Analysis of variancia (ANOVA)

A

B

C

Is there a difference between the groups?

No, There is only random deviation! This is the nullhypothesis.

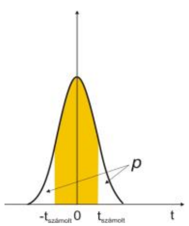
Why don't we do 2-sample t-tests?

The probability of the mistake increases rapidly with increasing number of groups!

Comparison of A – B and B – C and A – C. (not transitive)

No. of groups	No. of tests
3	3
4	6
5	10

How much is the chance of the mistake?



Suppose, that we reject the nullhypothesis in all cases.

p – probability being outside (**1-p**) – probability being inside <u>randomly</u>.

Question: How much is the probability to have mistake at least in one case?

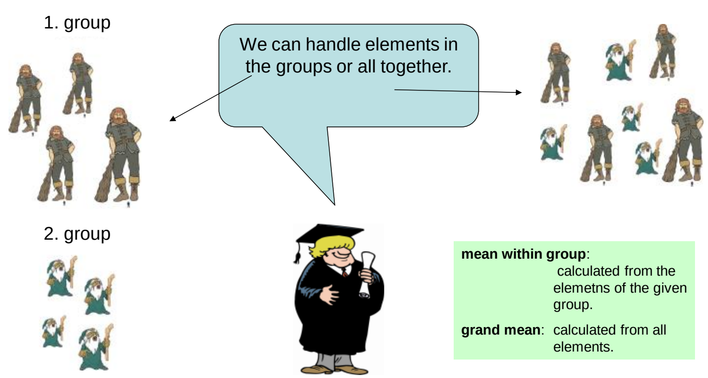
How much is the chance that at least one is outside randomly?

1 test: ***p*** (let it be 5%).
In the case of more than 1 test, the binomial distribution may be used to calculate.

$$1-(1-p)^3$$

In the case of 3 groups is about 15%!!!

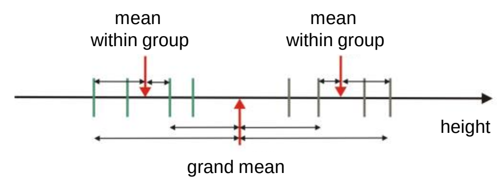
More than 2 groups



mean within group: calculated from the elemetns of the given group.

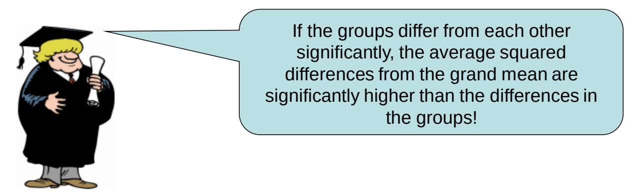
grand mean: calculated from all elements.

Components of the variance

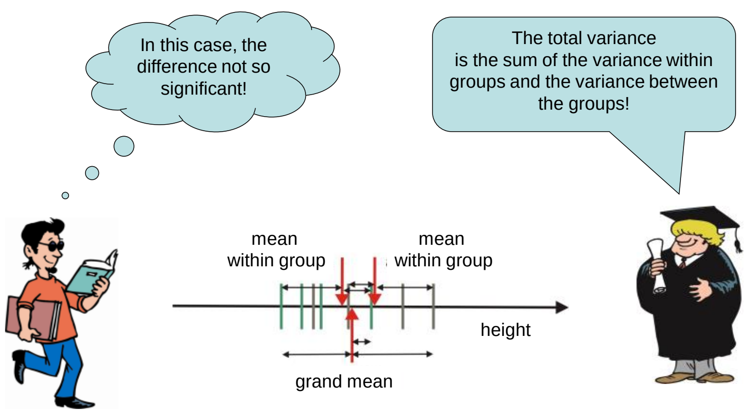


Remember:

The variance is proportional to the squared sum of the differences from the average!



The base of ANOVA



Components of the variance

	1. group	2. group	3. group
1	173	170	175
2	175	163	174
3	169	165	171
4	168		172
5			172
mean	171.25	166	172.8

grand mean = 170.58

(170-170.58) = (170-166) + (166-170.58)

(175-170.58) = (175-172.8) + (172.8-170.58)

$$(x_{i,j}-\bar{x})=(x_{i,j}-\bar{x}_j)+(\bar{x}_j-\bar{x})$$

between groups

within group (e.g. random) deviation

$\bar{x}$ - grand mean

$\bar{x}_j$ - mean in the jth group

Take the square!

$$(x-x_{ij})^2=(x-x_i)^2+(x_i-x_{ij})^2+2(x-x_i)(x_i-x_{ij})$$

$$\sum(x-x_{ij})^2=\sum(x-x_i)^2+\sum(x_i-x_{ij})^2+\sum 2(x-x_i)(x_i-x_{ij})$$

covariance = 0 (indepent events)

$$\sum 2(x-x_i)(x_i-x_{ij})=0$$

$$\sum(x-x_{ij})^2=\sum(x-x_i)^2+\sum(x_i-x_{ij})^2$$

between groups

within groups

(not necessary to know!)

We can decompose the variance!

Calculation of variances

Variance	Sum of squares	d.f.		F value
between groups	$SS_A=\sum_j n_j(\bar{x}_j-\bar{x})^2$	k-1	$MS_A=\frac{SS_A}{k-1}$	$F=\frac{MS_A}{MS_E}$
Within groups	$SS_E=SS_T-SS_A$	N-k	$MS_E=\frac{SS_E}{N-k}$	
Total	$SS_T=\sum_{i,j}(x_{i,j}-\bar{x})^2$	N-1		

$\bar{x}$ grand mean

$\bar{x}_j$ mean in the jth group

N – no. of all elements
k – no. of groups

Nullhypothesis

There is no difference between the groups.

The difference between the averages of the groups due to the random deviation.

Decision: On the base of the comparison of the variance between groups and within groups!

How can we compare them?

Comparison of variances? It was already discussed!

Really, in the case of 2-sample t-test.

$$F=\frac{MS_A}{MS_E}$$

Decision

- 1. If the probability of the random deviation is small (p ≤ α) – we ***reject*** the nullhypothesis.
- 2. If the probability of the random deviation is large (p > α) – we ***accept*** the nullhypothesis.

(After decision, if it is necessary,
we can do t-tests.)

Example

Groups	n. of elements	sum	mean	Variance
1	4	685	171.25	10.92
2	3	498	166	13
3	5	864	172.8	2.7

ANOVA

Factors	SS	df	MS	F	p-value	F crit.
Between groups	89.367	2	44.68	5.782	0.02427	4.257
Within group	69.55	9	7.728			
Total	158.92	11				

α = 0.05
p = 0.024

Decision:
we reject the nullhypothesis, there is a significant difference.

Conditions for ANOVA

- Task: comparison 3 or more independent groups.
- The variable has **normal distribution**.
- sd values are the same in the groups.

Kruskal-Wallis test

Rank data without separating into groups, than sum ranks in each group!

If the variable has no normal distribution!

Ranking

	1. group	2. group	3. group
1	173	170	175
2	175	163	174
3	169	165	171
4	168		172
5			172

value	163	165	168	169	170	171	172	172	173	174	175	175
rank	1	2	3	4	5	6	7.5	7.5	9	10	11.5	11.5

group	n. of elements	sum of the ranks
1	4	27.5
2	3	8
3	5	42.5

Nullhypothesis

There is no difference between the groups.

The difference between the averages of the groups due to the random deviation.

Which distribution is suitable?

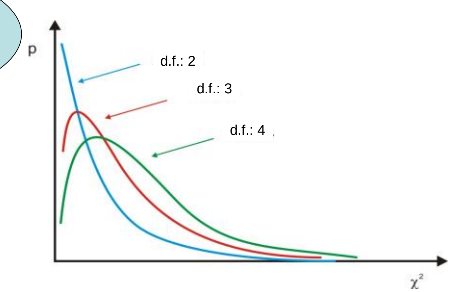
$$H = \frac{12}{N(N+1)} \sum_i \frac{R_i^2}{n_i} - 3(N+1)$$

N – no of elements
R_i – the sum of the ranks in the i-th group
n_i – no. of elements in the i-th group

χ^2-distribution

The value of $H \geq 0$!

H variable has χ^2-distribution!



Decision

- 1. If the probability of the random deviation is small ($p \leq \alpha$) – we ***reject*** the nullhypothesis.
- 2. If the probability of the random deviation is large ($p > \alpha$) – we ***accept*** the nullhypothesis.

Example

group	n_i	Sum of the ranks (R_i)
1	4	27.5
2	3	8
3	5	42.5

N = 12

$$H = \frac{12}{N(N+1)}\sum_i \frac{R_i^2}{n_i} - 3(N+1)$$

$$4.97 = \frac{12}{12(12+1)}\left(\frac{27.5^2}{4} + \frac{8^2}{3} + \frac{42.5^2}{5}\right) - 3(12+1)$$

df. = 3 – 1 = 2

α = 0.05
p = 0.083

Decision:

we accept the nullhypothesis, there is no significant difference.

Compare them!

	1. group	2. group	3. group
1	173	170	175
2	175	163	174
3	169	165	171
4	168		172
5			172

ANOVA

α = 0.05
p = 0.024

Decision:
we reject the nullhypothesis.

!!!

Kruskall-Wallis test

α = 0.05
p = 0.083

Decision:
we accept the nullhypothesis.

Hypothesis test?

- Set up the **nullhypothesis!**
- Look for a **variable with known distribution**.
- Calculate the **probability of the random deviation** on the base of the distribution.
- If it is smaller than the significance level **reject**, in opposite case **accept the nullhypothesis**.
- That's all!