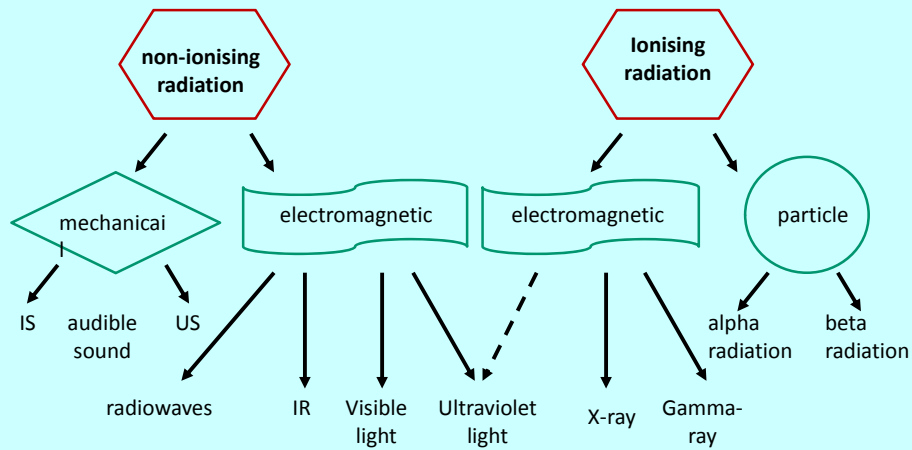


Radiation



1

Sound and

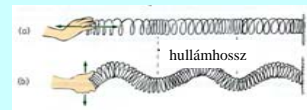
ultrasound



2

Physics of sound

Longitudinal vs. Transverse wave



longitudinal wave
(in the interior of liquids and gases only this type)



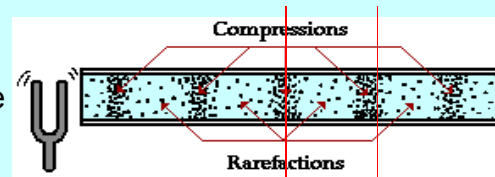
transverse wave
can generated in solid materials and at liquid surfaces

3

Physics of sound

Sound: mechanical wave (model)

whistle

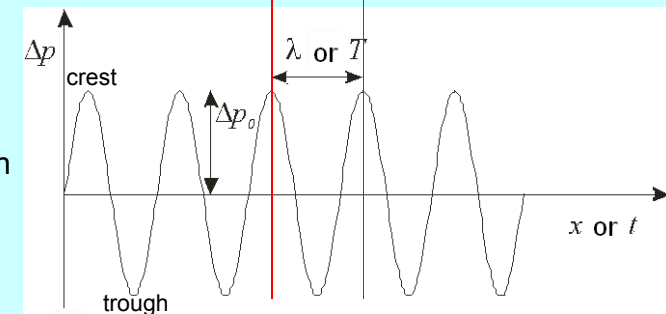


spring

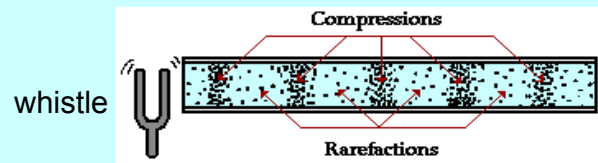


spatial and temporal periodicity

function



4



hydrostatic pressure pressure change, sound pressure

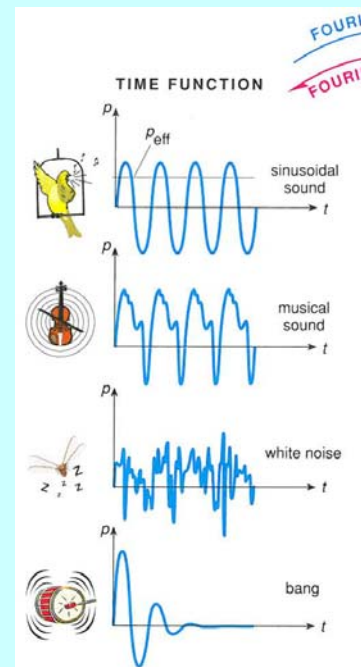
$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

pressure DC + AC amplitude phase

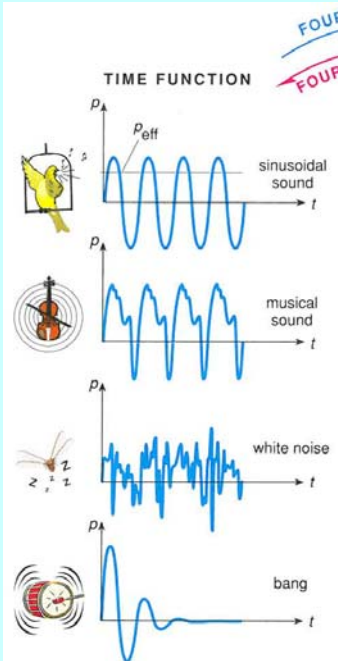
$$\Delta p(t, x) = \Delta p_{\text{max}} \sin \left[2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \right]$$


$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$

5



$$p(t) = p_1 \sin(\omega t) + p_2 \sin(2\omega t) + p_3 \sin(3\omega t) + \dots$$



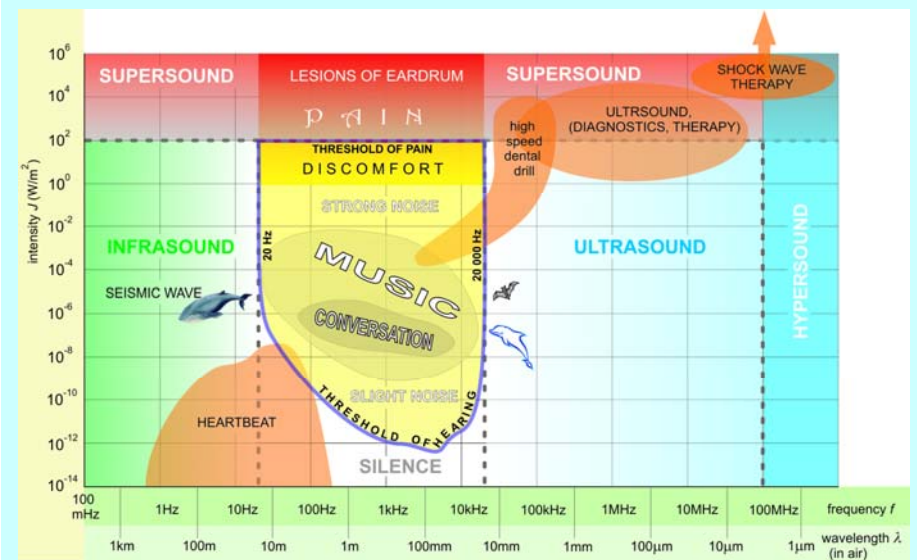
pitch
frequency of the fundamental high

timbre
relative strengths of overtones/harmonics (spectrum) tone colour

Intensity*
from pressure amplitude loudness

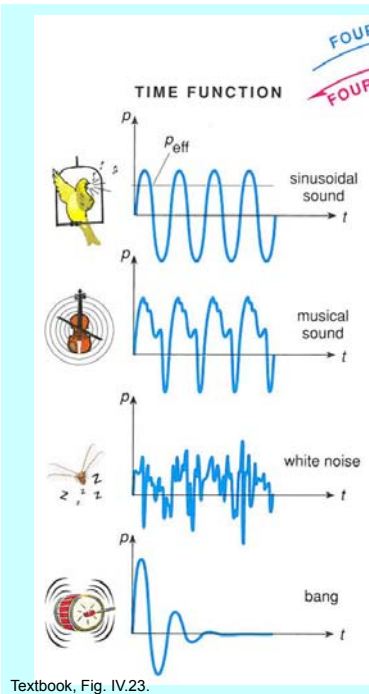
Textbook, Fig. IV.23.

Frequency and intensity regions of sounds



Lab. manual, Audiometry.

8



Textbook, Fig. IV.23.

pitch
frequency of the
fundamental

high

timbre
relative strengths of
overtones/harmonics
(spectrum)

tone colour

Intensity*
from pressure
amplitude

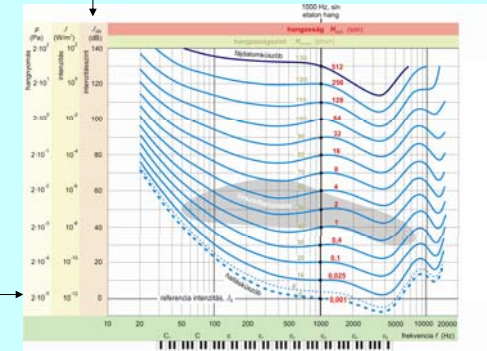
loudness

$$J_{dB} = 10 \lg \frac{J}{J_0}$$

$$J_0 = 10^{-12} \text{ W/m}^2$$

$$J_{dB} = 10 \lg \frac{J}{J_0}$$

$$J_0 = 10^{-12} \text{ W/m}^2$$



Curves of similar loudness levels

What is the intensity of a 300 Hz sound that a person, who has 25 dB hearing loss at this frequency can hear? (The average hearing threshold at this frequency is $3 \cdot 10^{-11} \text{ W/m}^2$)

$$J_{dB} = 10 \lg \frac{J}{J_0}$$

$$25 = 10 \lg \frac{J}{3 \cdot 10^{-11}}$$

$$10^{2.5} = \frac{J}{3 \cdot 10^{-11}}$$

$$J = 9,5 \cdot 10^{-9} \text{ Wm}^{-2}$$

Propagation of sound/ultrasound

The role of elastic medium – *speed* of propagation

$$c = f\lambda$$

$$c = \frac{1}{\sqrt{\rho\kappa}}$$



$$\kappa = -\frac{1}{V} \left(\frac{\Delta V}{\Delta p} \right) [\text{Pa}^{-1}]$$

compressibility
relative volume decrease
over pressure

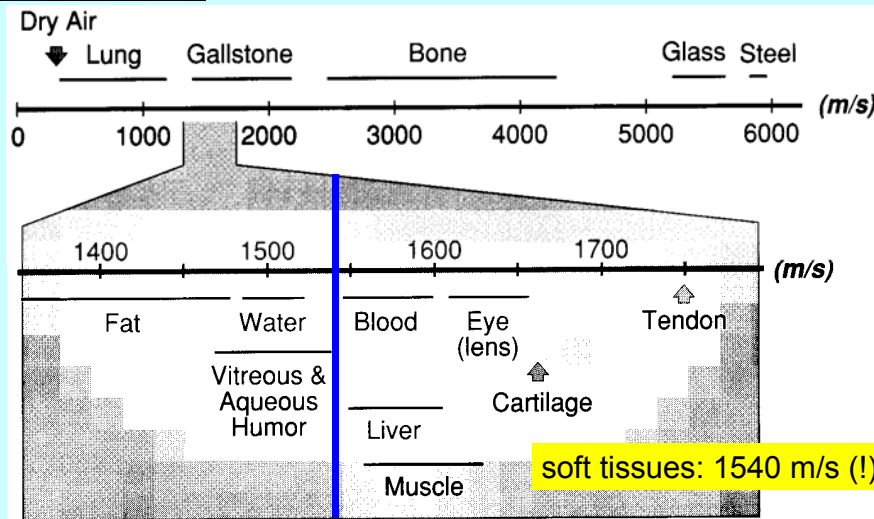
ρ : density of medium

Speed of propagation is higher in solids than in liquids.

$$\rho \uparrow \quad \kappa \downarrow$$



Speed of sound/US in various media



3

Propagation of sound/ultrasound

$$c = \frac{1}{\sqrt{\rho \kappa}}$$



$$\kappa = \frac{-\Delta V/V}{\Delta p} \quad [\text{Pa}^{-1}]$$

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

acoustic **impedance**
(definition)

$$Z_{\text{el}} = \frac{U}{I}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance**
(useful form)

$$[\text{kg} / \text{m}^2 \text{s}]$$

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Propagation of sound/ultrasound

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

$$\kappa = \frac{-\Delta V/V}{\Delta p}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

| material | ρ [kg/m ³] | κ [1/GPa] | c [m/s] | Z [kg/(m ² ·s)] |
|------------|--------------------------------|---------------------|--------------|---------------------------------|
| air | 1,3 | 7650 | 331 | 0,00043 · 10 ⁶ |
| water 20°C | 998 | 0,45 | 1492 | 1,49 · 10 ⁶ |
| aluminum | 2700 | 0,009 | 6400 | 17,28 · 10 ⁶ |
| quartz | 2650 | 0,011 | 5736 | 15,2 · 10 ⁶ |

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Problem:

A sound beam of 3 MHz frequency and 50 mW/cm² intensity propagates in blood.

What is the pressure? What is the maximal displacement and velocity of particles in this beam?

$$Z_{\text{ver}} = 1,66 \times 10^6 \text{ kg/m}^2 \text{s}$$

Solution:

Intensity:

$$J = \frac{p_{\max}^2}{2Z}$$

$$p = \sqrt{2JZ} = 40.74 \text{ kPa}$$

Velocity:

$$v = \frac{p}{Z} = \frac{40,74 \cdot 10^3}{1,66 \cdot 10^6} = 0,0245 \text{ m/s} = 24,5 \text{ mm/s}$$

Displacement:

$$A = \frac{v}{\omega} = \frac{24,5}{2 \cdot \Pi \cdot 3 \cdot 10^6} = 1,3 \cdot 10^{-6} \text{ mm} = 1,3 \text{ nm}$$

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Intensity of US

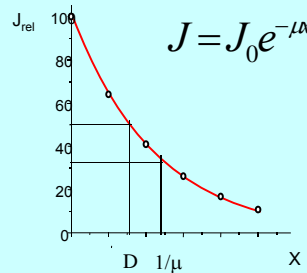
$$J = \frac{1}{Z} \Delta p_{\text{eff}}^2$$

intensity = energy/current density

$$P_{\text{el}} = \frac{1}{Z_{\text{el}}} U_{\text{eff}}^2$$

electric analogy

Loss of energy during propagation (absorption)

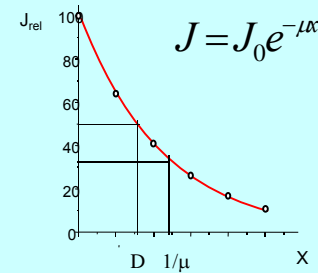


attenuation: $\alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

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Loss of energy during propagation (absorption)



attenuation: $\alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

μ is proportional to frequency in the diagnostic range

At $f = 1 \text{ MHz}$

$D_{\text{air}} \sim 1 \text{ cm}$

$D_{\text{water}} \sim 1 \text{ m}$

Specific attenuation: $\frac{\alpha}{f \cdot x}$

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μ is proportional to frequency in the diagnostic range

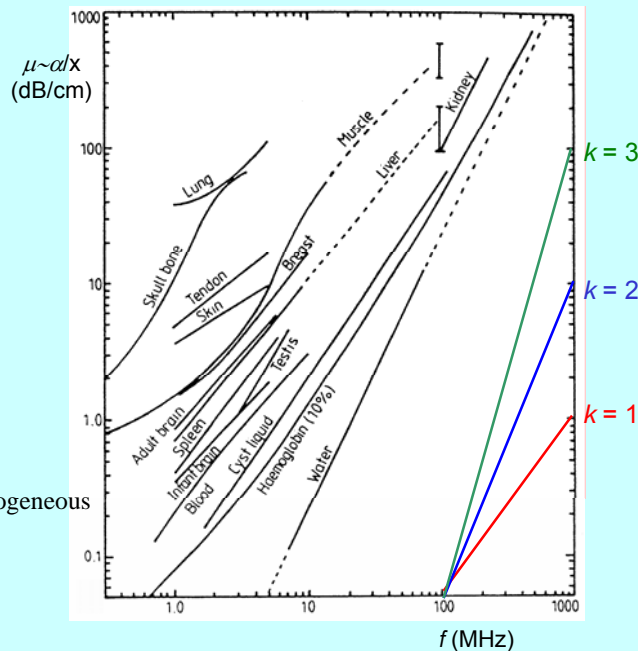
$$\mu \sim f^k, \quad k \sim 1(?)$$

$$\log \mu \sim k \log f$$

if the graph is a linear, the power function approximation is valid

specific attenuation for soft tissues (homogeneous tissue model):

$$\frac{\alpha}{f \cdot x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$



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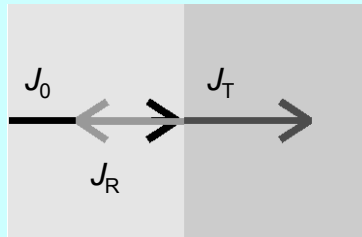
$$\frac{\alpha}{f \cdot x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$

| tissue | Specific attenuation |
|--------|----------------------|
| liver | 0,6 – 0,9 |
| kidney | 0,8 – 1,0 |
| fat | 1,0 – 2,0 |
| blood | 0,17 – 0,24 |
| bones | 16 – 23 |

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Phenomena at the boundary of different media

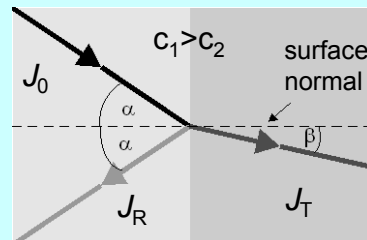
normal/perpendicular incidence



$$J_0 = J_R + J_T$$

reflection and transmission (penetration)

skew incidence

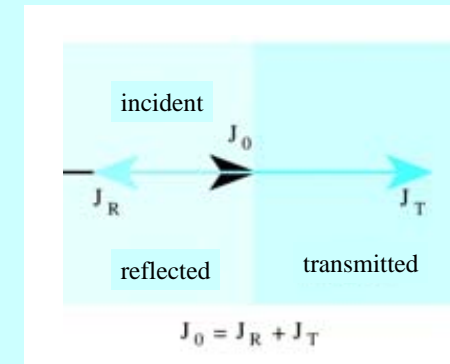


$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

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Reflection of ultrasound



$$R = \frac{J_R}{J_0}$$

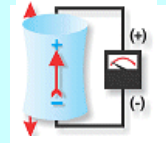
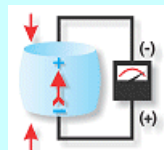
$$R = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

If $R \approx 1 \longrightarrow$ Total reflection

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Detection/Generation of US

Piezoelectric effect



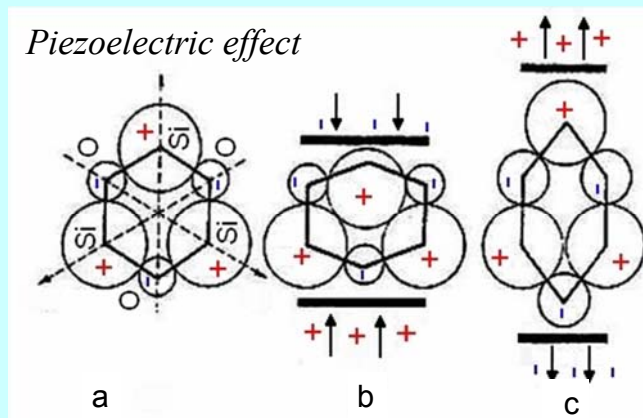
Pressure change

Mechanical deformation of crystal

Electric potential difference

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Piezoelectric effect



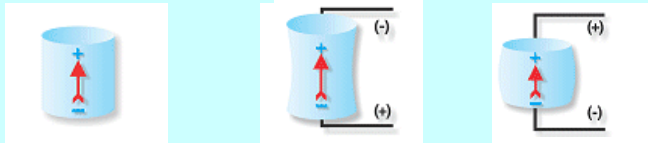
(a) Center of charge of positive and negative charges coincides.

(b) and (c) As a result of pressure, the charge centers are separated, i.e. a potential difference arises

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Detection/Generation of US

Inverse piezoelectric effect



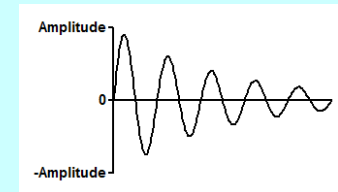
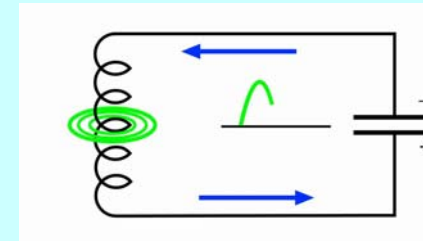
Periodic electric potential difference

The crystal is deformed when voltage is applied

Mechanical vibration

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Oscillator circuit – LC circuit



$$f = \frac{1}{2\pi\sqrt{LC}}$$

L : self inductance [$\text{S}\cdot\Omega^{-1}$]

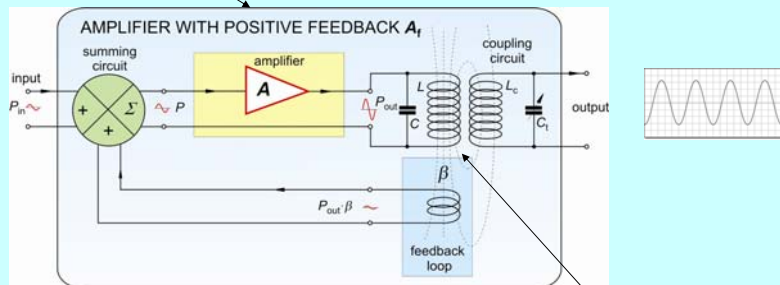
$$L \sim A N^2$$

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Source of electric signal: **sine wave oscillator**

$$A_{\text{feedback}} = \frac{U_{\text{out}}}{U_{\text{in}}} = \frac{A}{1 - A \cdot \beta}$$

$A\beta=1$
amplification = „infinity“
no input signal, output signal: sine voltage



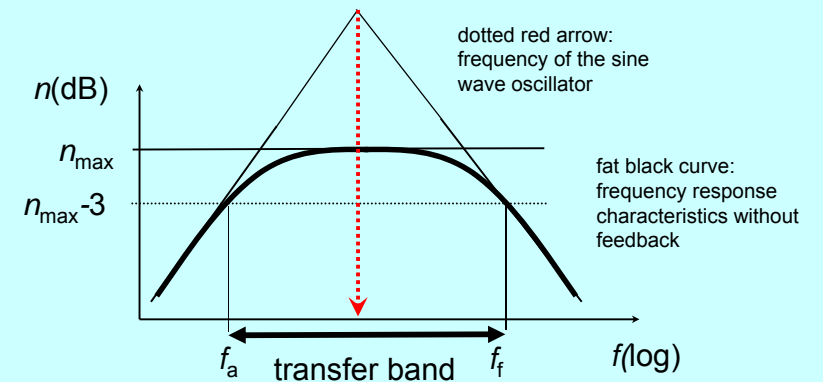
$$f = \frac{1}{2\pi\sqrt{LC}}$$

27

Source of electric signal: **sine wave oscillator**

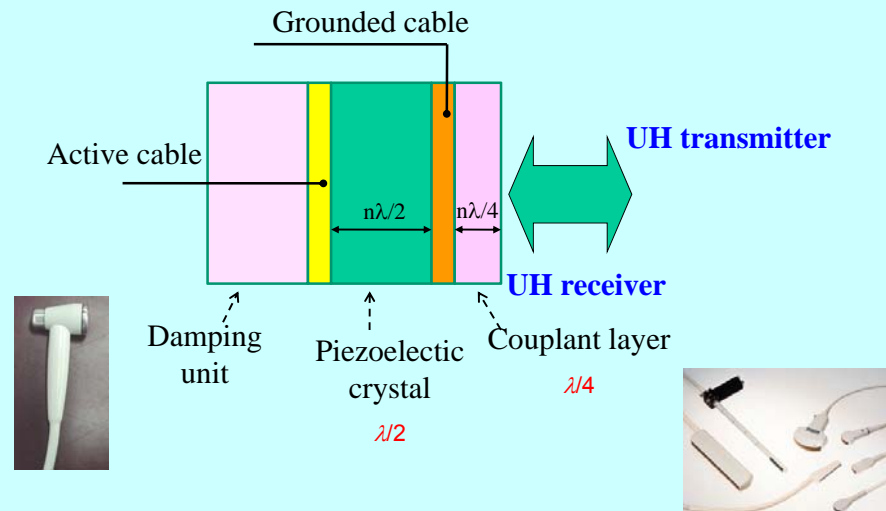
amplifier with positive feedback

$$A_{U, \text{feedback}} = \frac{A_U}{1 - \beta A_U}$$



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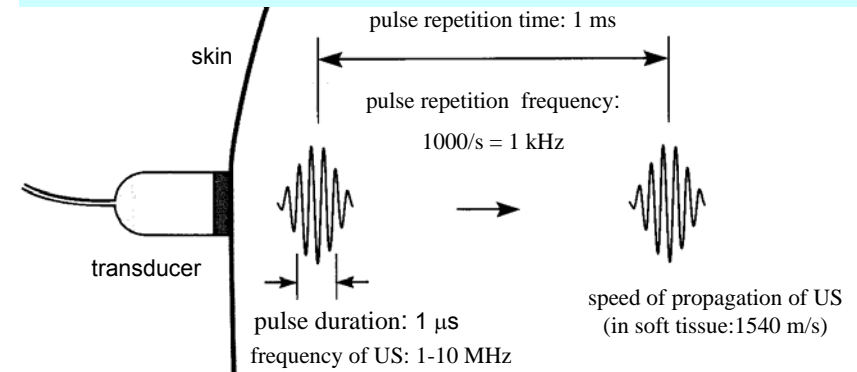
Detection/Generation of US - Ultrasound transducer



Characteristic of US pulses

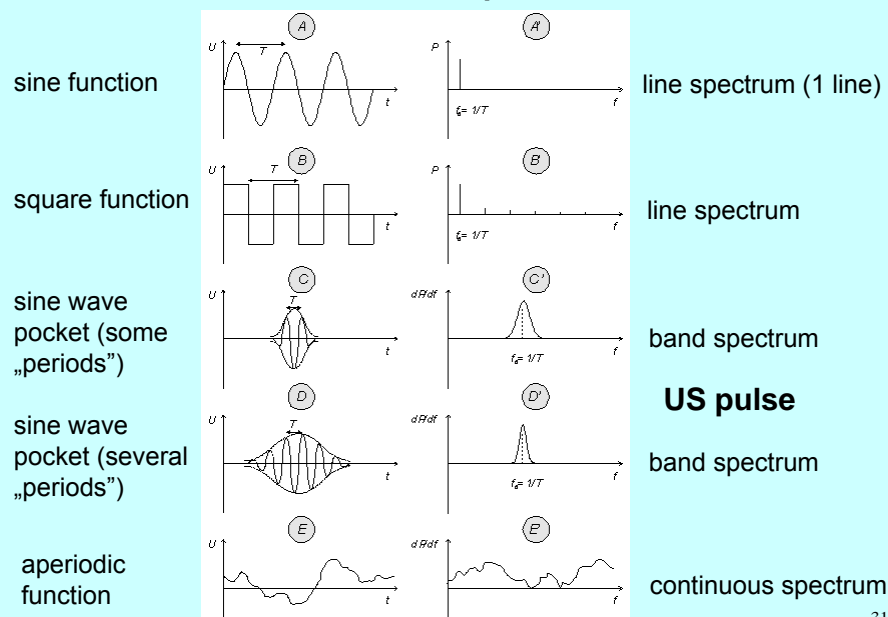
transducer: transmitter and receiver is the same unit

time sharing mode: pulses instead of continuous wave US



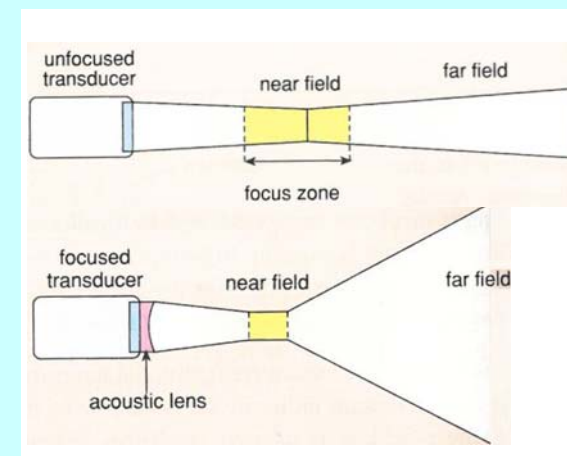
30

Time function Spectrum



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Focusing of the beam



Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

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