FLUID FLOW AND CIRCULATORY SYSTEM

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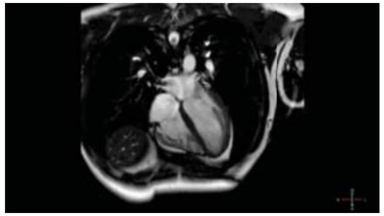
Importance of the physics of fluids

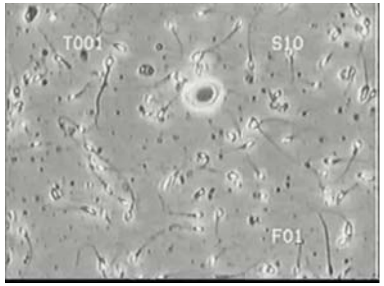
I. Hemodynamics

E.g.: What are the characteristics of blood flow in the circulatory system?



II. Motion in viscous fluids

E.g.: What is the force exerted by a single spermatocyte during its motion?

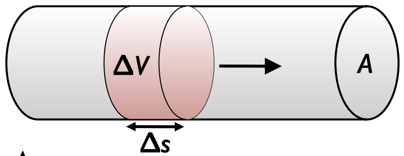


Biophysics of fluid flow and blood circulation

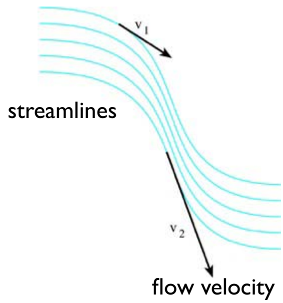
- Basic principles
- Types of fluids
- Types of fluid flow
- Laws of fluid flow
- Blood as a fluid; determinants of blood viscosity
- The circulatory system; blood vessels as elastic tubes
- Auxiliary forces of circulation

Basic principles I.

Volumetric flow rate (Q):

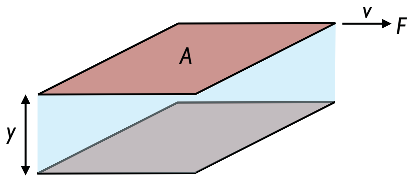


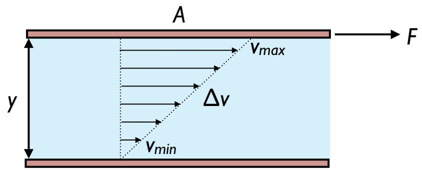
$$Q = \frac{\Delta V}{\Delta t} = A\frac{\Delta s}{\Delta t} = A\bar{v}$$



Average velocity: $\bar{v} = \frac{Q}{A}$

Basic principles II.
Viscosity (internal friction)





F	= shear force
A	= area of fluid layer
η	= viscosity
v	= flow velocity
y	= distance between fluid layers
F/A	= shear stress (τ)
Δ*v*/Δ*y*	= velocity gradient (*D*)

$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y}$$ (Newton's friction law)

$$\eta = \frac{\tau}{D}$$

Units of viscosity: $1 Pas = 1 \frac{Ns}{m^2} = 10 P(poise)$

Viscosity of distilled water (25 ˚C): 1 mPas (1 centipoise)

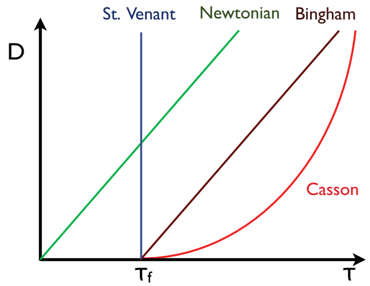
Types of fluids

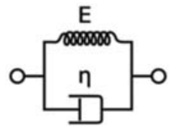
1. ***Ideal***
   frictionless, incompressible
   ρ = constant, η = 0

2. ***Non-ideal (real)***
   a. Newtonian (viscous)
   η independent of shear stress

   b. Non-newtonian (anomalous)
   η changes with shear stress

Relationship between velocity gradient and shear stress in ***real*** fluids



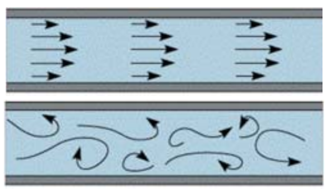


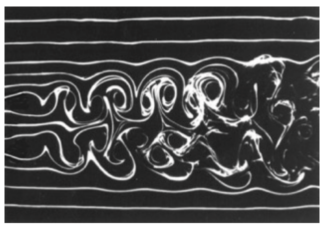
Model of viscoelastic body - Kelvin-body: spring and dashpot coupled in parallel

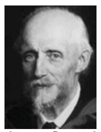
- τ_f = flow threshold
- Viscoelastic materials: combination of viscous and elastic properties (e.g., solution of polymers, macromolecules)
- Stress-relaxation: decay of stress in rapidly deformed viscoelastic body.
- Blood is a non-newtonian fluid; it displays viscoelastic properties.

Types of fluid flow

1. **Stationary**
   Volumetric flow rate stays constant (parameters characterizing flow remain unchaned).
2. **Laminar**
   Fluid layers do not mix.
3. **Turbulent**
   Fluid layers mix.





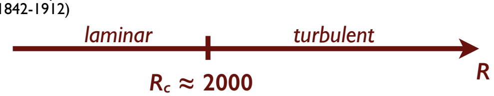


Osborne Reynolds (1842-1912)

Reynolds number (*R*):

$$R = \frac{v r \rho}{\eta}$$

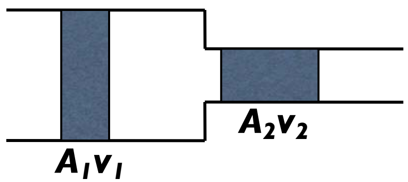
v = flow rate (m/s)
r = tube radius (m)
ρ = density of fluid (kg/m³)
η = viscosity (Ns/m²)



"When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first." (Werner Heisenberg)

Laws of flow in ideal fluids I.

Continuity equation

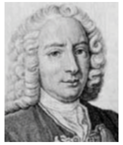


$$A_1 v_1 = A_2 v_2 = \text{constant}$$

A = cross-sectional area
v = flow velocity

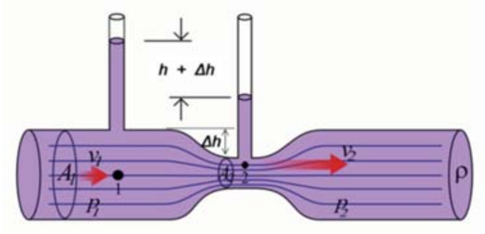
Laws of flow in ideal fluids II.

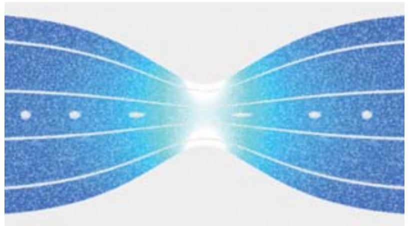
Bernoulli's law - law of conservation of energy


Daniel Bernoulli
(1700-1782)

$$p+\frac{1}{2}\rho v^2+\rho gh = konst$$

p = static pressure
$\frac{1}{2}\rho v^2$ = dynamic pressure
ρgh = hydrostatic pressure

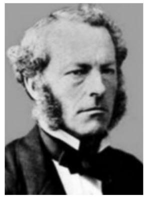


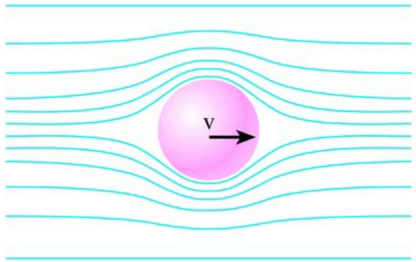


Venturi effect

Laws of flow in viscous fluids I.

Stokes' law


Georg Gabriel Stokes
(1819-1903)

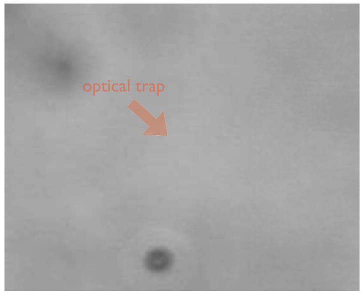


$$F = \gamma v = 6r\pi\eta v$$

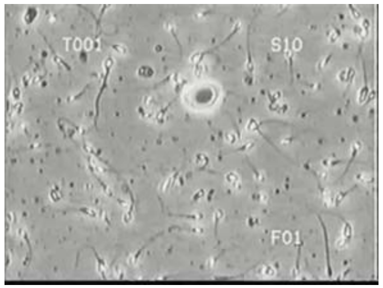
F = force
γ = drag coefficient (shape factor)
v = flow rate
r = radius of sphere
η = viscosity

Stokes force

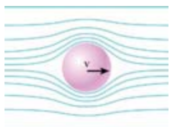
Hydrodynamic drag force (Stokes force): $F = \gamma v = 6r\pi\eta v$

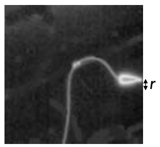


Affects stationary objects in moving fluid
(microbead captured in optical trap)



Affects objects moving in stationary fluid
(moving spermatocytes)





r = 1.6 μm = 1.6×10^{-6} m
v = 50 μm/s = 5×10^{-5} m/s
η = 10^{-3} Pas

How much force is exerted by a single spermatotyce during its movement?

$$\gamma = 6r\pi\eta = 6\cdot 1.6\times 10^{-6}\cdot \pi \cdot 10^{-3} = 3\times 10^{-8} Ns/m$$

$$F = \gamma v = 3\times 10^{-8} Ns/m \cdot 5\times 10^{-5} m/s = 1.5\times 10^{-12} N = \boxed{1.5\,pN}$$

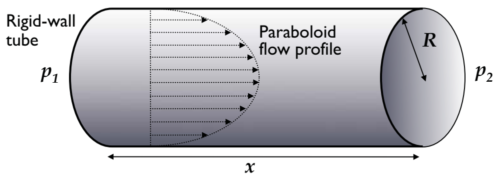
Laws of flow in viscous fluids II.

Hagen-Poiseuille's law


G.H.L. Hagen
(1797-1884)


J.-L.-M. Poiseuille
(1799-1869)

Thermodynamic current	Relevant intensive variable (its difference maintains current)	Current density	Physical law
Volumetric flow	Pressure (p)	$J_V = -\frac{R^2}{8\eta}\frac{\Delta p}{\Delta x}$	Hagen-Poiseuille



$$J_V = \frac{V}{tA} = \frac{R^2}{8\eta}\frac{\Delta p}{\Delta x}$$

$$A = R^2\pi$$

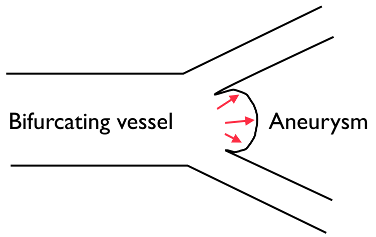
$$\frac{V}{t} = \frac{R^4\pi}{8\eta}\frac{\Delta p}{\Delta x}$$

V = volume
t = time
R = tube radius
η = viscosity
p = pressure
x = tube length

V/t = *Q* = volumetric vlow rate
$\Delta p/\Delta x$ = pressure gradient, maintained by p_1-p_2
A = cross-sectional area of tube
J_V = flow intensity

Medical significance of fluid flow

Bernoulli's law:



Formation of aneurysm (pathological expansion of blood vessel):

- Expansion of vessel: diameter increases
- Flow rate decreases, accoring to continuity equation
- Static pressure increases due to Bernulli's law
- Aneurysm pregrediates - positive fedback mechanism leading to catastrophy

Hagen-Poiseuille's law:

$$\frac{V}{t} = \frac{R^4 \pi}{8\eta} \frac{\Delta p}{\Delta x}$$

Flow intensity, hence the delivered oxygen quantity, may be **drastically reduced** in certain pathological conditions:

- constriction of blood vessels (e.g., diabetes, Bürger's disease)
- change in blood viscosity (e.g., fever, anaemia)
- Reduction of vessel diameter by half leads to a reduction of volumetric flow by 1/16!

Blood as a fluid

<table>
<tr><td colspan="3">55-60% of body mass is water
42 kg (70 kg body mass)</td></tr>
<tr><td>2/3 intracellular
28 kg</td><td colspan="2">1/3 extracellular
14 kg</td></tr>
<tr><td></td><td>1/3 plasma
4-5 kg</td><td>2/3 intersticium
9-10 kg</td></tr>
</table>

Blood:

Average volume: 5 l
Average viscosity: 5 mPas
Average density: 1.05 g/cm^3
Composition: 40-45 % corpuscular, 55-60 % plasma

Determinants of blood viscosity

1. Hematocrit (htc, φ):

$$htc = \frac{V_{cells}}{V_{total}}$$

Normal range: 0.4-0.5.

Viscosity of blood as suspension
(in the physiologically relevant htc range):

$$\lg \eta_s = A + B\phi$$

η_s=suspension viscosity
A, B=empirical constants

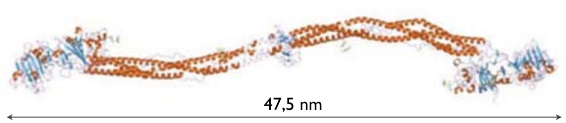
Determinants of blood viscosity

2. Plasma viscosity

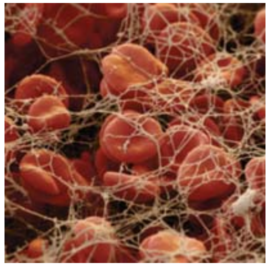
- Depends on plasma proteins.
- In *paraproteinaemias* (e.g. myeloma multiplex or plasmocytoma) the concentration of immunoglobulins is high, leading to increased viscosity.

Plasma protein	Normal concentration	% ratio	Function
Albumin	35-50 g/l	55%	maintenance of colloind osmotic pressure, transport
Globulins	20-25 g/l	38%	Part of the immune system
Fibrinogen	2-4.5 g/l	7%	Blood coagulation

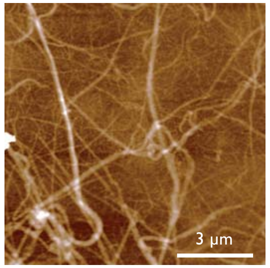
Fibrinogen, fibrin



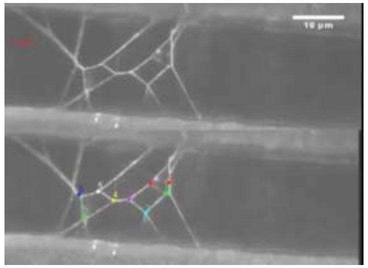
Fibrinogen:
MW = 340.000 Da
In plasma: 2-4 g/l ≈ 10 μM
Average nearest-neighbor distance ≈ 55 nm



Red blood cells in fibrin meshwork



Fibrin polymerized in vitro (AFM)

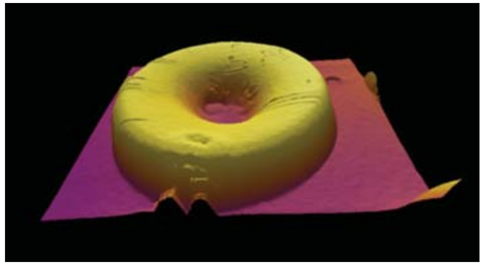


Extensibility of fluorescently labeled fibrin fibrils

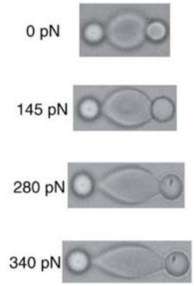
Determinants of blood viscosity

3. Plasticity of red blood cells

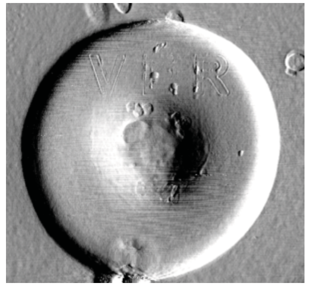
- 65% suspension of blood-cell-size particles is rock hard.
- By contrast, a 95% blood suspension if fluid, with viscosity of ~20 mPas!
- Deformation of red blood cells: droplet, parachute, arrowhead shapes.



Disc-shaped cell with 7-11 μm diameter



Deformation of a RBC with optical tweezers

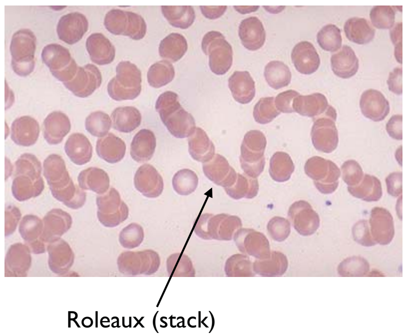


Fixed RBC maintaining impression (AFM)

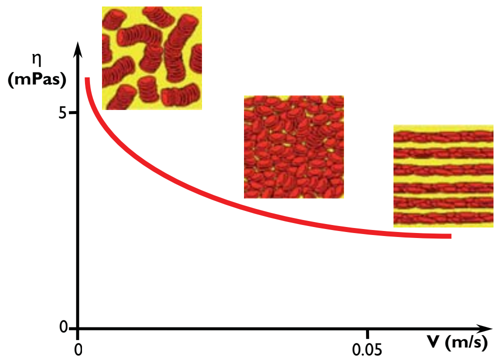
Determinants of blood viscosity

4. Aggregation of red blood cells

- Stack or roleaux formation.
- More pronounced at low flow rates

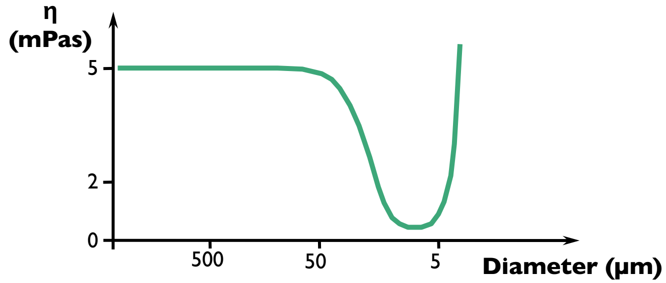


5. Flow rate, velocity gradient



Determinants of blood viscosity

6. Blood vessel diameter



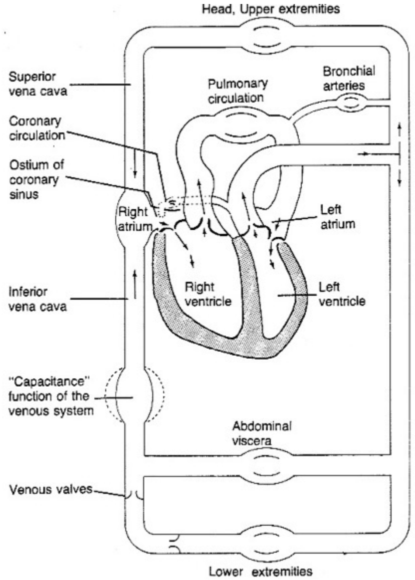
N.B.:

- With a decrease of vessel diameter, the anomalous (non-newtonian) behavior of blood becomes more pronounced.
- *Axial migration*: the red blood cells line up in the axis of the vessel (Bernoulli's law). In the axis the velocity gradient decreases, and near the vessel wall it increases. Increase in velocity gradient decreases apparent viscosity (Fåhraeus-Lindquist effect).

The vascular system is a closed vessel system returning into itself

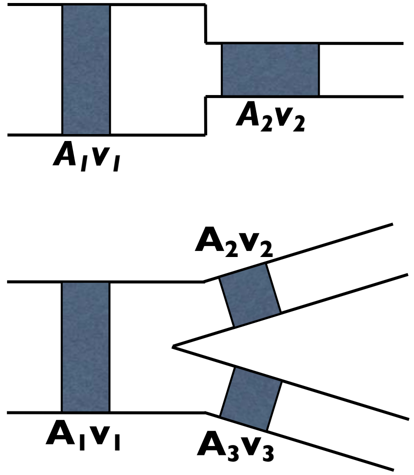
A. Function:
Maintenance of environmental parameters of cells ("steady state")
Transport:
Gases
Metabolites
Hormones, signal transmitters
Immunoglobulins
Heat

B. Hemodynamic requirements:
Slow (matches diffusion-driven processes)
Steady (no fluctuations)
Unidirectional



Fluid flow in bifurcating vessel system

Continuity equation



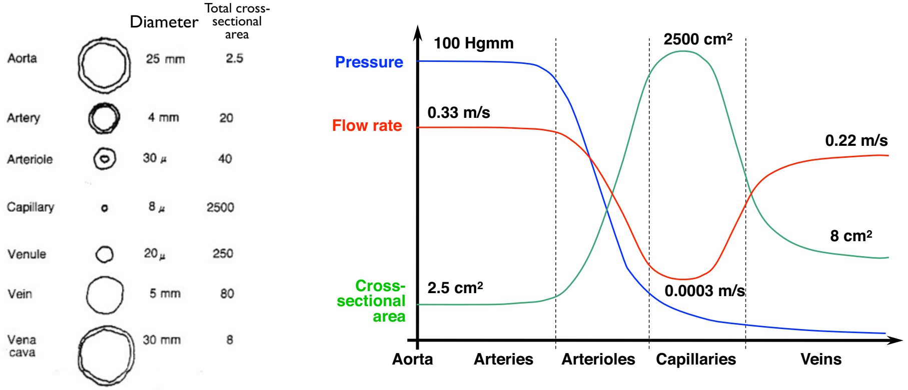
$$A_1v_1 = A_2v_2 = const$$

A = cross-sectional area
v = flow rate

$$A_1v_1 = A_\Sigma(v)_{average} = const$$

A_Σ = total cross-sectional area

Structure and physical properties of the vascular system



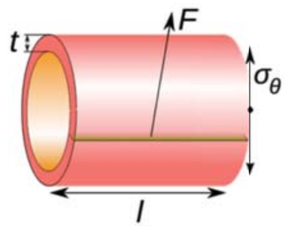
- **Pressure** on blood vessel wall: ***"blood pressure"***. Pressure drop along vessel maintains blood flow.
- Reason of **pressure drop**: flow resistance - most of energy is converted to heat.
- **Flow rate** and total **cross-sectional area** change inversely (based on equation of continuity, *Av* = constant).
- Flow rate typically does not exceed the **critical** (see Reynolds number), and flow remains laminar. (Exceptions: behind aortic valve, constricted vessels, low-viscosity conditions, Korotkoff sound).
- **Arterioles** (vessels containing smooth muscle, under vegetative innervation) are pressure-regulators: ***"resistance vessels."***
- Most of blood volume in veins: ***"capacitance vessels."***

Wall tension and bood pressure

Circumferential stress (σ_θ) depends on blood pressure: (Young-Laplace - equation)

$$\sigma_\theta = \frac{P \cdot r}{t}$$

P = blood pressure
r = radius of tube
t = wall thickness

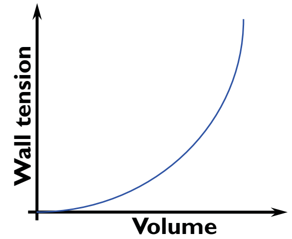


$$\sigma_\theta = \frac{F}{t \cdot l}$$

F = force
l = tube length

Wall tension or circumferential stress is the average force exerted circumferentially (perpendicular to both the axis and the radius) in the cylinder wall.

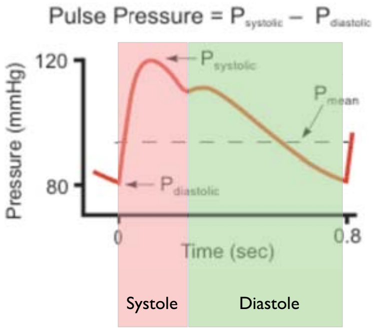
Vessel wall displays non-linear elastic properties

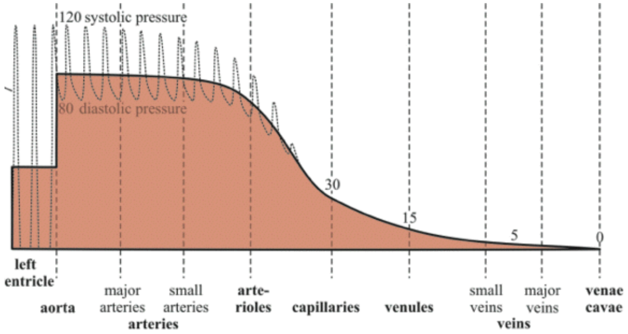


Determinants of vascular elasticity:
Elastin
Collagen
Smooth muscle

Implications of vascular easticity:
Storage of potential (elastic) energy
Dampening of pressure pulses
Constant flow rate

Dynamic pressure-changes in the arterial system



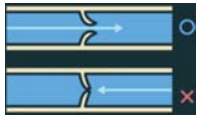


Because of vessel wall elasticity, pressure fluctuations are dampened.

Auxiliary factors of circulation

1. Arterial elasticity
elastic fibers ➛ storage of potential energy

2. Venous valves (Harvey's experiment)
"On the Circulation of the Blood" (1628).



3. Muscle action

4. Negative intrathoracic pressure

5. "Up-and-down" movement of **atrioventricular plane**

Harvey's experiment (1628)