Dynamics

Motion, deformation and their background

Newton: **motion** is natural **state**.

One of the fundamental physical quantities of the topic is the momentum (p).

In classical case it is the product of the mass (m) and the velocity (υ) of the body.

$$p = m\upsilon$$

vector quantity

Newton's laws of motion

II. For the change of momentum needs **force** (F).

$$\Delta p = \Delta m\upsilon = F\Delta t \text{ (impulse of a force)}$$

or

$$\frac{\Delta m\upsilon}{\Delta t} = m\frac{\Delta\upsilon}{\Delta t} = ma = F$$

If no forces are exerted (or $F = 0$)

$\Delta m\upsilon = 0$, means $p = m\upsilon =$ constant.

I. Momentum is conserved (momentum conservation)

law of inertia

III. $F = -F_{\text{versus}}$ interaction

A single force cannot exist.

Forces are always directed to contrary parts.

law of action and reaction

Application e.g.:

at the pressure of ideal gases, (see later)

at the explanation of annihilation (see in 2$^{\text{nd}}$ semester PET).

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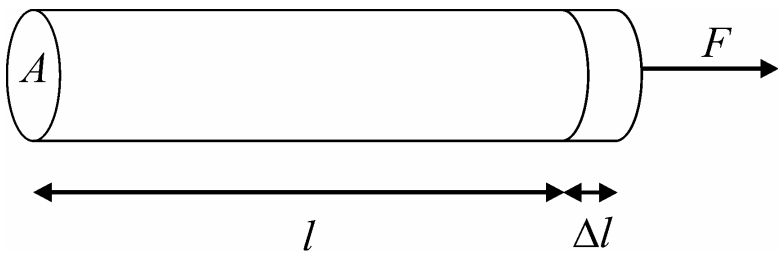
But
force may result in **deformation**.

Simplest deformation is the **elongation.**
tensile strain: $\Delta l/l$
Hooke's law



$$F = AE\frac{\Delta l}{l}$$

$$\frac{F}{A} = E\frac{\Delta l}{l}$$

F/A the stress (tensile stress), but

it could be compressive stress or pressure (p[Pa])

Coefficient: Young's modulus (E[Pa])

e.g.

collagen fiber 0,3–2,5 GPa, bone 10–20 GPa

Similar to the case of spring: $F_{spring} = Dx$ (if $x \equiv \Delta l$, and $D \equiv AE/l$)

More general (compressive stress):

$$\Delta p = -K\frac{\Delta V}{V}$$

K a bulk modulus,
$1/K = \kappa$ compressibility (e.g. $\kappa_{steel} = 0{,}006\ \mathrm{GPa}^{-1}$)

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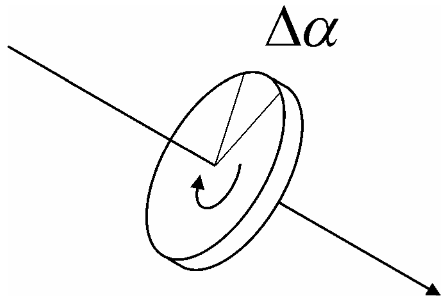
Newton's laws for rotation

similarly to the momentum ($m\upsilon$) here the fundamental physical quantity is the angular momentum ($\Theta\omega$), where

Θ is moment of inertia, rotational analog of the mass,
ω is angular velocity,

$$\omega = \frac{\Delta\alpha}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

period (T), frequency (f)
(ω angular frequency)



I. $\Theta\omega$ = constant (conservation of angular momentum)
(see: rotating skater)

II. For the change of angular momentum needs **torque** (M).

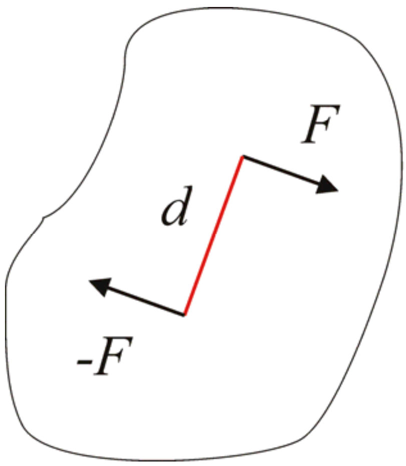
$$\frac{\Delta\Theta\omega}{\Delta t} = M$$

Equilibrium, if

$F_{resultant} = 0$ **and** $M_{resultant} = 0$ simultaneously.

Then: $m\upsilon$ = constant

and $\Theta\omega$ = constant

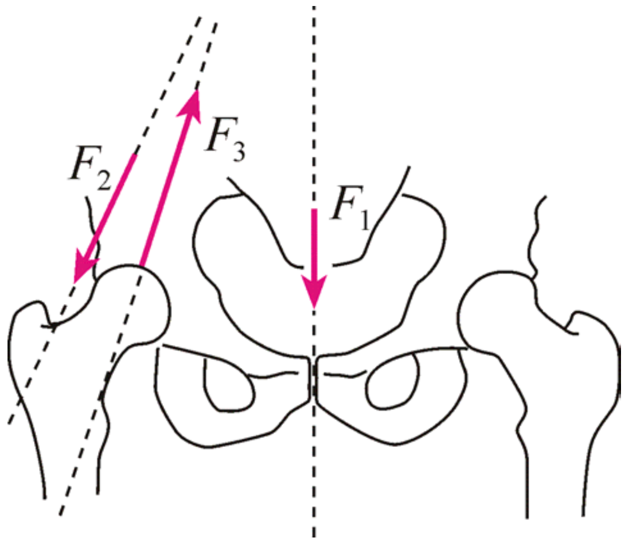


$F_{resultant} = 0$

$M = Fd$

Statics

See: orthopaedy



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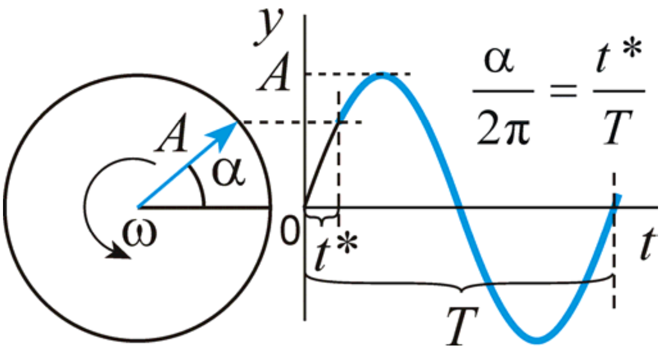
Uniform circular motion

Direction of the velocity (or momentum) changes only.
The body accelerates (a_{cp} [m/s^2]), but its speed does not increase.

$$a_{cp} = \frac{\upsilon^2}{r} = \omega^2 r$$

Dynamical condition: $\upsilon \perp F = ma_{cp}$

Harmonic motion



Projection of
uniform circular motion
($\alpha = \omega t = 2\pi t/T = 2\pi f t$)

$$y = A \sin \omega t$$

Dynamical condition: $F = -Dx = ma$

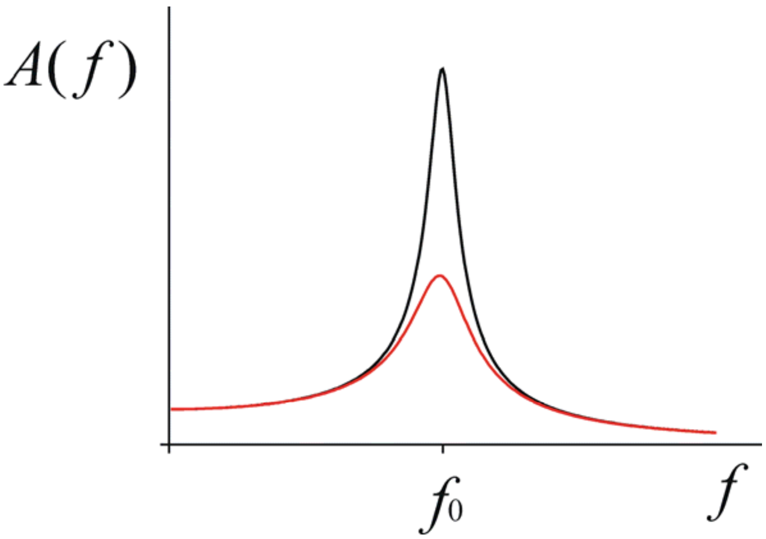
$$\omega = \sqrt{\frac{D}{m}}$$

Forced oscillation, resonance

($\omega = 2\pi f$)

$$A(f) \sim \frac{1}{(f - f_0)^2 + K}$$

K is characteristic for damping



Application e.g.: at the interpretation of different spectra (ESR, NMR); at the explanation of AFM and MRI (see in 2nd semester).

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Waves (harmonic waves)

Propagation of oscillation
(Waving sea, or waving public in a stadium during a boring football match.)

$y = A \sin \omega t$ $\qquad$ $y = A \sin (\omega t + \varphi)$ $\qquad$ $y = A \sin (\omega t + kx)$

Phase (angle, φ) depends not only on time (t), but on space (x) as well
Two important parameters: wavelength (λ), period (T)

$$\varphi(x) = \frac{2\pi}{\lambda} x = kx \qquad \varphi(t^*) = \frac{2\pi}{T} t^* = \omega t^*$$

(k wave number)

Connection between them:

$$c = \frac{\lambda}{T} = \lambda f$$

c is the speed of propagation.

The most important phenomenon in connection with waves is the **interference**.
(We will study it in physical optics part more detailed.).

Application e.g.: at the discussion of different radiations
US, EMR (see in 2nd semester).

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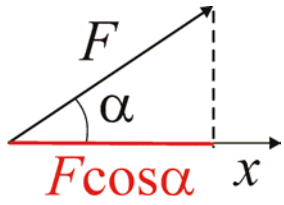
Work

Work (W) is a product of displacement (Δx) and the projection of force (F) to the direction of displacement.



$W = \Delta x \, F\cos\alpha$ [Nm] or [J]

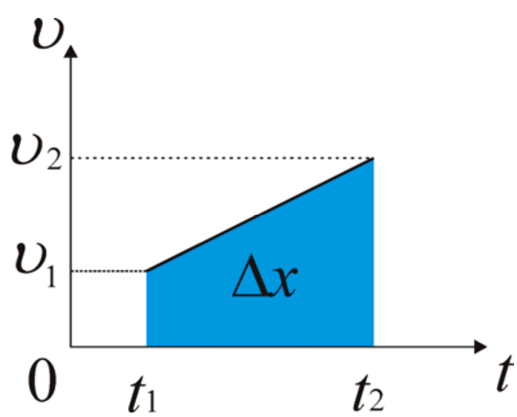
Permanent acting force without displacement ($\Delta x = 0$);
or $\alpha = \pi/2$ (means $\cos\alpha = 0$), then $W = 0$ (in mechanics)

Work-energy theorem

Force is constant (and $\alpha = 0$).

$$F = ma = m\frac{\Delta \upsilon}{\Delta t} = m\frac{\upsilon_2 - \upsilon_1}{t_2 - t_1}$$

The displacement would be $\Delta x = \upsilon \Delta t$, but υ also changes.



$$\Delta x = \frac{(\upsilon_1 + \upsilon_2)(t_2 - t_1)}{2}$$

$$W = F\Delta x = m\frac{\upsilon_2 - \upsilon_1}{t_2 - t_1}\frac{(\upsilon_1 + \upsilon_2)(t_2 - t_1)}{2} = m\frac{(\upsilon_2 - \upsilon_1)(\upsilon_1 + \upsilon_2)}{2}$$

$$W = m\frac{(\upsilon_2{}^2 - \upsilon_1{}^2)}{2} = \frac{1}{2}m\upsilon_2{}^2 - \frac{1}{2}m\upsilon_1{}^2 = E_{\mathrm{kin}\,2} - E_{\mathrm{kin}\,1} = \Delta E_{\mathrm{kin}}$$

kinetic energy (E_{kin})
Result of work → bigger E_{kin}.

Application e.g.: at the discussion of x-ray tube or electron-microscope (see in 2nd semester).

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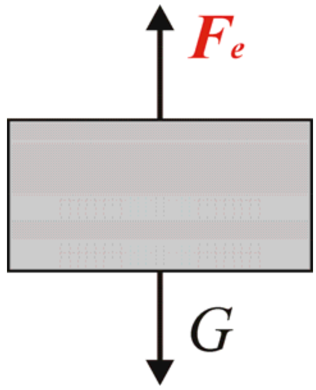
Work done against another force

e.g. elevation against force of gravity (G)
(g acceleration of gravity)

$F_e = G = mg$



Result of work → „storable”

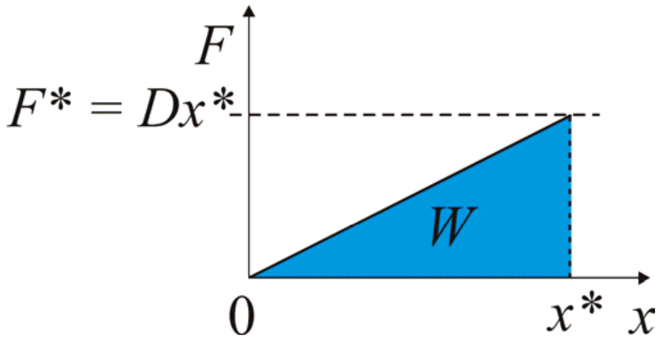
potential energy (E_{pot})

In gravitational field: $\Delta E_{pot} = mg\Delta h$;

Potential energy of a spring

$$\Delta E_{pot} = W$$

$$W = \frac{Dx^* x^*}{2} = \frac{1}{2} D(x^*)^2$$



e.g. elastic blood vessels (conservation of mechanical energy)

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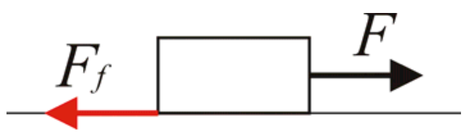
Power (“speed” of work done):

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

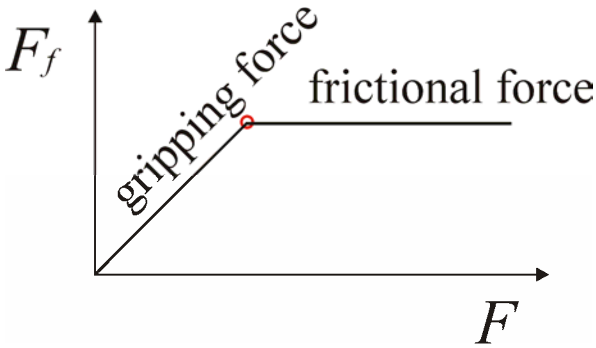
[W] = [J/s]

Friction



Is **not** the energy conservation valid?

(see thermodynamics)



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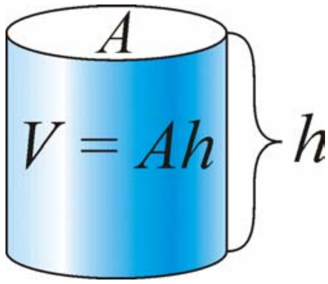
Static fluids (and gases) → **hydrostatics**

Pascal's principle
Pressure is transmitted undiminished in fluids because they are incompressible. ($\kappa_{\text{water}} = 0{,}5$ GPa^{-1})
(hidraulic jack, brakes)

Hydrostatic pressure (originates from the weight of fluid)
In a static fluid on the Earth (simplest case):

$mg = V\rho g = Ah\rho g = F_{\text{weight}}$
(ρ density)



$p = F_{\text{weight}}/A = \rho gh$

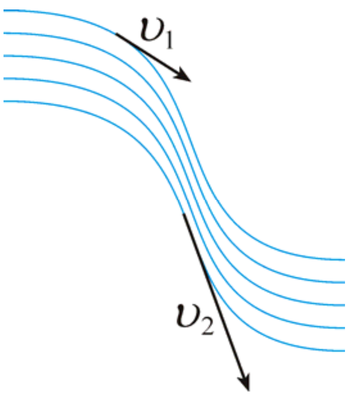
Its consequence is the buoyant force (F_{b}):
Archimedes' principle

A body that is submerged in a fluid is buoyed up by a force:

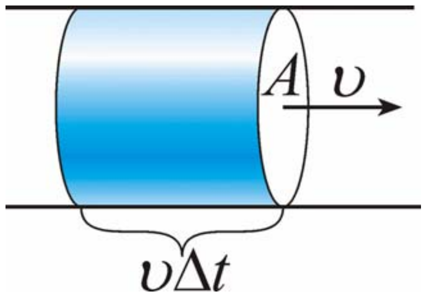
$$F_{\text{b}} = \rho_{\text{fluid}} g V$$

Flow of fluids (and gases) → **hydrodynamics**
(constant streamlines in time: **stationary flow**)

Flow velocity and **volumetric flow-rate**:



$$I_V = \frac{\Delta V}{\Delta t} = \frac{A\upsilon\Delta t}{\Delta t} = A\upsilon$$

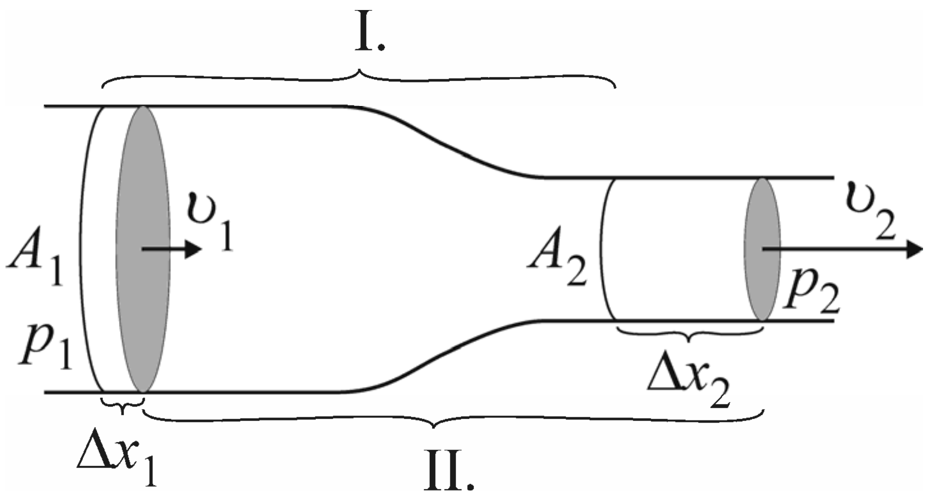


Law of continuity $\rightarrow$ I_V = constant (conservation of matter)
e.g. blood vessels

blood vessel	*diameter* (cm)	*number of branches*	A_{total} (cm^2)	υ (cm/s)
aorta	2.4	1	4.5	23
arteries	0.4	160	20	5
arterioles	0.003	$5.7 \cdot 10^7$	400	0.25
capillaries	0.0007	$1.2 \cdot 10^{10}$	4500	0.022
venules	0.002	$1.3 \cdot 10^9$	4000	0.025
veins	0.5	200	40	2.5
venae cavae	3.4	2	18	6

Bernoulli's law (application of work-energy theorem)
for **ideal** fluids

$F_1 \Delta x_1 \quad F_2 \Delta x_2$



I_V

$$p_1 A_1 \upsilon_1 \Delta t - p_2 A_2 \upsilon_2 \Delta t = (p_1 - p_2)\Delta V = \frac{1}{2} m \upsilon_2^2 - \frac{1}{2} m \upsilon_1^2$$

$$p_1 + \frac{1}{2}\rho \upsilon_1^2 = p_2 + \frac{1}{2}\rho \upsilon_2^2 = \text{constant} \qquad \text{(consequences)}$$

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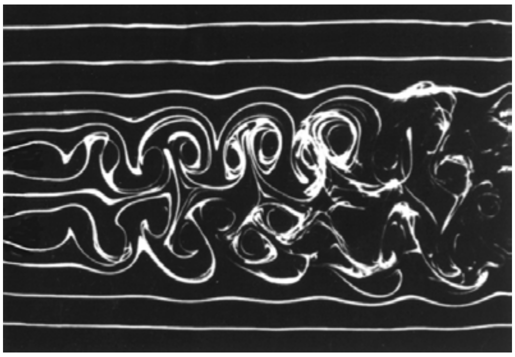
Internal friction

real fluids

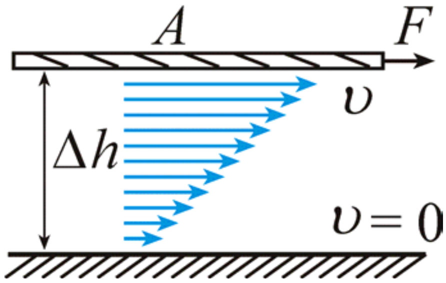
turbulent flow →

laminar flow →



Newton's law of friction:

$$F = \eta A \frac{\Delta \upsilon}{\Delta h}$$



$\Delta\upsilon/\Delta h$ velocity drop

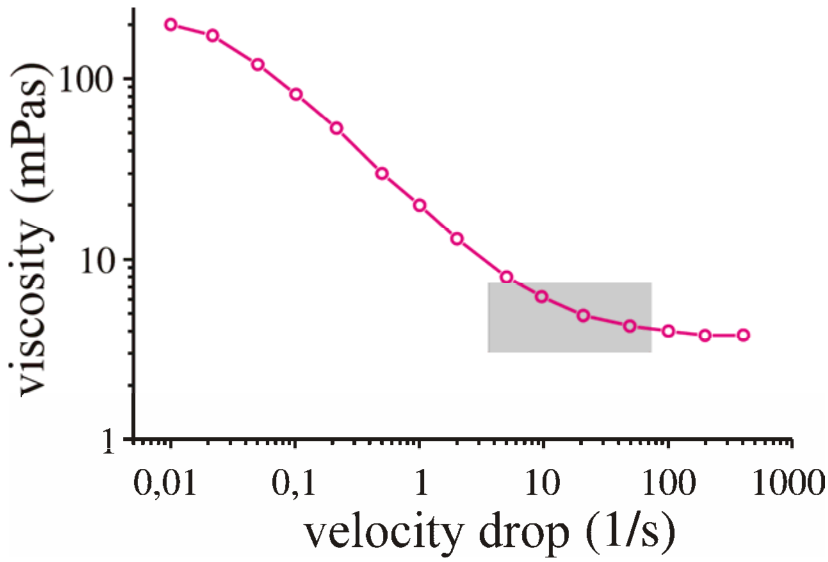
η[Pas] **internal friction coefficient** or **viscosity**

Viscosity of a few substances (measured at normal conditions):

substance	viscosity [Pas]
air	10^{-5}
water	$\mathbf{10^{-3}}$
honey	10^{1}
bitumen	10^{8}
glass	10^{40}

Validity: **newtonian fluid** **non-newtonian fluid**

(e.g. change of viscosity of blood)



Application e.g.:
blood circulation
(see in 2nd semester).