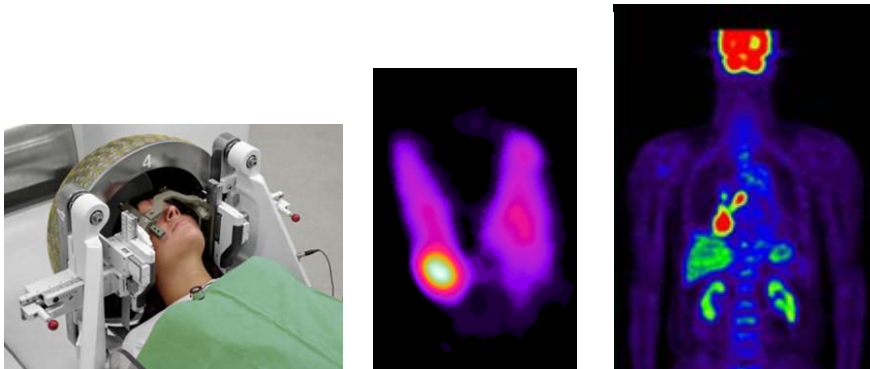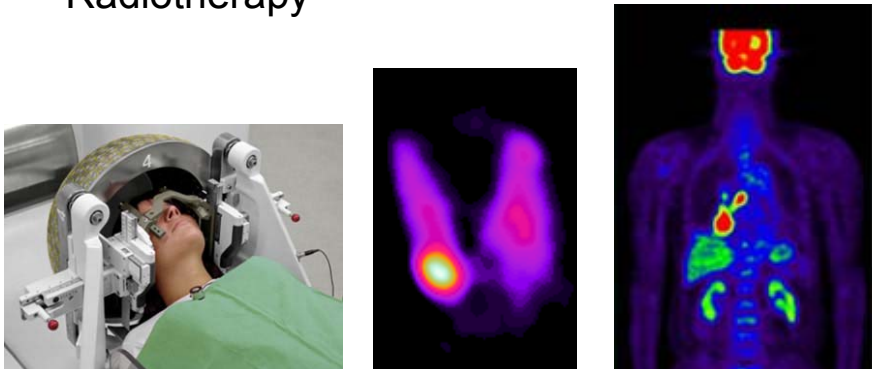# The atomic nucleus.
Radioactivity. Nuclear radiations

László Smeller
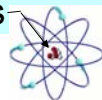
Why?

Medical applications of the nuclear radiation:
- Nuclear imaging
- Radiotherapy

Length scale of the nature
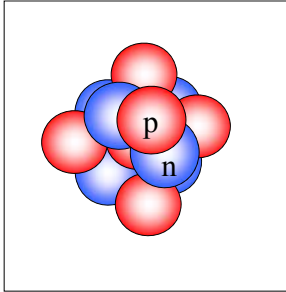
m		
10^0	meter	men
10^{-3}	millimeter	size of letters you can read
10^{-6}	micrometer	size of a cell (e.g. erytrocyte) $\varnothing\ 7\mu m$
10^{-9}	nanometer	protein
10^{-10}	– angstrom	diameter of an atom, bond length
		H atom $\varnothing \approx 1$ angstrom (Å)
10^{-12}	picometer	wavelength of the X-ray
10^{-15}	femtometer	size of the nucleus

The electrons and the nucleus

10^{-10} m

10^{-15} m

Electrons:
=> chemical properties

Nucleus:
=> radioactivity
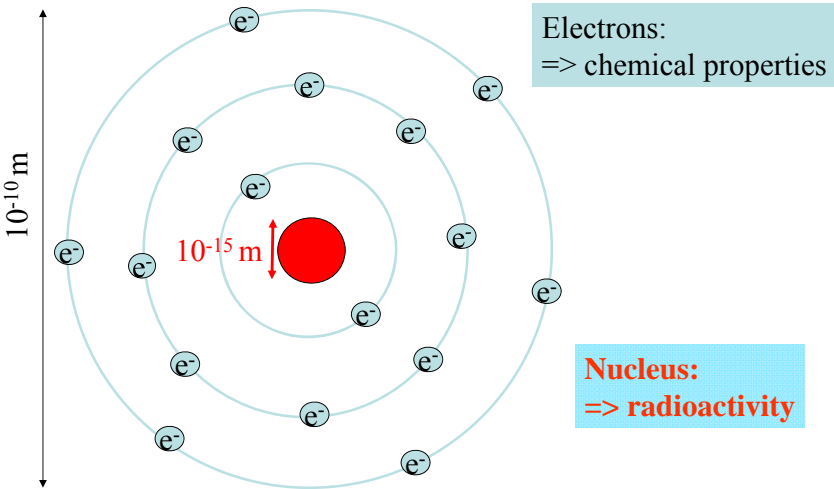
Structure of the nucleus

Elementary charge=$1.6 \cdot 10^{-19}$ C

	charge	mass
proton	**+1 e**	**1 atomic mass unit**
neutron	**0**	**1 atomic mass unit**

A (mass number) = number of protons + number of neutrons

Z (atomic number) = number of protons

$$^{99}_{43}\text{Tc}$$
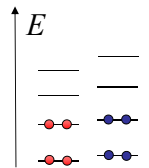
99 nucleon, 43 proton and 56 neutron

Stability of the nucleus

- Coulomb force: destabilization
  (electrostatic repulsion between the protons)

- Nuclear force: very strong
  attractive force
  acts only on short range (~fm)
  independent on the charge

- Quantized energy levels for the nucleus.

- Typical binding energy is in the MeV range
  eV=$1.6 \cdot 10^{-19}$ J

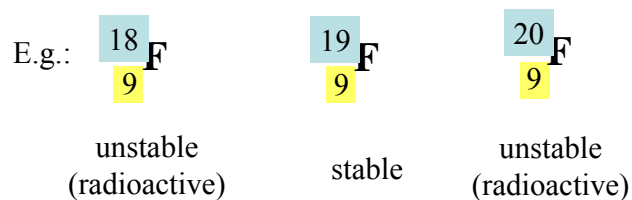# Isotopes

Variants of an element, where the
- number of protons is the same
- number of neutrons is different

$\Rightarrow$ the chemical properties are identical.

E.g.:

$$^{18}_{9}\text{F}$$

unstable (radioactive)

$$^{19}_{9}\text{F}$$

stable

$$^{20}_{9}\text{F}$$

unstable (radioactive)

isotope <-> radioactive isotope

Table of isotopes

Number of protons

Number of neutrons

Z=0-28 Part 1 of 2

Decay Q-value Range
Q(??)
Q(β-)>0
Q(β-)-S$_n$>0
Q(β-)>0 + Q(EC)>0
Stable to Beta Decay
Q(EC)>0
Q(EC)-S$_p$>0
Q(P)>0
Naturally Abundant

Number of protons

Number of neutrons
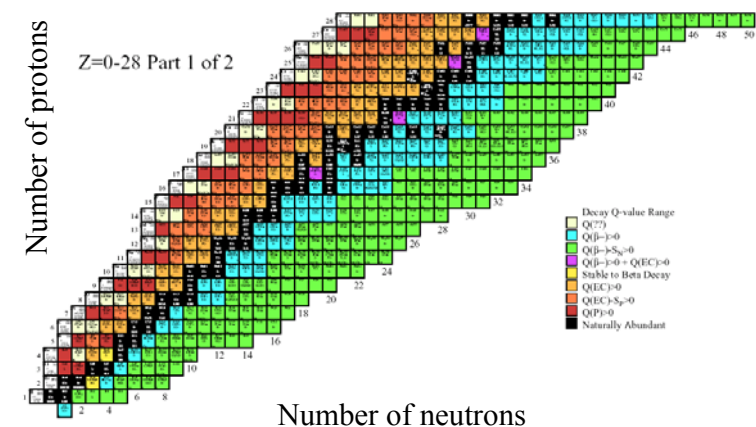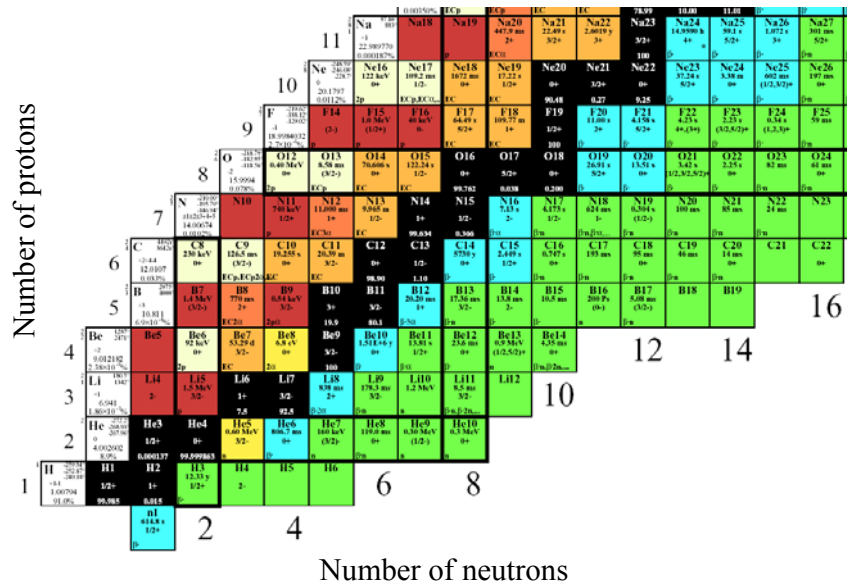
Number of protons

Number of neutrons
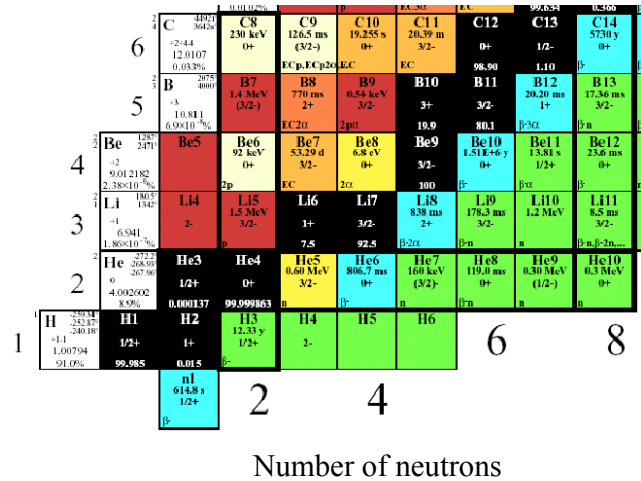
Radioactive decays and particles

α - decay   α - particle = $_2^4$He nucleus

β - decay : β⁻   β⁻ particle = electron
              β⁺   β⁺ particle = positron

Isomeric transition   γ-ray

K-electron capture   characteristic x-ray photon

α - decay

α - decay: an α particle (⁴He nucleus) will be emitted
             typical for the heavy atoms

$$_Z^A X \longrightarrow {}_{Z-2}^{A-4} Y + {}_2^4 \alpha$$

e.g. $_{88}^{226} Ra \longrightarrow {}_{86}^{222} Rn + {}_2^4 \alpha$

Line spectrum
$E_\alpha \sim$ MeV

β⁻ - decay

neutron surplus

$$^A_Z X \longrightarrow ^A_{Z+1} Y + ^0_{-1}\beta + \bar{\nu}$$

$$^1_0 n \rightarrow ^1_1 p + ^0_{-1}\beta + \bar{\nu}$$
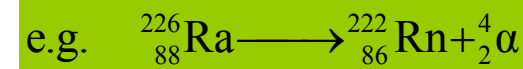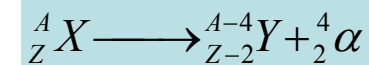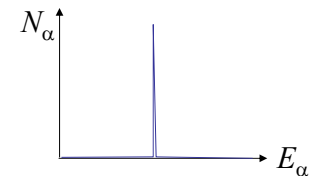
remains in the nucleus   leave the nucleus

e.g.: $^{20}_9 F$ $^{32}_{15} P$ $^{59}_{26} Fe$ $^{131}_{53} I$
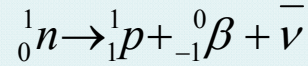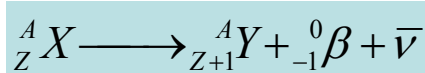
e.g.: $^{32}_{15}\text{P} \longrightarrow ^{32}_{16}\text{S} + ^0_{-1}\beta + \bar{\nu}$

β⁻-ray

continous energy spectrum

$\beta^- = ^0_{-1}\beta = e^-$

N_β E_{max} E_β

β⁺ - decay

proton surplus

$$^A_Z X \longrightarrow ^A_{Z-1} Y + ^0_{+1}\beta + \nu$$

$$^1_1 p \rightarrow ^1_0 n + ^0_{+1}\beta + \nu$$
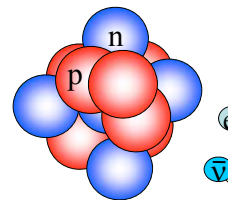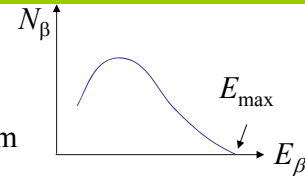
remains in the nucleus   leave the nucleus

e.g.: $^{11}_6 C$ $^{15}_8 O$ $^{18}_9 F$ $^{52}_{26} Fe$

e.g.: $^{30}_{15}\text{P} \longrightarrow ^{30}_{14}\text{Si} + ^0_{+1}\beta + \nu$

β⁺-ray
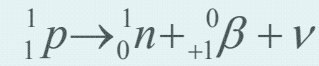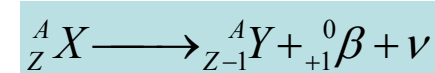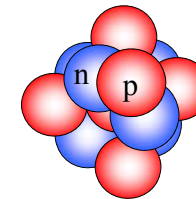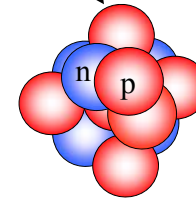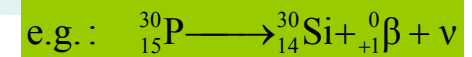
continous energy spectrum

These isotopes must be produced artificially (e.g. in cyclotron)

Prompt γ-radiation

The daughter nucleus has an energetically unfavoured arrangement of nucleons.

The surplus energy will normally be emitted immediately (<ps) in form of the γ radiation

E α or β   γ
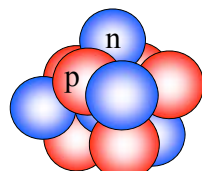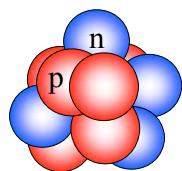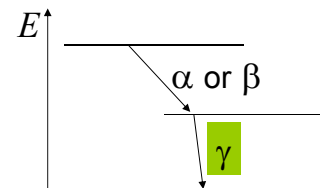
Atomic number, mass number are unchanged.

Isomeric transition

If the excited state of the daughter nucleus is metastable, the γ-radiation will be emitted later.

The parent and daughter atoms can be separated: the daughter atom emits **only γ-radiation!**

=> **Isotope diagnostics (nuclear imaging)**

E.g.: ^{99m}Tc

$$^{99}_{42}Mo \xrightarrow{\beta^-} ^{99m}_{43}Tc \xrightarrow{\gamma} ^{99}_{43}Tc$$

66 h   6 h

K-capture (inverse β-decay)

$$_1^1 p + {}_{-1}^0 \beta \rightarrow {}_0^1 n + \nu$$
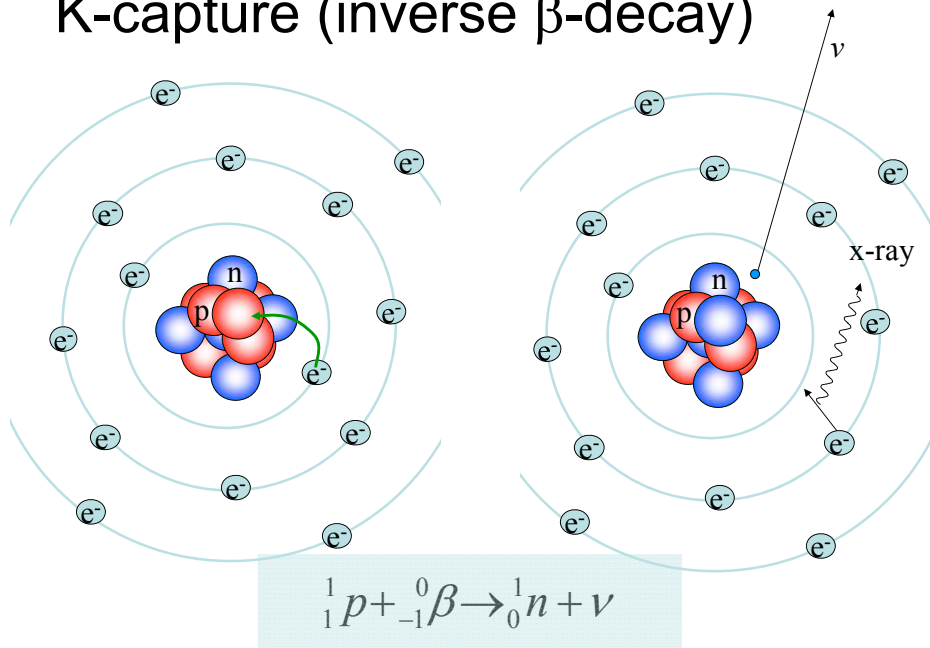
Some examples of the decay paths
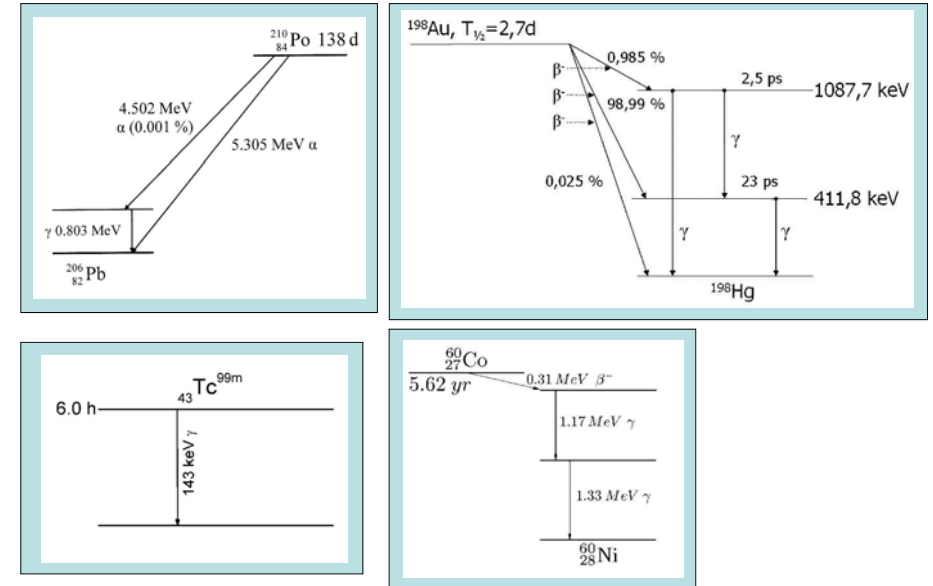
Characteristics of radioactive decays in general

activity — characterizes the source

half life time — characterizes the speed of the decay*

particle energy — characterizes the radiation*

*depends on the type of the isotope

Activity (Λ)

$$\Lambda = \left| \frac{dN}{dt} \right| \quad \left(= \left| \frac{\Delta N}{\Delta t} \right| \right)$$
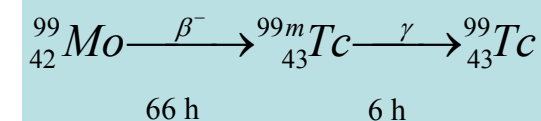
N = Number of undecayed atoms
t = time
ΔN = Number of decays during Δt time

Activity= number of decays in a unit time

unit: becquerel Bq
1 Bq= 1 decay/sec

kBq, MBq, GBq, TBq

immeasurably small

level of natural activity

in vivo diagn.

careful work with it!

activity applied in radiotherapy
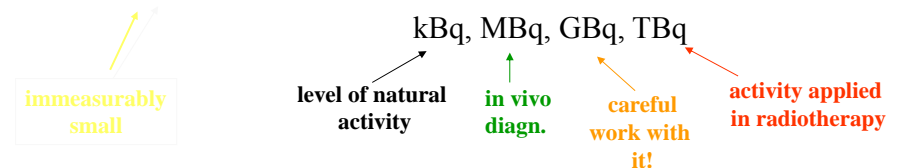
Law of radioactive decay

$$\Delta N = -\lambda N \Delta t$$

N: Number of undecayed nuclei

λ: decay constant (probability of the decay [1/s])

$1/\lambda = \tau$ average lifetime

$$\frac{dN}{dt} = -\lambda N$$

Differential equation

solution:

$$N(t) = N_0 e^{-\lambda t}$$ Exponential decrease

number of undecayed atoms at $t=0$

An example

- At start: N_0=10000   (number of atoms
  λ=0.1 $^1/_S$ (decay probability)

- after 1 sec: 9000   (10000x0.1=1000 decayed)
- after 2 sec: 8100   (9000x0.1=900 decayed)
- after 3 sec: 7290   (8100x0.1=810 decayed)
- after 4 sec: 6561   (7290x0.1=729 decayed)
- ….

An example

- At start: N_0=10000   (number of atoms
  λ=0.1 $^1/_S$ (decay probability)

- after 1 s: 9000
- after 2 s: 8100
- after 3 s: 7290
- after 4 s: 6561
- ….
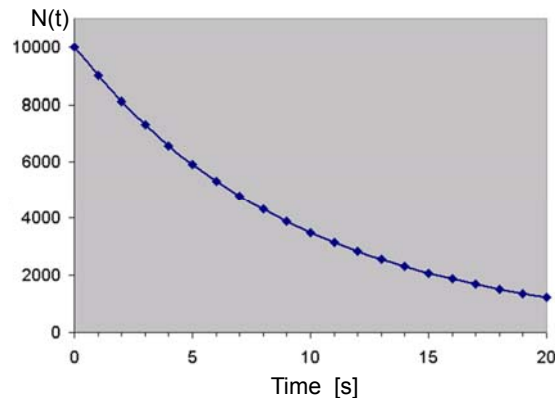
Law of radioactive decay

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}}$$
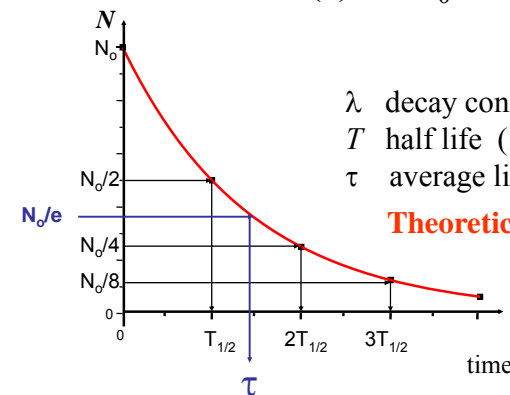
λ decay constant (probability of the decay)
T half life $(T_{1/2})$
τ average lifetime

Theoretically never decreases to zero !

$$\lambda = \frac{\ln 2}{T} = \frac{0,693}{T}$$

Decrease of the activity as a function of time

$$\Lambda = \left| \frac{dN}{dt} \right| \qquad \frac{dN}{dt} = -\lambda N \qquad N(t) = N_0 e^{-\lambda t}$$

$$\Lambda = \lambda N$$

$$\Lambda(t) = \Lambda_0 e^{-\lambda t} = \Lambda_0 2^{-\frac{t}{T}}$$

Λ decreases on the same mode as N!

Theoretically never decreases to zero!

During about 10 T the activity decreases to its 1/1000 (e.g GBq->MBq)

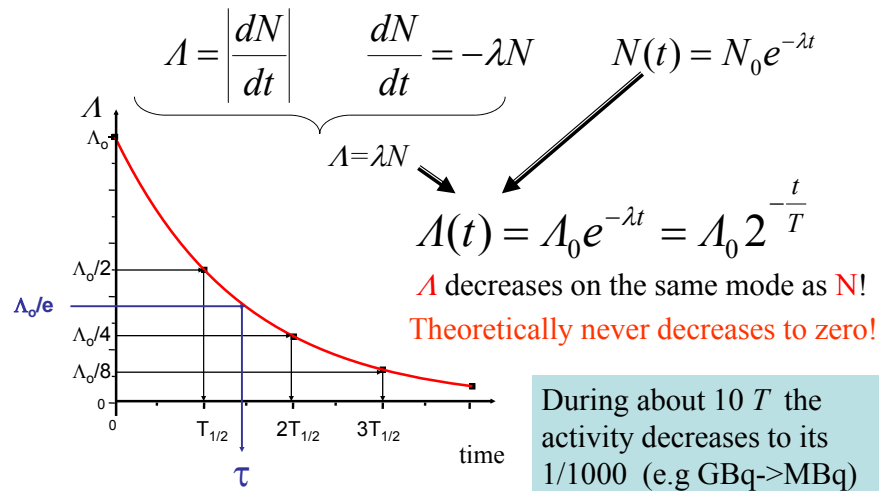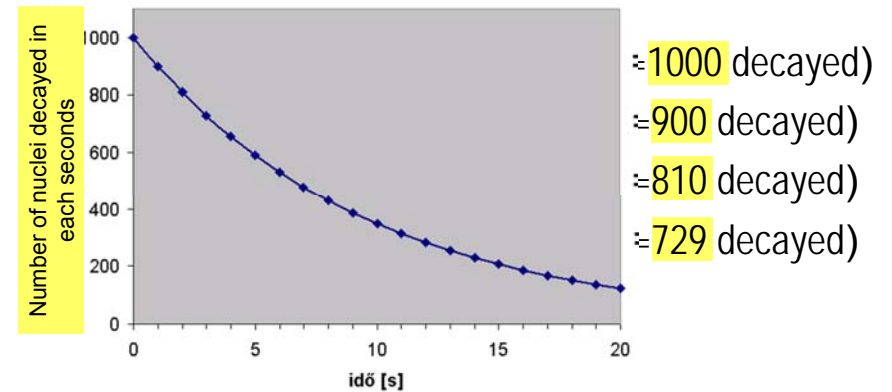(y-axis: Λ, values Λ_o, $\Lambda_o/2$, Λ_o/e, $\Lambda_o/4$, $\Lambda_o/8$, 0; x-axis: 0, $T_{1/2}$, $2T_{1/2}$, $3T_{1/2}$, time; τ)

An example

• At start: N_0=10000 (number of atoms
λ=0.1 $^1/_s$ (decay probability)

≑1000 decayed)
≑900 decayed)
≑810 decayed)
≑729 decayed)

(graph: Number of nuclei decayed in each seconds vs idő [s])

Few examples for half life

^{232}Th	1.4·10^{10} y	^{60}Co	5.3 y
^{238}U	4.5 ·10^9 y	^{59}Fe	1.5 m
^{40}K	1.3 ·10^9 y	^{56}Cr	1 m (28 d)
^{14}C	5736 y	^{131}I	8 d
^{137}Cs	30 y	^{99m}Tc	6 h
^{3}H	12.3 y	^{18}F	110 min
		^{11}C	20 min
		^{15}O	2 min
		^{222}Th	2.8 ms

Don't learn these numbers!

Typical energy levels in the microworld

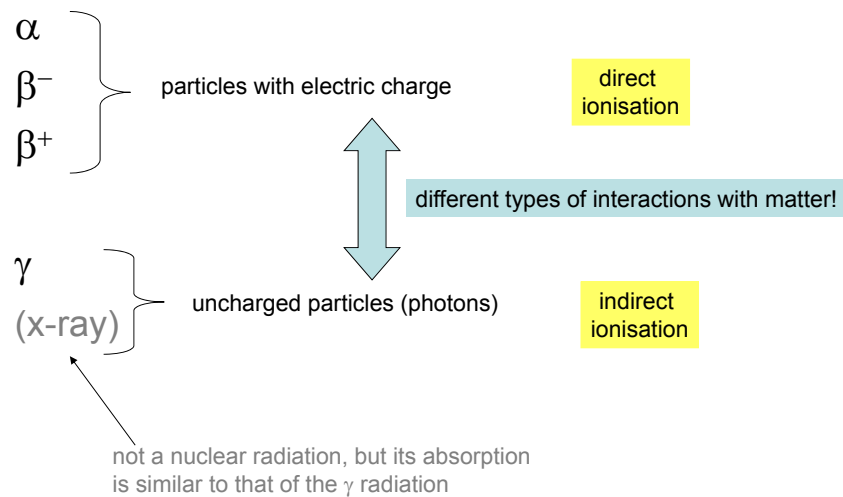| Excitation of the outer electrons	Electron transition between inner electrons	Transformation of the nucleus (decay)
eV (aJ)	keV (fJ)	MeV (pJ)
light	X-ray	Nuclear radiation α, β, γ

Absorption of the nuclear radiation

α
β^- particles with electric charge   direct ionisation
β^+

different types of interactions with matter!

γ
(x-ray)   uncharged particles (photons)   indirect ionisation

not a nuclear radiation, but its absorption
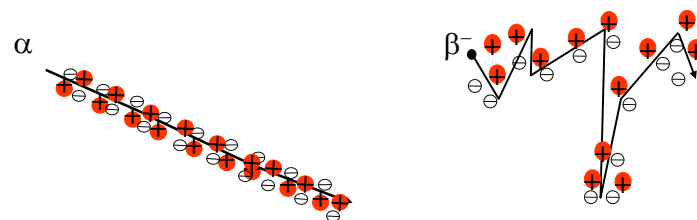is similar to that of the γ radiation
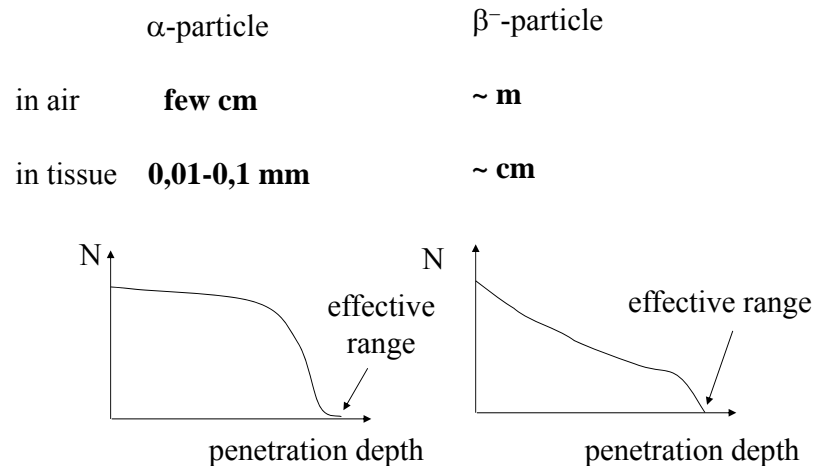
Absorption of the charged particles

Ionizing during the path => continuous decrease of the particle energy
The energy after a given path length decreases to the thermal value
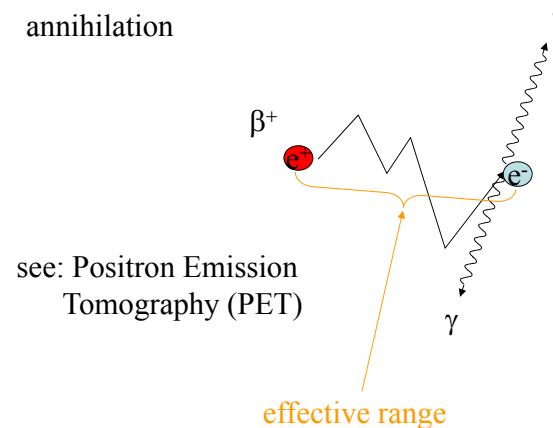
effective range

α

β^-

Effective range

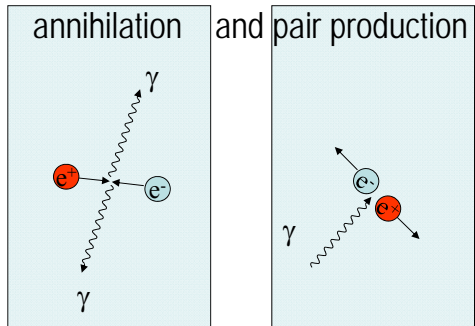|	α-particle	β^--particle
in air	**few cm**	**~ m**
in tissue	**0,01-0,1 mm**	**~ cm**

N

effective range

penetration depth

N

effective range

penetration depth

β^+-radiation

annihilation

γ

β^+

e⁻

see: Positron Emission
Tomography (PET)

γ

effective range

Electron and positron

- particle - antiparticle
- same mass,
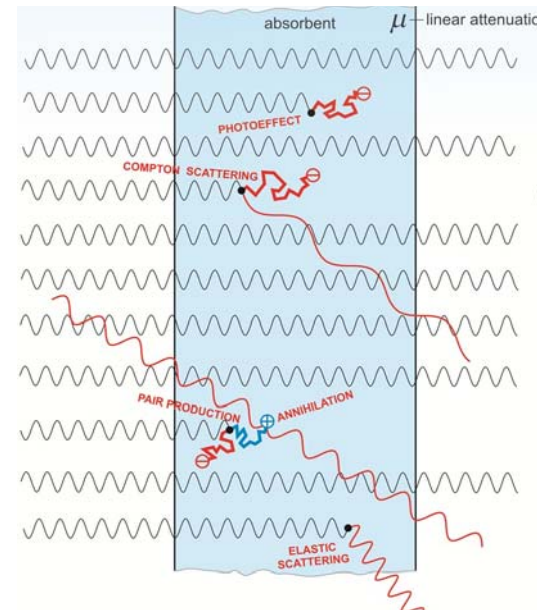- charge: same value, but differen

annihilation and pair production

Einstein:

mass-energy ekvivalence

$$E=mc^2$$
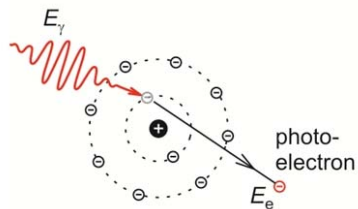
$m_e c^2 = 511 \text{ keV} \approx 0.5 \text{ MeV}$
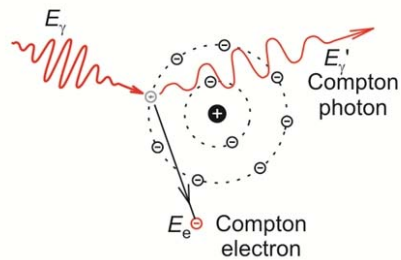
Absorption of the γ-radiation (and x-ray)

Absorption pricesses happen accidentally :

Photoeffect,
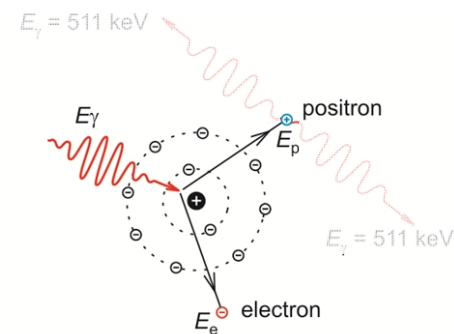Compton-effect,
Pair production,
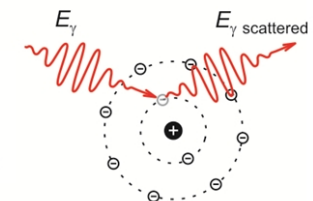(elastic scattering)

Photoeffect

E_γ

photo-electron

E_e

**Compton effect
Compton-scattering**

E_γ

E_γ' Compton photon

E_e Compton electron

Pair production

E_γ = 511 keV

E_γ

positron

E_p

E_γ = 511 keV

E_e electron

elastic scattering

E_γ

$E_{\gamma \text{ scattered}}$

Attenuation of the γ-radiation and x-ray

never reaches zero!

$J_0 \rightarrow \boxed{} \rightarrow J$

$$J = J_0 e^{-\mu x}$$

$\underset{\text{x}}{\longleftrightarrow}$

no finite effective range!
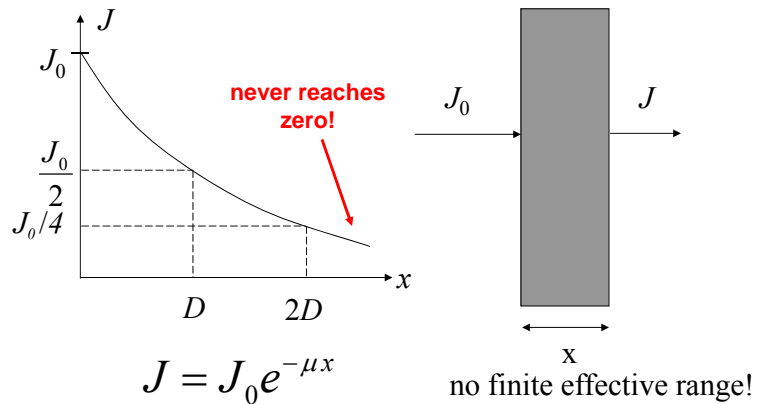
few rules of thumb: $x_{1/10} = 3{,}33\,D$ $x_{1/1000} = 10\,D$
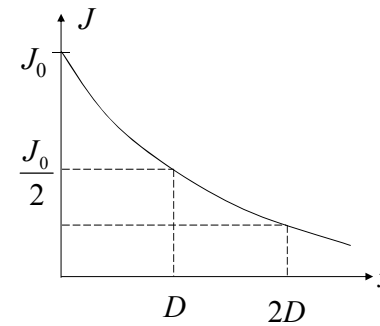
$$J = J_0 e^{-\mu x}$$
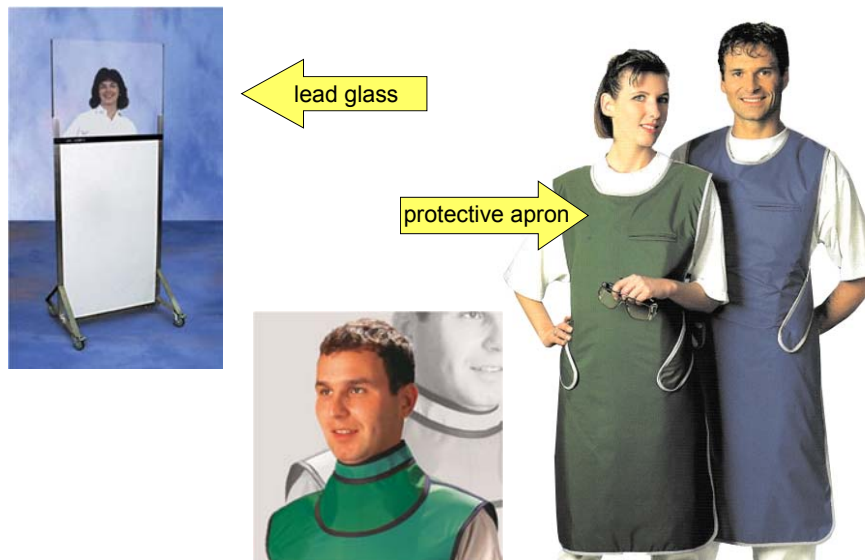
μ: (linear) attenuation coefficient

its units are: 1/m, 1/cm

$\delta = \dfrac{1}{\mu}$ „penetration depth"

Intensity decreases to the e-th part (c.a. 37%)

μ(material, number of absorbing centers, energy of the radiation)
$= \mu(\text{material}, \rho, E_{photon}) \sim \rho$

$\mu_m = \dfrac{\mu}{\rho}$ mass attenuation coefficient

Applications (attenuation)

lead glass

protective apron

Applications: isotopes and nuclear radiation

Gamma rays