## Slide 1

# Medical signal processing

KAD 2015.12.09

Slide 2
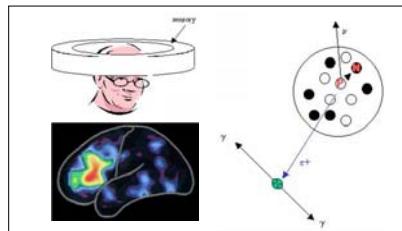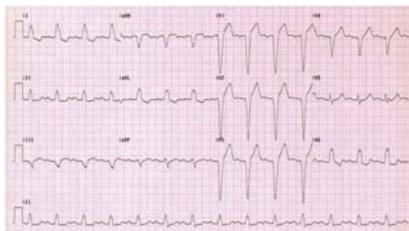
A **signal** is any kind of physical quantity that conveys/transmits/stores information

e.g. (1)
electrical voltage, that can be measured on the surface of the skin/head as a result of the heart-/muscle-/brain activities (ECG/EMG/EEG)

e.g. (2)
gamma photon detection in radioisotope diagnostics

(1)   (2)

1mV/cm

25 mm/s

Impulsamplitude (Volt)

t

Slide 3
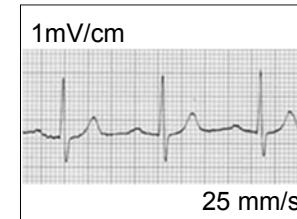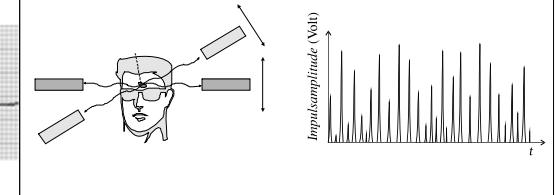
Classification of signals

static	–	time-dependent
periodic	–	non-periodic
random	–	deterministic
pulsed	–	continuous
electric	–	non-electric
analog	–	digital

1mV/cm

25 mm/s

Impulsamplitude (Volt)

t

Slide 4
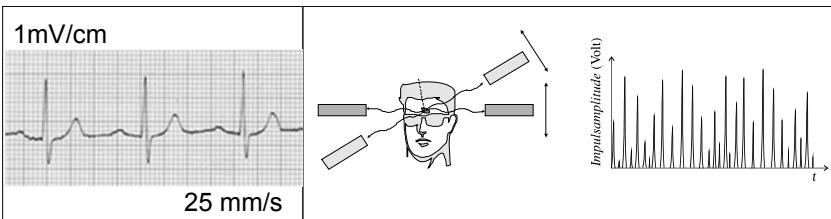
in a very special role

electric signals

non-electric signals are transferred to electric ones

advantages of **electric** signals: they are easy to transform, amplify, transmit

digital signals

analog signals are transferred to digital ones

advantages of **digital** signals: they are easy to store, the noise can be engineered and influence can be reduced
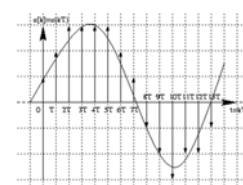
quantity that compares the magnitudes of two signals:

<mark>**Signal level** or **Bel**-number (or **Decibel**-number): n</mark>

(named after A. Bell)

unit of n : Bel (B) or decibel (dB)

$$n = \lg \frac{P_2}{P_1}\, B = \lg \frac{J_2}{J_1}\, B = \lg \frac{E_2}{E_1}\, B$$

decimal logarithm of ratio of two powers (intensities, energies)

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cf. **radian**

$$\Theta = \frac{arc}{radius}$$

$$[\Theta] = \frac{m}{m} = rad = 1$$

radius

arc

θ
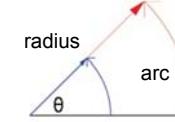
cf. **pH** (power of Hydrogen)

$$pH = -\lg \frac{\left[H^+\right]}{1M}$$

e.$g.$: $\left[H^+\right] = 10^{-7} M$

$$\Rightarrow pH = -\lg 10^{-7} = -1 \cdot (-7) = 7$$

instead of Bel number we are using <mark>**decibel-number**</mark>

$$n = 10 \cdot \lg \frac{P_2}{P_1}\, dB \qquad\qquad (10d = 1)$$

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the **characteristic** unit: **power** (or intensity/energy),
the **practical** unit: (electric) **voltage**

the relation between power and voltage:

$$P = U \cdot I = \frac{U^2}{R} \quad (Ohm: U = R \cdot I)$$

signal level with voltages:

$$\boxed{n} = 10 \cdot \lg \frac{P_2}{P_1}\, dB = 10 \cdot \lg \frac{\dfrac{U_2^{\,2}}{R_2}}{\dfrac{U_1^{\,2}}{R_1}}\, dB \overset{R_2 \approx R_1}{=}$$

$$= 10 \cdot \lg \frac{U_2^{\,2}}{U_1^{\,2}}\, dB = \boxed{20 \cdot \lg \frac{U_2}{U_1}}\, dB$$

$$\frac{P_2}{P_1} = 2 \Leftrightarrow 10 \lg 2\, dB =$$

$$= 10 \cdot 0{,}3\, dB = 3 dB$$

$$\frac{P_2}{P_1} = \frac{1}{2} \Leftrightarrow -3 dB$$

cf. half life,
half value thickness

$$\frac{P_2}{P_1} = 10 \Leftrightarrow 10 \cdot \lg 10\, dB =$$

$$= 10 \cdot 1 dB = 10 dB$$

$$\frac{P_2}{P_1} = 100 \Leftrightarrow 10 \lg 100\, dB =$$

$$= 10 \cdot 2 dB = 20 dB$$

U_2/U_1	P_2/P_1	dB
1,414	2	3
2	4	6
	8	9
3,16	10	10
	20	13
10	100	20
	$1000 = 10^3$	30
$100 = 10^2$	$10000 = 10^4$	40
$1000 = 10^3$	10^6	60

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$\frac{\Delta n}{\Delta h}$

(1/10 cm)

10cm *1/10cm = 1

n: number of data

1

body height, h (cm)

$\frac{\Delta H}{\Delta h}$

10cm *16.5 = 165 cm

205 cm

H

body height, h (cm)

H

h
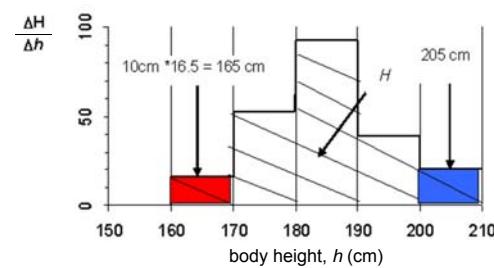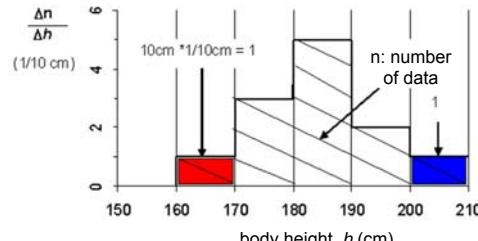
spectrum, as a special density function

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Density function   **Spectrum**

$\frac{\Delta n}{\Delta h}$

$\left(\frac{1}{10\text{cm}}\right)$

area under the curve: n

$\frac{\Delta H}{\Delta h}$

area under the curve: H

160 170 180 190 200 210
h (cm)

160 170 180 190 200 210
h (cm)

10

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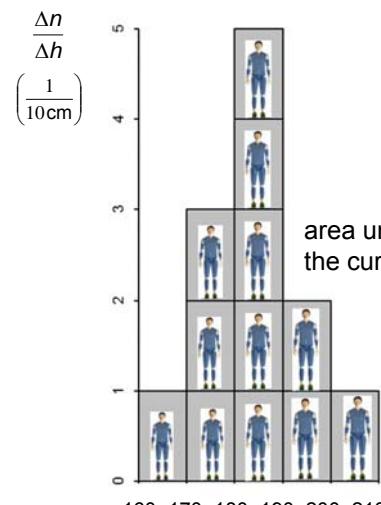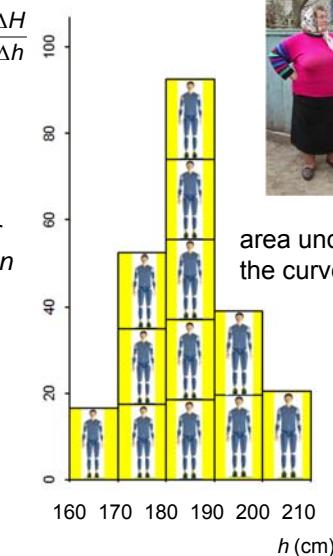
all (usual) periodic functions can be expressed as a sum of sine (and cosine) functions from the fundamental frequency and the overtones

periodic function:
there is a period, T

t

$\frac{1}{T} = f,$ where f is the frequency T

the sine function, which has the same frequency as the periodic function:
   **fundamental frequency**

$2f$, $3f$, $4f$, ... : **overtones**

(line spectrum)

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function   **spectrum**
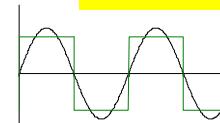
square pulse train
fundamental fr(equency)
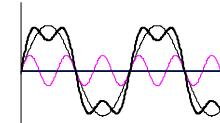
0   5   10   15 f 20

fundamental fr.+ 3rd overtone

0   5   10   15 f 20

fundamental fr.+ 3rd overtone + 5th overtone

0   5   10   15 f 20
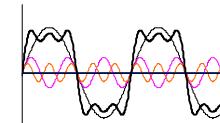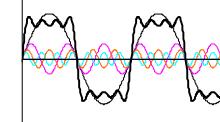
fundamental fr.+ 3rd overtone + 5th overtone + 7th overtone

0   5   10   15 f 20

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function | spectrum

fundamental fr.+
3rd overtone +
+ ... +
9th overtone

fundamental fr.+
3rd overtone +
+ ... +
11th overtone

fundamental fr.+
3rd overtone +
+ ... +
13th overtone

fundamental fr.+
3rd overtone +
+ ... +
15th overtone

13

Slide 14

function | spectrum

fundamental fr.+
3rd overtone +
+ ... +
17th overtone

.
.
.

fundamental fr.+
3rd overtone +
+ ... +
17th overtone +
+ ...

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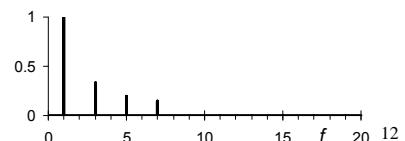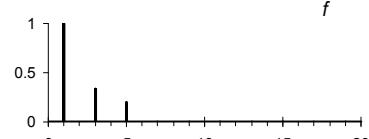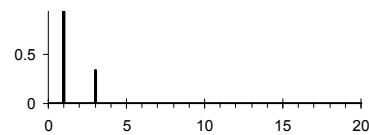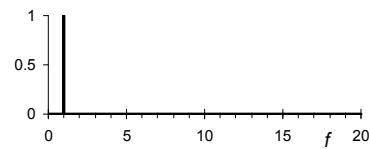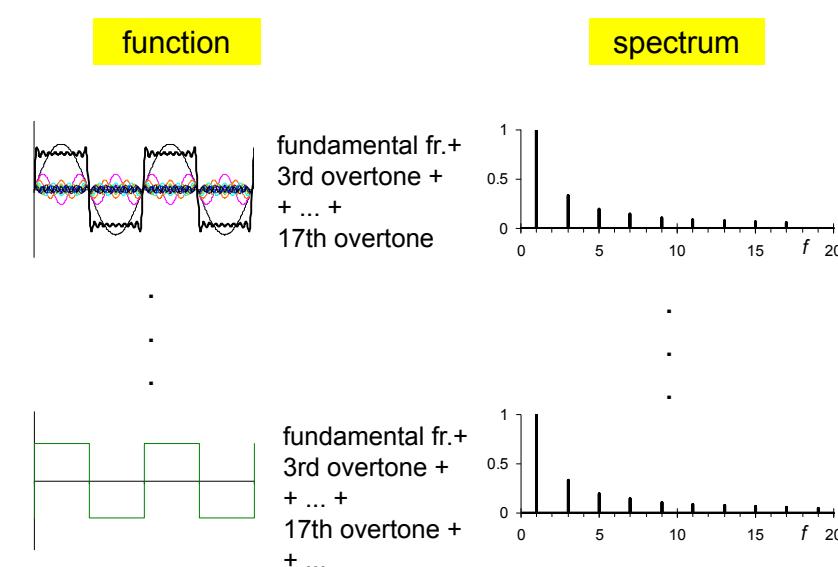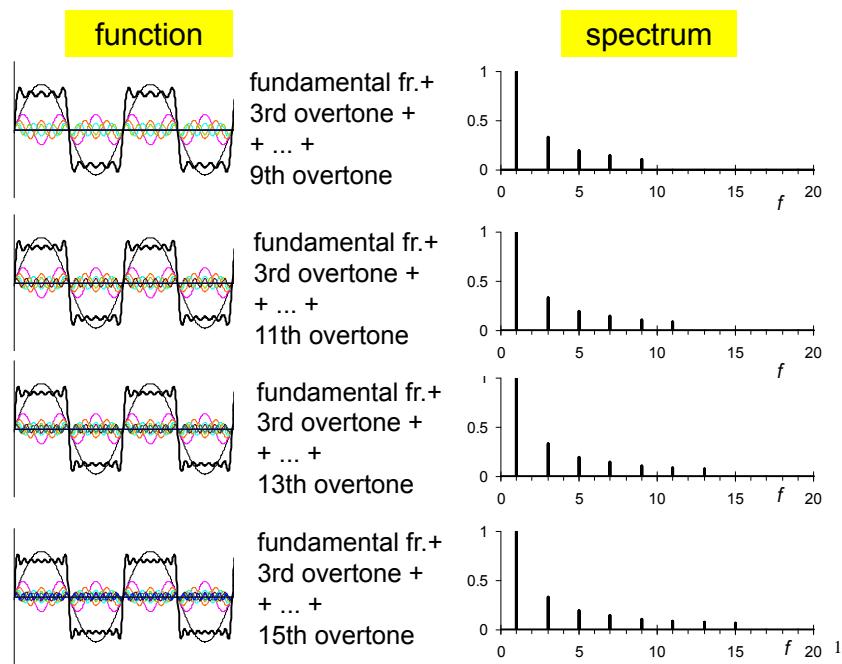
cf. infinite **series**

$$\sum_{k=0}^{\infty}\frac{1}{2^k} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + ... = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... = 2$$

0   1   2   3

Einzelne Summanden bis zur Ordnung 0

−1a   1a   2a   3a

Überlagerung

−1a   1a   2a   3a

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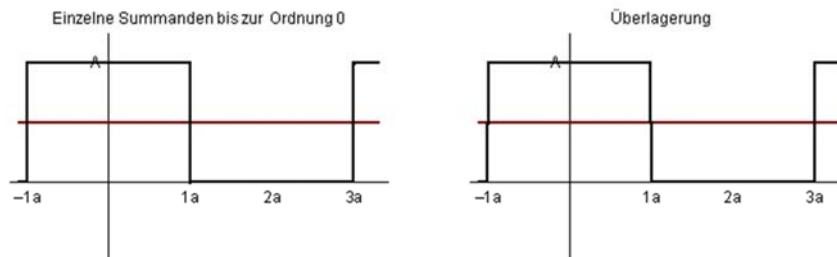
Creating an ECG signal from sine functions

signal voltage

The original signal and the signal reconstructed from the first 3 Fourier component

time

signal voltage

The original signal and the signal reconstructed from the first 10 Fourier component

time

signal voltage

The original signal and the signal reconstructed from the first 64 Fourier component

time
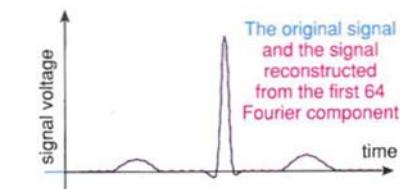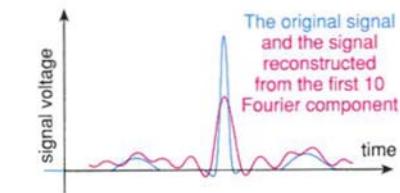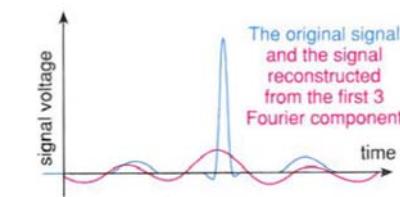
Textbook, Figure VII.3.

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Measured spectra

flute

clarinet

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all (usual) functions can be expressed as a sum of sine (and cosine) functions

spectrum: continuous

cf. emission spectra of incandescent light sources

6000 K

5000 K

4000 K

3000 K

$\frac{\Delta J}{\Delta \lambda}$

400   800

λ (nm)

UV | VIS | IR

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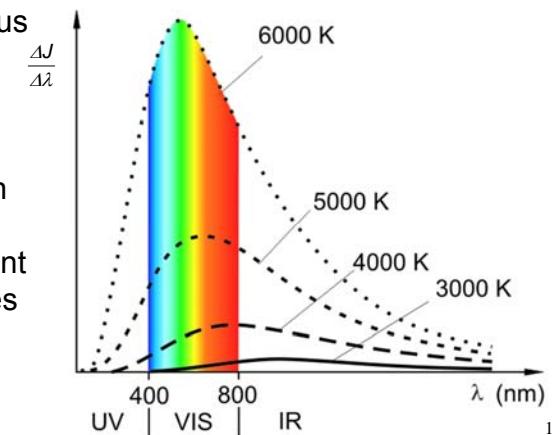
function

spectrum

sine function
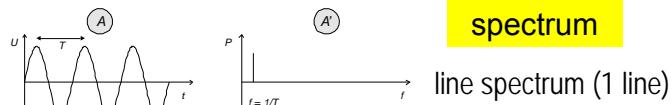
line spectrum (1 line)

periodic function

line spectrum
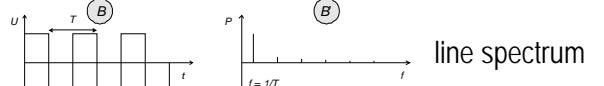
a few periods
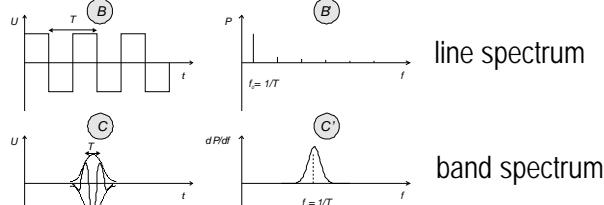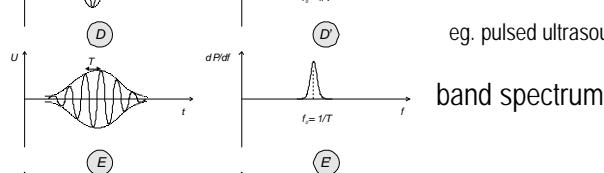
band spectrum

eg. pulsed ultrasound

more periods

band spectrum

non-periodic function
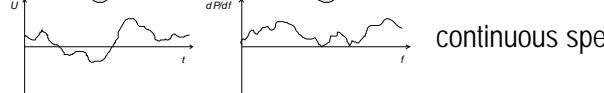
continuous spectrum

Music in time-frequency representation

Inisheer

Penny Whistle

Traditional

Air

f_{sine}

t

Voiceprint

oscillogram (sound intensity vs. time)

time – frequency representation

pronounced consonants and vowels

http://www.nrips.go.jp/org/fourth/info3/index-e.html

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Heart beats in time-frequency representation
(+ oscillogram)

U

t (s)

f_{sine} (Hz)

t (s)

systole   diastole
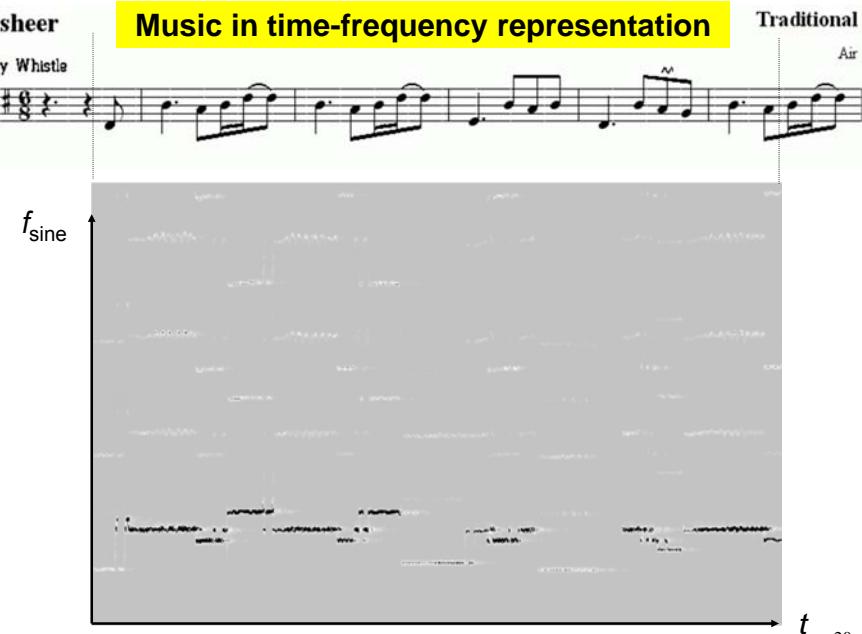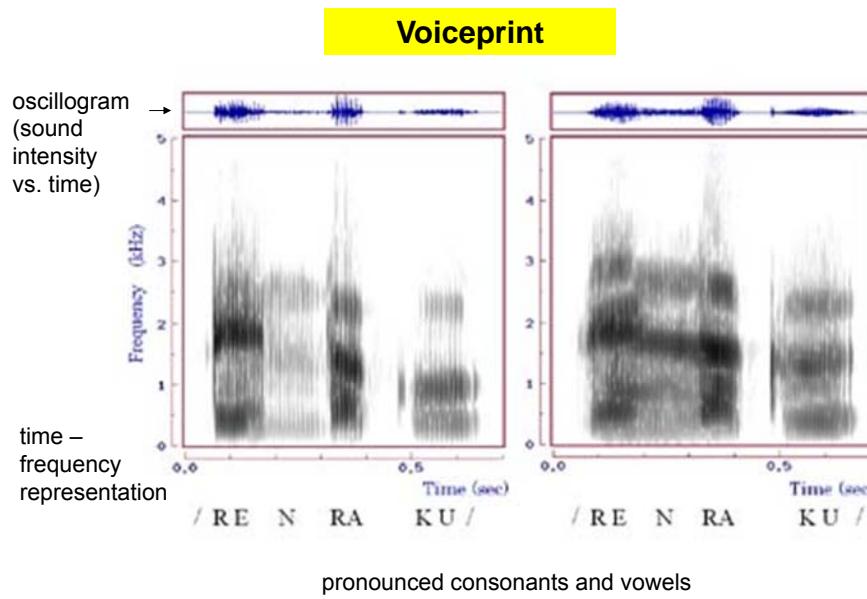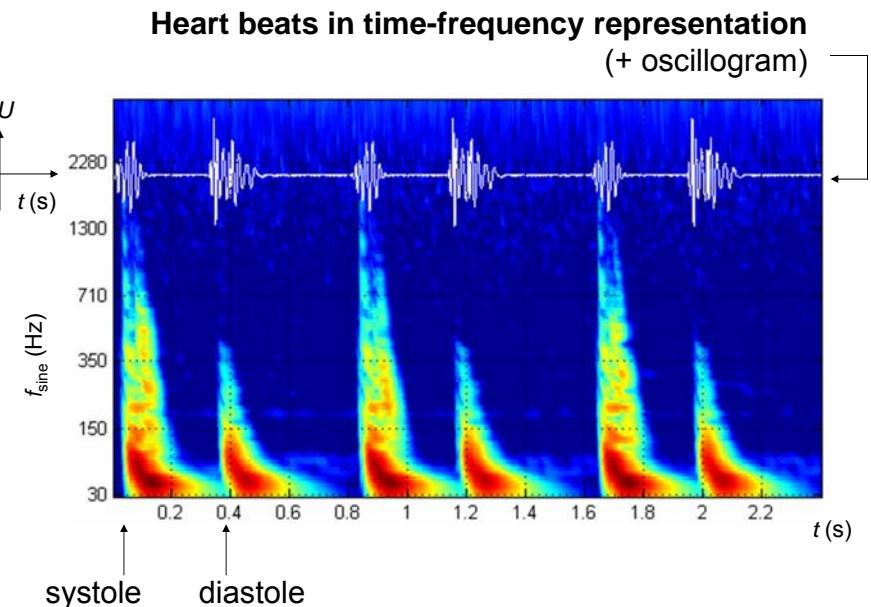
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Frequency and amplitude ranges of biological signals
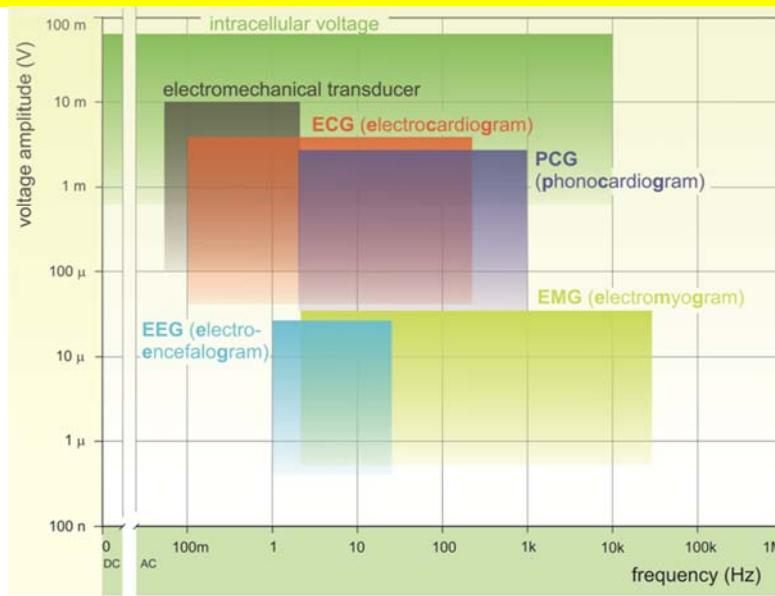
Practical manual, titel page of meas. 17

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Frequency dependent unit: Electronic amplifier

(1) $P_{in} < P_{out}$

(2) P_{in} and P_{out} : same functions

same: „fundamentalist" requirement
similar: realistic requirement

$(1) + (2)$ $A_P \cdot P_{in}(t) \equiv P_{out}(t)$, where $A_P > 1$

$$A_P = \frac{P_{out}}{P_{in}},$$ **power gain** (amplification)

$$A_U = \frac{U_{out}}{U_{in}},$$ **voltage gain** (amplification)

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(frequency independent) voltage-divider
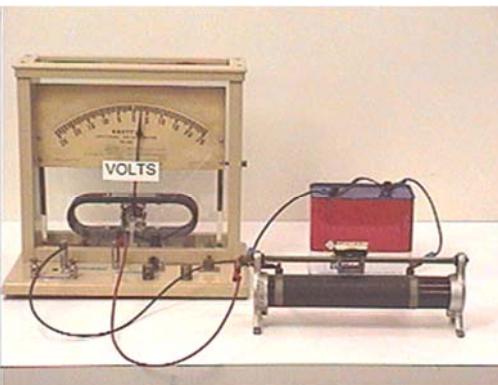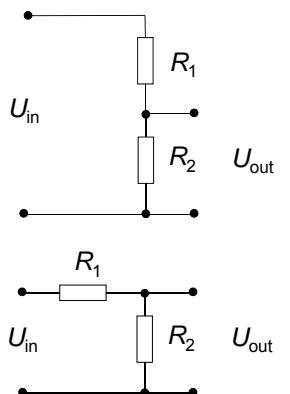
$$U_{\text{out}} = \frac{R_2}{R_1 + R_2} U_{\text{in}}$$

frequency dependent voltage-divider: with capacitor

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High-pass/low-cut filter

$$R_C = \frac{1}{C\omega}$$ at high frequencies the capacitor is a shortcut
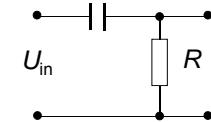
because of the phase difference, the sum should be calculated as vectors

$$U_{\text{out}} = \frac{R}{\sqrt{\dfrac{1}{C^2\omega^2} + R^2}} U_{\text{in}} = \frac{RC\omega}{\sqrt{1 + R^2 C^2 \omega^2}} U_{\text{in}}$$

at very low frequencies: if $\omega << \omega_0$ ($\omega \approx 0$), $U_{\text{out}} = 0$

at low frequencies: if $\omega << \omega_0$, $U_{\text{out}} = RC\omega\, U_{\text{in}}$ ↔ 6 dB/octave

at high frequencies : if $\omega \approx \infty$, $U_{\text{out}} = U_{\text{in}}$

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Low-pass/high-cut filter
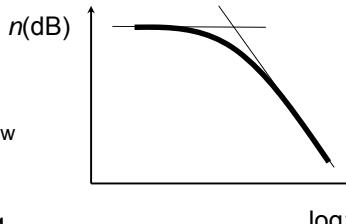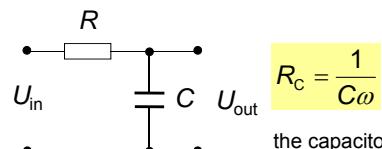
$$R_C = \frac{1}{C\omega}$$

the capacitor at low frequencies is a discontinuity

$$U_{\text{out}} = \frac{\dfrac{1}{C\omega}}{\sqrt{R^2 + \dfrac{1}{C^2\omega^2}}} U_{\text{in}} = \frac{1}{\sqrt{R^2 C^2 \omega^2 + 1}} U_{\text{in}}$$

at low frequencies: if $\omega << \omega_0$ ($\omega \approx 0$), $U_{\text{out}} = U_{\text{in}}$

at high frequencies: if $\omega >> \omega_0$, $U_{\text{out}} = \dfrac{1}{RC\omega} U_{\text{in}}$ ↔ -6 dB/octave

at very high frequencies :   if $\omega >> \omega_0$ ($\omega \approx \infty$), $U_{\text{out}} = 0$

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for (1): $A_P > 1$,

$n = 10 \lg A_P = 20 \lg A_U > 0$ dB

for (2): **frequency characteristics**

transfer band (passband)
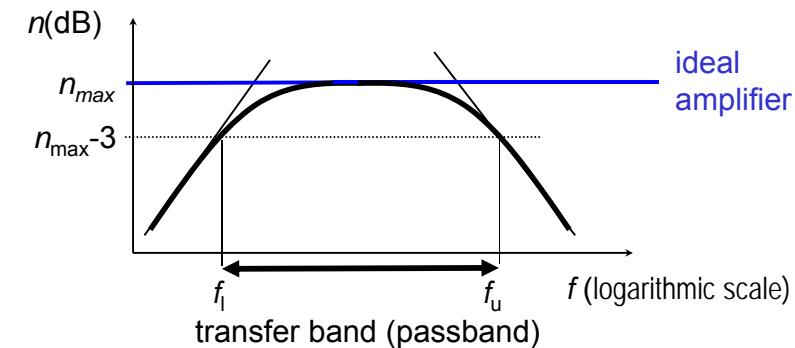
f_l : lower frequency limit f_u : upper frequency limit

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Amplifier with feedback
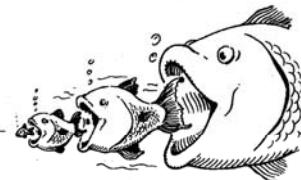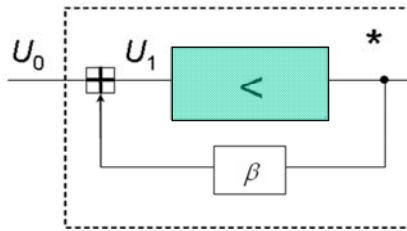
Without appropriate perspective we may not recognize our real position.

(a) $U_1 = U_0 + \beta U_2$ (b) $A_U = \dfrac{U_2}{U_1}$

(c) $A_U^{*} = \dfrac{U_2}{U_0} = \dfrac{U_1 A_U}{U_0} = \dfrac{(U_0 + \beta U_2) A_U}{U_0} = A_U + \beta \dfrac{U_2}{U_0} A_U = A_U + \beta A_U^{*} A_U$

$A_U^{*} - \beta A_U^{*} A_U = A_U$ $\boxed{A_U^{*} = \dfrac{A_U}{1 - \beta A_U}}$

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$\boxed{A_U^{*} = \dfrac{A_U}{1 - \beta A_U},}$ A_U^{*}: voltage gain with feedback

A_U: voltage gain without feedback

$\beta > 0$, **positiv feedback** (same phase), $A_U^{*} > A_U$ (advantage)

$\beta < 0$, **negativ feedback** (in opposite phase), $A_U^{*} < A_U$ (disadv.)

positiv feedback:
(a) $\beta A_U = 1$, amplification: „infinite"
    – sine wave oscillator
    e.g.: ultrasound generator, heat theraphy
(b) $\beta A_U \leq 1$, amplification: very big
    – regenerative amplifier
    e.g.: hearing, outer haircells

negativ feedback: „all" amplifier

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n(dB)

positiv feedback   ⇕ 3 dB

without feedback

negativ feedback

$\log f$
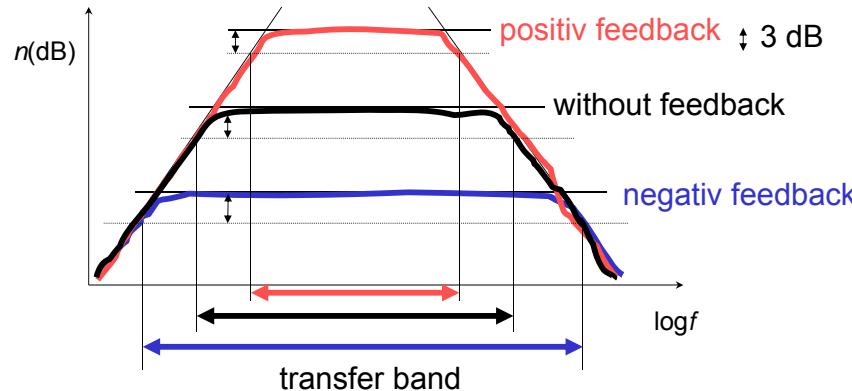
transfer band

positiv feedback: transfer band – narrrower (big disadvantage)
                   higher gain (advantage)

negativ feedback: transfer band – broader (advantage)
                   less gain (small disadvantage)

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Analog signal – digital signal

analog signal: time- and value-contin.

time-discrete value-contin.

time-cont. value-discrete

digital signal: time- and value-discrete

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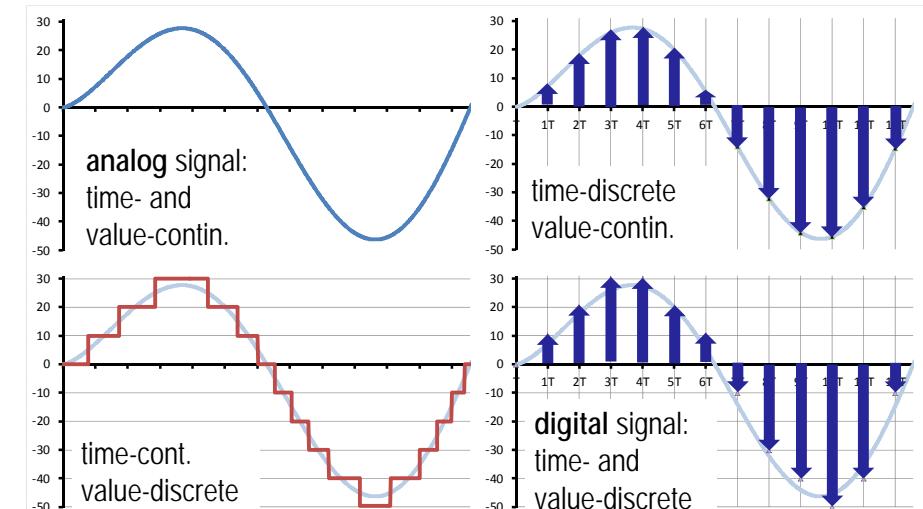
time-discrete: the value of the signal is not known for all moments in time

highest frequency component of the signal

$f_{sampl} = f_{max}$, reconstructed signal is constant

$f_{sampl} = 1.5\, f_{max}$, freq. of reconstructed signal is wrong

$f_{samp} = 2\, f_{max}$, freq. of reconstructed signal is correct
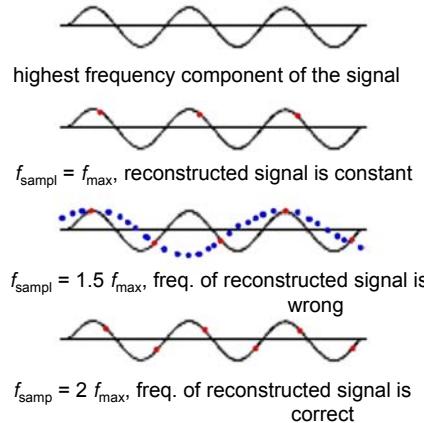
Nyquist–Shannon sampling theorem:

for complete reconstruction the minimum sampling frequency should be twice the frequency of the highest overtone of the signal

e.g.: hifi, f_{max} = 20 kHz

f_{sampl} = 44.1 kHz > 2*20 kHz

value discrete: the value of the signal can not be arbitrary

e.g.: hifi, 16 bit = 2^{16} = 65 536 (CD standard)

24 bit = 2^{24} = 16 777 216 ("best" audio card)

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Pulse processing

integral discrimination   differential discrimination
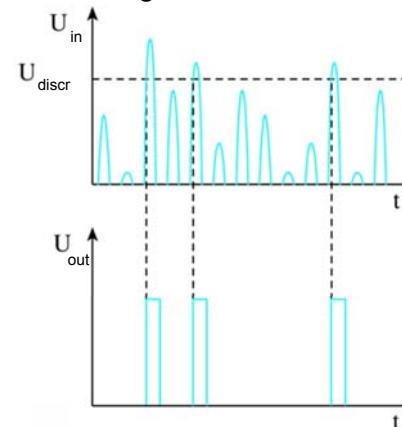
to select only those pulses that are larger than a preset amplitude
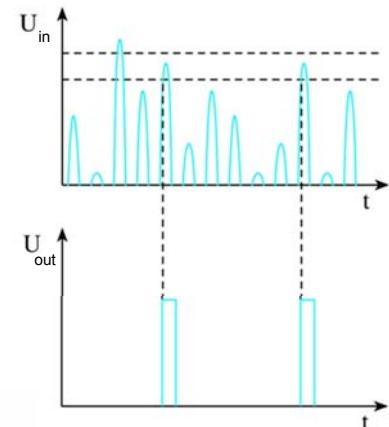
to select only those pulses whose amplitudes lie within a preset window

Textbook, Figure VII.32.

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Distribution functions and ID/DD "spectra"

$\dfrac{\Delta N}{\Delta h}\left(\dfrac{1}{10cm}\right)$

density function

DD-"spectrum"

$M = N_0 - N$

"cumulative-distribution-function"

ID-"spectrum"

how many pulses are larger than h?

N

cumulative-distribution-function

how many pulses are smaller than h?

h (cm)

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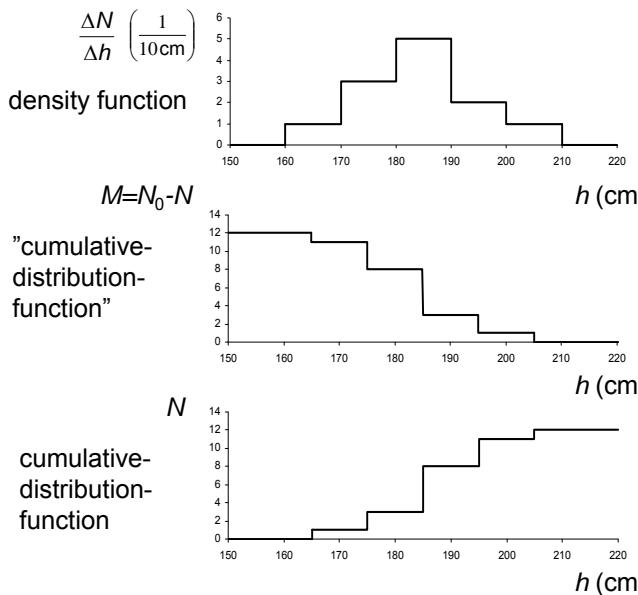
No.   3524
DATE: 93/ 3/30 09:22
MODE: WHOLE BLOOD

WBC   7.5x10³/μℓ
RBC   3.64x10⁶/μℓ
HGB   11.8   g/dℓ
HCT   33.1   %
MCV   90.9   fℓ
MCH   32.4   pg
MCHC   35.6   g/dℓ
PLT   158x10³/μℓ
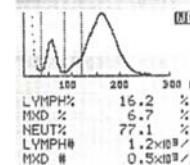
LYMPH%   16.2   %
MXD %   6.7   %
NEUT%   77.1   %
LYMPH#   1.2x10³/μℓ
MXD #   0.5x10³/μℓ
NEUT#   5.8x10³/μℓ
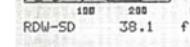
RDW-SD   38.1   fℓ

PDW   14.0   fℓ
MPV   10.5   fℓ
P-LCR   31.1   %

Concentration of white blood cells

Coulter counter

LYMPH%	16.2	%
MXD %	6.7	%
NEUT%	77.1	%
LYMPH#	1.2x10³/μℓ
MXD #	0.5x10³/μℓ
NEUT#	5.8x10³/μℓ