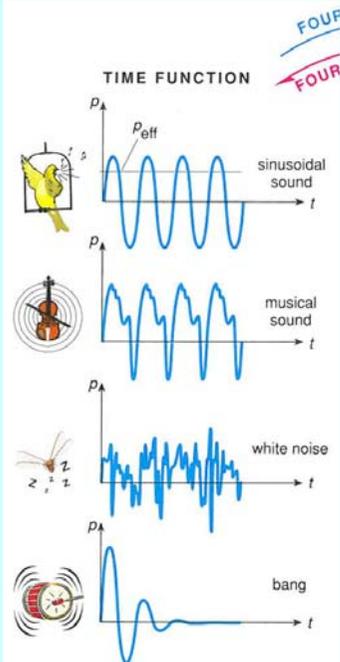
$$\Delta p(t, x) = \Delta p_{\text{max}} \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$



$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$



$$p(t) = p_1 \sin(\omega t) + p_2 \sin(2\omega t) + p_3 \sin(3\omega t) + \dots$$

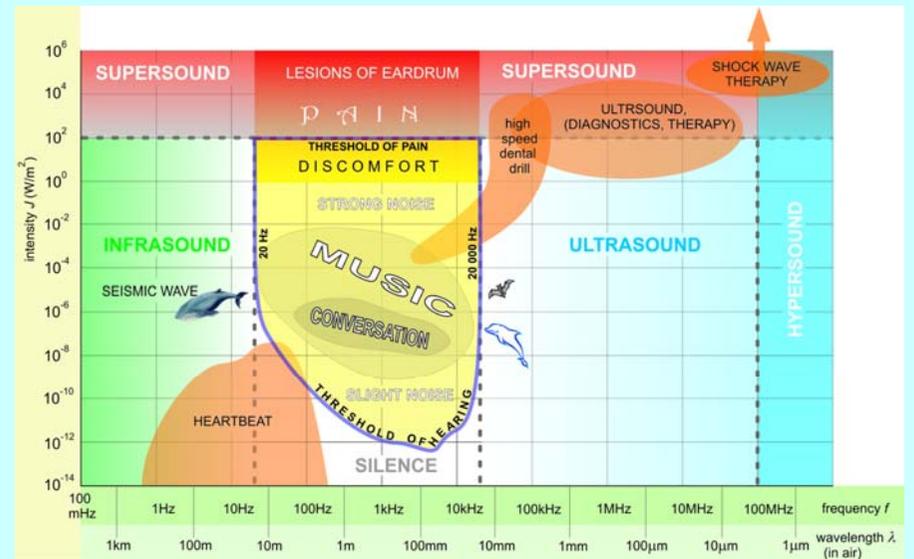


**pitch**  
frequency of the fundamental high

**timbre**  
relative strengths of overtones/harmonics (spectrum) tone colour

**Intensity\***  
from pressure amplitude loudness

### Frequency and intensity regions of sounds



**TIME FUNCTION**

**pitch**  
frequency of the fundamental      high

**timbre**  
relative strengths of overtones/harmonics (spectrum)      tone colour

**Intensity\***  
from pressure amplitude      loudness

$$J_{dB} = 10 \lg \frac{J}{J_0} \qquad J_0 = 10^{-12} \text{ W/m}^2$$

Textbook, Fig. IV.23.

$$J_{dB} = 10 \lg \frac{J}{J_0}$$

$J_0 = 10^{-12} \text{ W/m}^2$

Curves of similar loudness levels

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What is the intensity of a 300 Hz sound that a person, who has 25 dB hearing loss at this frequency can hear? (The average hearing threshold at this frequency is  $3 \cdot 10^{-11} \text{ W/m}^2$ )

$$J_{dB} = 10 \lg \frac{J}{J_0} \qquad 25 = 10 \lg \frac{J}{3 \cdot 10^{-11}}$$

$$10^{2.5} = \frac{J}{3 \cdot 10^{-11}}$$

$$J = 9,5 \cdot 10^{-9} [\text{Wm}^{-2}]$$

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### Propagation of sound/ultrasound

The role of elastic medium – *speed* of propagation

$$c = f\lambda$$

$$c = \frac{1}{\sqrt{\rho\kappa}}$$

$\rho$ : density of medium

$$\kappa = -\frac{1}{V} \left( \frac{\Delta V}{\Delta p} \right) [\text{Pa}^{-1}]$$

**compressibility**  
relative volume decrease over pressure

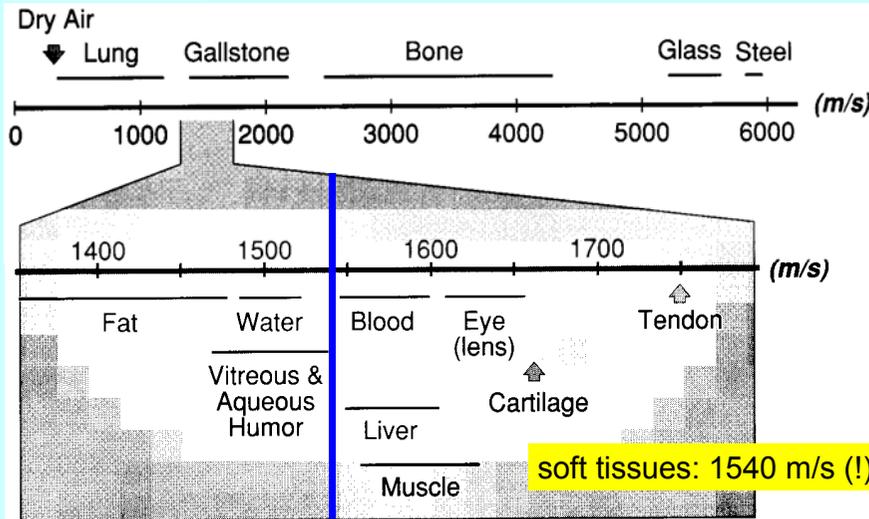
Speed of propagation is higher in solids than in liquids.

$\rho \uparrow \qquad \kappa \downarrow$

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## Speed of sound/US in various media



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## Propagation of sound/ultrasound

$$c = \frac{1}{\sqrt{\rho \kappa}}$$



$$\kappa = -\frac{\Delta V/V}{\Delta p} \quad [\text{Pa}^{-1}]$$

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

acoustic **impedance**  
(definition)

$$Z_{\text{el}} = \frac{U}{I}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance**  
(useful form)

$$[\text{kg}/\text{m}^2\text{s}]$$

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## Propagation of sound/ultrasound

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

$$\kappa = \frac{-\Delta V/V}{\Delta p}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

material	$\rho$ [kg/m <sup>3</sup> ]	$\kappa$ [1/GPa]	$c$ [m/s]	$Z$ [kg/(m <sup>2</sup> ·s)]
air	1,3	7650	331	0,00043 · 10 <sup>6</sup>
water 20°C	998	0,45	1492	1,49 · 10 <sup>6</sup>
aluminum	2700	0,009	6400	17,28 · 10 <sup>6</sup>
quartz	2650	0,011	5736	15,2 · 10 <sup>6</sup>

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### Problem:

A sound beam of 3 MHz frequency and 50 mW/cm<sup>2</sup> intensity propagates in blood.

What is the pressure? What is the maximal displacement and velocity of particles in this beam?

$$Z_{\text{vér}} = 1,66 \times 10^6 \text{ kg/m}^2\text{s}$$

### Solution:

Intensity:

$$J = \frac{p_{\max}^2}{2Z}$$

$$p = \sqrt{2JZ} = 40,74 \text{ kPa}$$

Velocity:

$$v = \frac{p}{Z} = \frac{40,74 \cdot 10^3}{1,66 \cdot 10^6} = 0,0245 \text{ m/s} = 24,5 \text{ mm/s}$$

Displacement:

$$A = \frac{v}{\omega} = \frac{24,5}{2 \cdot \Pi \cdot 3 \cdot 10^6} = 1,3 \cdot 10^{-6} \text{ mm} = 1,3 \text{ nm}$$

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## Intensity of US

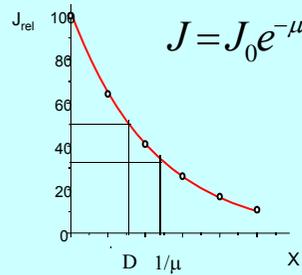
$$J = \frac{1}{Z} \Delta p_{eff}^2$$

intensity = energy/current density

$$P_{el} = \frac{1}{Z_{el}} U_{eff}^2$$

electric analogy

### Loss of energy during propagation (absorption)

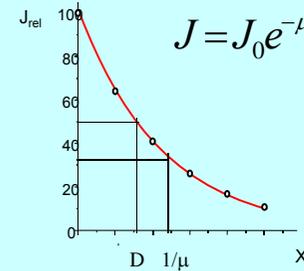


attenuation:  $\alpha = 10 \cdot \lg \frac{J_0}{J}$  dB

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

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### Loss of energy during propagation (absorption)



attenuation:  $\alpha = 10 \cdot \lg \frac{J_0}{J}$  dB

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

$\mu$  is proportional to frequency in the diagnostic range

At  $f = 1$  MHz

$D_{air} \sim 1$  cm

$D_{water} \sim 1$  m

Specific attenuation:  $\frac{\alpha}{f \cdot x}$

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$\mu$  is proportional to frequency in the diagnostic range

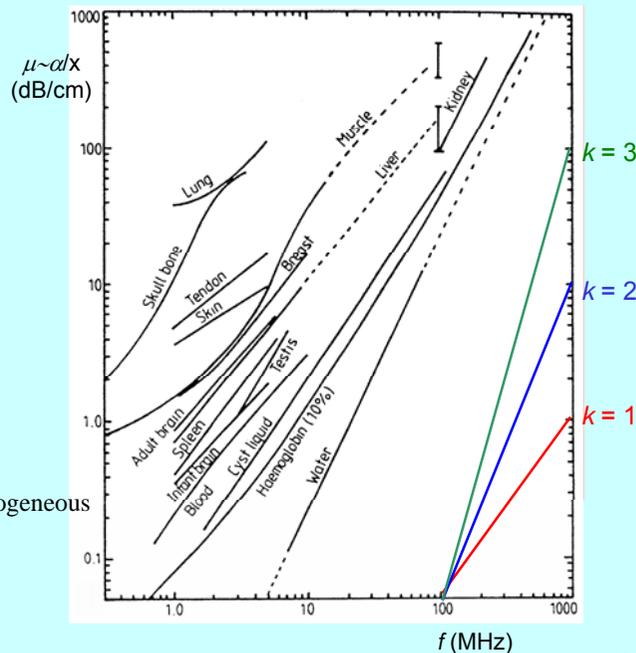
$$\mu \sim f^k, \quad k \sim 1(?)$$

$$\lg \mu \sim k \lg f$$

if the graph is a linear, the power function approximation is valid

specific attenuation for soft tissues (homogeneous tissue model):

$$\frac{\alpha}{f \cdot x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$



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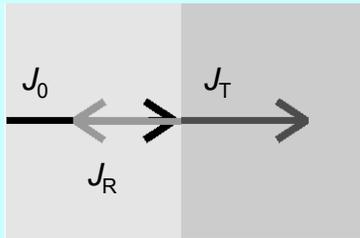
$$\frac{\alpha}{f \cdot x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$

tissue	Specific attenuation
liver	0,6 – 0,9
kidney	0,8 – 1,0
fat	1,0 – 2,0
blood	0,17 – 0,24
bones	16 – 23

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## Phenomena at the boundary of different media

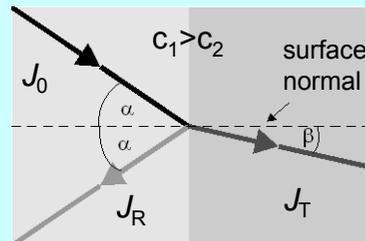
normal/perpendicular incidence



$$J_0 = J_R + J_T$$

reflection and transmission (penetration)

skew incidence

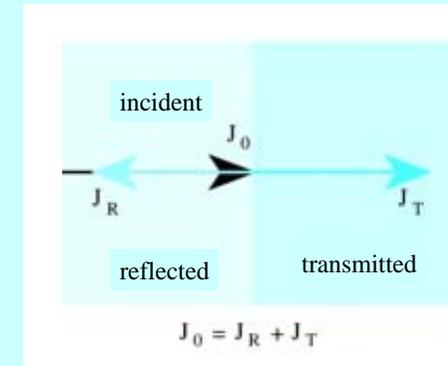


$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

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## Reflection of ultrasound



$$R = \frac{J_R}{J_0}$$

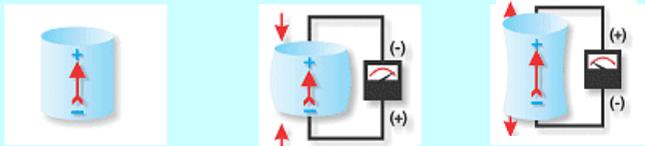
$$R = \left( \frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

If  $R \approx 1$   $\longrightarrow$  Total reflection

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## Detection/Generation of US

Piezoelectric effect



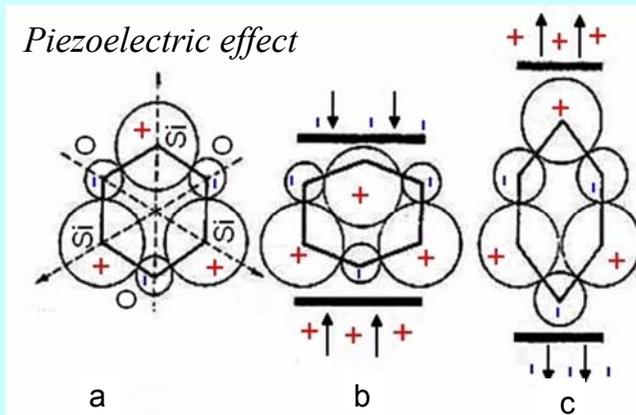
Pressure change

Mechanical deformation of crystal

**Electric potential difference**

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Piezoelectric effect



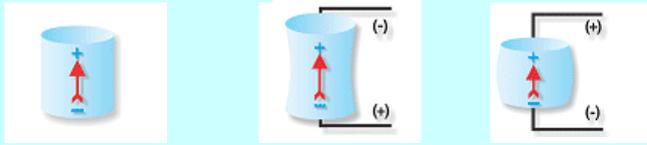
(a) Center of charge of positive and negative charges coincides.

(b) and (c) As a result of pressure, the charge centers are separated, i.e. a potential difference arises

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## Detection/Generation of US

### Inverse piezoelectric effect



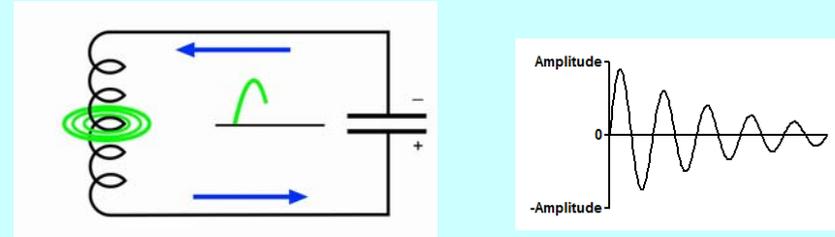
Periodic electric potential difference

The crystal is deformed when voltage is applied

**Mechanical vibration**

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## Oscillator circuit – LC circuit



$$f = \frac{1}{2\pi\sqrt{LC}}$$

L : self inductance [ $S \cdot \Omega^{-1}$ ]

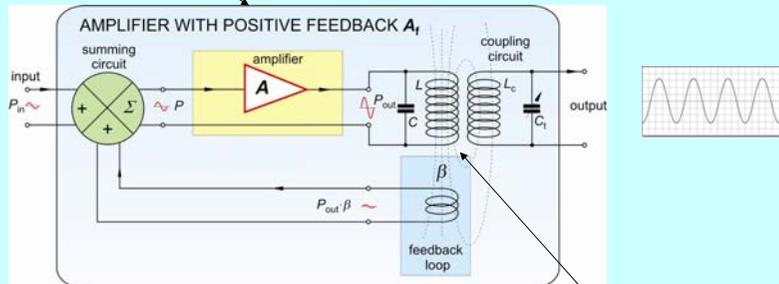
$$L \sim A N^2$$

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## Source of electric signal: sine wave oscillator

$$A_{\text{feedback}} = \frac{U_{\text{out}}}{U_{\text{in}}} = \frac{A}{1 - A \cdot \beta}$$

$A\beta=1$   
amplification = „infinity“  
no input signal, output signal: sine voltage

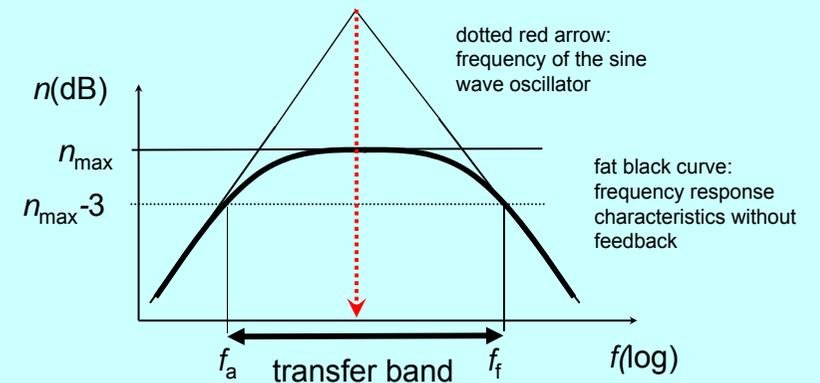


$$f = \frac{1}{2\pi\sqrt{LC}}$$

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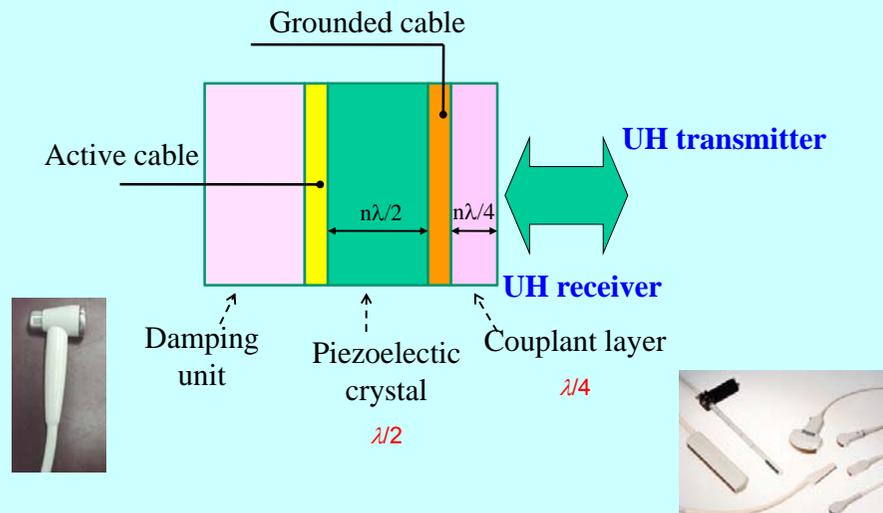
## Source of electric signal: sine wave oscillator

amplifier with positive feedback  $A_{U, \text{feedback}} = \frac{A_U}{1 - \beta A_U}$



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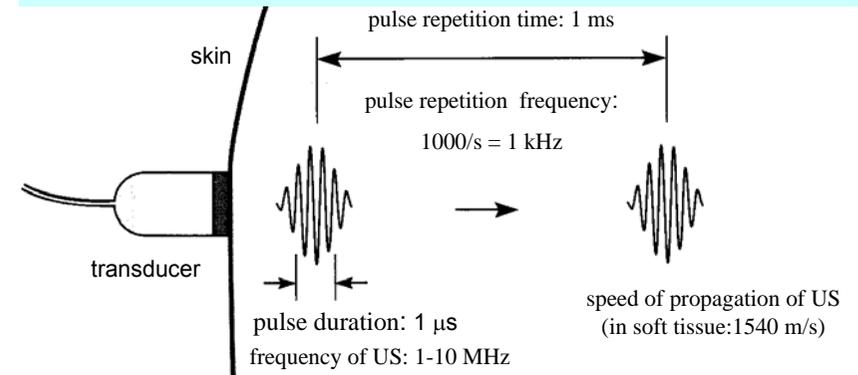
## Detection/Generation of US - Ultrasound transducer



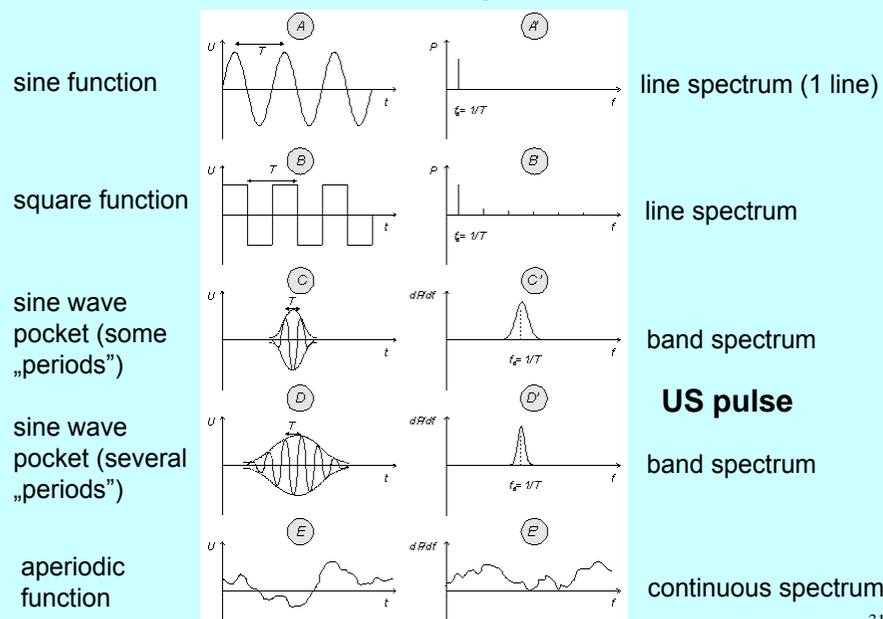
## Characteristic of US pulses

transducer: transmitter and receiver is the same unit

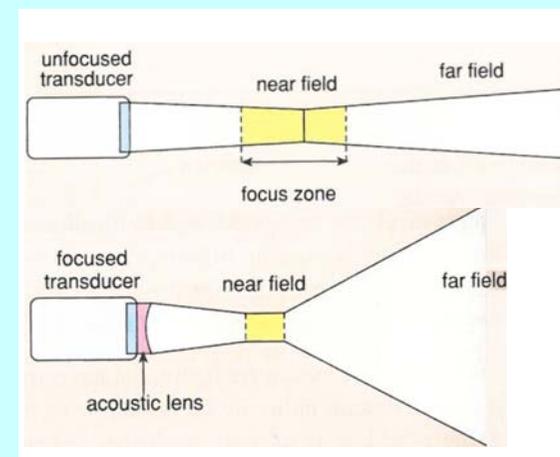
**time sharing mode:** pulses instead of continuous wave US



### Time function      Spectrum



## Focusing of the beam



Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.