

# MEDICAL STATISTICS

Physiology

Anatomy

Chemistry

...

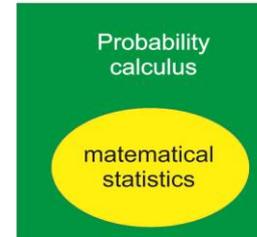
Statistics



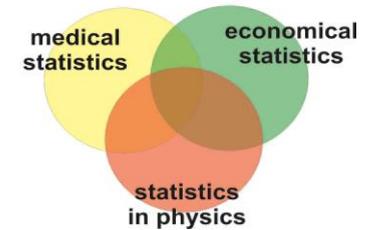
No any doubt

# Medical statistics

Theory:  
matematics



Practice:  
applied statistics  
(examples)



# Example: body temperature

36.7 °C

36.7 °C

36.9 °C

36.9 °C

36.6 °C

36.5 °C



1. Inaccuracy of the measurement.

2. Daily fluctuation!!!

3. Biological variability!!!

The measured value is not constant!

Measured value: 37.0 °C.

Is it healthy or not?

# Another examples

RBC:  $4.5 \times 10^{12} \text{ 1/l}$  ( $3.9\text{-}5 \times 10^{12} \text{ 1/l}$ ) → normal range?

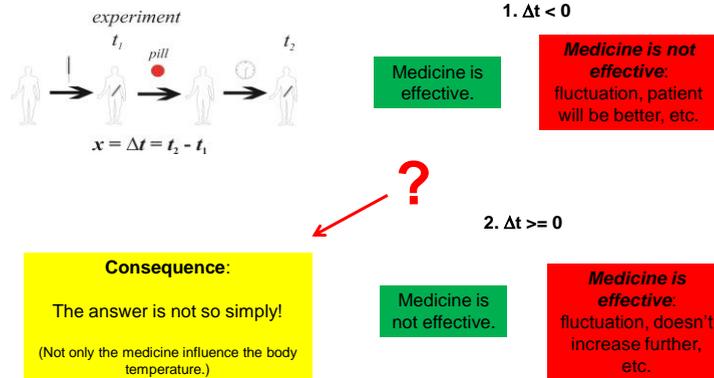
The new method in therapy is better then the old one or not?

How can we prove that a medicine decreases the fever or not?



**Questions!**

## How can we answer?



## Variables

variable	range	type	variable type	
height	~50 cm ... ~250 cm	real number	numerical	continuous
no. of teeth	0 .. 32	integer		discrete
blood type	A, B, AB, 0	letters	categorical	nominal
severity of cancer	1 ... 4	integer		ordinal

**Descriptive statistics!**

## Description of a variable

- Type
- Possible values
- Occurrence of the values

## Numerical variables

Name	<i>Continuous</i>	<i>Discrete</i>
Definition	Infinitely large no. of values in a certain range	Only finite number of values
Example	Height, temperature, pressure ...	No. of teeth, no. of children ...

## Categorical variables

Name	<b>Nominal</b>	<b>Ordinal</b>
Definition	No order among the values	There is a certain order
Example	Gender, blood-type ...	Severity of the illness, strength of pain ...

## Determination of the possible values

- Continuous : giving a possible range.  
» e.g.: height from ~50 cm - to ~ 250 cm
- Another : listing the values, if it is possible  
» E.g.: blood type: A, B, AB, 0

## Occurrence

**Observation**: The occurrence of the values are not the same!



Trial: experiment, observation, data collection.

*Deal with only the case, when the trial may be repeated!*

Outcome: result of one trial. (e.g.: height of a student)

## Population

How many people?



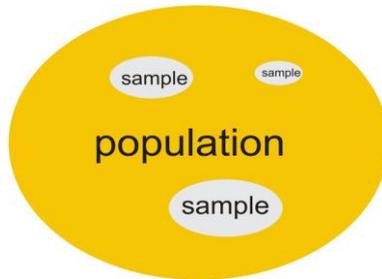
As many as possible.



Ideal case: All of the people → **population**

## Sample

A smaller portion of the population.



$n$ : no. of the elements (people) in the sample.

$x$ : the tested variable (quantity)

$x_i$ :  $i$ -th element from the sample

## Selection of the sample

Main principle: **Random sample**

Medical statistics: if there is no any reason to exclude,  
must be random!

## Occurrence

**Frequency** ( $k$ ): no. of occurrence in the sample.

$k_i$ : no. of occurrence of the  $i$ -th value in the sample.

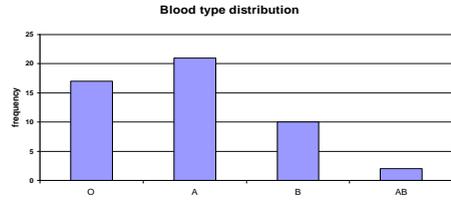
$$n = \sum_i k_i$$

## Frequency distribution

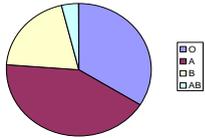
Frequency as the function of the possible values.

Blood-type	<b>0</b>	<b>A</b>	<b>B</b>	<b>AB</b>	total
frequency	17	21	10	2	50

## Presentation



Bar-chart



Pie-chart

## Relative frequency, proportion

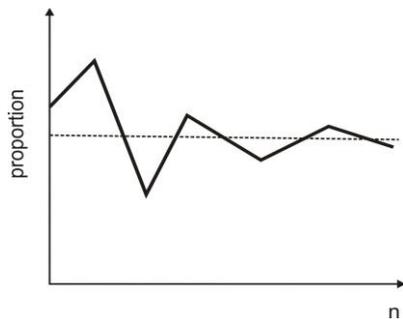
The ratio of the frequency and the total no. of the elements.

$$\sum_i \frac{k_i}{n} = \frac{1}{n} \sum_i k_i = \frac{1}{n} \times n = 1$$

Frequently it is given as percentage:

$$\frac{k_i}{n} \times 100\%$$

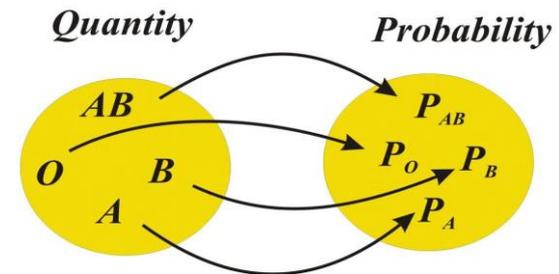
## Probability ( $P$ )



If  $n$  is infinite the name of the proportion is the probability.

Probability ( $P$ ): proportion in the population.

## Probability distribution



## Properties of the probability

$$0 \leq P \leq 1$$



P = 0 - never occur  
P = 1 - always occur

Example: blood- type

$$P_A + P_B + P_{AB} + P_O = 1$$



$$\sum_i P_i = 1$$

(exclusive events)

## Probability and proportion

**Sample**

n is finite

proportion

**Population**

$n = \infty$

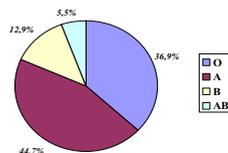
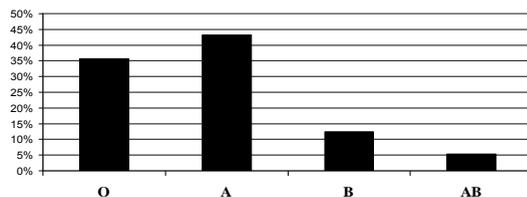
probability

**Probability very frequently is unknown!**

We usually use proportion instead of the probability.

## Presentation

Blood type distribution



## Continuous quantity

Infinite no. of possible values!!!

**Class:** a short interval in the whole range.

**Class-width:** the length of the class.

Frequency: no. of elements in the given class.



Like a discrete value!

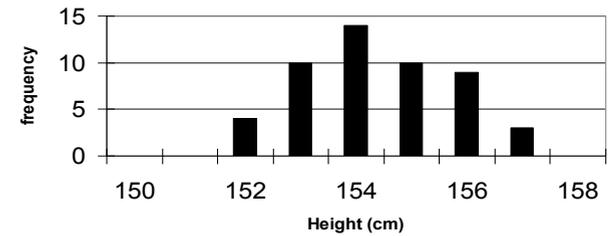
## Example

1	160 cm
2	181 cm
3	175 cm
4	163 cm
5	165 cm
6	179 cm
7	164 cm
8	185 cm
9	177 cm
10	168 cm

class	$k_i$
160-164	3
165-169	2
170-174	0
175-179	3
180-184	1
185-189	1

## Presentation

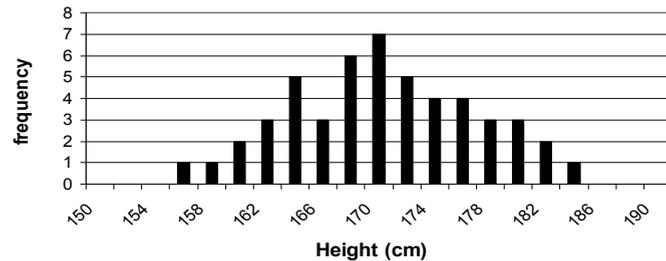
Frequency distribution (class width = 5 cm)



5 cm is too large!

## Decrease the width!

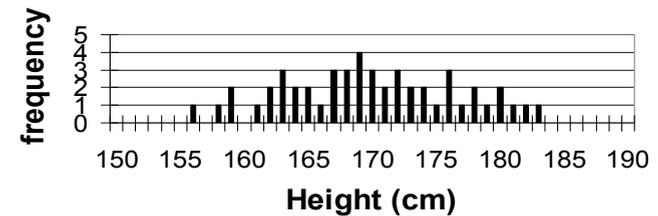
Frequency distribution (width = 2 cm)



Observation: frequency decreases!

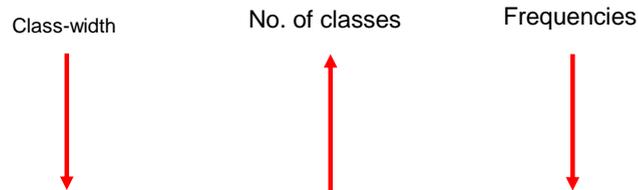
## Presentation

Frequency distribution (class width = 1 cm)



Reason: n is too small!

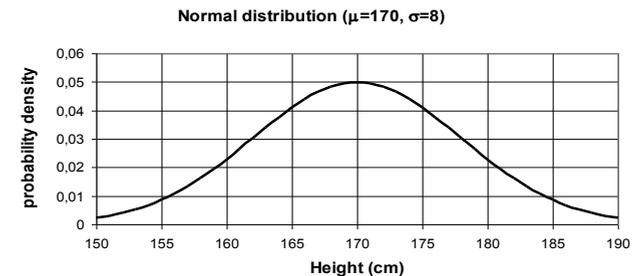
## Consequence



We must increase the no. of the elements!

## Normal distribution

If  $n$  and no. of classes are infinite!



## Theoretical description

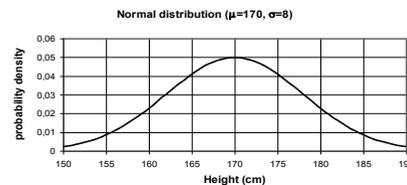
Normal or Gauss-distribution

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Parameters:

$\mu$  – expected value or mean

$\sigma$  – theoretical standard deviation

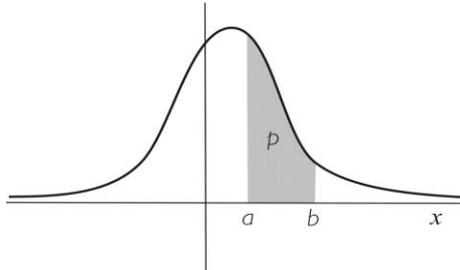


## Meaning of the parameters

$\mu$  **(mean):**  
the value belonging to the maximum of the curve.

$\sigma$  **(theoretical standard deviation):**  
the average deviation of the data from the  $\mu$ .

## Probability



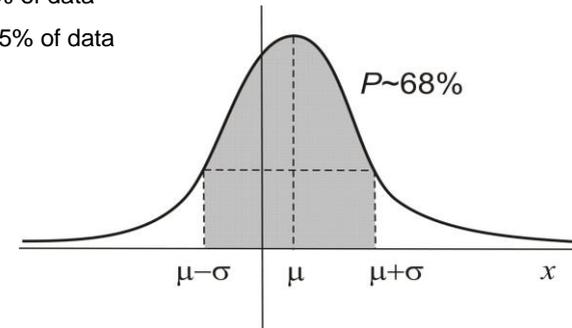
$P$  is the probability that  $x$  is in the  $(a,b)$  interval.

## Standard deviation

$(\mu \pm \sigma)$  ~ 68% of data

$(\mu \pm 2\sigma)$  ~ 95% of data

$(\mu \pm 3\sigma)$  ~ 99.5% of data



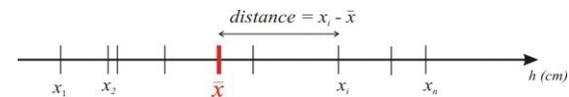
## Normal distribution

**Theoretical distribution** that describes the population. In practice usually we don't know the parameters of this.



We usually have a **random sample** from the population.  
We must estimate the parameters!

## Estimation of the $\mu$



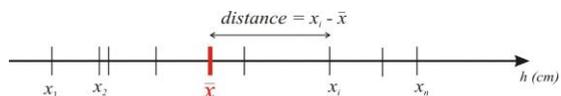
**average:** must be in the center of the data range.

$$\sum_i (x_i - \bar{x}) = 0 \quad \longrightarrow \quad \bar{x} = \frac{\sum_i x_i}{n}$$

## Estimation of the $\sigma$

$\sigma$  = average deviation of the data from the  $\mu$ .

**s (standard deviation)** = average deviation of the elements from the average.



$$Q_x = \sum_i (x_i - \bar{x})^2 \geq 0$$

## Standard deviation

$$s = \sqrt{\frac{Q_x}{n-1}}$$

s: the average deviation of the elements from the average.

$$(\bar{x} \pm s) \sim 68\%$$

$$(\bar{x} \pm 2s) \sim 95\%$$

$$(\bar{x} \pm 3s) \sim 99.5\%$$

## Relation of parameters

Sample	$n \rightarrow \infty$	Population
average	—————→	$\mu$
s	—————→	$\sigma$

## Question of the week!

How can we estimate the  $\mu$  and the  $\sigma$ ?