Physical basis of medical ultrasound

G.Schay 2016

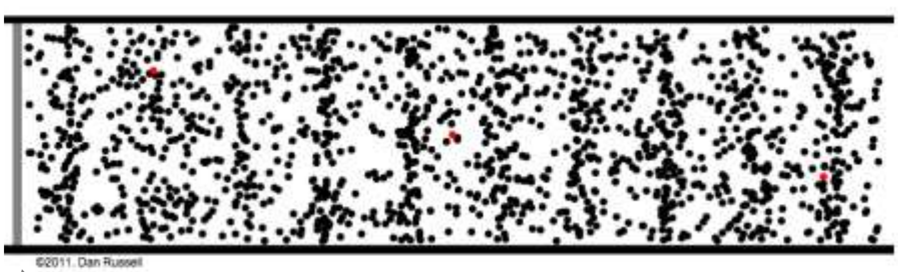
Topics :

Sound as a mechanical wave
Frequency ranges - ultrasound
Generation of ultrasound
Ultrasound transducers – technical questions
Imaging by ultrasound
Doppler method
Medical imaging

Transversal wave – such as light, or sound in some cases in solids

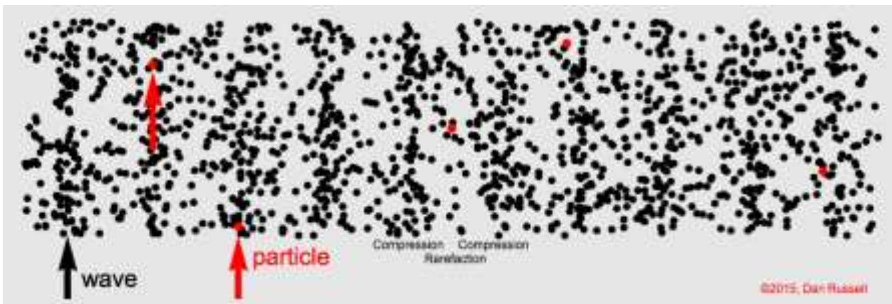
Transversal: wave propagation is perpendicular to the "motion"

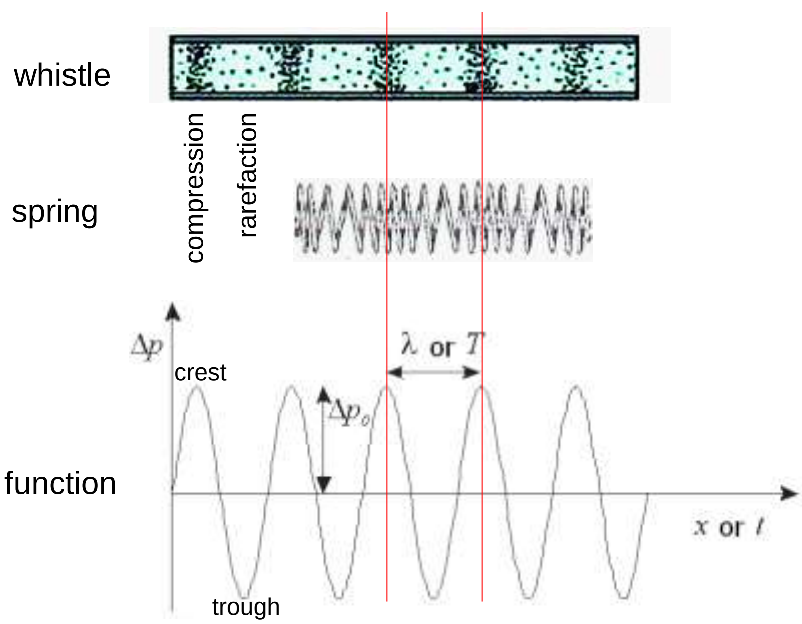
Longitudinal waves:
propagation direction is parallel to the "motion"



Moving surface (wave "source")

Compression: pressure increase, density increase
Rarefaction: pressure drop, density drop





hydrostatic pressure

pressure change, sound pressure

$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

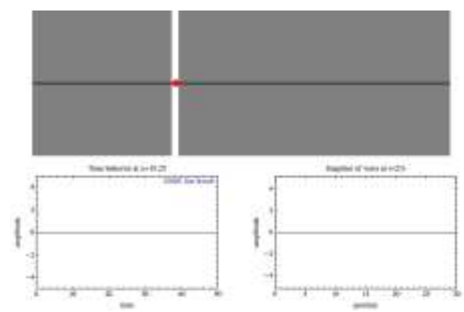
pressure DC + AC

amplitude phase

$$\Delta p(t,x) = \Delta p_{\max} \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

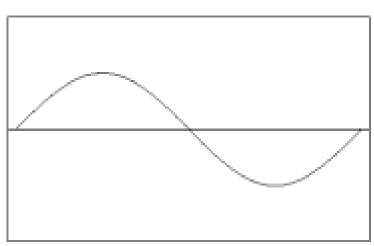
$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$

(see elecrical analogy:
DC=direct current,
AC=alternating current)
Usually $p_{\text{hydrostat}} >> \Delta p$

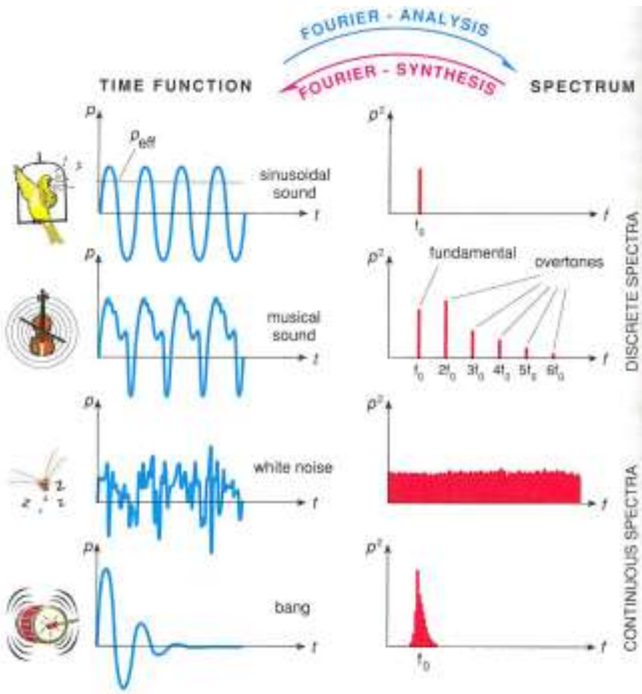


Frequency ranges – Fourier theorem

$$Signal(t) \longleftrightarrow \sum_i A_i \cdot \sin(\omega_i t) + B_i \cos(\omega_i t)$$

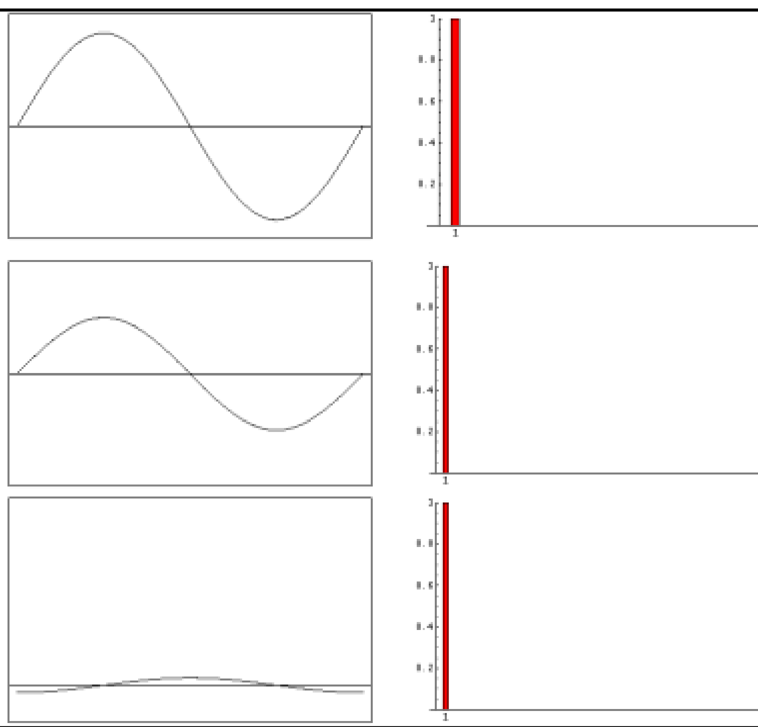


The more sin/cos signals we add, the better the accuracy



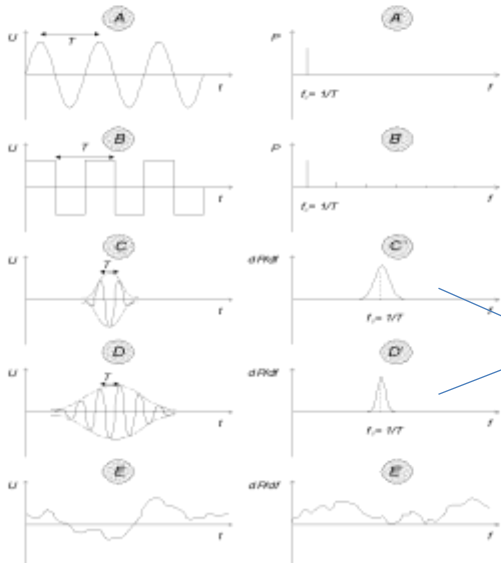
pitch:
frequency of the fundamental

timbre (tone colour):
relative strengths of overtones/harmonics (spectrum)

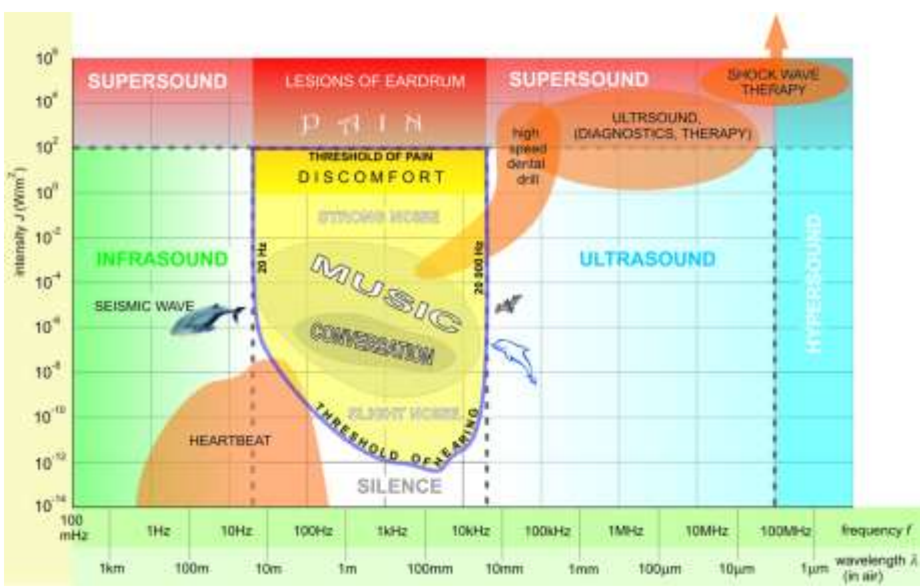


$$F(\omega) = \frac{1}{\sqrt{(2\pi)}} \cdot \int_{-\infty}^{+\infty} f(t) e^{i\omega t} dt$$

If the signal is non-periodic, we have an integral instead of the summation



If the pulse gets shorter, then the frequency spectrum spans a broader range!



Propagation of sound waves

$$\Delta p(t,x) = \Delta p_{\max} \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$

$$c = \sqrt{\frac{E}{\rho}} = \frac{1}{\sqrt{K\rho}}$$

E is called the elastic (or Young's) modulus of the material and is a measure of the stiffness of the material. (See Hooke's law!)

Some important equations – the role of the elastic medium

$$\kappa = \frac{-\frac{\Delta V}{V}}{\Delta p}$$

compressibility
relative volume decrease
over pressure

$$c = \frac{1}{\sqrt{\rho\kappa}}$$

speed of sound

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

acoustic **impedance**
(definition)

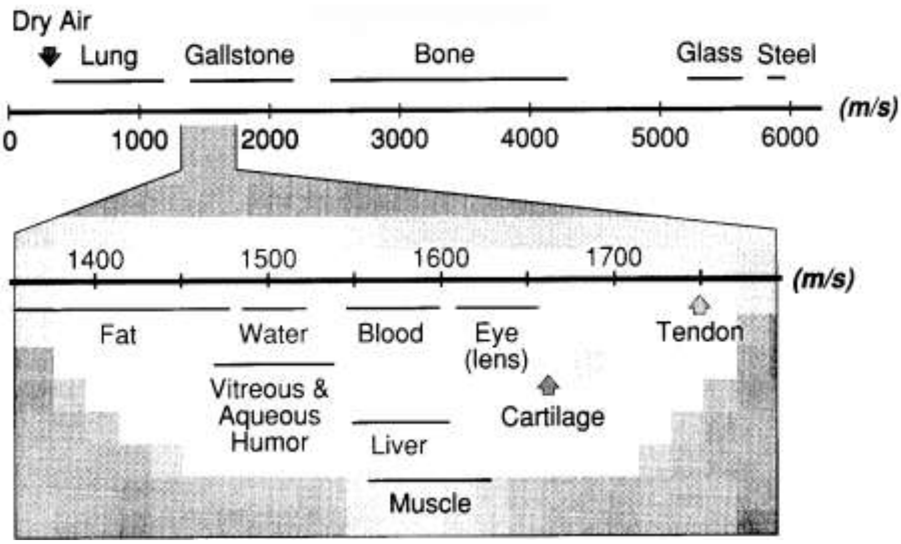
Here v is the volume flow, not the speed!

$$Z_{\mathrm{el}} = \frac{U}{I}$$

$$\boxed{Z = c\rho} = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance**
(useful form)

Speed of ultrasound in various materials.
The soft tissue median is 1540 m/s

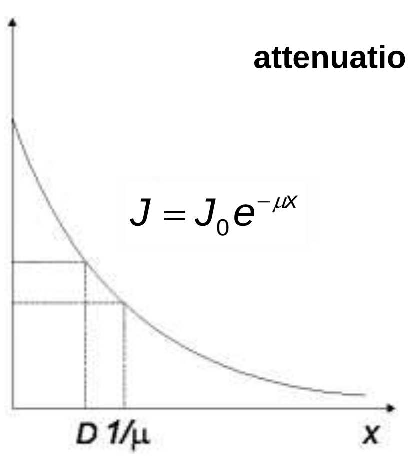


$$J = \frac{1}{Z} \Delta p_{eff}{}^{2}$$ intensity = energy-current density

Intensity obeys the absorption law, such as any wave

attenuation: $\alpha = 10 \cdot \lg \frac{J_0}{J}$ dB

$\alpha = 10 \cdot \mu \cdot x \cdot \lg e$ dB



μ is proportional to frequency in the diagnostic range!

μ is proportional to frequency in the diagnostic range

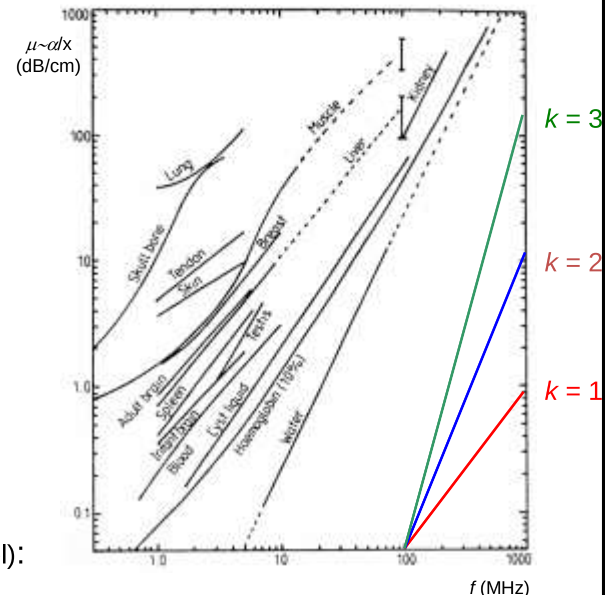
$\mu \sim f^k, \quad k \sim 1\,(?)$

$\log\mu \sim k\log f$

double-log graph: if the graph is a linear, the power function approximation is valid

specific attenuation for soft tissues (homogeneous tissue model):

$\boxed{\dfrac{\alpha}{f\,x} \sim 1\dfrac{\text{dB}}{\text{cm MHz}}}$

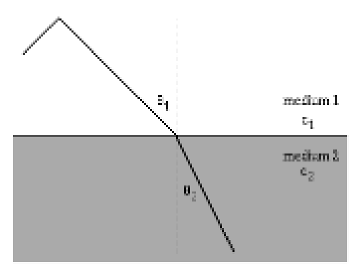


specific attenuation: $\boxed{\dfrac{\alpha}{f \cdot x}}$

Reflection and refraction – again at the boundaries (as with light)

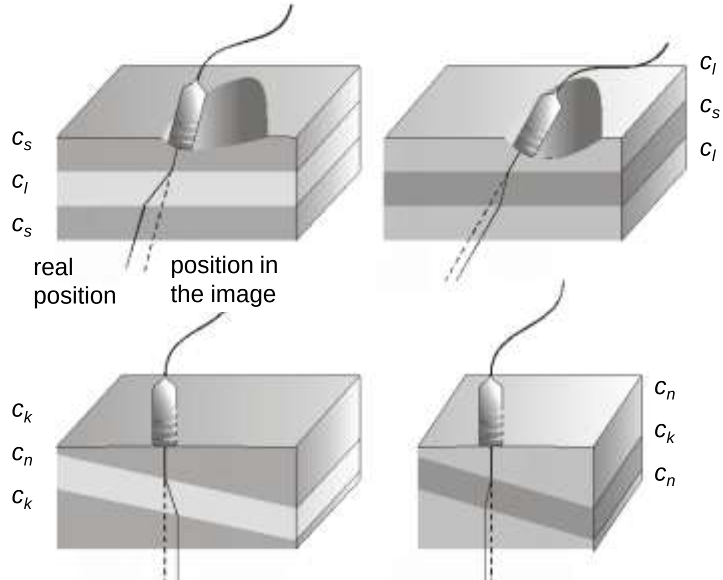
$$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}$$

Snellius-Descartes



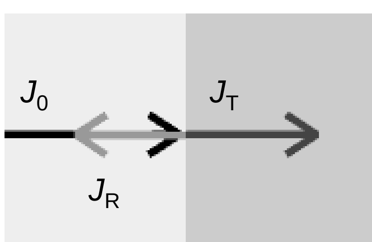
Frequency remains constant!

Ultrasound “beams” will change direction by refraction – just as light does



We need to take this into account in the imaging!

Reflection of ultrasound (normal incidence)



$J_0 = J_R + J_T$

reflection and transmission (penetration)

reflectivity:

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

Analogy to light: Z stands here instead of refractive index

boundary surface	*R*
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

"full" reflection:

$$Z_1 \ll Z_2, \quad R \approx 1$$

optimal coupling:

$$Z_{connecting} \approx \sqrt{Z_{source} Z_{skin}}$$

Coupling medium is required for medical ultrasound imaging!

Generation of ultrasound - Piezoelectric effect

source of electric signal

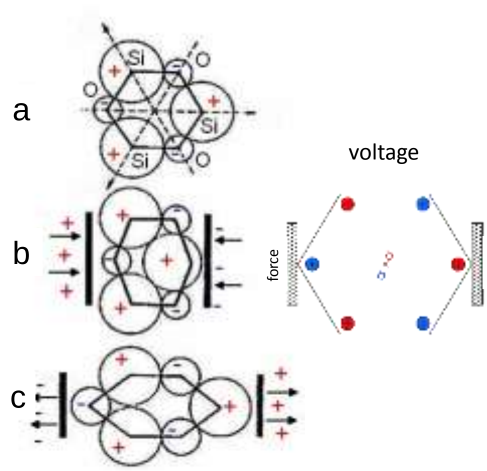
(sine wave oscillator)

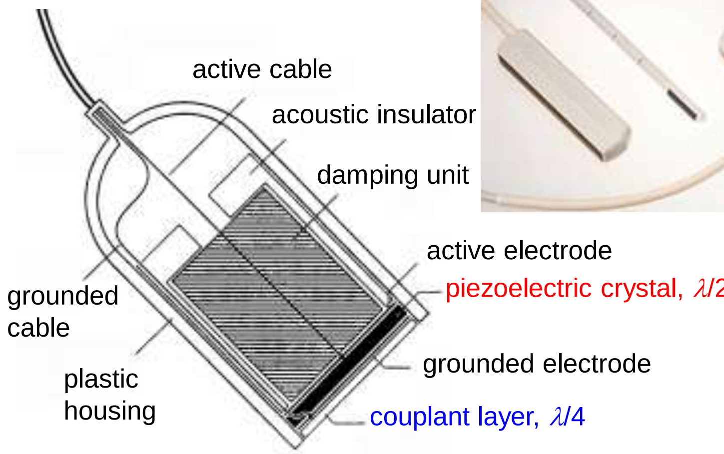
+

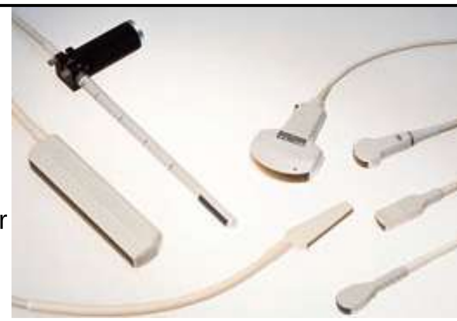
transducer (piezo-crystal)

(a) Center of charge of positive and negative charges coincides.

(b) and (c) As a result of pressure , the charge centers are separated, i.e. a potential difference arises (direct ~). The crystal is deformed when voltage is applied (inverse ~).

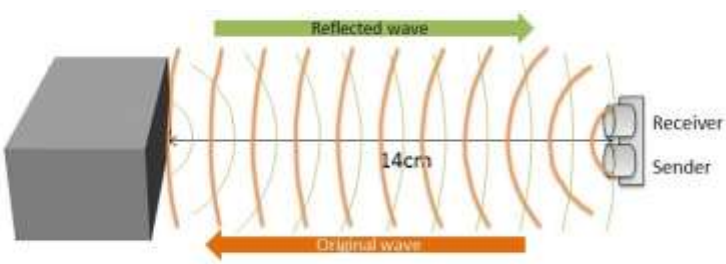






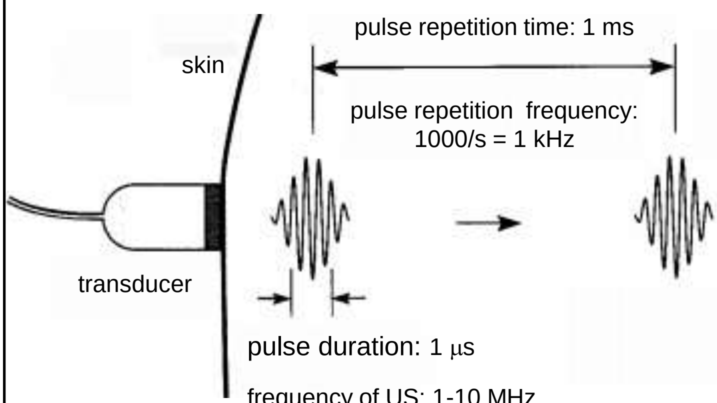
It acts as both reciever and transmitter of ultrasound pulses

Principle of ultrasound imaging:
We detect the reflections from various surfaces

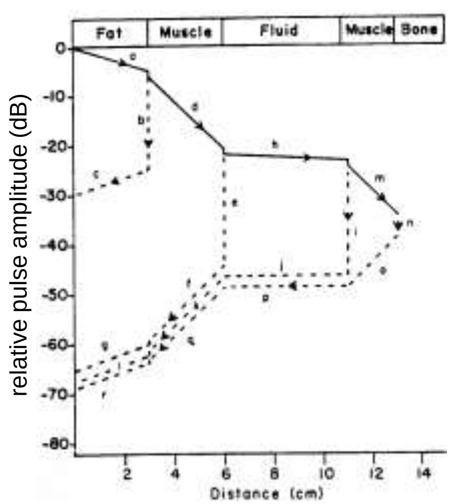


time sharing mode: **pulses** instead of continuous wave ultrasound

This enables the usage of the same transducer, and improves resolution



speed of propagation of US (in soft tissue): 1540 m/s



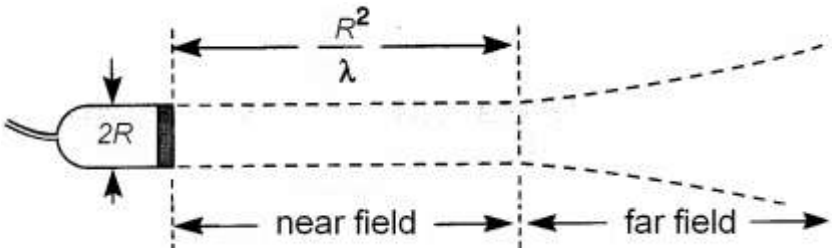
We need to deal with absorption: the deeper the reflection comes from, the weaker it will be

TGC: time gain compensation

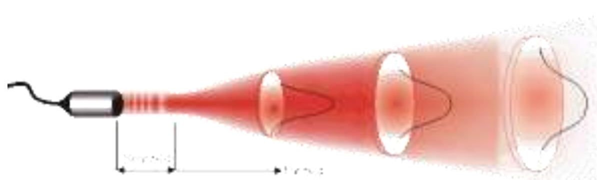
DGC: depth gain control

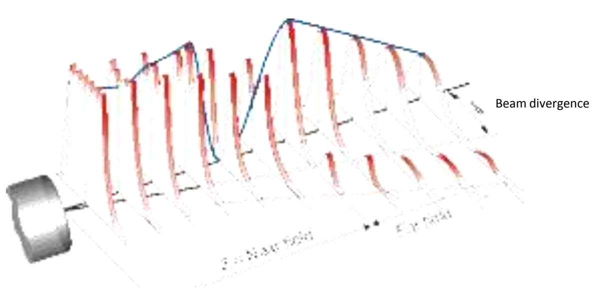
boundary surface	*R*	10lg*R* (dB)	*T*	10lg*T* (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

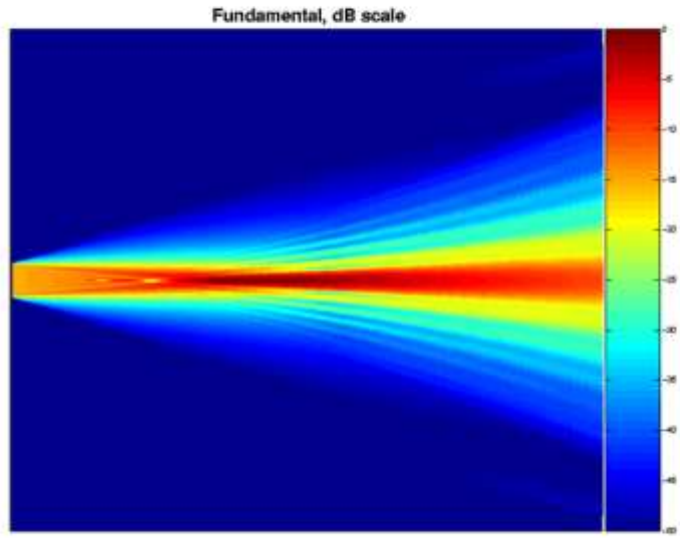
Technical details – beam shape, resolution, etc.



(Fresnel zone) (Fraunhofer zone)







Resolving limit is the distance between two object details which can be just resolved as distinct objects (the smaller the better).
Resolution (resolving power): the reciprocal of the resolving limit (the greater the better)

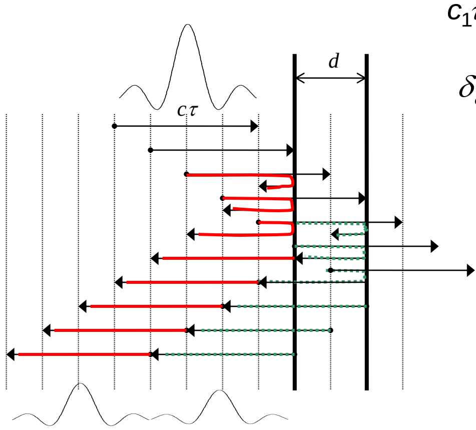
Axial resolving limit depends on the pulse length. Pulse length is inversly proportional to the frequency.

Lateral resolving limit is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

Typical values

frequency (MHz):	2	15
wavelength (in muscle) (mm):	0.78	0.1
penetration depth (cm):	12	1.6
lateral resolving limit (mm):	3.0	0.4
axial resolving limit (mm):	0.8	0.15

Axial resolving limit – depends on the pulse shape



τ : pulse duration

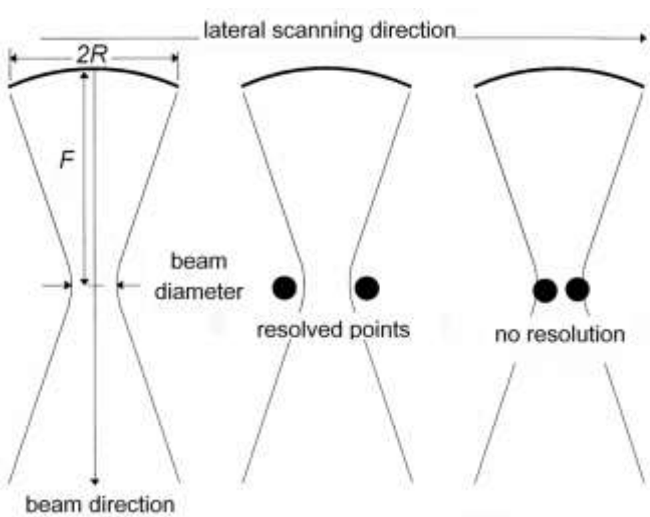
$c_1\tau \cong c_2\tau = c\tau$ pulse length

$\delta_{ax} = d = \dfrac{c\tau}{2}$ resolving limit

The axial resolving limit is the half of the pulse length. The echoes from the adjacent surfaces in this case just hit each other.

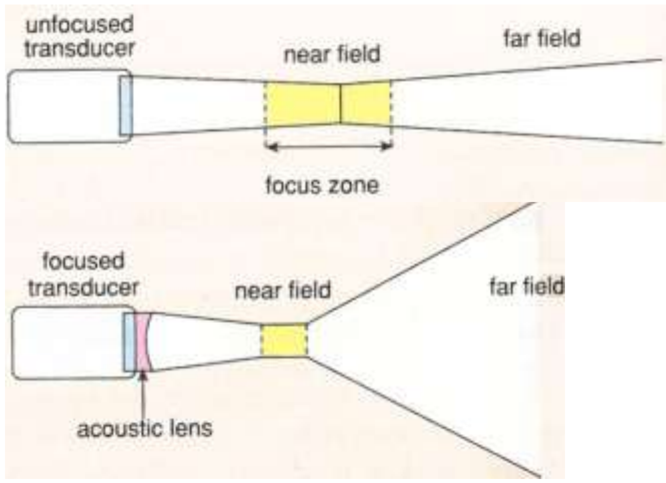
$$\tau \sim T = \frac{1}{f}$$

Lateral resolution – depend on the beam profile, or beam shape



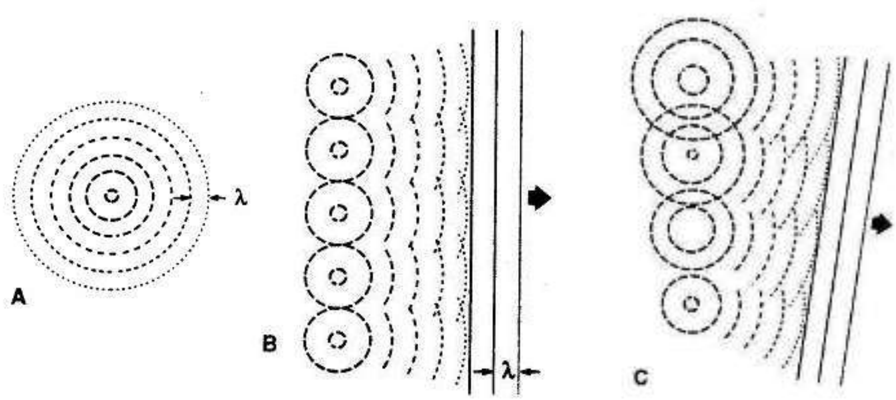
$$\left(\delta_{\text{lat}} \sim \frac{F}{2R} \cdot \lambda\right)$$

F: focal length
2*R*: diameter of the transducer
λ: wavelength



Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

Point sources and Huygens' principle



Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.

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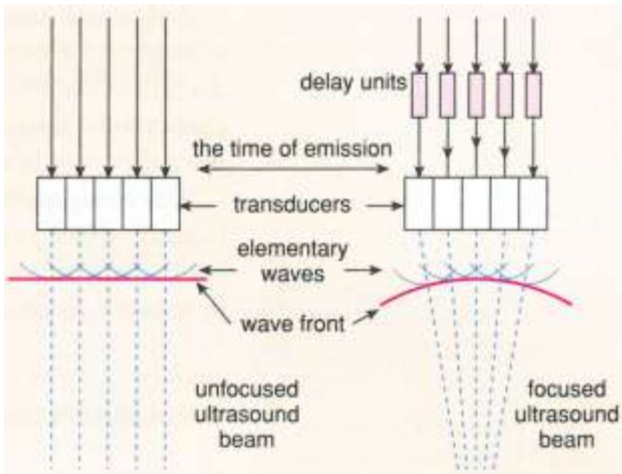
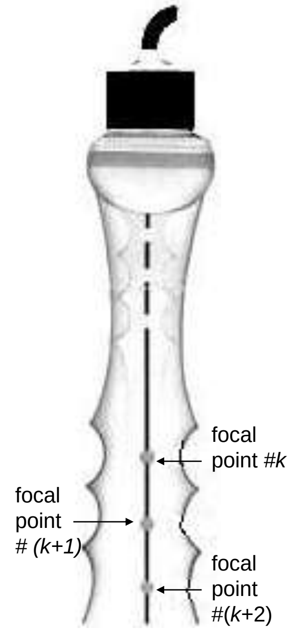
Electronic focusing – using Huygens





cf. Textbook Fig. on p.507

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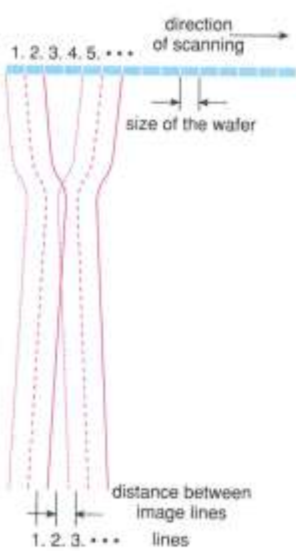
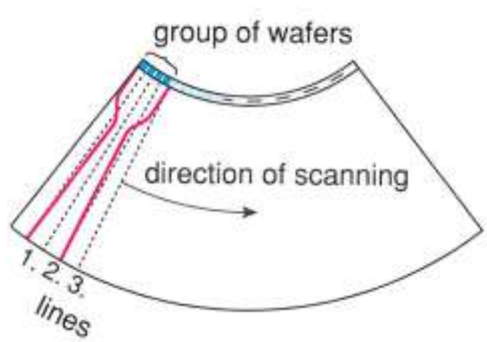
Scannig, moving the beam direction

multi unit linear array



multi unit curved array



cf. Textbook Fig. VII. 36-37

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Doppler-effect : how to measure velocity of targets

Standing source
frequency is constant
f

Moving source
frequency changes
("observed freq.")
f'

If the target, which reflects ultrasound is moving, then it also acts as a moving source…

---

f': **observed frequency**, f : original frequency

(a) standing source and moving observer (v_O)
+: observer approches the source
–: observer moves away from the source

$$\boxed{f' = f\left(1 \pm \frac{v_O}{c}\right)}$$

(b) moving source and standing observer
(if $v_S \ll c$, then „same" as (a))

$$f' = \frac{f}{1 \mp \frac{v_S}{c}}$$

(c) moving source and moving observer

$$f' = f\,\frac{1 \pm \frac{v_O}{c}}{1 \mp \frac{v_S}{c}}$$

(d) moving reflecting object (surface),
(if $v_R \ll c$)

$$\boxed{f' = f\left(1 \pm \frac{2v_R}{c}\right)}$$

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Doppler frequency = frequency change = frequency shift

if v_i, v_R<<c (i= S or O)

rearranging equation (a)
moving source or observer:

$$\Delta f = f_D = \pm \frac{v_i}{c} f$$

rearranging equation (d)
moving reflecting object or surface:

$$\Delta f = f_D = \pm 2 \frac{v_R}{c} f$$

if v and c are not parallel, then $v \cos\Theta$ should be used instead of v (remark: if $\Theta = 90°$, $f_D = 0$)

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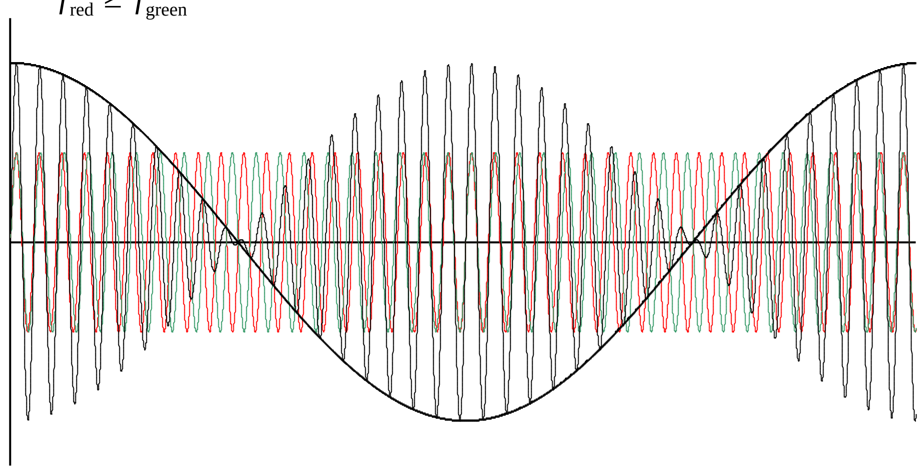
Beating phenomenon

$f_{\text{red}} \geq f_{\text{green}}$

the beating frequency equals to the difference of the two interfering frequency



reminder: $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

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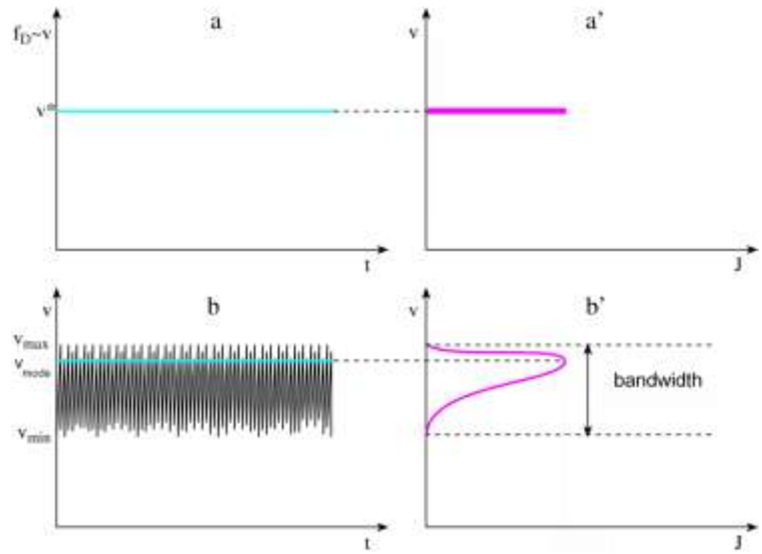
Doppler curves

one constant velocity (v*)

frequency distribution (with v_{mode})



velocity distribution

velocity distribution at a certain time

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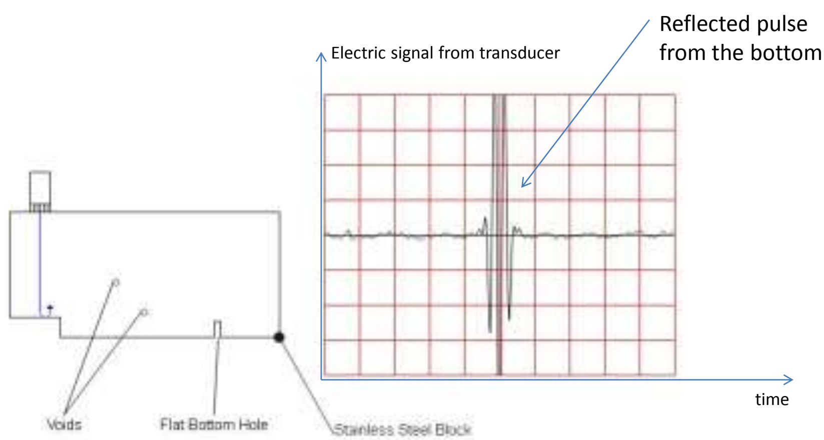
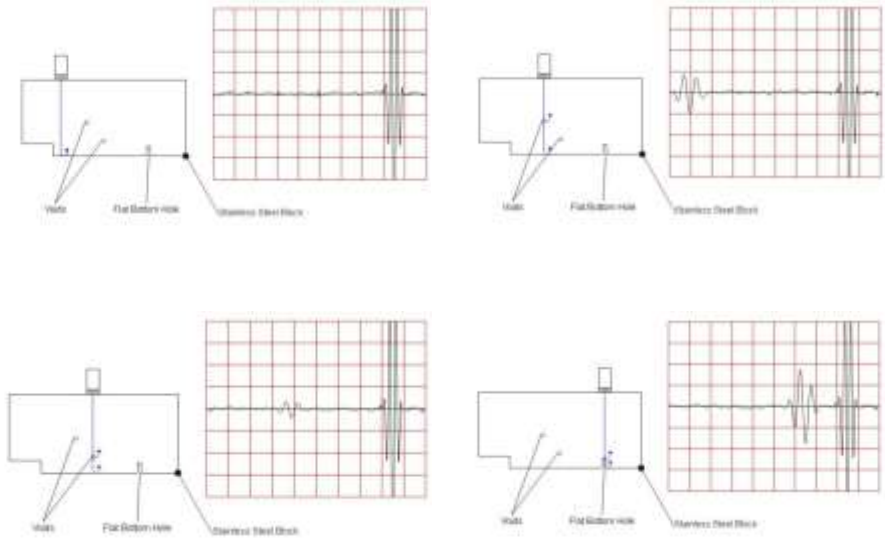
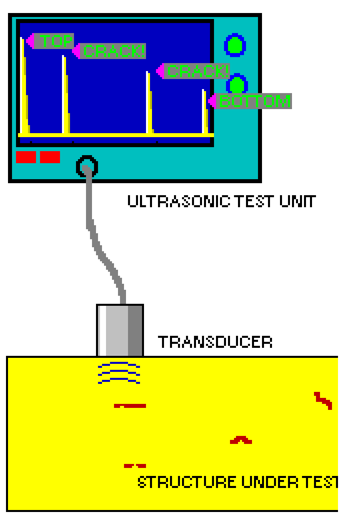
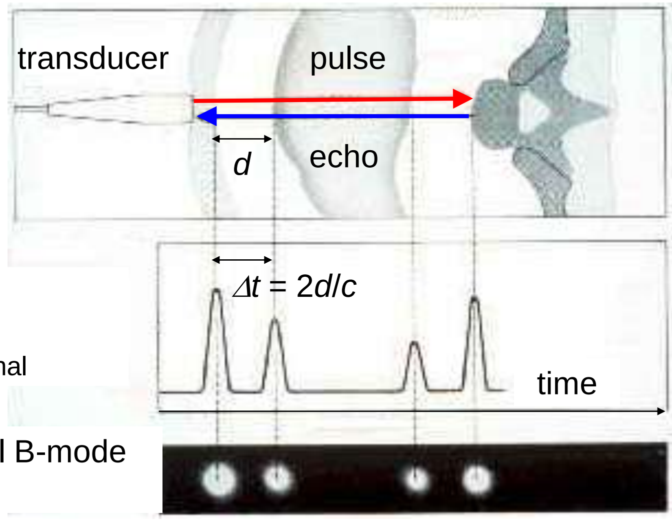
Imaging surfaces by observing pulse echoes





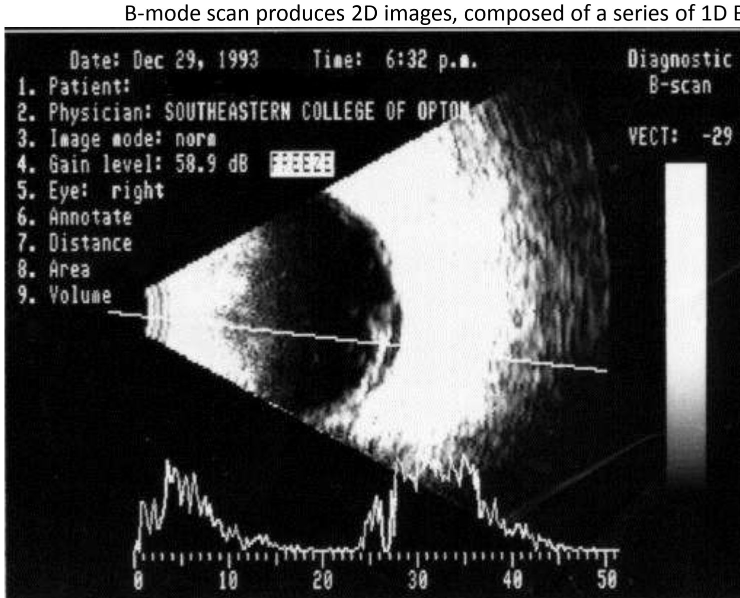

TOP
CRACK!
CRACK!
BOTTOM
ULTRASONIC TEST UNIT
TRANSDUCER
STRUCTURE UNDER TEST



A-mode
(***A***mplitude)
only 1-dimensional

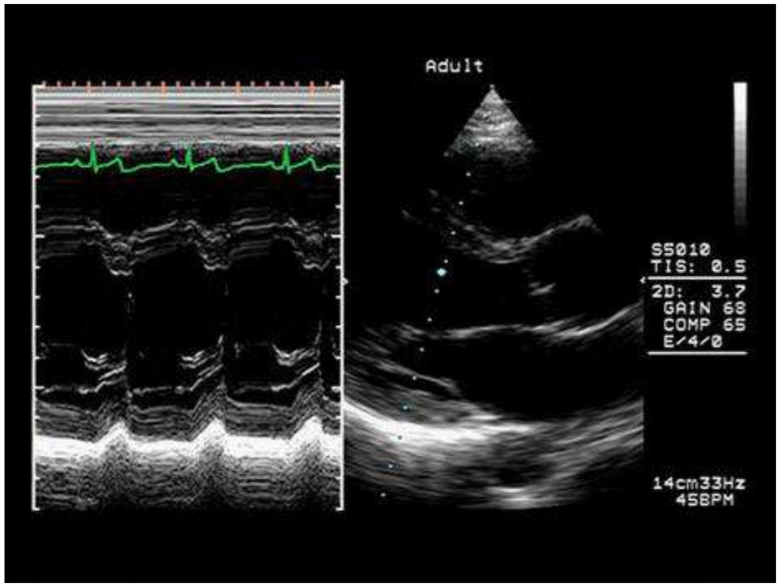
1-dimensional B-mode
(***B***rightness)

A-mode and B-mode images of the same structure
B-mode scan produces 2D images, composed of a series of 1D B-mode data

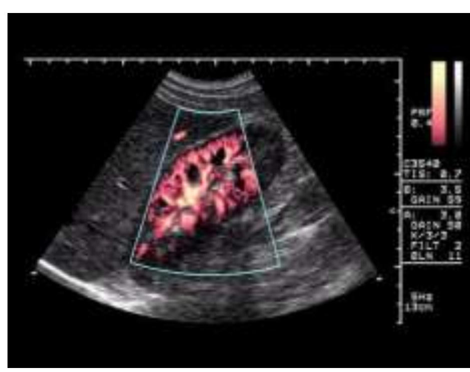


TM-mode B-mode

Time-Motion mode: multiple 1D lines as the time goes on.



Doppler modes: calculate velocity from the freq. change.



BART: Blue Away Red Towards

Color-code the velocity calculated from f' or from the beating

power Doppler

Show the intensity of the doppler signal

In the doppler method the angle is important!

Example: 1D CW doppler with beating detection

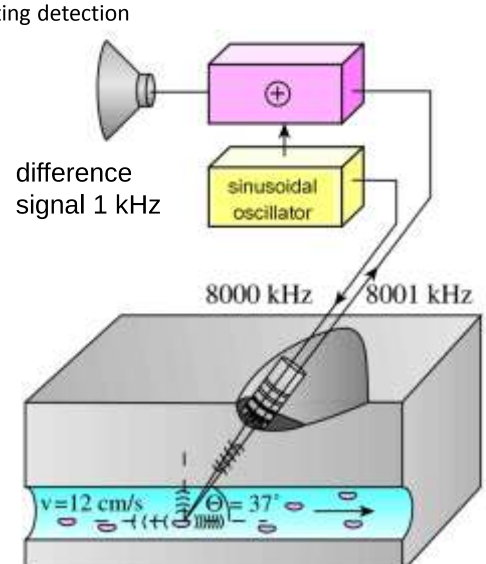
CW: continuous wave

source and detector are separated

$$|f_D| = 2\frac{v_R \cos\theta}{c} f$$

e.g. f=8000 kHz
v=12 cm/s
c=1600 m/s
Θ = 37º

⇨f_D=1 kHz
(beating phenomenon)

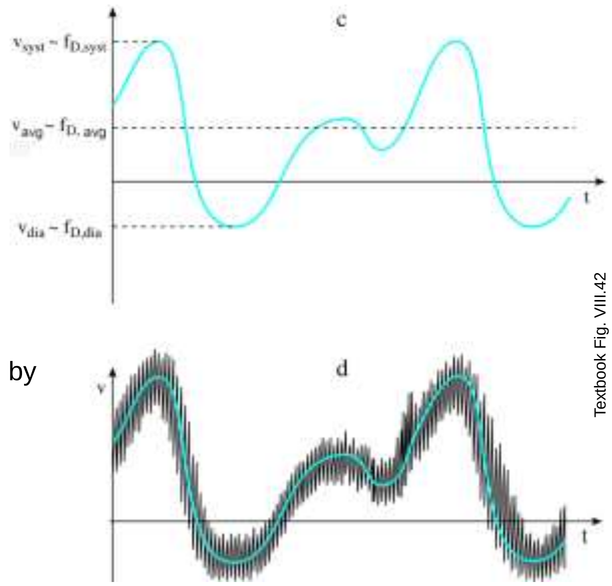


Textbook Fig. VIII.41

Doppler curves

flow can be represented by one velocity in each moment

flow can be represented by a velocity distribution in each moment



Textbook Fig. VIII.42

velocity distribution in TM-mode

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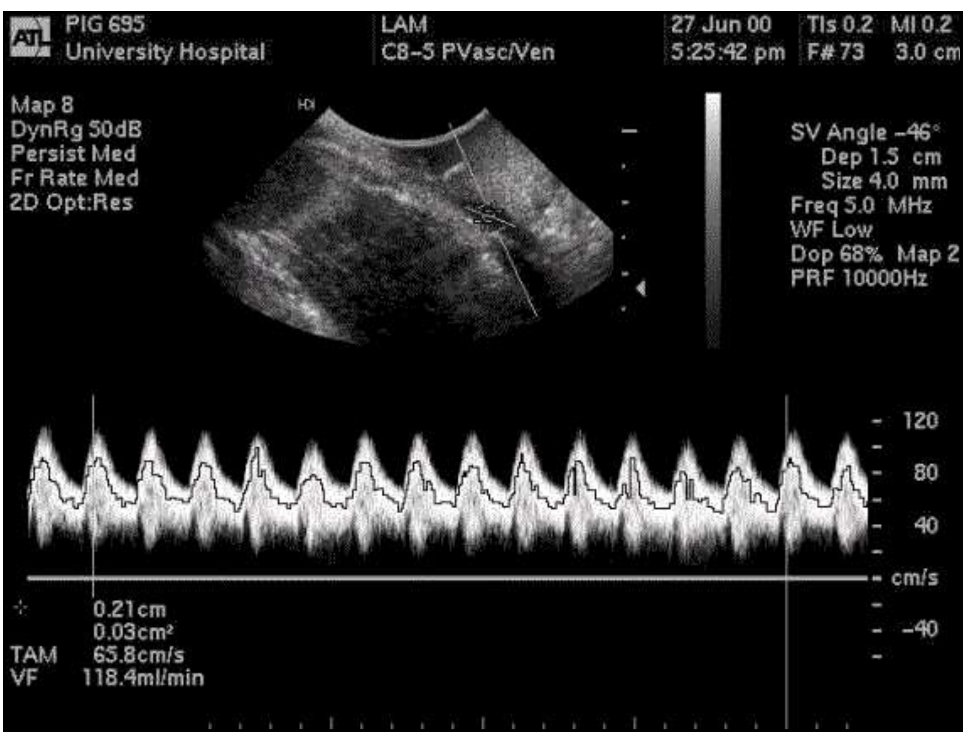
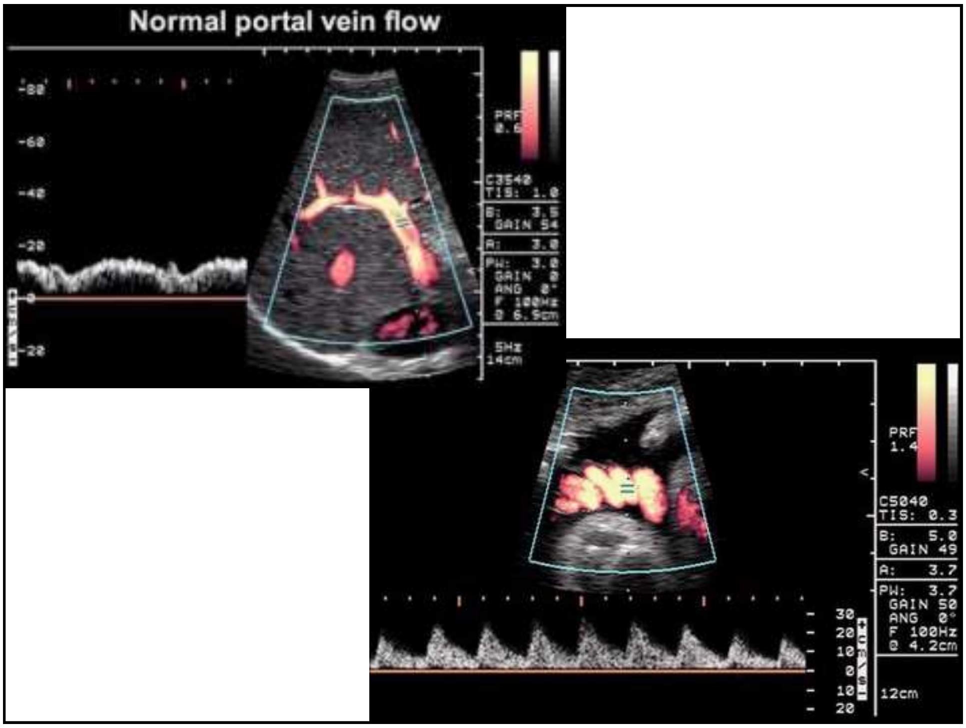


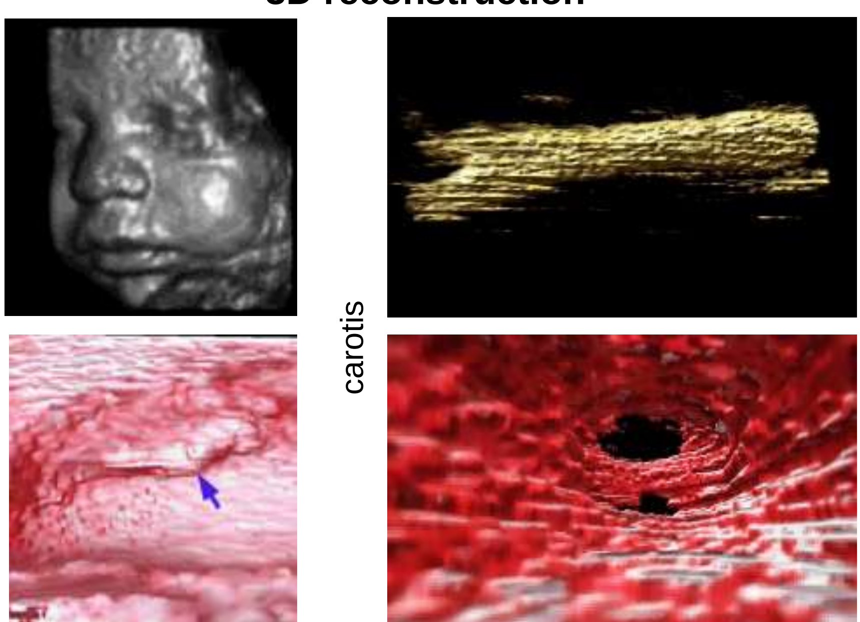


3D reconstruction

face of a fetus

bladder

carotis



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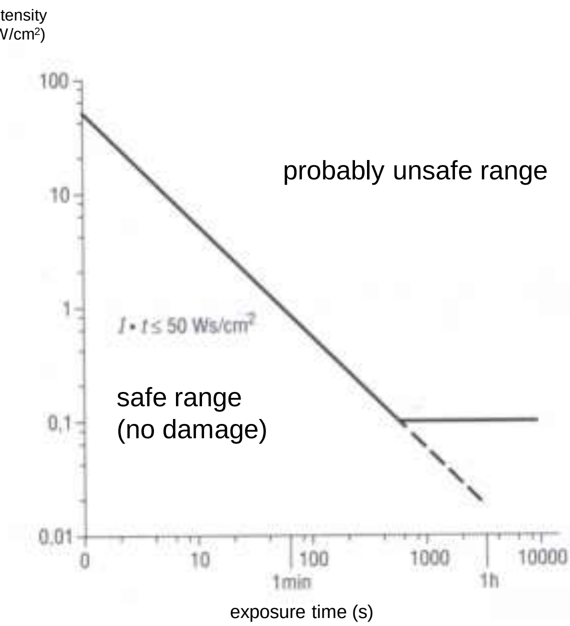
Safety

in the diagnostics:

10 mW/cm^2 = 100 W/m^2

spatial average temporal average (SATA) intensity;
spatial peak temporal peak (SPTP) intensity;
spatial peak temporal average (SPTA) intensity;
spatial peak pulse average (SPPA) intensity
spatial average pulse average (SAPA) intensity



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Mechanical index = peak negative pressure / SQRT(center frequency of the US beam)

Thermal index = W_p / W_{deg}
W_p: relevent (attenuated) acoustic power at the depth of interest
W_{deg}: estimated power necessary to raise the tissue equilibirum temperature by one degree C

in the therapy: 1 W/cm^2