Wave optics

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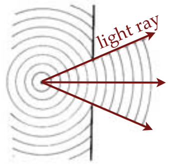
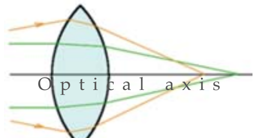
Geometric optics and wave optics

Geometric optics

If light propagates through a slit much larger than its wavelength, then the spreading of the wavefront (phase) may be simplified into a line ("light ray").



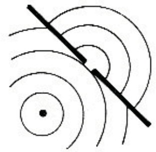


Ray diagram

- Optical (light) ray ("light beam"): abstraction, mathematical line.
- Arrows represent the direction of energy propagation.
- Optical axis: line connecting the midpoint of optical components (e.g., lenses).
- Principle of reversibility: the direction of energy propagation (arrows) may be reversed.

Wave optics

If light propagates through a slit comparable or smaller than its wavelength, then its wave properties ust be taken into account.



Important parameters of the propagating wave:

- **Period** (T)
- **Frequency** (f=1/T)
- **Velocity** (v, c)
- **Wavelength** (λ): distance covered in a period:

$$\lambda = cT = \frac{c}{f}$$

Speed of propagation of light in ***vacuum***: c=2,99792458 x 10^8 ms^{-1}
In optically denser media the speed of propagation is reduced (c_1).
This may be expressed with the ***absolute refractive index*** (n_1):

$$n_1 = \frac{c}{c_1}$$

Wave optics

- Oscillations; harmonic oscillation
- Types of waves
- Wave phenomena
- Detectable parameters of the light wave
- Phase; phase contrast microscopy
- Polarization; polarimetry, polarization microscopy
- Resolving power of the human eye
- Color vision

Wave: propagating oscillation

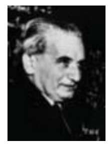
What is an oscillation?

Example:
Tacoma Narrows Bridge

Tacoma Narrows Bridge ("Gallopin' Gertie")
- (*"Gertie the Dinosaur"* (1914), cartoon, Winsor McCay)
- Opening: July 1, 1940.
- During wind (50-70 km/h): oscillation for hours
- Oscillation amplitude initially 0.5 m, then, after snapping of a suspension cable, up to 9 m!
- Collapse: November 7, 1940.

(Explanation of the effect)

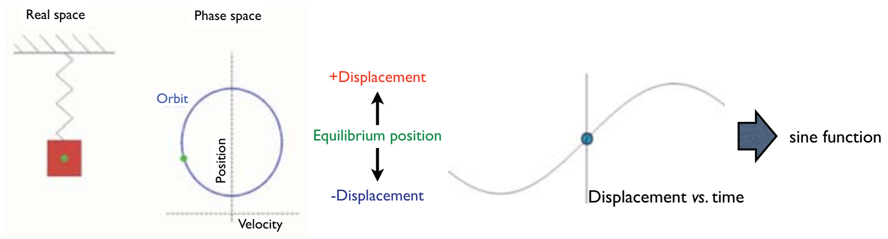
Kármán vortex street

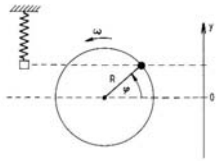


Theodore von Kármán
1881-1963

Harmonic oscillation

Restoring force acts on a system displaced out of equilibrium (e.g., mass on a spring).





ϕ = phase angle at time *t*
y = displacement at time *t*
ω = angular velocity (ϕ/t)
R = length of rotating unit vector
= maximal displacement (amplitude)

$$y = R\sin\varphi$$

Because ϕ = ωt: $y = R\sin(\omega t)$

If the initial phase angle (ϕ_0) differs from 0: $y = R\sin(\omega t + \varphi_0)$

Because angular velocity (ω) is the full circular orbit (2π) per period (*T*): $y = R\sin(\frac{2\pi}{T}t + \varphi_0)$

Important parameters of the propagating wave:

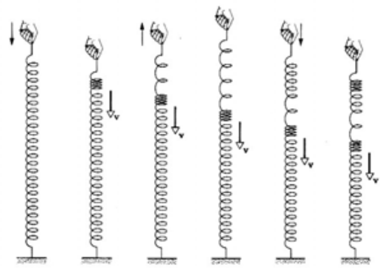
- **Period** (*T*)
- **Frequency** (*f*=1/*T*)
- **Velocity** (*v*, *c*)
- **Wavelength** (λ): distance covered in a period:

$$\lambda = cT = \frac{c}{f}$$

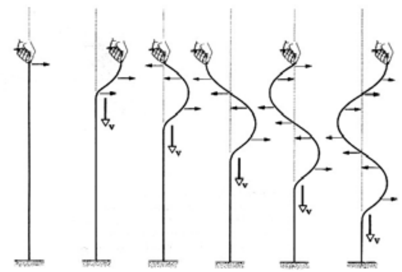
Types of waves

- According to source:
  1. Mechanical: elastic deformation propagating through elastic medium
  2. Electromagnetic: electric disturbance propagating through space (vacuum)
- According to propagation dimension:
  1. One-dimensional (rope)
  2. Surface waves (pond)
  3. Spatial waves (sound)
- According to relative direction of oscillation and propagation:

1. Longitudinal



2. Transverse



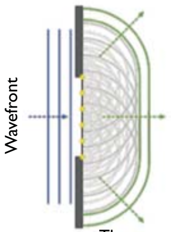
Wave phenomena I.

Diffraction

Huygens-Fresnel principle:
every point of a wavefront is the source of further waves

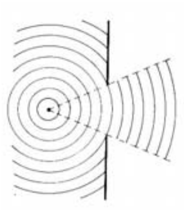
Christiaan **Huygens** (1629-1695)

Augustin-Jean **Fresnel** (1788–1827)

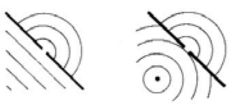


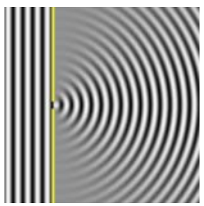
The wave appears in the "shaded" areas, too.

Slit much greater than the wavelength (λ)



Slit much smaller than wavelength (λ)





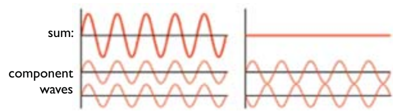
Wave phenomena II.

interference

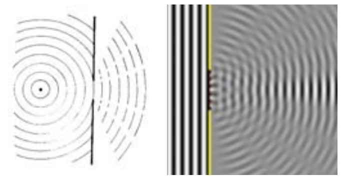
Principle of superposition

Waves in phase (φ=0): amplification

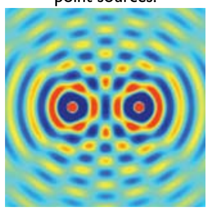
If φ=π : cancellation



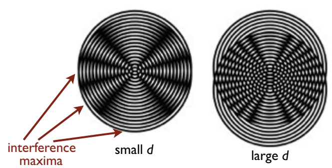
Slit comparable to wavelength
(=pointlike slits separated by distance *d*, where *d*~λ)



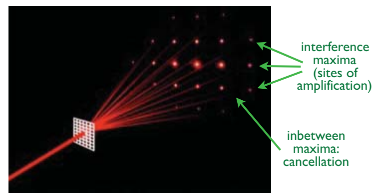
Interference of waves emerging from two point sources.



Interference pattern depends on distance (*d*) separating the pointlike slits



Diffraction pattern of a 2D optical grating



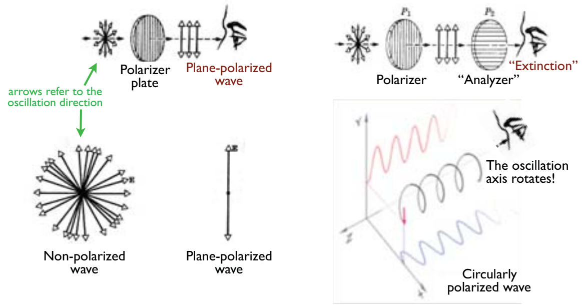
Wave phenomena III.
Polarization

- **Polarization:** oscillation is oriented in some *preferred* direction
- **Birefringence** is related to polarization: anisotropic propagation velocity
- Only *transverse* waves can be polarized.

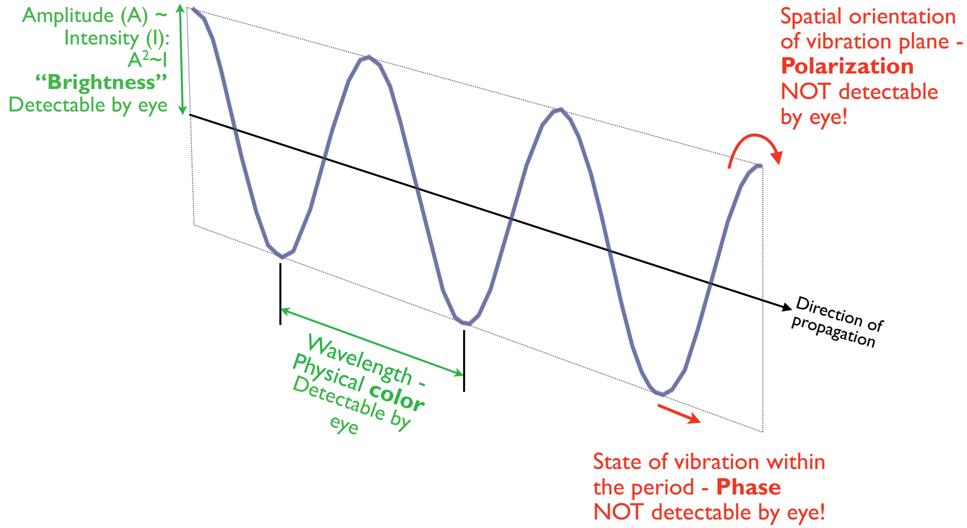
Polarization of Mechanical waves

Polarization of Electromagnetic waves

Polarization can be understood by observing the head-on view of the wave:



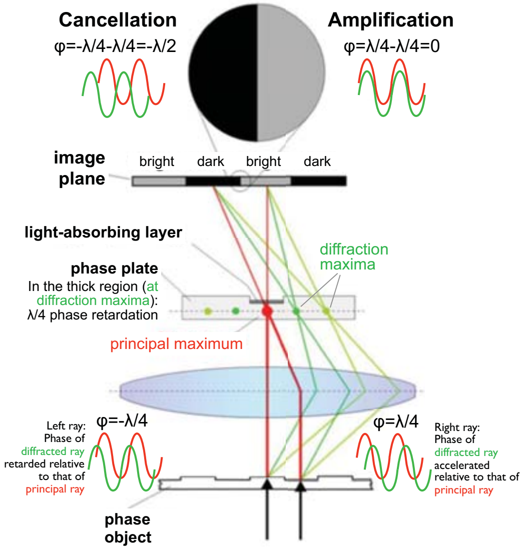
Detectable parameters of the light wave



Origin and nature of wave: next week!

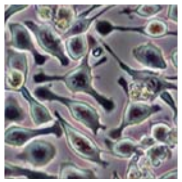
Phase, phase contrast microscopy

Frits Zernike (1888-1966) Nobel-prize



Live (unstained) cells

Bright-field microscopic image



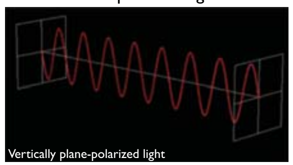
Phase-contrast microscopic image

- **Phase**: shows the state of vibration within the entire period (2π).
- Expressed with the **phase angle** (φ).
- Phase difference between waves: **phase shift** (retardation or acceleration)

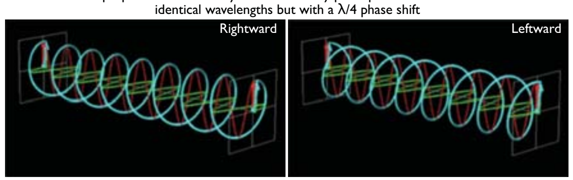
Polarized light and its interactions

Direction of the vibration (electric or magnetic field) has preferred orientation

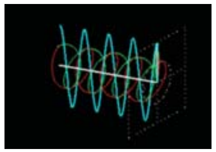
Plane-polarized light



Circularly polarized light:
Superposition of vertically and horizontally plane-polarized waves with identical wavelengths but with a λ/4 phase shift

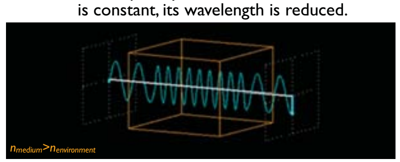


Superposition of a right- and leftward circularly polarized wave results in plane-polarized light.

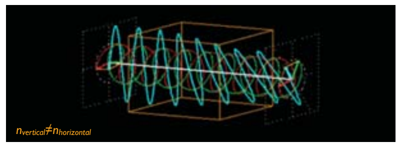


Orientation of polarization plane depends on the relative phase of the two circularly polarized waves

Light decelerates in optically dense medium; because its frequency is constant, its wavelength is reduced.



In an anisotropic* medium a phase shift occurs between the circularly polarized components: the polarization plane of the emerging wave rotates.

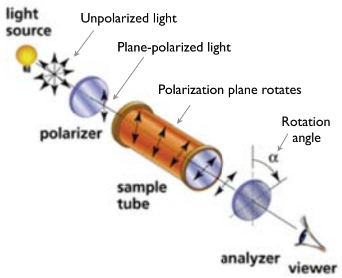


Anisotropy (birefringence): refractive index (~light speed) is orientation dependent (i.e., in different directions within the sample, light propagates with different speeds).

Movies - http://www.enzim.hu/~szia/cddemo/demo0.htm

Applications of polarization

Polarimetry



Rotation angle depends on the concentration (c) of the optically active* material:

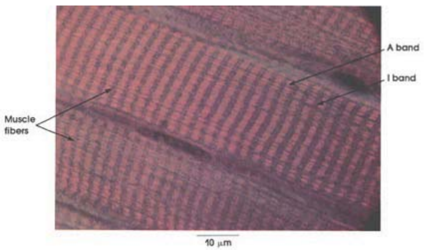
$$\alpha = [\alpha]_D^{20} \cdot c \cdot l$$

[α] = specific angle of rotation ("20": room temperature; "D": emission spectral line of Na λ=589 nm)
l = length of sample tube

*Optically active material: contains *chiral* (mirror-symmetric) molecules tat rotate the plane of polarization.

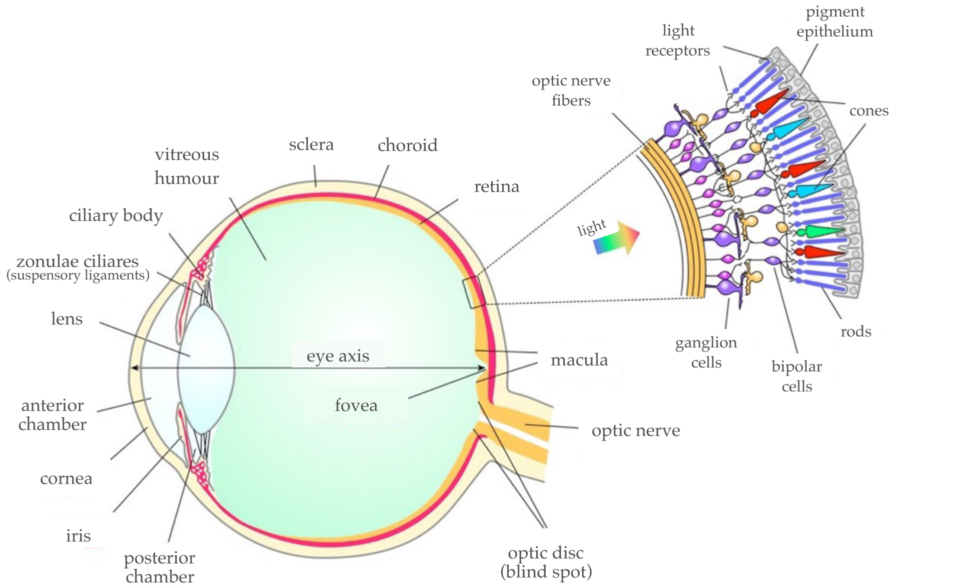
Polarization microscopy

Cross-striated skeletal muscle in the polarization microscope



- A-band: anisotropic (birefringent) region (contains myosin helically ordered myosin molecules)
- I-band: isotropic region

Optics of the human eye

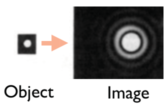


Horizontal section of the human eye

Resolution of the human eye I.

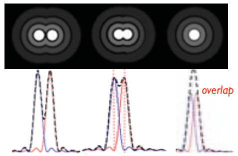
Diffraction limit

Because of diffraction: image of a point object is an Airy disk



Sir George Biddel Airy (1801-1892)

Rayleigh criterion: objects may be resolved if their corresponding Airy disks do not overlap



Lord Rayleigh (1842-1919)

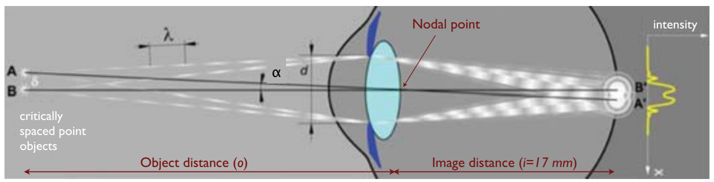
Smallest resolved distance has a limit (Abbe equation):

$$d = \frac{0.61\lambda}{n \sin\alpha}$$

λ = wavelength
n = refractive index of medium
α = angle between axis and outermost ray

Ernst Abbe (1840-1905)

Diffraction limit of the human eye



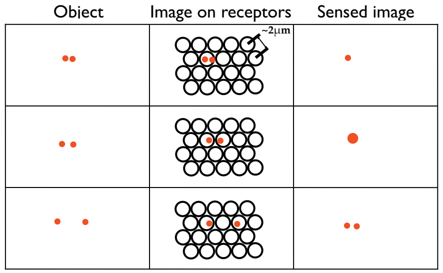
Reduced eye model

Limiting visual angle: $\alpha_H = 1.22\frac{\lambda}{d}$

Smallest angle of view at which two closely spaced objects may be resolved.
At average wavelength (550 nm) and pupil diameter (4 mm): **0.6'** (angular minutes)

Resolution of the human eye II.

Biological limit: receptor cell density



- Condition of resolution: at least one inactivated receptor cell falls in between two activated ones. The limiting angle of view under this condition is (α_B)≈ **0.8'**
- The diffraction and biological limits of the human eye are **comparable**!

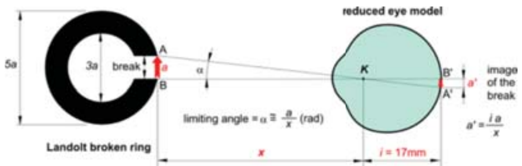
Visual Acuity ("visus", vision):

$$visual\_acuity = \frac{1'}{\alpha}100\%$$

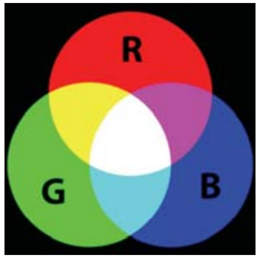
α = experimental (measured) visual angle

Average visual angle in healthy humans: 1' (= 100% vision)

Measurement of visual acuity



Color coding, color vision



Additive color coding

Any color may be generated by mixing three basic colors (R=red, G=green, B=blue) with varying weighing factors.

$$X = rR + gG + bB$$

In the human eye:

- 3 different color-sensitive receptors.
- Each receptor absorbs in different regions of the visible spectrum (R=64%, G=32%, B=2%).

Absorption spectra of the human color-sensitive receptors (cones)