Mechanics, dynamics

Motion, deformation and their background

One of the fundamental physical quantities of the topic is the momentum (p).

In classical case it is the product of the mass (m) and the velocity (υ) of the body.

$$p = m\upsilon$$

vector quantity

Newton's laws of motion

II. For the change of momentum needs **force** (F).

$$\frac{\Delta m\upsilon}{\Delta t} = m\frac{\Delta \upsilon}{\Delta t} = ma = F$$

If no forces are exerted (or $F = 0$)

$\Delta m\upsilon = 0$, means $p = m\upsilon =$ constant.

I. Momentum is conserved (momentum conservation)

law of inertia

III. $F = -F_{\text{versus}}$ interaction

A single force cannot exist.

Forces are always directed to contrary parts.

law of action and reaction

Application e.g.:

at the pressure of ideal gases, (see later)

at the explanation of annihilation (see in PET).

***

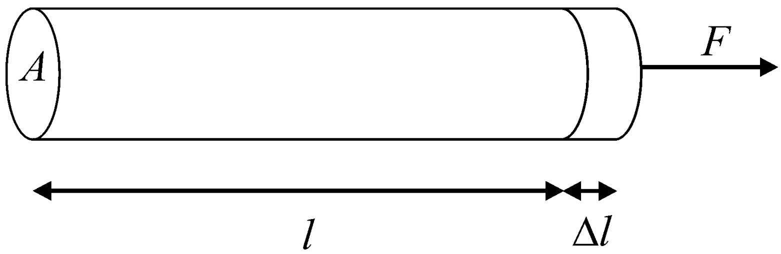
But

force may result in **deformation**.

Simplest deformation is the **elongation.**

tensile strain: $\Delta l/l$

Hooke's law



$$F = AE\frac{\Delta l}{l}$$

$$\frac{F}{A} = E\frac{\Delta l}{l}$$

F/A the stress (tensile stress), but

it could be compressive stress or pressure (p[Pa])

Coefficient: Young's modulus (E[Pa])

Similar to the case of spring: $F_{\text{spring}} = Dx$ (if $x \equiv \Delta l$, and $D \equiv AE/l$)

***

Newton's laws for rotation

similarly to the momentum ($m\upsilon$) here the fundamental physical quantity is the angular momentum ($\Theta\omega$), where

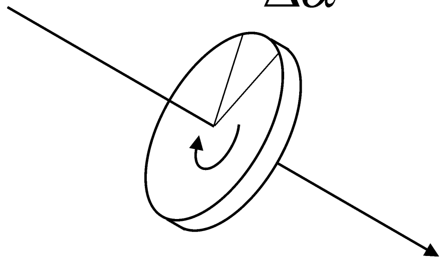
Θ is moment of inertia, rotational analog of the mass,

ω is angular velocity,

$$\omega = \frac{\Delta\alpha}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

period (T), frequency (f)

(ω angular frequency)



I. $\Theta\omega$ = constant (conservation of angular momentum)

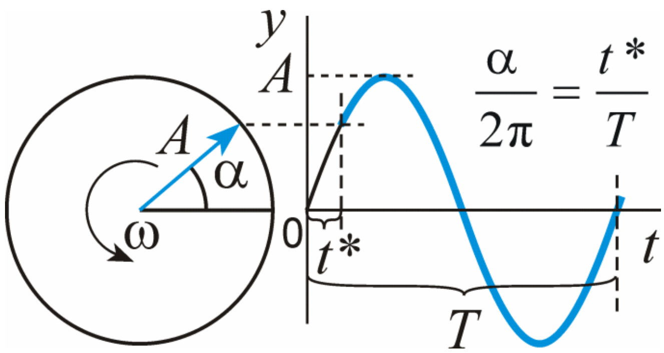
(see: rotating skater)

II. For the change of angular momentum needs **torque** (M).

$$\frac{\Delta\Theta\omega}{\Delta t} = M$$

Harmonic motion

Projection of
uniform circular motion
($\alpha = \omega t = 2\pi t/T = 2\pi f t$)



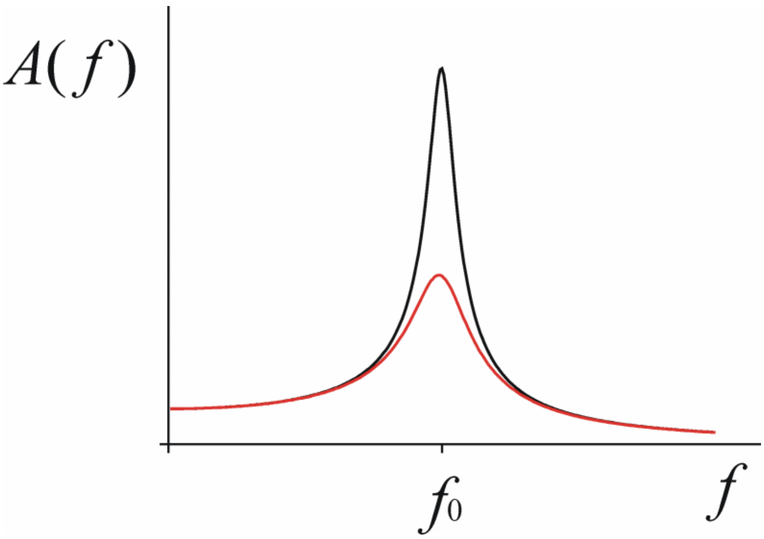
$$y = A \sin \omega t$$

Dynamical condition: $F = -Dx = ma$

$$\omega = \sqrt{\frac{D}{m}}$$

Forced oscillation, resonance

($\omega = 2\pi f$)



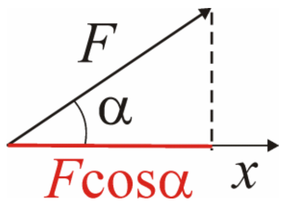
$$A(f) \sim \frac{1}{(f - f_0)^2 + K}$$

K is characteristic for damping

Application e.g.: at the interpretation of different spectra (ESR, NMR); at the explanation of AFM and MRI

Work

Work (W) is a product of displacement (Δx) and the projection of force (F) to the direction of displacement.



$W = \Delta x\, F\cos\alpha$ [Nm] or [J]

Permanent acting force without displacement ($\Delta x = 0$);
or $\alpha = \pi/2$ (means $\cos\alpha = 0$), then $W = 0$ (in mechanics)

Work-energy theorem

Force is constant (and $\alpha = 0$).

$$W = F\Delta x = \frac{1}{2}m\upsilon_2^{\,2} - \frac{1}{2}m\upsilon_1^{\,2} = \Delta E_{\mathrm{kin}}$$

kinetic energy (E_{kin})
Result of work → bigger E_{kin}.

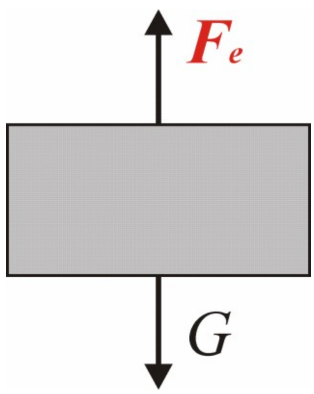
Application e.g.: at the discussion of x-ray tube or electron-microscope.

***

Work done against another force

e.g. elevation against force of gravity (G) (g acceleration of gravity)

$F_e = G = mg$



Result of work → „storable"

potential energy (E_{pot})

In gravitational field: $\Delta E_{\mathrm{pot}} = mg\Delta h$;

Power ("speed" of work done):

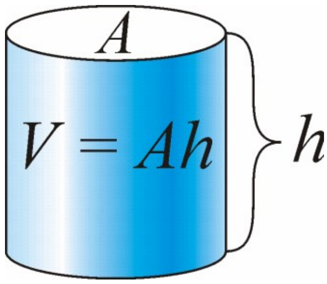
$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$ [W] = [J/s]

Static fluids (and gases) → **hydrostatics**

Pascal's principle
Pressure is transmitted undiminished in fluids because they are incompressible.
(hidraulic jack, brakes)

Hydrostatic pressure (originates from the weight of fluid)
In a static fluid on the Earth (simplest case):

$mg = V\rho g = Ah\rho g = F_{\text{weight}}$
(ρ density)



$p = F_{\text{weight}}/A = \rho g h$

Its consequence is the buoyant force (F_b):

Archimedes' principle

A body that is submerged in a fluid is buoyed up by a force:

$$F_b = \rho_{\text{fluid}} g V$$