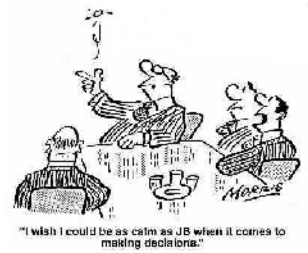
Information theory

Concept of information (through an example)

Information content of data streams, information rate

Entropy and information



Concept of information (through an example)

Intuitive concept:

"informare" (Lat.) : „to give form to the mind", or to teach, instruct somebody

Thus: „We can only change our minds, when we recieve ***information***."

Or:
„a type of input to an organism or designed device" : Ecology, sensory input
(Smell of food → movement of animal)

Or:
„information is any type of pattern that influences the formation or transformation of other patterns."
(RNA sequence → Protein structure)

Transmitting information – information content

Event and information:
What happened?

„Information content" of events:

-It is light traffic this morning

-It will rain tomorrow.

-I have won the lottery!

How can we *encode* information?

Transmitting information – information **coding**

in general

Information source — Which event occured from a set of possibilities?

↓

Information reciever
destination — (news)

Transmitting information – information **coding**

in general

Information source

Which event occured from a set of possibilities?

Transmission channel

Information reciever
destination

(news)

Transmitting information – information **coding**

in general

Information source

Which event occured from a set of possibilities?

encoding

Encoding: We represent **events** with **NUMBERS**

Transmission channel

decoding

Decoding: We reconstruct **events** from **NUMBERS**

Information reciever
destination

(news)

Transmitting information – information **coding**

in general

Information source

encoding

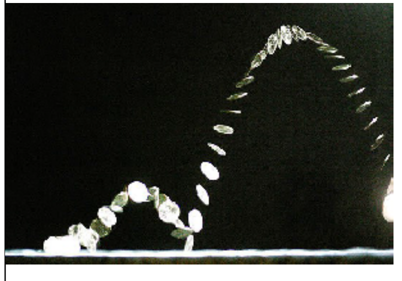
Transmission channel

decoding

Information reciever
destination

an example

Tossing a dime



Head or Tail?

Transmitting information – information **coding**

in general

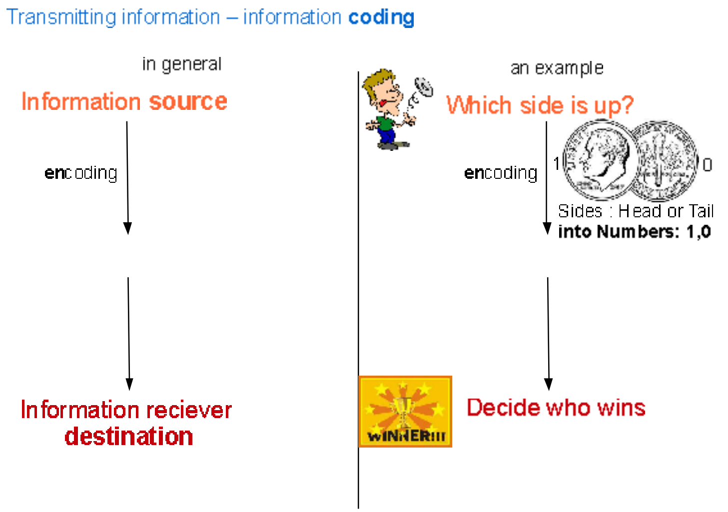
Information source

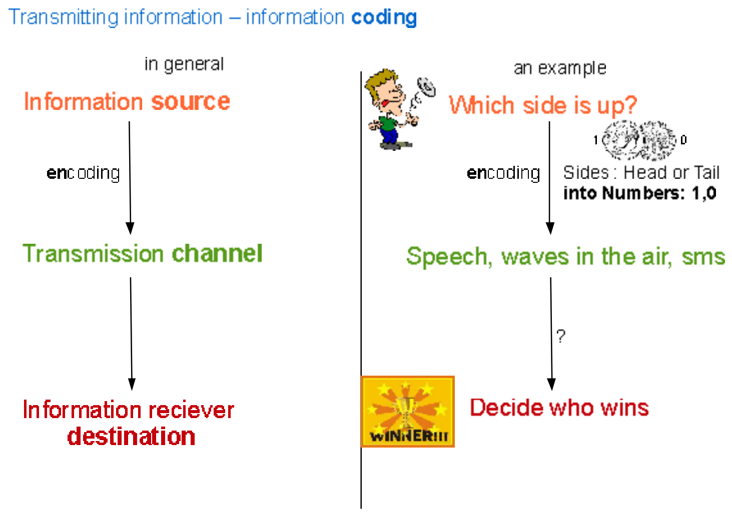
Information reciever
destination

an example

Which side is up?

Decide who wins


Transmitting information – information coding
in general
Information source
encoding
Information reciever
destination
an example
Which side is up?
encoding
1
0
Sides : Head or Tail
into Numbers: 1,0
WINNER!!!
Decide who wins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Speech, waves in the air, sms
?
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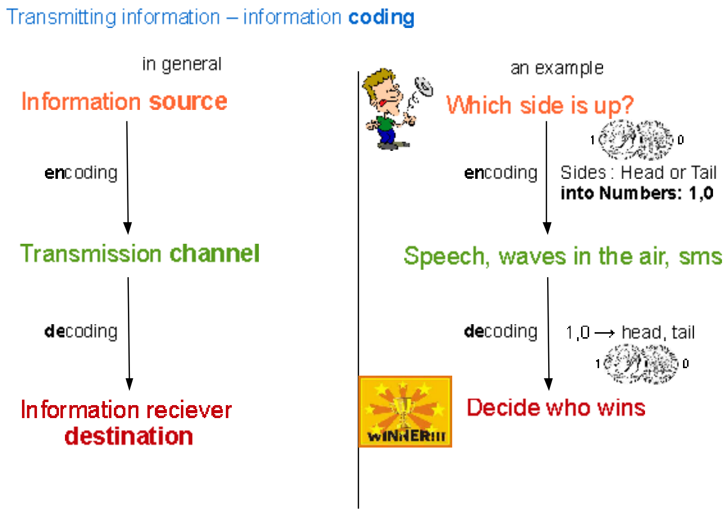
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1,0 → head, tail
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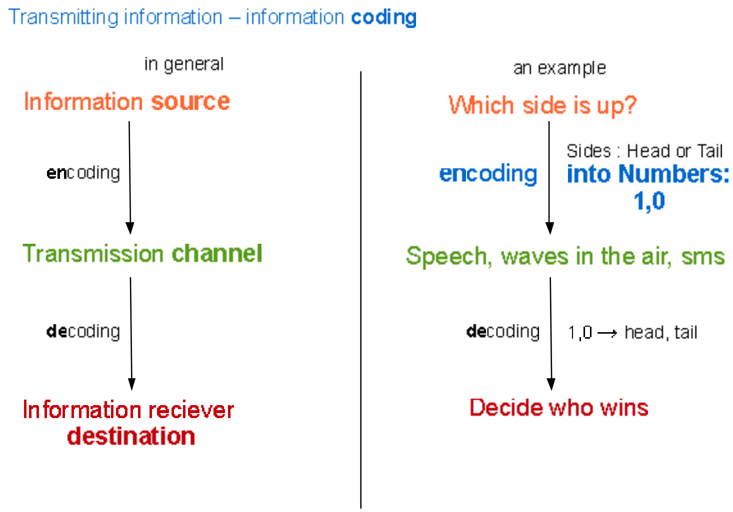
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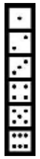
Decide who wins

Transmitting information – *digital* **coding**

Event	Number	Digital code
(heads)	1	1
(tails)	0	0



Event	Number	Digital code
	1	001
	2	010
	3	011
	4	100
	5	101
	6	110

2-base numbers: example: $101_2 = 1*2^2+0*2^1+1*2^0 = 5_{10}$

bit = „binary digit"

Transmitting information – *digital* **coding**

How many **bits** we need?

Bit: **b**inary dig**it**

0 or 1

Transmitting information – *digital* **coding**

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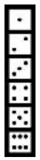
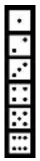


Event	Number	Digital code	Bits needed
	1	1	**1**
	0	0	
	1	001	
	2	010	
	3	011	**3**
	4	100	
	5	101	
	6	110	

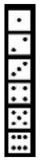
Transmitting information – coding ***efficiency***



Event	Number	Digital code	Bits needed	Maximum number of events
	1	1	**1**	**2**
	0	0		
	1	001		
	2	010		
	3	011	**3**	**8**
	4	100		
	5	101		
	6	110		
	7	111		
	0	000		

Here we only have 6 events, but could encode 8 in 3 bits!

Transmitting information – coding ***efficiency***



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	0	000		

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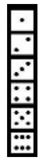
A better encoding:

$\{X_1X_2X_3\}$ group 3 events together

Classic coding
3x3 bits = **9** bits

Transmitting information – coding ***efficiency***



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:	6	110		
	7	111		
	0	000		

Here we only have 6 events,
but could encode 8 in 3 bits!

A better encoding:

$\{X_1X_2X_3\}$ group 3 events together : number of possibilities = $6^3 = 216$

$256 = 2^8$

Classic coding
3x3 bits = **9** bits ⟶ It is possible to encode 3 events in **8** bits

1 bit less!

Transmitting information – information content

Information content = how many bits do we *minimally* need to encode

(This also gives the encoding efficiency limit)

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How does this connect with intuitive information content?

-I have tossed a dime. Head or Tail?

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Transmitting information – information content

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(This also gives the encoding efficiency limit)

How does this connect with intuitive information content?

	p	q
-I have tossed a dime. Head or Tail?	½	½
-It is light traffic this morning	¼	¾
-It will rain tomorrow.	1%	99%
-I have won the lottery!	1/13,983,816	0.999....

Transmitting information – information content

Information content = how many bits do we *minimally* need to encode

(This also gives the encoding efficiency limit)

How does this connect with intuitive information content?

	p	q	
-I have tossed a dime. Head or Tail?	½	½	No idea
-It is light traffic this morning	¼	¾	
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Gained information is inverse proportional to the probability (p)

Transmitting information – measure of information

Fair	P	probability	code example	bits needed	p*(number of bits needed)
⚀	1/6	0,17	000	3	0,5
⚁	1/6	0,17	001	3	0,5
⚂	1/6	0,17	010	3	0,5
⚃	1/6	0,17	011	3	0,5
⚄	1/6	0,17	100	3	0,5
⚅	1/6	0,17	101	3	0,5

Expected number of bits needed: 3

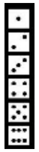
Loaded P_i

We can encode more efficiently here:

Loaded	P_i	probability	code example	bits needed	p*(number of bits needed)
⚀	1/2	0,5	0	1	0,5
⚁	1/4	0,25	10	2	0,5
⚂	1/8	0,13	110	3	0,38
⚃	1/16	0,06	1110	4	0,25
⚄	1/32	0,03	11110	5	0,16
⚅	1/32	0,03	11111	5	0,16

Expected number of bits needed: 1,94

Transmitting information – measure of information



Fair P	probability	code example	bits needed	p*(number of bits needed)
1/6	0,17	000	3	0,5
1/6	0,17	001	3	0,5
1/6	0,17	010	3	0,5
1/6	0,17	011	3	0,5
1/6	0,17	100	3	0,5
1/6	0,17	101	3	0,5

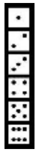
Here we do NOT Expect anything

Maximal uncertainity

Expected number of bits needed: **3**

Gained information is proportional to the number of bits needed

Loaded



P	probability	code example	bits needed	p*(number of bits needed)
1/2	0,5	0	1	0,5
1/4	0,25	10	2	0,5
1/8	0,13	110	3	0,38
1/16	0,06	1110	4	0,25
1/32	0,03	11110	5	0,16
1/32	0,03	11111	5	0,16

Here we *expect* „one" (most probable)

On average we **learn less**

Expected number of bits needed: **1,94**

Here the information content is less.

Transmitting information – measure of information

How should be information content **matematically** specified? (Shannon 1948)

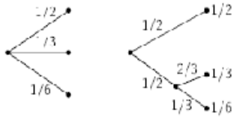
1.: H should be *continuous* in the p_i (small change in $p_i \rightarrow$ small change in H)

2.: Unlikely events carry a high information content:
H should be in some way inverse proportional to p
If all the p_i are equal, (p_i = 1/n)
then H should be a monotonic increasing function of n.

With equally likely events there is more choice, or uncertainty, when there are more possible events.

3.: Branching Chioces:
If a choice can be broken down into two successive choices,
the original H should be the weighted sum of the individual values of H.

$$H\left(\frac{1}{2},\frac{1}{3},\frac{1}{6}\right)=H\left(\frac{1}{2},\frac{1}{2}\right)+\frac{1}{2}\cdot H\left(\frac{2}{3},\frac{1}{3}\right)$$



Transmitting information – measure of information

Shannon : define measure as: $H = p \cdot \log_2\left(\frac{1}{p}\right)$

$\log_2$: 2-base logarithm

Examples:

$\log_2 (2) = 1$
$\log_2 (4) = 2$
$\log_2 (8) = 3$

Transmitting information – measure of information

Shannon $H = p \cdot \log_2\left(\frac{1}{p}\right)$ [bit]

If we have multiple events in the set, then it is a sum for every possible event:

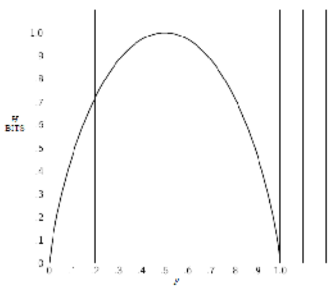
$$H = \sum_i p_i \cdot \log_2\left(\frac{1}{p_i}\right) = \sum_i -p_i \cdot \log_2 p_i$$

other log-bases:
$\log_e$ (ln) : [nat]
$\log_{10}$ (lg) : [ban]

measure of information - entropy

Dime tossing

p q = 1-p



$$H=\sum_i -p_i\cdot\log_2 p_i = -p\cdot\log_2 p - q\cdot\log_2 q = -p\cdot\log_2 p-(1-p)\cdot\log_2(1-p)$$

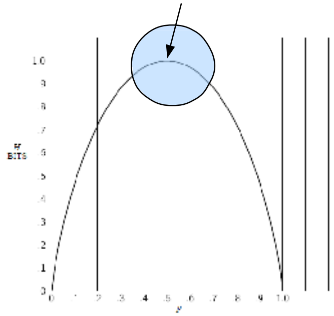
measure of information - entropy

Fair dime: p = ½

no expectations
maximal uncertainity

Dime tossing

p q = 1-p



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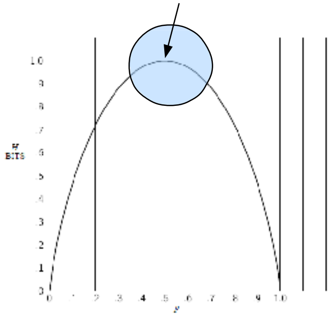


H has another name: **Shannon-entropy**

H has a maximum when we know nothing in advance (all p_i-s are equal, p_i = 1/n)

Expected outcomes are maximized: each state is equally probable

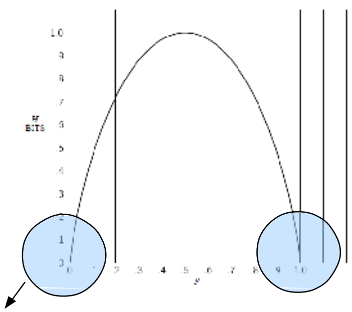
⇕

Physical entropy (S) has a maximum if the number of microstates is maximal.

measure of information - entropy

Dime tossing

p q = 1-p



H has another name: **Shannon-entropy**

H vanishes ONLY if we are absolutely certain of the outcome: p=0 or p=1

⇕

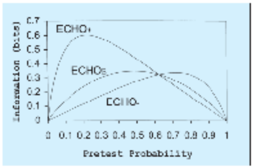
Physical entropy (S) vanishes ONLY if there is exactly 1 microstate

Examples of usage in medicine

Bayes-theorem based methods:

The amount of information gained by performing a diagnostic test can be quantified by calculating the relative entropy between the posttest and pretest probability distributions

Application:
- Diagnostic tests
- expert systems



a_i: pretest probability
b_i: post test probability

$$D(b||a) = \sum_{i=1}^{n} b_i \log_2(b_i/a_i)$$

Testing Situation	Pretest Probability of Disease	Test Operating Characteristics Sensitivity/Specificity	Test Result	Posttest Probability of Disease	Information Gained
Breast cancer screening with mammography	0.01	0.75/0.94	Positive Negative	0.11 0.003	0.25 bits 0.006 bits
Mammography given palpable breast mass	0.2	0.80/0.90	Positive Negative	0.67 0.05	0.74 bits 0.13 bits
Screening for HIV with antibody test	0.001	0.99/0.998	Positive Negative	0.33 0.00001	2.4 bits 0.001 bits
Presence of tonsillar exudate in diagnosing infection with group A streptococci	0.1	0.45/0.84	Positive Negative	0.24 0.07	0.11 bits 0.01 bits
Colon cancer screening by fecal occult blood testing	0.005	0.40/0.90	Positive Negative	0.02 0.003	0.02 bits 0.0005 bits

Examples of usage in medicine

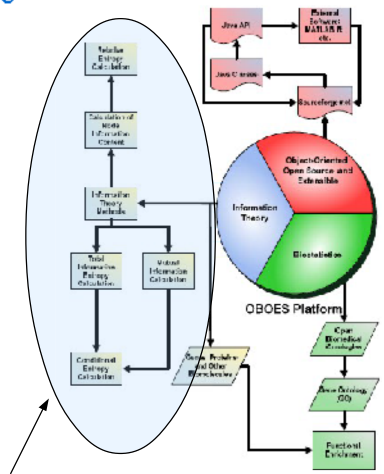
Gene technology,sequencing

Gene interactions

Gene and function identification

Proteome mapping

Systems biology



Information theory methods in a recent publication
(eg.: G. Alterovitz, M. Xiang, M. Mohan, et al., Nucleic Acids Res, 2006)

Databases

Databases store information:

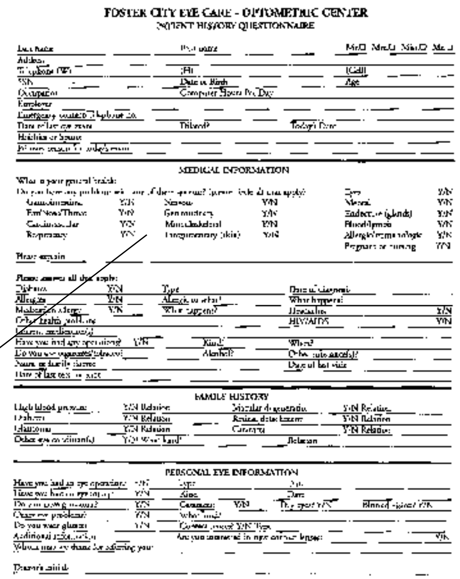
Databases are used for:
storage, structuring and extraction of ***information*** gathered previously.

Databases

Databases store information:

Databases are used for:
storage, structuring and **extraction** of ***information*** gathered previously.

It is hard to ***extract*** or modify information stored on paper


FOSTER CITY EYE CARE - OPTOMETRIC CENTER
PATIENT HISTORY QUESTIONNAIRE

MEDICAL INFORMATION

FAMILY HISTORY

PERSONAL EYE INFORMATION

Databases – storing information

Name	Telephone	Insurance	Diagnosis	Physician	Room	Bed
Sam Sma						

Instead of paper, one could start typing the data into a spreadsheet (Excel, OpenOffice, etc)

Databases – storing information

Name	Telephone	Insurance	Diagnosis	Physician	Room	Bed
Sam Small	(763) 865 345	Medicaid	Influenza	Dr. Barkins	37	2
Sara Goldmann	(691) 579 467	Medicare	Ascites	Dr. Magenheim	21	1
Dan Trideman	(691) 556 322	Medicaid	Malaria	Dr. Haydens	17	2
Bill Hardy	(691) 654 321	Medicare	Diabetes	Dr. Haydens	43	1
Bob Mindy	(691) 143 613	Tricare	Not yet known	Dr. Barkins	33	1
Yo Him	(691) 244 567	Medicare	Colitis	Dr. Magenheim	27	3
Kim Suhan	(691) 379 788	Tricare	Pneumonia	Dr. Barkins	37	1

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Table : ordered set of data (information)

Databases – storing information

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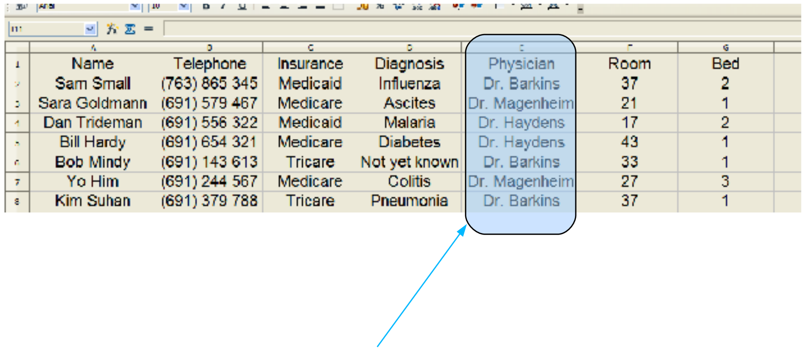
Record : Information *grouped together (one ROW in a Table)*

Each row is a selected set of data

Every row has the same structure

Databases – storing information



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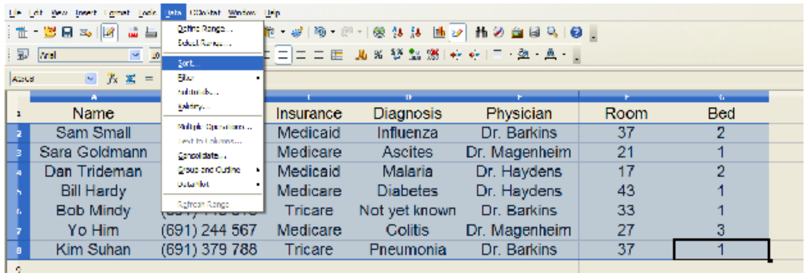
Column: data type

Databases – manipulating information

Sorting data



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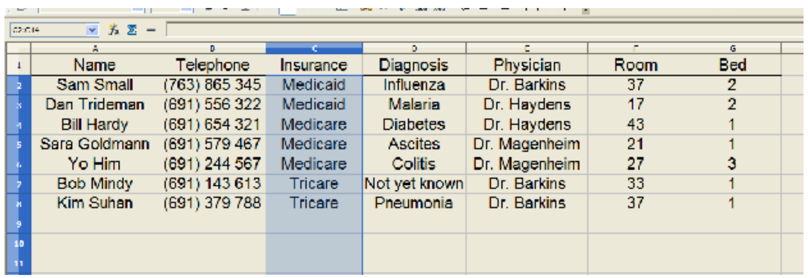

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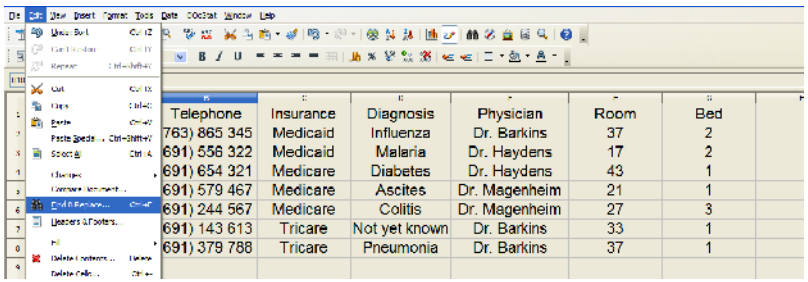
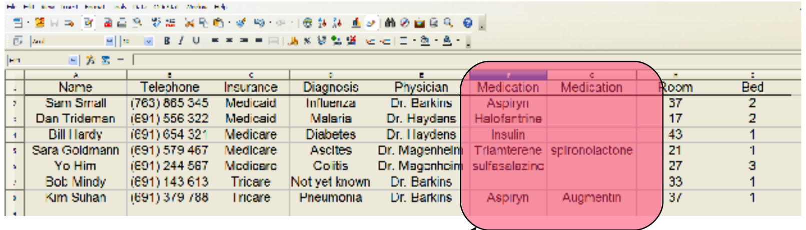
Kim Suhan	(691) 379 788	Tricare	Pneumonia	Dr. Barkins	37	1

Databases – retrieving information



Telephone	Insurance	Diagnosis	Physician	Room	Bed
763) 865 345	Medicaid	Influenza	Dr. Barkins	37	2
691) 556 322	Medicaid	Malaria	Dr. Haydens	17	2
691) 654 321	Medicare	Diabetes	Dr. Haydens	43	1
691) 579 467	Medicare	Ascites	Dr. Magenheim	21	1
691) 244 567	Medicare	Colitis	Dr. Magenheim	27	3
691) 143 613	Tricare	Not yet known	Dr. Barkins	33	1
691) 379 788	Tricare	Pneumonia	Dr. Barkins	37	1

Databases – problems with simple methods

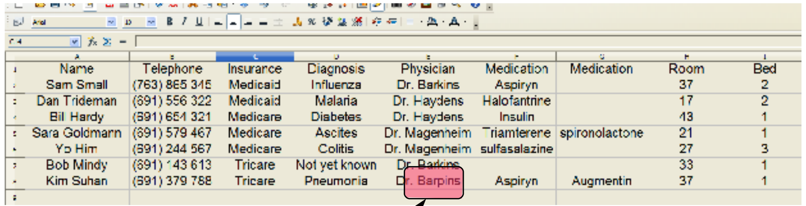


Name	Telephone	Insurance	Diagnosis	Physician	Medication	Medication	Room	Bed
Sam Small	(763) 865 345	Medicaid	Influenza	Dr. Barkins	Aspiryn		37	2
Dan Trideman	(691) 556 322	Medicaid	Malaria	Dr. Haydans	Halofartrine		17	2
Bill Hardy	(691) 654 321	Medicare	Diabetes	Dr. Haydens	Insulin		43	1
Sara Goldmann	(691) 579 467	Medicare	Ascites	Dr. Magenheim	Triamterene	spironolactone	21	1
Yo Him	(691) 244 567	Medicare	Colitis	Dr. Magenheim	sulfasalazine		27	3
Bob Mindy	(691) 143 613	Tricare	Not yet known	Dr. Barkins			33	1
Kim Suhan	(691) 379 788	Tricare	Pneumonia	Dr. Barkins	Aspiryn	Augmentin	37	1

Records do not have the same size

Waste of space
Adding new data types tedious
Inconsistency : is a field empty by error?

Databases – problems with simple methods



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Kim Suhan	(691) 379 788	Tricare	Pneumonia	Dr. Barpins	Aspiryn	Augmentin	37	1

Entering the same data multiple times:

Typos
Redundancy
Later change almost impossible – too many items
...

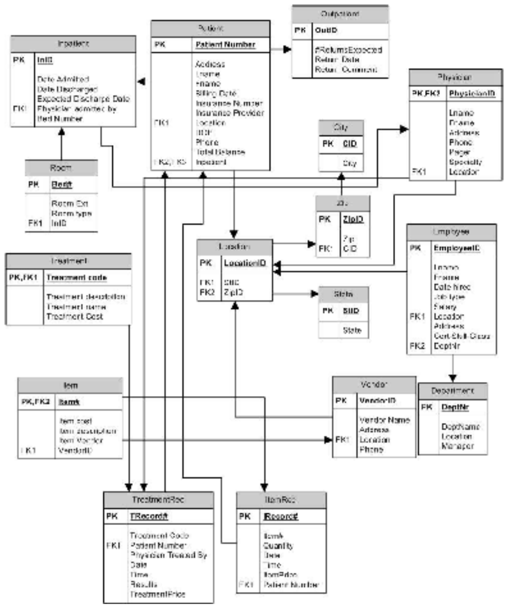
Databases – SQL

Information Retrieval — P. BAXENDALE, Editor

A Relational Model of Data for Large Shared Data Banks

E. F. Codd
IBM Research Laboratory, San Jose, California

Databases

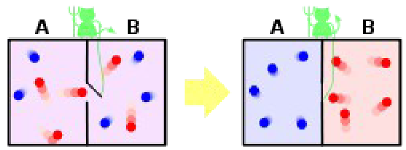


Extension material: information entropy and physical entropy

Information entropy and physical entropy

„ in an isolated system, entropy never decreases." Second Law of Thermodynamics

The Maxwell demon

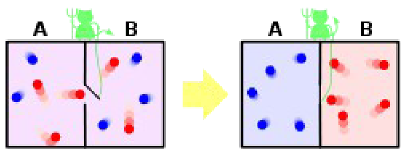


Temperature of A decreses, B increases → Violation of the Second Law ?

Information entropy and physical entropy

„ in an isolated system, entropy never decreases." Second Law of Thermodynamics

The Maxwell demon

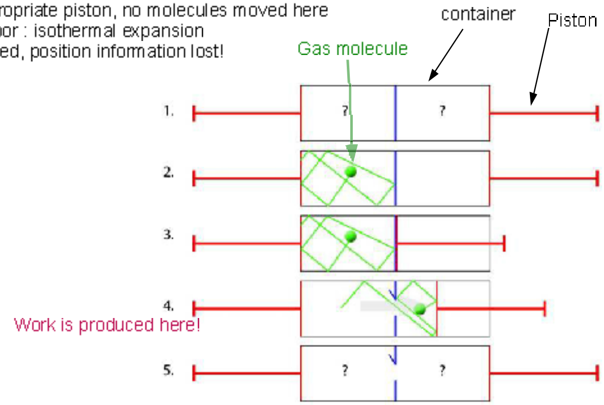


Temperature of A decreses, B increases → Violation of Law II. ?

Solution: NO, since the demon interacts with the system, it must be considered.
The demon acquires ***information***, and this changes it's state!

Information entropy and physical entropy

1. : molecule's position unknown
2. : measure position, information = 1 bit
3. : move appropriate piston, no molecules moved here
4. : release door : isothermal expansion
5. : door opened, position information lost!



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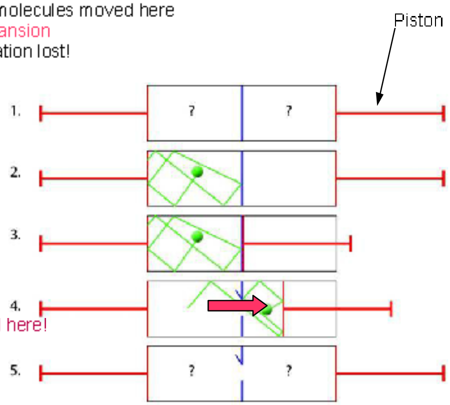
Isothermal expansion:

$$W_{A\to B} = NkT\ln\left(\frac{V_A}{V_B}\right)$$

In this case:
N=1
$V_A/V_B = 2$

Hence

$W = kT\ln(2)$ Work is produced here!



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Hence

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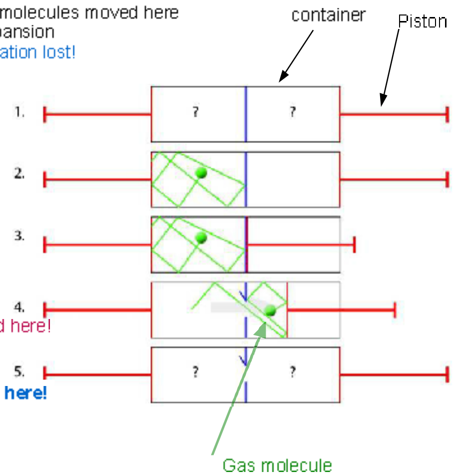
Information is lost here!



Information entropy and physical entropy

Leo Szilárd:

From Law II. taking into account that $W = T\Delta S$

$$W_{\text{produced by piston}} = W_{\text{loss of information}}$$

$$T\Delta S_{bit} = kT\ln 2$$

$$\Delta S_{1\,bit} = k\ln 2$$

Ereasing 1 bit of information increases physical entropy by *k·ln2*

(Landauer 1971, logically irreversible processes, eg. AND-gate)