Biophysik für Pharmazeuten II.

20. 04. 2015.

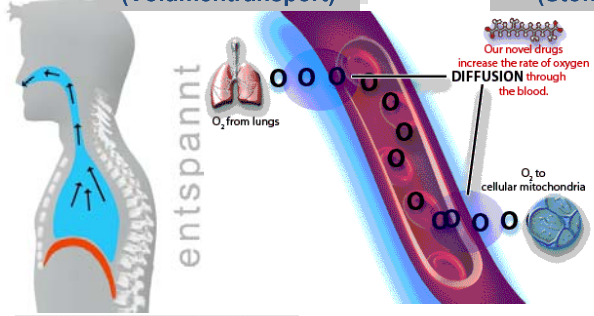
Transportprozesse 2.

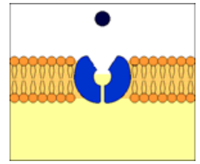
Strömungen, Diffusion, Wärmeleitung

Transportprozesse

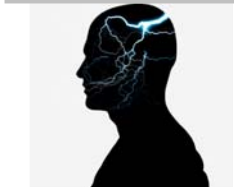
II. Strömung (Volumentransport)

III. Diffusion (Stofftransport)





I. Elektrischer Strom (el. Ladungstransport)



IV. Wärmeleitung (Energietransport)

V. Verallgemeinerung

VI. Energetische Aspekte

II. Strömungen (Fortsetzung)

- **Transportgesetz** (Hagen-Poiseuille-Gesetz):

$\Delta p = p_2 - p_1 < 0$

p_1 I p_2 $(p_1 > p_2)$

Δl

Volumen-stromstärke

Radius des Rohres

$$\frac{\Delta V}{\Delta t} = -\frac{\pi}{8}\frac{1}{\eta}R^4\frac{\Delta p}{\Delta l}$$

Viskosität

Druckgradient

G. H. L. Hagen 1797-1884 Wasseringenieur

J. L. M. Poiseuille 1799-1869 Physiologe

Bedingungen:
- inkompressible Fl.
- laminare Str.
- stationäre Str.
- newtonsche Fl.

Zur Erinnerung

Volumen-stromdichte

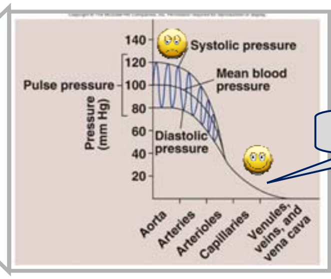
Alternativform: $\frac{\Delta V}{A \cdot \Delta t} = \frac{R^2}{8\eta}\frac{\Delta p}{\Delta l}$

„Strömungs-leitfähigkeit"

Ist das H-P-Gesetz anwendbar für die Blutströmung?

Gültigkeitsbedingungen?

- inkompresible Fl.?
- laminare Strömung?
- stationäre Strömung?
- newtonsche Fl.?

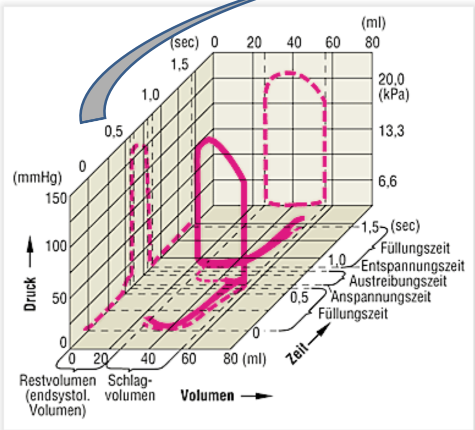


elastische Aderwände

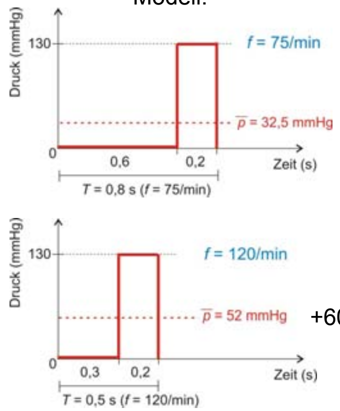
Folgerung: H-P nur qualitativ anwendbar!

Blutströmung

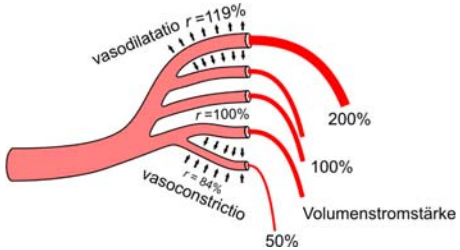
- Regulation der Volumenstromstärke laut Hagen-Poiseuille-Gesetzes:
  - ➢ **Druck**



Modell:

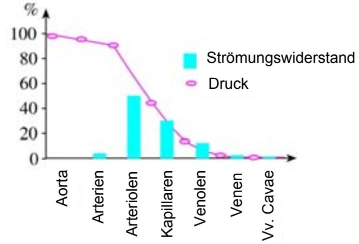


- ➢ **Radius** (r^4 !)

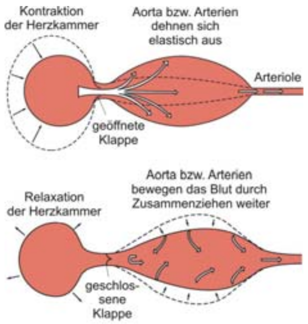


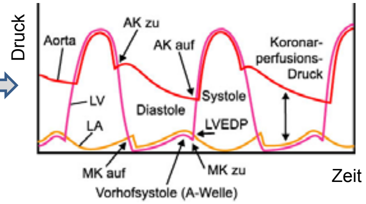
⇨ Elastizität!

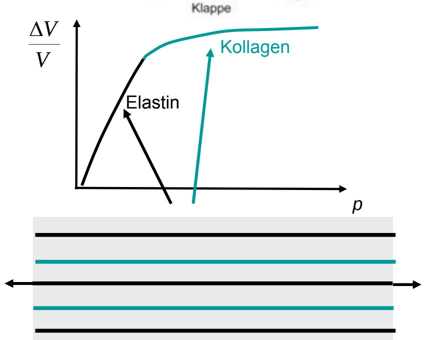
- Druck und Strömungswiderstand im Kreislauf:

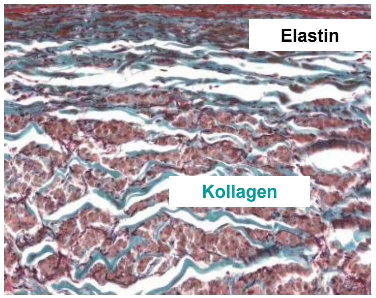


- Rolle der Elastizität von Aorta und Arterien (Windkesselfunktion):

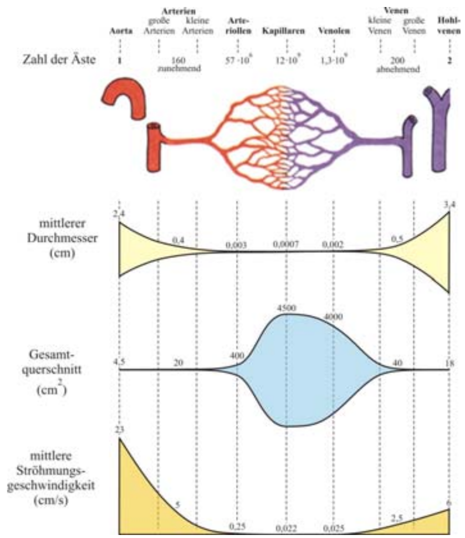


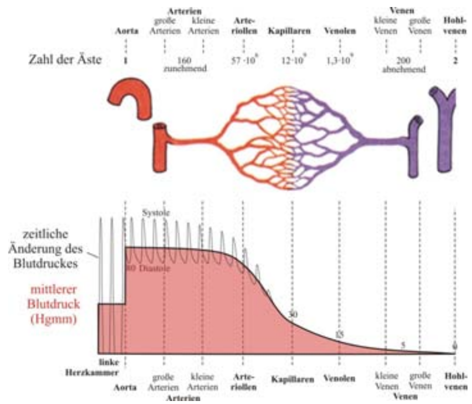






Zusammenfassend:



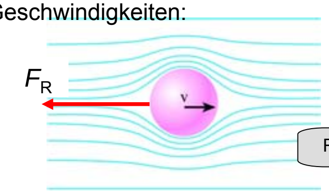


Analogie

	Was strömt?	Stärke?	Was treibt die Strömung?		Zusammenhang?
Ladungs-transport	q	$J_q = \dfrac{\Delta q}{A \cdot \Delta t}$	φ	$-\dfrac{\Delta\varphi}{\Delta l}$	$J_q = -\sigma\dfrac{\Delta\varphi}{\Delta l}$
Volumen-transport	V	$J_V = \dfrac{\Delta V}{A \cdot \Delta t}$	p	$-\dfrac{\Delta p}{\Delta l}$	$J_V = -\dfrac{R^2}{8\eta}\dfrac{\Delta p}{\Delta l}$

4. Bewegung von Teilchen in reellen Flüssigkeiten

Bei kleineren Geschwindigkeiten:



stokessches Reibungsgesetz:

$$F_R = 6\pi\eta\, r\, v$$

Reibungskraft

Radius des Teilchens

Viskosität

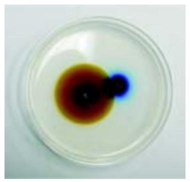
Geschwindigkeit des Teilchens

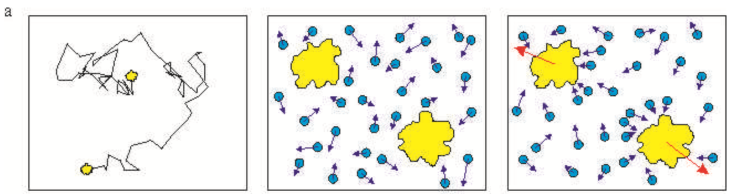
G. G. Stokes
1819-1903
Mathematiker
Physiker

Bei gleichmäßigen Bewegung: $F_{\text{Bewegung}} = F_R$

Beweglichkeit (u) eines Teilchens: $u = \dfrac{v}{F_{\text{Bewegung}}} \Rightarrow \quad u = \dfrac{1}{6\pi\eta r} \Longrightarrow$ s. Diffusion

III. Stofftransport (Diffusion)





1. Grundbegriffe

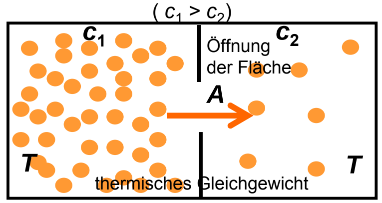
- Diffusion: Tendenz zur gleichmäßigen Verteilung von Molekülen durch die thermische Bewegung

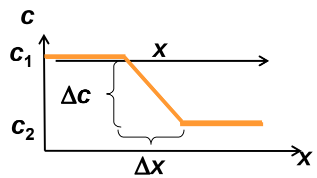
- Stoffstromstärke (I): $I = \dfrac{\Delta\nu}{\Delta t}$ $\left(\dfrac{\text{mol}}{\text{s}}\right)$
- Stoffstromdichte (J): $J = \dfrac{\Delta\nu}{A \cdot \Delta t}$ $\left(\dfrac{\text{mol}}{\text{s}\cdot\text{m}^2}\right)$
- stationäre Diffusion: zeitlich konstant

Adolf Fick
1829-1901
Physiologe

2. Transportgesetz — 1. ficksches Gesetz

$\Delta c = c_2 - c_1 < 0$
$(c_1 > c_2)$





$$\frac{\Delta\nu}{\Delta t} = -DA\frac{\Delta c}{\Delta x}$$

$$J = -D\frac{\Delta c}{\Delta x}$$

Konzentrationsgradient

Stromdichte

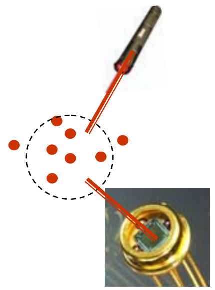
Diffusionskoeffizient

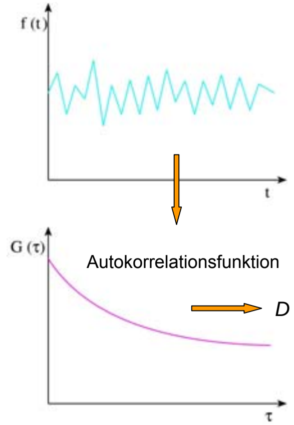
- **Diffusionskoeffizient:**
  - stoffspezifisch
    - diffundierende Moleküle – Größe (r) – Form
    - Medium (η)
  - temperaturabhängig $D \sim e^{-\frac{\Delta E}{RT}}$
  - $D = ukT$ (Beweglichkeit des Teilchens)
  - **Einstein-Stokes-Gleichung** (für kugelförmige Teilchen)

$$D = \frac{kT}{6\pi\eta r}$$

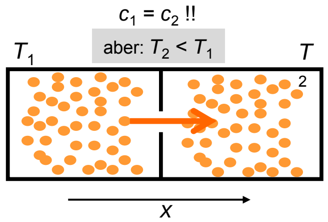
Diffundierendes Teilchen (Molmasse)	Medium	D (m²/s)
H_2 (2)	Luft	$6{,}4\cdot10^{-5}$
O_2 (32)	Luft	$2\cdot10^{-5}$
CO_2 (44)	Luft	$1{,}8\cdot10^{-5}$
H_2O (18)	Wasser	$2{,}2\cdot10^{-9}$
O_2 (32)	Wasser	$1{,}9\cdot10^{-9}$
Glyzin (75)	Wasser	$0{,}9\cdot10^{-9}$
Serum Albumin (69 000)	Wasser	$6\cdot10^{-11}$
Tropomiozin (93 000)	Wasser	$2{,}2\cdot10^{-11}$
Tabakmosaik-virus (40 000 000)	Wasser	$4{,}6\cdot10^{-12}$

- Messung:
  eine Möglichkeit – dynamische Lichtstreuungsmessung





- Im thermischen Nichtgleichgewicht:



Konzentration (c) $\Rightarrow$ chemisches Potenzial (μ)

$$\mu = \mu_0 + RT\ln\frac{c}{c_0}$$

Die Triebkraft der Diffusion ist: $-\frac{\Delta\mu}{\Delta x}$

3. Das 2. ficksche Gesetz:

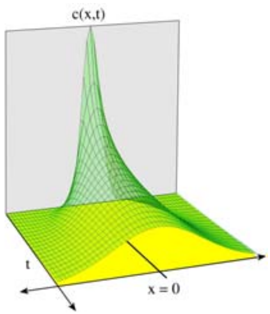
$$D\frac{\Delta\left(\frac{\Delta c}{\Delta x}\right)}{\Delta x} = \frac{\Delta c}{\Delta t} \qquad D\frac{\partial^2 c}{\partial x^2} = \frac{\partial c}{\partial t}$$

Lösungen:

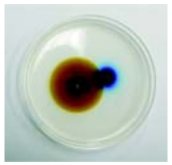
- Für eindimensionale Diffusion:

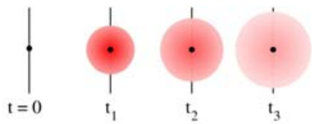
$$c(x) = \frac{c_0\Delta x}{\sqrt{2\pi\sigma_x^{\,2}}}\, e^{-\frac{x^2}{2\sigma_x^{\,2}}}$$

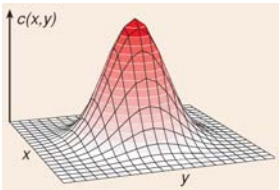
$$\sigma_x = \sqrt{2Dt}$$



- Für zweidimensionale Diffusion:

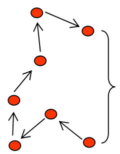




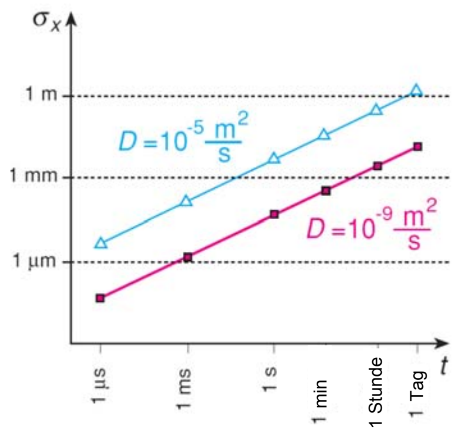


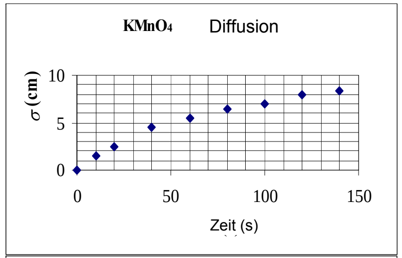
Siehe auch Praktikum!

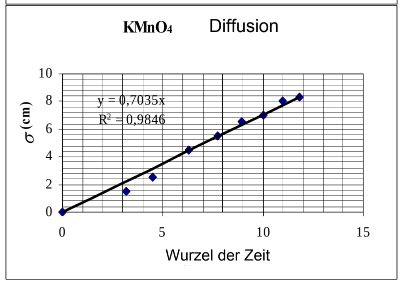
4. Diffusion als Random Walk



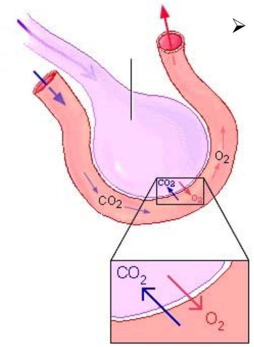
$\sigma = ?$ $\quad \boxed{\sigma \approx \sqrt{2D\cdot t}}$



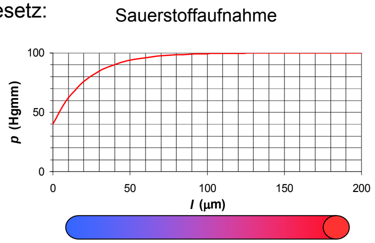


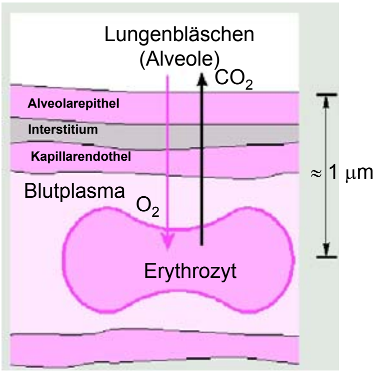


5. Anwendungen ▪ O_2/CO_2-Diffusion Lunge-Blut



➢ 1. ficksches Gesetz:





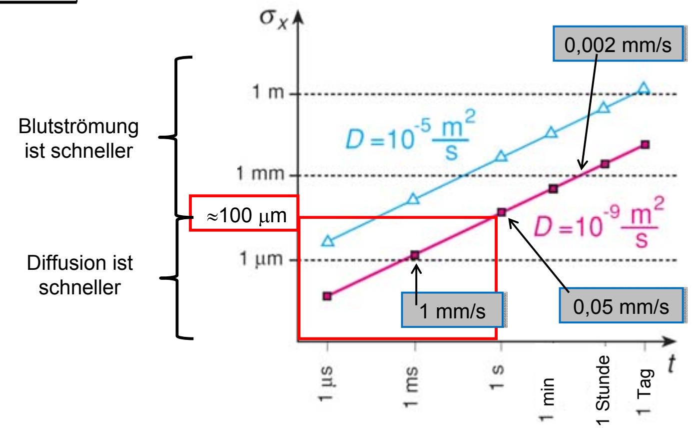
➢ Random Walk:

1 µm ⇒ 1 ms

▪ Welche ist „schneller” für O_2-Transport im Gewebe? Diffusion ↔ Blutströmung

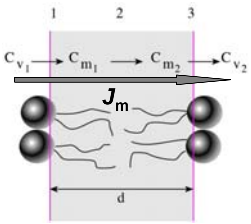
Gefäß	Kapillaren
A (cm²)	4500
v (cm/s)	0,022

(= 0,22 mm/s)



▪ Diffusion durch eine Membran (passiver Transport)

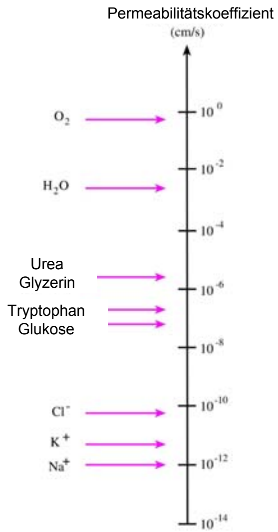
Für neutrale Teilchen:



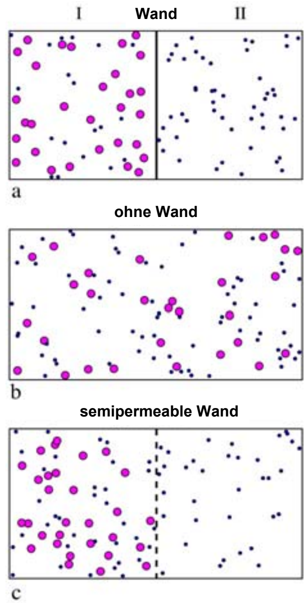
$$J_m = -D \cdot \frac{\Delta c}{\Delta x} = -D \cdot \frac{c_{m2} - c_{m1}}{d} = -p(c_{v2} - c_{v1})$$

$$J_m = -p(c_{v2} - c_{v1})$$

Permeabilitätskoeffizient (m/s)



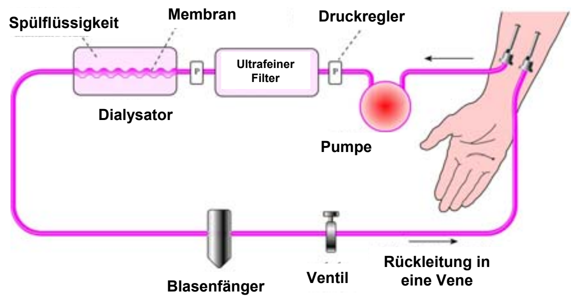
▪ Osmose



J. H. van't Hoff
1852-1911
Chemiker

Van't Hoff-Gesetz:

$$p_{\text{Osmose}} = cRT$$

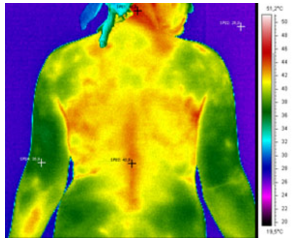


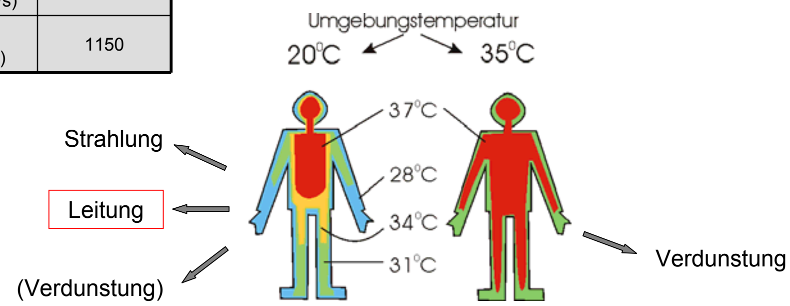
Analogie

	Was strömt?	Stärke?	Was treibt die Strömung?		Zusammenhang?
Ladungs-transport	q	$J_q=\frac{\Delta q}{A\cdot\Delta t}$	φ	$-\frac{\Delta\varphi}{\Delta l}$	$J_q=-\sigma\frac{\Delta\varphi}{\Delta l}$
Volumen-transport	V	$J_V=\frac{\Delta V}{A\cdot\Delta t}$	p	$-\frac{\Delta p}{\Delta l}$	$J_V=-\frac{R^2}{8\eta}\frac{\Delta p}{\Delta l}$
Stoff-transport	ν	$J_\nu=\frac{\Delta\nu}{A\cdot\Delta t}$	c	$-\frac{\Delta c}{\Delta x}$	$J_\nu=-D\frac{\Delta c}{\Delta x}$

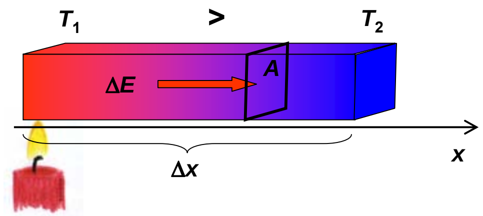
Wärmebildung und -abgabe

Aktivität	Wärme-bildung (W)
In Ruhe	115
Langsames Spazieren	260
Radfahren (15 km/h)	420
Treppen-steigen (2/s)	700
Laufen (15 km/h)	1150





IV. Wärmeleitung (Energietransport)



$$\boxed{\frac{\Delta E}{A\cdot\Delta t}=-\lambda\frac{\Delta T}{\Delta x}}$$

J. B. J. Fourier
1768-1830
Mathematiker
Physiker

Stoff	λ (W/(m·K))
Silber	420
Glas	1
Wasser	0,6
Muskel	0,4
Fett	0,2
Luft	0,025

V. Zusammenfassung

	Was strömt?	Stärke?	Warum?		Zusammenhang?
Ladungs-transport	q	$\frac{\Delta q}{A\cdot\Delta t}$	φ	$-\frac{\Delta\varphi}{\Delta l}$	$\frac{\Delta q}{A\cdot\Delta t}=-\sigma\frac{\Delta\varphi}{\Delta l}$
Volumen-transport	V	$\frac{\Delta V}{A\cdot\Delta t}$	p	$-\frac{\Delta p}{\Delta l}$	$\frac{\Delta V}{A\cdot\Delta t}=-\frac{r^2}{8\eta}\frac{\Delta p}{\Delta l}$
Stoff-transport	ν	$\frac{\Delta\nu}{A\cdot\Delta t}$	c^*	$-\frac{\Delta c}{\Delta x}$	$\frac{\Delta\nu}{A\cdot\Delta t}=-D\frac{\Delta c}{\Delta x}$
Energie-transport	E	$\frac{\Delta E}{A\cdot\Delta t}$	T	$-\frac{\Delta T}{\Delta x}$	$\frac{\Delta E}{A\cdot\Delta t}=-\lambda\frac{\Delta T}{\Delta x}$
allgemein	x_{ext}	$J=\frac{\Delta x_{\text{ext}}}{A\cdot\Delta t}$	y_{int}	$X=-\frac{\Delta y_{\text{int}}}{\Delta x}$	$J=LX$
	extensive Gr.	**Strom-dichte**	**intensive Gr.**	**termo-dynamische Kraft**	**onsagersche Beziehung**

* Im allgemeinen Fall μ